OVERVOLTAGE REDUCTION EFFECT OF LINE RESISTANCES IN POWER SYSTEMS

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(Received May 31, 1965)

1. Introduction

A power transmission system can be called to be effectively grounded when the following requirements are always satisfied for a single-line-to-ground fault, no matter where the fault may occur,

$$0 \leqq X_0/X_1 \leqq 3, \quad 0 \leqq R_0/X_1 \leqq 1, \tag{1}$$

where X_0 and X_1 are the zero- and positive-sequence reactances respectively, and R_0 the zero-sequence resistance all viewed from the point of fault, the positive- and negative-sequence reactances being assumed to be equal. Then, the fundamental-frequency line-to-ground voltage on the other two lines would not exceed the 75 percent threshold of the normal line-to-line voltage, that is, 130 percent of the highest line-to-ground voltage. In such a system, considerable merit of lowering the arrester rating is expected which leads, in turn, to the favourable aspects on the establishment of system insulation co-ordination. However, it must be born in mind that the above conclusion is obtained with the positive- and negative-sequence resistances neglected.

Recently, it is a general tendency in our country to raise the distribution system voltage up to 10 kv from the existing lower level, say 6 kv. The 10 kv distribution system is preferable to be grounded effectively. As is not the case with transmission lines, the magnitude of resistances of distribution feeder is usually one or two times greater than that of the reactances concerned and can never be neglected now. System grounding resistances of lower value will generally satisfy the requirements shown in equation (1). For the distribution system, the line resistances might also be available to secure a system as "effectively grounded".

The simple transformations proposed in this paper give us a clear information on the overvoltage reduction effect of line resistances of positive- and negative-sequences.

2. Theoretical point of view

For a single-line-to-ground fault on phase a of a symmetrical three phase system, the sustained overvoltages to ground on phase b and c are, in per unit, given as follows,

$$\frac{V_b}{E_a} \equiv K_b = \varepsilon^{j\frac{4\pi}{3}} - \frac{Z_0 - Z_1}{Z_0 + 2Z_1 + 3R_f}, \tag{2}$$

$$\frac{V_c}{E_a} \equiv K_c = \varepsilon^{j\frac{2\pi}{3}} - \frac{Z_0 - Z_1}{Z_0 + 2Z_1 + 3R_f}. \tag{3}$$

where the sequence of the phase voltage is $(a-b-c)$ and R_f is the actual fault resistance, the normal line-to-ground voltage E_a of phase a being conventionally selected as reference.

$$Z_0 = R_0 + jX_0 \tag{4}$$

and

$$Z_1 = R_1 + jX_1 \tag{5}$$

are the zero- and positive-sequence impedances of the system respectively, viewed from the fault point. The latter is generally equal to the negative-sequence impedance and so treated in this paper. Each component may be related to X_1 with the following notations,

$$X_0/X_1 = x, \quad R_0/X_1 = y, \tag{6}$$

$$R_f/X_1 = y_0, \quad R_1/X_1 = h. \tag{7}$$

The second term in equations (2) and (3), substituting equations (4), (5), (6) and (7), becomes

$$\frac{Z_0 - Z_1}{Z_0 + 2Z_1 + 3R_f} = \frac{R_0 + jX_0 - (R_1 + jX_1)}{R_0 + jX_0 + 2(R_1 + jX_1) + 3R_f} = \frac{y - h + j(x-1)}{y - h + 3(y_0 + h) + j(x+2)}. \tag{8}$$

Here, the following simple but ingenious replacements are proposed,

$$y - h = y', \tag{9}$$

and

$$y_0 + h = y_0'. \tag{10}$$

Substituting equations (9) and (10) in (8),

$$Eq.\ (8) = \frac{y' + j(x-1)}{y' + 3y_0' + j(x+2)}. \tag{11}$$

Accordingly,

$$K_b = \epsilon^{j\frac{4\pi}{3}} - \frac{y' + j(x-1)}{y' + 3y_0' + j(x+2)}. \tag{12}$$

$$K_c = \epsilon^{j\frac{2\pi}{3}} - \frac{y' + j(x-1)}{y' + 3y_0' + j(x+2)}. \tag{13}$$

It is noteworthy that the term $h = R_1/X_1$, signifying the line resistive component, is implicitly contained in the above equations. These equations are formally equal to those obtained under the neglect of the line resistive components. Therefore, we can develop the above equations by the usual method,[1] if the former y and y_0 are only replaced by y' and y_0'.

Our purpose is to obtain the circle diagrams estimating K_b and K_c on the plane with x and y' as horizontal and vertical axes. The drawing method of such

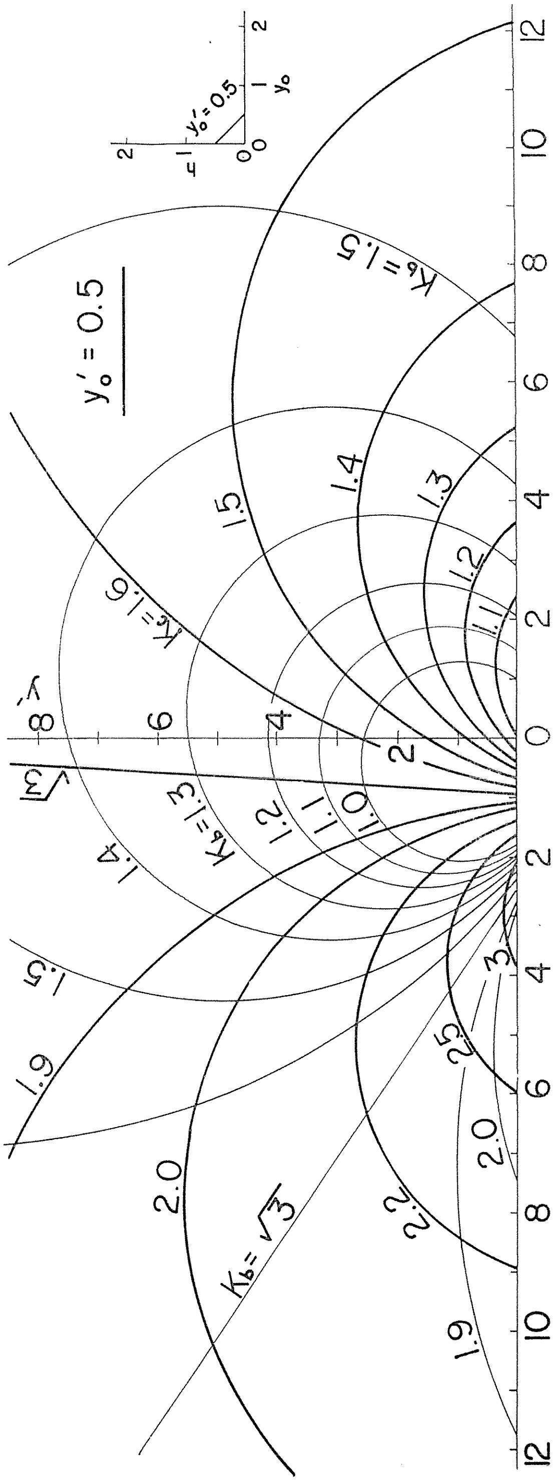

FIG. 1. Line-to-ground voltage chart ($y_0' = y_0 + h = 0.5$)

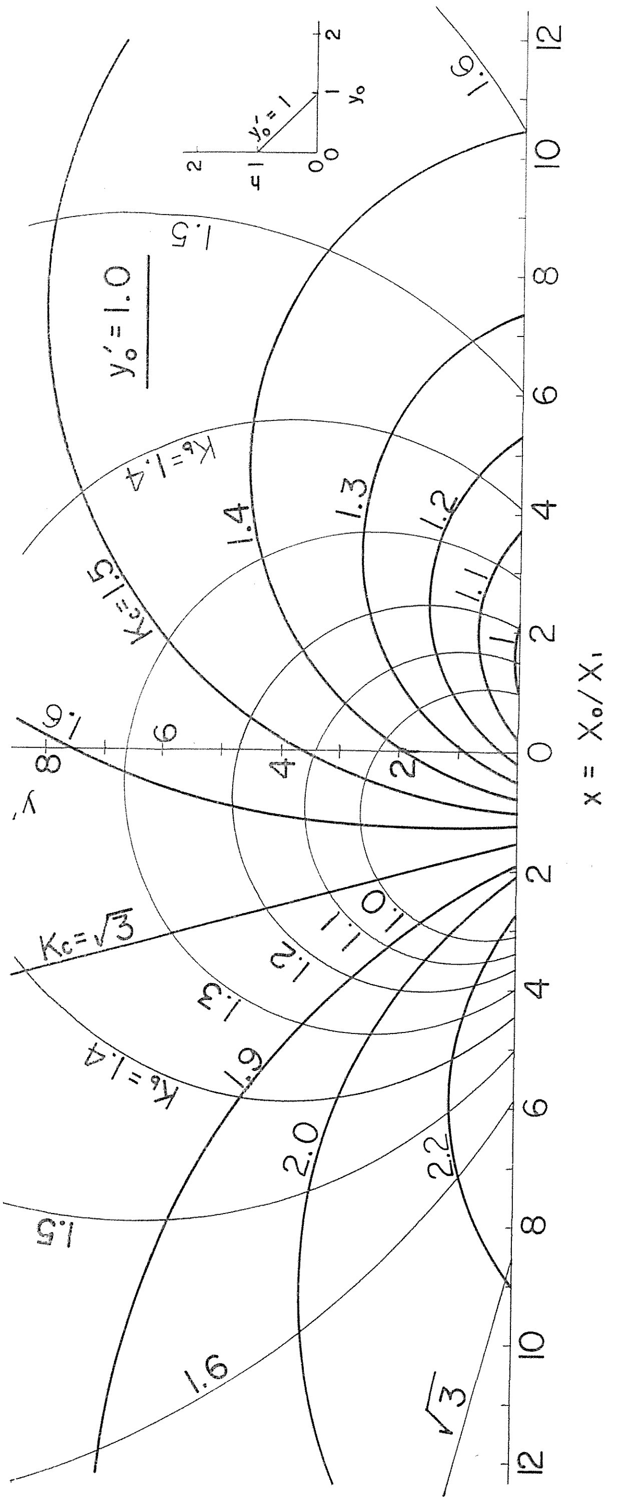

FIG. 2. Line-to-ground voltage chart ($y_0' = y_0 + h = 1$)

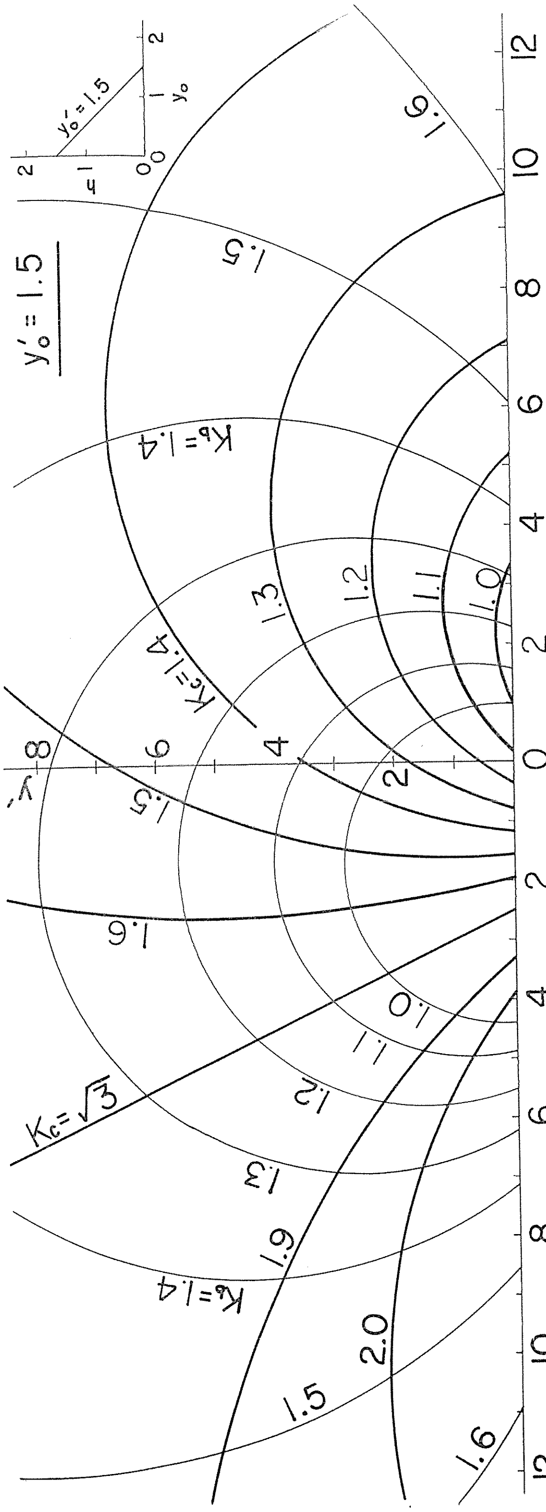

FIG. 3. Line-to-ground voltage chart ($y_0'=y_0+h=1.5$)

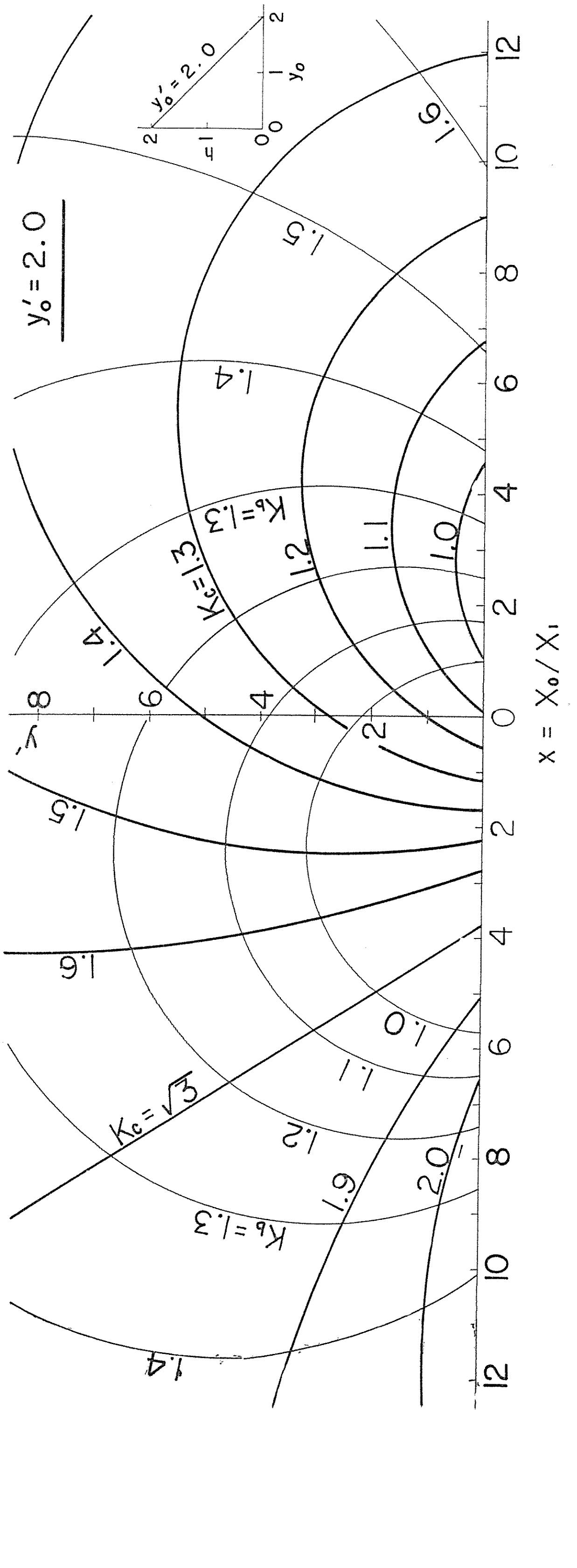

FIG. 4. Line-to-ground voltage chart ($y_0' = y_0 + h = 2$)

a diagram is referred to in Appendix I. The typical four line-to-ground voltage charts are shown in Figs. 1, 2, 3 and 4, each of which takes a different value of y'_0 as a parameter. One chart for a given magnitude of y'_0, say $y'_0=2$, can be applied to all cases where the sum of y_0 and h takes a constant value, that is, $y_0+h=2$. Because the equation (10) shows a linear relation, it is graphically designated as a straight line with the falling slope of 45° as shown in Fig. 5. Therefore, the aspects concerning the line-to-ground voltage chart for any combination of (y_0, h) with the unique value of $y'_0=2$ will fall in one and the same expression as in Fig. 4. For any other magnitude of y'_0, equation (10) designates another straight line of the same inclination with that of $y'_0=2$. Each line attached to the right-hand side of each chart shows the possible combination of (y_0, h) concerned.

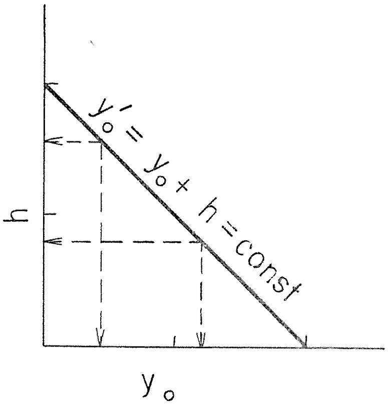

FIG. 5. Straight line $y_0+h=$constant shown in equation (10)

3. Contribution of line resistances to effective grounding

The selected domain can be finally found so as to be enveloped by a set of circles for $|K_b|=|K_c|=1.3$ chosen from line-to-ground voltage charts shown in the previous section. Such a set of 1.3-times circles can be newly obtained according to the indication given in Appendix II, which shows the position of the center and the radius of each 1.3-times circle with the magnitude of y'_0 as a parameter. Thus, the effects of the fault resistance R_f and the positive and negative-sequence resistance R_1 are fully taken account of in the obtained set of circles. From such a diagram, it is possible to select each minimum voltage-domain for a given manitude of h enveloped by a number of K_b- and K_c-circles. Fig. 6 shows the final result which represents the minimum overvoltage-domain, taking h as the parameter. In this chart, however, the vertical axis takes the usual scale of $y=R_0/X_1$, in place of y', and the domain for $h=0$ is also given, in order to make it easy to compare with the usual results neglecting R_1.[2] The scale of the vertical axis may be easily changed from y' to y as shown in Fig. 8 in Appendix I. If the system impedance viewed from the fault point falls in one of these domains figured by a magnitude of h for a given condition, such a system can be called as effectively grounded, because the sustained overvoltages to ground on the other two ungrounded lines are expected to be kept under 1.3 times the normal line-to-ground voltage during fault. Now, it is possible to designate both extended magnitudes of X_0/X_1 and R_0/X_1 in a preceding form as shown in equation (1), while the original domain in Fig. 6 takes a shape sorrounded by two curves. Table 1 gives such an escalading designation. According to the overvoltage reduction effect of line resistances, referring to numerical values shown in Table 1, it is permissible for the magnitude of R_0/X_1 to be set in rather higher values than formerly required by equation (1) without loosing any favourable condition.

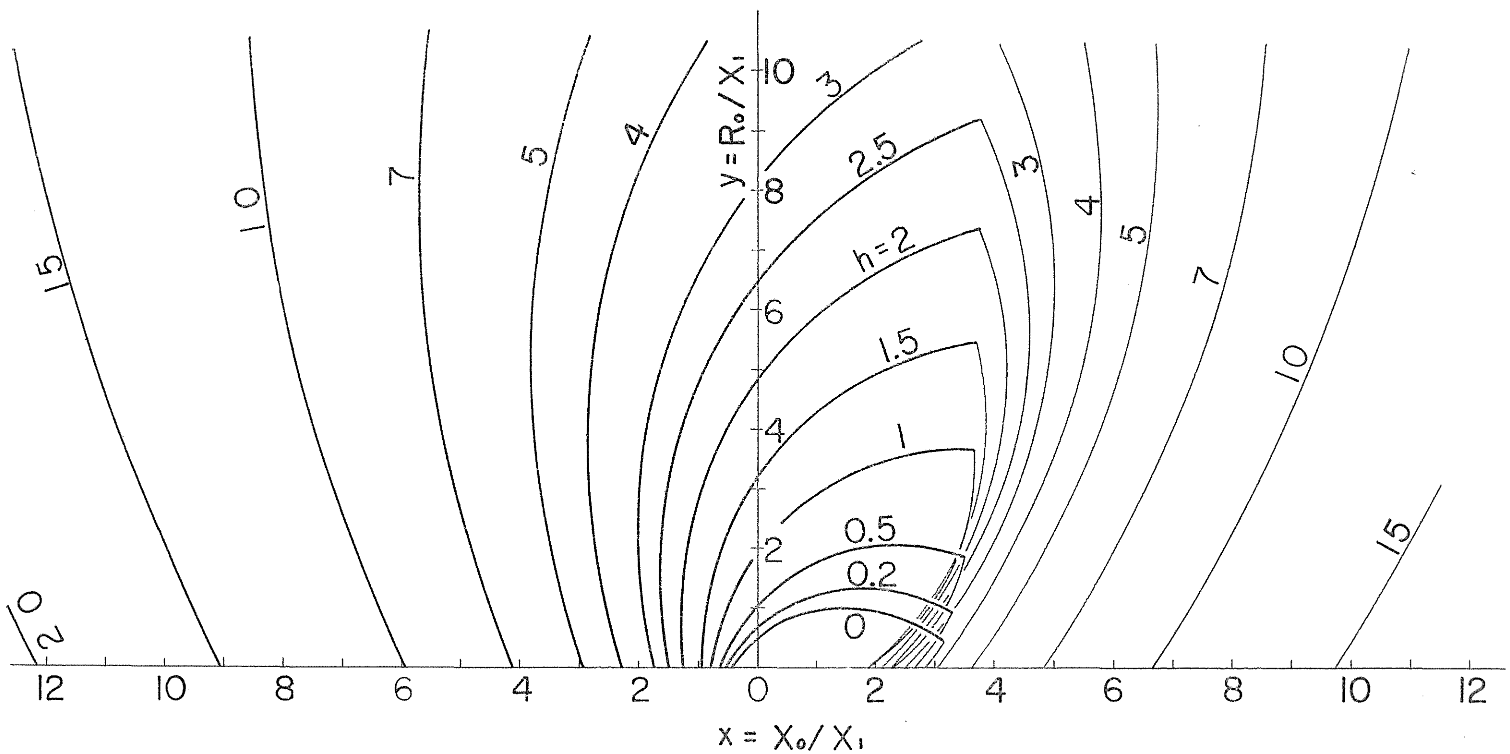

FIG. 6. Minimum overvoltage-domain enveloped by every particular set of circles for $|K_b| = |K_c| = 1.3$, where designated number on each curve shows the magnitude of h

TABLE 1. Effectively grounded conditions in terms of R_0/X_1, X_0/X_1 and line resistance contribution h

h	The first quadrant		The second quadrant	
	$X_0/X_1 \leqq$	$R_0/X_1 \leqq$	$X_0/X_1 \leqq$	$R_0/X_1 \leqq$
0	3	1	0	0
0.3	3	1.5	−0.5	0.5
0.5	3	2	−0.5	1
1	3.5	3.5	−0.5	1.5
1.5	3.5	5	−1	2.5
2	4	6.5	−1	3.5
2.5	4.5	8.5	−1.5	5
3	5	10	−2	7

4. Conclusion

(1) The positive- and negative-sequence resistive components in a power system yield some favourable improvements in the requirements for effective grounding. It is effected so readily through the proposed simple transformations between three variables that an ordinary computation method can be available for obtaining the fundamental-frequency line-to-ground voltages on the ungrounded line during a single line-to-ground fault.

(2) The replacement given in equation (9), $y-h=y'$, shows that the zero-sequence resistance is equivalently reduced by the magnitude of h which signifies the positive- and negative-sequence resistive components as described in this paper.

(3) The replacement given in equation (10), $y_0+h=y_0'$, shows that the fault resistance is equivalently added to by the magnitude of h.

(4) From newly obtained line-to-ground voltage charts, we can easily find the improved requirements for the sustained overvoltages on the ungrounded line not to exceed 1.3 times the normal line-to-ground voltage during a single line-to-ground fault. Such a process finally reveals the method of reducing overvoltages and makes possible the rearrangement of the combination of R_0/X_1 and X_0/X_1 referred to the values of h as shown in Table 1.

(5) According to Table 1, the positive- and negative-sequence resistive components keep a system free from the feasible overvoltages during a single line-to-ground fault, even if the magnitude of R_0/X_1 attains appreciably beyond 1.

References

1) E. CLARKE: Circuit Analysis of AC Power Systems (Book), 1950, pp. 177.
2) Westinghouse Electric Co.: Electrical Transmission and Distribution Reference Book, 1950, pp. 626.

Appendix I: Drawing method of line-to-ground voltage charts

The following notations are referred to Fig. 7.

$$\phi' = \tan^{-1} 1/y_0' = \tan^{-1} 1/(y_0 + h)$$

$$OC' = 3\sqrt{3}\, A'/(3 - |K_c|^2)$$

$$r' = 3|K_c|/(3 - |K_c|^2)$$

$$A' = \sqrt{1 + y_0'^2} = \sqrt{1 + (y_0 + h)^2}$$

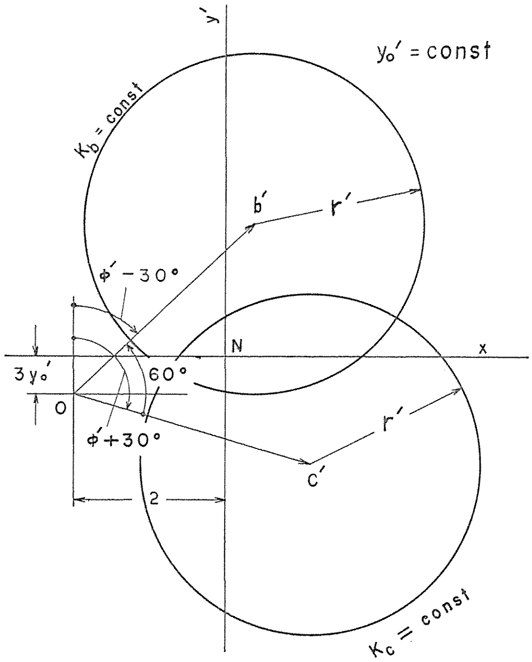

FIG. 7. Drawing method of line-to-ground voltage chart

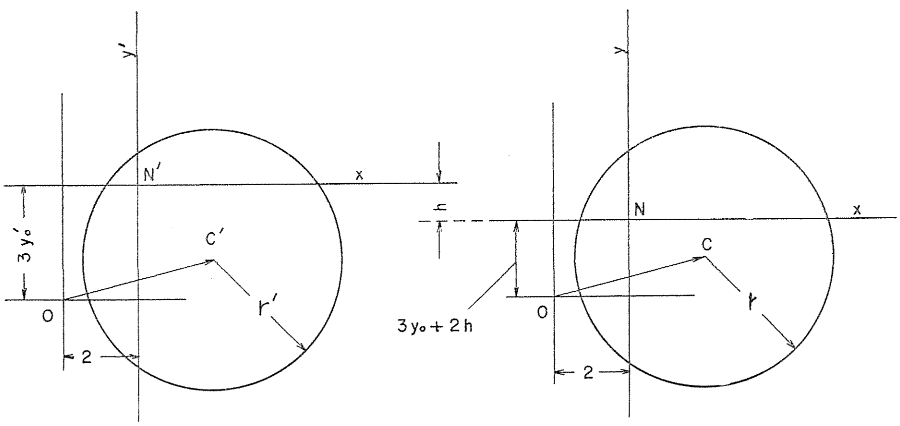

FIG. 8. Changing the scale of the vertical axis

K_b-circle is obtained by turning K_c-circle counter-clockwise by 60°.

It is possible to change the vertical scale from y' to the ordinary y, when the horizontal axis is moved downward by the magnitude of h as shown in Fig. 8.

Appendix II: Drawing the 1.3-times-voltage-circles

The centers of overvoltage circles and the radii, (P'_{0b}, Q'_{0b} and r'_{0b}) for K_b-circle and (P'_{0c}, Q'_{0c} and r'_{0c}) for K_c-circle, are given as follows:

$$\left.\begin{aligned} P'_{0b} &= \frac{3\sqrt{3}(\sqrt{3}-y'_0)}{2(3-|K_b|^2)} - 2 \\ Q'_{0b} &= \frac{3\sqrt{3}(\sqrt{3}y'_0+1)}{2(3-|K_b|^2)} - 3y'_0 \\ r'_{0b} &= \sqrt{1+y_0'^2}\cdot\frac{3|K_b|}{3-|K_b|^2} \end{aligned}\right\}$$

$$\left.\begin{aligned} P'_{0c} &= \frac{3\sqrt{3}(\sqrt{3}+y'_0)}{2(3-|K_c|^2)} - 2 \\ Q'_{0c} &= \frac{3\sqrt{3}(\sqrt{3}y'_0-1)}{2(3-|K_c|^2)} - 3y'_0 \\ r'_{0c} &= \sqrt{1+y_0'^2}\cdot\frac{3|K_c|}{3-|K_c|^2} \end{aligned}\right\}$$

Putting the magnitude of $|K_b|$ and $|K_c|$ as 1.3, the above equations are reduced to the following ones with y'_0 as a parameter.

$$\left.\begin{aligned} P'_{0b} &= -1.98y'_0 + 1.44 \\ Q'_{0b} &= 0.435y'_0 + 1.98 \\ r'_{0b} &= 8.86(1+y_0'^2) \end{aligned}\right\}$$

$$\left.\begin{aligned} P'_{0c} &= 1.98y'_0 + 1.44 \\ Q'_{0c} &= 0.435y'_0 - 1.98 \\ r'_{0c} &= 8.86(1+y_0'^2) \end{aligned}\right\}$$

When y'_0 is eliminated from the equations of P'_{0b} and Q'_{0b}, or of P'_{0c} and Q'_{0c}, the following linear relations are obtained as the straight line locii of the centers. We obtain

$$\frac{P^0_{\prime b}}{10.48} + \frac{Q'_{0b}}{2.30} = 1$$

for K_b-circle, and

$$\frac{P'_{0c}}{10.48} - \frac{Q'_{0c}}{2.30} = 1$$

for K_c-circle.