PLASMA DIFFUSION IN MAGNETIC FIELD

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Preface

The diffusion phenomena across magnetic field have been investigated by measuring the diffusion length. The measurement is made in a narrow tube placed behind the anode. The result shows that the classical ambipolar diffusion model is adequate if the magnetic field is not strong.

I. Introduction

A number of discussions have been extensively made about the diffusion phenomena across magnetic field from the point of view of the plasma confinement[1)~9)].

Some of the experimental investigations have been carried out by examining the electric field along the direction of the magnetic field, in a cylindrical positive column[3)5)~7)]. On the basis of the classical diffusion theory and the energy balance condition for the column, the rate of the radial diffusion is related to the axial electric field. The experiments pointed out that the rate of the radial diffusion and hence the axial electric field decreases with increasing the magnetic field in agreement with the classical theory up to a certain critical magnetic field, beyond which the rate is anomalously large.

Some works have been made in a rather short tube instead of the long cylindrical discharge tube[1)2)4)8)9)]. In these works, the lateral density distribution was concerned to see what mechaim is essential.

Most of those investigators confirmed that the diffusion rate perpendicular to the magnetic field decreases proportionally to the square of the magnetic field. However, the question concerning the magnitude of the diffusion coefficient has been unsolved still. Some of them proved that it is determined by the ionic diffusion, *i.e.*, the so-called short-circuit diffusion proposed by Simon[2)4)], while the other asserted that it is determined by the ambipolar diffusion[5)~7)]. One of the experiments supporting the latter has been carried out by Sato and Hatta with using a dark plasma which is formed between a transparent grid mesh and an anode[9)].

In our present work, a diffusing plasma extending along the direction of the magnetic field is used. The merit of the use of this diffusing plasma is the absence of the electric field or the ionization by collision. It makes the situation simple and hence the problem may be solved easier.

Now, the diffusion length characteristic to the mechanism involved is measured as a function of the magnetic field by a movable probe.

II. Diffusion Length in Longitudinal Magnetic Field

In this section, we will derve the fromula for the diffusion length in a longitudinal magnetic field. Since there exist no electric field and ionization in the diffusing plasma formed behind the anode used in the present experiment, the radial flow of the charged particle is determined only by the diffusion. Here, it is assumed that the recombination is neglected. This assumption is justified if the density is low, according to Solunskii and Timan's result[10] as follows:

$$\frac{N_{re}}{N_{di}} = 0.11\,\frac{\beta R^2 n}{D} \tag{1}$$

where N_{re} is the number of the charged particle lost due to the recombination per unit time, N_{di} that due to the diffusion, β the recombination coefficient, D the diffusion coefficient, R the radius of the tube, n the density of the charged particle. For example, if we take $\beta=10^{-8}$ cm^3/sec., $D=10^4$ cm^2/sec., $R=0.2$ cm, we find the ratio to be $4.4.10^{-5}$.

We start with the following continuity and flow equations:

$$\frac{\partial n}{\partial t} + div\Gamma = 0 \tag{2}$$

$$\Gamma = -D_{\perp}\frac{\partial n}{\partial r} - D_z\frac{\partial n}{\partial z} \tag{3}$$

where $D_{\perp}$ is the transverse diffusion coefficient, D_z the longitudinal diffusion coefficient.

Considering the steady state condition, the combination of eqs. (2) and (3) leads to

$$D_z\frac{\partial^2 n}{\partial z^2} + D_{\perp}\left(\frac{1}{r}\frac{\partial n}{\partial r} + \frac{\partial^2 n}{\partial r^2}\right) = 0 \tag{4}$$

If we use the following new variables, α and ξ,

$$D_{\perp} = \alpha D_z \tag{5}$$

$$r = \sqrt{\alpha}\,\xi \tag{6}$$

then eq. (4) can be rewritten as

$$\frac{\partial^2 n}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial n}{\partial \xi} + \frac{\partial^2 n}{\partial z^2} = 0 \tag{7}$$

Taking a form e^{-Kz} as a solution to the above equation, we obtain

$$\frac{\partial^2 n}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial n}{\partial \xi} + K^2 n = 0 \tag{8}$$

The solution of eq. (8) is then given by

$$n = n_0 J_0(K\xi)e^{-Kz} \tag{9}$$

The boundary condition is that $n = n_0$ at $z = 0$ and $n = 0$ at $r = R$. From these, we have

$$K\xi_0 = 2.405 \tag{10}$$

where

$$\xi_0 = \frac{R}{\sqrt{\alpha}} \tag{11}$$

The diffusion length is defined as the *e*-folding length, so that

$$L = \frac{1}{K} \tag{12}$$

From eqs. (10), (11) and (12), the diffusion length is finally expressed in terms of the depletion factor α and the tube diameter as

$$\frac{1}{L^2} = \alpha\left(\frac{2.405}{R}\right)^2 \tag{13}$$

The above expression can be used to estimate the depletion factor from the measurement of the diffusion length.

Sato and Hatta also analysed the general problem involving the radial and axial electric fields and obtained the same result as ours, for the special case where no electric field is present[9]. They also gave an expression for the voltage-current characteristic of the dark plasma as a function of the magnetic field, from which the diffusion length is derived. However, their derivation of the diffusion coefficient across the magnetic field is different from others, because they neglected the term of the azimuthal velocity in the equation of motion.

III. Experimental Arrangements and Method

A discharge tube shown in Fig. 1 is placed between the poles of a permanent magnet in such a way that the axis of the discharge tube is directed along the

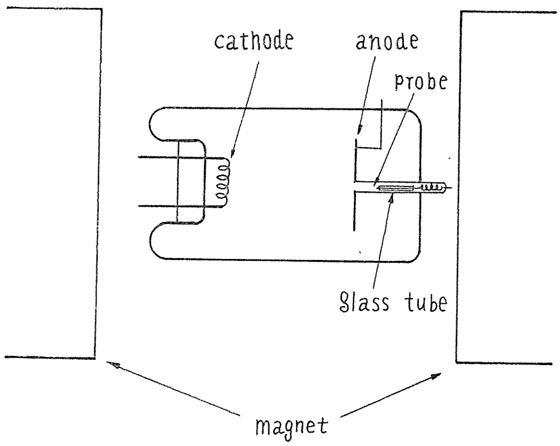

FIG. 1. Discharge tube

direction of the magnetic field. The discharge tube has an oxide-coated cathode, a plane titanium anode, a cylindrical glass tube having the inner diameter of 0.4 cm and a thin movable probe of diameter of 0.02 cm.

A diffusing plasma is formed in the cylindrical glass tube and the diffusion length is measured by the movable probe as a function of the magnetic field up to 5,000 gauss.

The numerical value of the electron temperature is required for estimating the collision frequency of the electron, because the latter is dependent on the electron energy. The measurement of the electron temperature is done by use of the Langmuir probe technique. For this purpose, the movable probe is displaced untill it is immersed in the plasma between the anode and the cathode.

IV. Experimental Results and Discussion

The measurements of the diffusion length are carried out in He, Ne and A. The discharge current is less than 100 mA. The semi log plot of the ion current flowing into the probe v.s. the distance from the anode is fairly linear if the pressure is not low. In fact, the lack of the linearity is observed at a lower pressure, say, 0.1 mmHg. This may be due to the existence of accelerated electron produced in the anode fall region.

In Table 1, the measured values of the diffusion length, the electron temperature and the estimated values of the electron-neutral and ion-neutral collision frequencies, ν_e and ν_i, are tabulated.

The electron-neutral collision frequency is estimated from the published data of the electron-neutral collision cross section together with the measured electron temperature[11], while the ion-neutral collision frequency is calculated from the mobility data[11].

Now, neglecting the contribution of the Coulomb interaction between the electron and ion to the diffusion coefficient[12], the diffusion coefficients across

TABLE 1. (a) Diffusion length and depletion factor for Ne of 1 mmHg. $i=80$ mA.

Magnetic field (gauss)	105	140	230	500	1000	2000	5000
Electron temperature (10^4 K)	6.2	5.97	7.95	5.73	5.03	3.74	2.22
Diffusion length (cm)	0.09	0.11	0.13	0.15	0.12	0.13	0.11
Electron-neutral collision frequency (10^9/sec)	1.4	1.4	1.7	1.4	1.3	0.92	0.64
Ion-neutral collision frequency (10^7/sec)	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Electron cyclotron frequency (10^9/sec)	1.2	1.6	2.7	5.9	12	24	59
Ion cyclotron frequency (10^5/sec)	1.7	2.2	3.7	8.0	16	32	80
Depletion factor α (ambipolar)	1.0	1.0	1.0	0.96	0.83	0.47	0.09
$(2.405R)^2\,\alpha L^2$ (ambipolar)	1.2	1.7	2.5	3.3	1.7	1.2	0.16
Depletion factor α (short-circuit)	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	$0.99\times T_i/T_e$
$(2.405/R)^2\alpha L^2$ (short-circuit)	$1.2\times T_i/T_e$	$1.7\times T_i/T_e$	$2.5\times T_i/T_e$	$3.3\times T_i/T_e$	$1.7\times T_i/T_e$	$2.4\times T_i/T_e$	$1.7\times T_i/T_e$

TABLE 1. (b) Diffusion length and depletion factor for He of 1 mmHg. $i=20$ mA.

Magnetic field (gauss)	105	140	230	500	1000	2000	5000
Electron temperature (10^4 K)	6.32	5.03	4.10	6.08	4.33	3.63	3.16
Diffusion length (cm)	0.07	0.07	0.09	0.08	0.14	0.13	0.10
Electron-neutral collision frequency (10^9/sec)	2.3	2.3	2.0	2.1	2.0	2.0	1.8
Ion-neutral collsion frequency (10^7/sec)	2.6	2.6	2.6	2.6	2.6	2.6	2.6
Electron cyclotron frequency (10^9/sec)	1.2	1.6	2.7	5.9	12	24	59
Ion cyclotron frequency (10^5/sec)	1.7	2.2	3.7	8.0	16	32	80
Depletion factor α (ambipolar)	1.0	1.0	0.98	0.92	0.73	0.45	0.09
$(2.405/R)^2\ \alpha L^2$ (ambipolar)	0.72	0.72	1.1	0.78	2.1	1.1	0.13
Depletion factor α (short-circuit)	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	$0.98\ T_i/T_e$	$0.91\times T_i/T_e$
$(2.405/R)^2\alpha L^2$ (short-circuit)	$0.72\times T_i/T_e$	$0.72\times T_i/T_e$	$1.13\times T_i/T_e$	$0.86\times T_i/T_e$	$2.88\times T_i/T_e$	$2.40\times T_i/T_e$	$1.3\times T_i/T_e$

TABLE 1. (c) Diffusion length and depletion factor for A of 1 mmHg. $i=20$ mA.

Magnetic field (gauss)	105	140	230	500	1000	2000	5000
Electron temperature (10^4 K)	1.29	1.67	1.67	1.75	1.75	1.75	1.75
Diffusion length (cm)	0.15	0.15	0.18	0.11	0.11	0.15	0.18
Electron-neutral collision frequency (10^8/sec)	3.5	4.7	4.7	5.7	5.7	5.7	5.7
Ion-neutral collision frequency (10^7/sec)	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Electron cyclotron frequency (10^9/sec)	1.2	1.6	2.7	5.9	12	24	59
Ion cyclotron frequency (10^5/sec)	1.7	2.2	3.7	8.0	16	32	80
Depletion factor α (ambipolar)	1.0	1.0	0.98	0.93	0.78	0.48	0.13
$(2.405/R)^2\ \alpha L^2$ (ambipolar)	3.3	3.3	4.7	1.6	1.4	1.6	0.64
Depletion factor α (short-circuit)	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e	T_i/T_e
$(2.405/R)^2\alpha L^2$ (short-circuit)	$3.3\times T_i/T_e$	$3.3\times T_i/T_e$	$4.7\times T_i/T_e$	$1.7\times T_i/T_e$	$1.7\times T_i/T_e$	$3.3\times T_i/T_e$	$4.6\times T_i/T_e$

magnetic field are given according as the type of diffusion as follows:

$$D_\perp = D_{a\perp} = \frac{D_a}{1+\left(\frac{\omega_e\omega_i}{\nu_e\nu_i}\right)} \text{ (Ambipolar)},^{13)} \quad D_\perp = D_{i\perp} = \frac{D_i}{1+\left(\frac{\omega_i}{\nu_i}\right)^2} \text{ (Short-circuit)}^{4)} \tag{14}$$

The depletion factor are

$$\alpha = \frac{1}{1+\left(\frac{\omega_e\omega_i}{\nu_e\nu_i}\right)^2} \text{ (Ambipolar)}, \quad \alpha = \frac{D_\perp}{D_a} \doteqdot \frac{T_i}{T_e}\frac{1}{1+\left(\frac{\omega_i}{\nu_i}\right)^2} \text{ if } T_e \gg T_i \text{ (Short-circuit)} \tag{15}$$

In the above equations, ω_e and ω_i are the cyclotron frequencies of the electron and ion, respectively.

The use of eqs. (13) and (15) with aid of the measured L allows us to examine the validity of the theoretical value of the depletion factor. In the eighth column, the values of $(2.405/R)^2\alpha L^2$ for the case of the ambipolar diffusion are tabulated. If the value is equal to the nuity, it is said that the ambipolar diffusion mechanism is valid. As seen in the table, the discrepancy from the unity is not too large as long as the magnetic field is not strong.

Also, the fact that the value of $(2.405/R)^2\alpha L^2$ remains within the same order of magnitude makes us to exclude the so-called Bohm's type diffusion from the possible mechanism[1].

The smallness of the value of $(2.405/R)^2\alpha L^2$ at strong magnetic field means that the theoretical value of the depletion factor is too small to explain the measured diffusion length.

Furthermore, for comparison, the value of $(2.405/R)^2\alpha L^2$ for the case of the short-circuit diffusion is calculated. The large deviation from the unity may lead to the conclusion that the short-circuit diffusion mechanism is inadequate.

However, the question why the diffusion is enhanced in strong magnetic field still remains unsolved in our knowledge. Solution to this problem has been attacked by several authors. These are based on either a fluctuation[14] or a helical instability[15]. As regard to the latter, it has been understood that in a short plasma as in the present case the excitation of the helical instability is not likely. However, there may be only possibility of excitation of a higher mode.

Unfortunately, at the present time, we have no direct evidence identifying such instability. According to the noise measurement, the noise in the frequency range higher than 100 Kc/sec. becomes pronounced at a magnetic field of several hundreds gauss.

V. Conclusion

In this article, we proposed a simple way for examining the diffusion mechanism across magnetic field by use of a thin cylindrical diffusing plasma behind the anode. In the diffusing plasma, there are no electric field and collisional ionization. It makes the situation quite simple. Only disadvantage is that the diffusion may be affected by the accelerated electron which may exist in the biffusing plasma if the pressure is low and the anode fall is positive. However, under the condition where the ambipolar biffusion takes place or the pressure is high and the area of the anode is large enough not to develop the positive anode fall, the effect of the accelerated electron is negligible.

The experimental data on the diffusion length presented by us provide an evidence confirming the validity of the classical ambipolar diffusion mechanism.

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