THE IMPEDANCE FORMULA FOR A PARTIALLY IONIZED, COLLISION-DOMINATED PLASMA

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In the present paper, the impedance formula for a partially ionized, collision-dominated plasama is derived on the basis of the transport equation, taking the temperature peturbation into account. Several attempts are made. In the first attempt, the density perturbation is completly neglected but the inertia term is retained. The second is based on the ignorance of all space derivatives of the quantities involved. The third attempt is made by use of the quasi-steady approximation, but without exception of the term related to the space charge field. The fourth corresponds to the quasi-steady treatment of the second case.

1. Introduction

When an electric field is applied to a plasma, the number density, the velocity and the temperature of the charged particles may accordingly be perturbed. As a consequence of the combined effect, an applied a.c. field causes an a c. component of current to flow through the plasma, which is mostly the electron current. The current is proportional to the applied a.c. field, provided that the amplitude of the field is sufficiently small compared to that of the steady d.c. field. The reciprocal of the coefficient of proportinality is defined as the impedance of plasma.

From this point of view, the plasma impedance should be evaluated considering all the perturbations described above. The plasma impedance has been considered on the basis that only the velocity is peturbed.[1] According to this elementary theory, the plasma impedance is expressed as a function of the plasma density n, the collision frequency ν_c and the angular frequency of the applied field ω. The real component of the impedance or the resistance is given by $R=m\nu_c/\mathrm{ne}^2$ and the inductance deduced from the imaginary part is given by $L=m/\mathrm{ne}^2$, both of which are independent on the angular frequency.

However, the earlier experiments showed that the resistance as well as the inductance was frequency-dependent in low frequency range, where most of the experiments were carried out.[2] [3] An effort has been made to understand the frequency dependence by use of the quasi-steady approach, taking the ionizing collision into account.[2] The quasi-steady approach is based upon the assumption that the d.c. equations for velocity and energy are employed even when a time-dependent problem is concerned. The result provided an explanation for the frequency dependence, in which the density perturbation due to the ionizing collision was considered to be essential.

We are interested in general formulation of the plasma impedance in which all the perturbations of the associated quantities are taken into consideration. In particular, our attention is directed towards the effect of the perturbation of electron temperature on the plasma impedance.

The plasma considered here has a drift due to a steady electric fleld, which may exist to compensate for the loss of the electron energy through various processes. We also assume the plasma to be partially ionized, in other words, the collision between the charged particle and the neutral molecule to be dominant.

In the present analysis, we shall make use of the macroscopic transport equation describing the relation among the density, velocity and temperature.

The first paragragh is devoted to the formulation of the problem and the second to the derivation of the impedance formula.

II. Basic Equations

We shall start with one dimensional transport equations representing the continuity of the number density and the conservation of the momentum and energy of the charged particle, in order to obtain the relation among these quantities.[4] Here, we make the assumption that the ions are rest and do not influence on the response of plasma. The plasma considered here is partially ionized as encountered in ordinary laboratory condition, so that all the collision processes are performed between the electron and the neutral molecule. The encounter between the electrons is assumed to be negligibly small.

With these assumptions, the basic equations describing the relation among the associated quantities for the electron are expressed as follows;

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=n\nu_i-W, \tag{1}$$

$$\frac{\partial}{\partial t}(nu)+\frac{\partial}{\partial x}\left\{n\left(u^2+\frac{kT}{m}\right)\right\}+\frac{eEn}{m}=-\nu_c nu, \tag{2}$$

$$\frac{\partial}{\partial t}\left\{n\left(u^2+\frac{kT}{m}\right)\right\}+\frac{\partial}{\partial x}\left\{n\left(u^3+\frac{3\,ukT}{m}\right)\right\}+\frac{2\,eEnu}{m}$$
$$=-\left(\frac{2\,m\nu_c n}{M}\right)\frac{kT}{m}-\frac{\nu_i eV_i n}{m}, \tag{3}$$

where n is the electron density, u the drift velocity, ν_i the ionization frequency, W the loss in the number density, k the Boltzmann constant, T the electron temperature, E the electric field, m the mass of the electron, e the electronic charge, ν_c the collision frequency of the electron with the neutrals and V_i the ionization potential of the gas used.

The use of eq. (1) enables us to rewrite eq. (2) as

$$n\frac{\partial n}{\partial t}+nu\frac{\partial u}{\partial x}+u(n\nu_i-W)+\frac{\partial}{\partial x}\left(\frac{nkT}{m}\right)+\frac{eEn}{m}=-\nu_c nu. \tag{4}$$

Combining eqs. (3) and (4) to eliminate E, we obtain

$$\frac{\partial}{\partial t}\left(\frac{nkT}{m}\right)+u\frac{\partial}{\partial x}\left(\frac{nkT}{m}\right)+\left(\frac{3\,nkT}{m}\right)\frac{\partial u}{\partial x}+u^2(-2\,\nu_c n-\nu_i n+W)$$
$$=-\frac{2\,n\nu_c kT}{M}-\frac{\nu_i e V_i}{m}. \tag{5}$$

The conduction current J is given by

$$J=-neu. \tag{6}$$

The external current is defined as the sum of the conduction current J and the displacement current arising from the time derivative of E. Thus, the external current is expressed as

$$J_e=-neu+\frac{1}{4\pi}\frac{\partial E}{\partial t}. \tag{7}$$

III. Derivation of the Impedance Formula

Now, it is assumed that the a.c. components of the associated quantities are smaller than these in the steady state and change with time as $e^{j\omega t}$, where ω is the angular frequency of the applied a.c. field. The smallness of the a.c. components allows us to linearize the basic equations described above.

To proceed the formulation of the linearized a.c. equations, the temperature dependence of ν_i, ν_c and W wust be known beforehand. First, the ionization frequency ν_i is a function of the electron temperature as follows:

$$\nu_i=A\left(\frac{kT}{e}\right)^{1/2}e^{-eV_i/kT}, \tag{8}$$

where A is a constant.

Since the collision frequency is a product of the collision probability and the mean velocity, ν_c is proportional to T, provided that the collision probability is invariant over the energy range considered here. As regards to W, if the ambipolar diffusion to the surrounding is dominant it is proportional to T, because the loss W is proportional to $D_a\ (2.4/R)^2\approx(kT\mu_i/e)\ (2.4/R)^2$ for a cylindrical tube, where R is the tube radius, D_a the ambipolar diffusion coefficient and μ_i the ionic mobility.

From the above consideration, we obtain the expressions for the a.c. component of ν_i, ν_c and W as a function of the temperature perturbation as follws:

$$\Delta\nu_i=\nu_{i_0}\frac{\Delta T}{T}\left(\frac{eV_i}{kT_0}+\frac{1}{2}\right), \tag{9}$$

$$\Delta\nu_c=\nu_{c_0}\frac{\Delta T}{2\,T_0}, \tag{10}$$

$$\Delta W=W_0\frac{\Delta T}{T_0}, \tag{11}$$

where Δ denotes the a.c. component and the subscript 0 the unperturbed part.

Thus, eqs. (1), (4), (5) and (6) or (7) are transformed into the a.c. equations determining the impedance, with the help of the linealization and by use of eqs. (9), (10) and (11). They are given by

$$\frac{\partial \Delta n}{\partial t} + \frac{\partial}{\partial x}(u_0 \Delta n + n_0 \Delta u) = n_0(\Delta \nu_i - \Delta W) \approx \frac{n_0 \nu_{i_0}}{T_0} \frac{eV_i}{kT_0} \Delta T, \tag{12}$$

$$n_0 \frac{\partial \Delta u}{\partial t} + n_0 u_0 \frac{\partial \Delta u}{\partial x} + (n_0 u_0 \nu_{i_0} T_0) \Delta T \cdot \frac{eV_i}{kT_0} + \frac{kT_0}{m} \frac{\partial \Delta n}{\partial x} + \frac{kn_0}{m} \frac{\partial \Delta T}{\partial x} + \frac{eE_0}{m} \Delta n$$
$$+ \left(\frac{en_0}{n}\right) \Delta E = -\left(\nu_{c_0} u_0 \Delta n + n_0 u_0 \nu_{c_0} \frac{\Delta T}{2 T_0} + n_0 \nu_{c_0} \Delta u\right), \tag{13}$$

$$\frac{kn_0}{m} \frac{\partial \Delta T}{\partial t} + \frac{kT_0}{m} \frac{\partial \Delta n}{\partial t} + \frac{u_0 k n_0}{m} \frac{\partial \Delta T}{\partial x} + \frac{u_0 k T_0}{m} \frac{\partial \Delta n}{\partial x} + \frac{3 n_0 k T_0}{m} \frac{\partial \Delta u}{\partial x}$$
$$-2\left(2 u_0 n_0 \nu_{c_0} \Delta u + u_0^2 \nu_{c_0} \Delta n + n_0 u_0^2 \nu_{c_0} \frac{\Delta T}{2 T_0}\right) - \left(n_0 u_0^2 \nu_{i_0} \frac{\Delta T}{T_0}\right) \frac{eV_i}{kT_0}$$
$$= \frac{-2k}{M}\left(3 n_0 \nu_{c_0} \frac{\Delta T}{2} + \nu_{c_0} T_0 \Delta n\right) - \frac{eV_i}{m}\left\{\frac{n_0 \nu_{i_0} \Delta T}{T_0} \cdot \frac{eV_i}{kT_0} + \nu_{i0} \Delta n\right\}.$$

$$\Delta J = -n_0 e \Delta u - u_0 e \Delta n, \tag{14}$$

$$\Delta J_e = \Delta J + \frac{1}{4\pi} \frac{\partial \Delta E}{\partial t}. \tag{15}$$

In deriving eq. (12), it is assumed that $eV_i/kT_c \gg 1/2$ and $n_0 \nu_{i_0} = W_0$. If the a.c. quantities Δu, Δn, ΔT, $\partial \Delta u/\partial x$, $\partial \Delta n/\partial x$ and $\partial \Delta T/\partial x$ are all eliminated from eqs. (12) through (16), the impedance formula can be obtained as the ratio $\Delta E/\Delta J$ or $\Delta E/\Delta J_e$. The former refers to the internal impedance, while the latter the external impebance. However, the number of the equation available now is not enough for this purpose. For this reason, the formulation is possible only when an appropriate assumption is made. In this paper, we restrict ourselves to the following four cases:

(a) $\Delta n = 0$ and $\partial \Delta T/\partial x = 0$,
(b) $\partial \Delta n/\partial x = 0$, $\partial \Delta u/\partial x = 0$ and $\partial \Delta T/\partial x = 0$,
(c) $\partial \Delta n/\partial x = 0$, $\partial \Delta u/\partial x = 0$ and $\partial \Delta T/\partial x = 0$, but the quasi-steady approach,
(d) $\partial \Delta n/\partial x = 0$ and $\partial \Delta T/\partial x = 0$, but the quasi-steady approach.

(a) The case $\Delta n = 0$ and $\partial \Delta T/\partial x = 0$

In this case, the density perturbation and the thermal conduction are disregarded, so that the resulting a.c. equations are written in terms of Δu, ΔT and $\partial \Delta u/\partial x$.

After rearranging and substituting $\partial/\partial t$ with $i\omega$, eqs. (12) through (15) or (16) become respectively

$$\frac{\partial \Delta u}{\partial x} = \frac{\nu_{i_0} V_i \Delta T}{kT_0^2}, \tag{17}$$

$$i\omega \Delta u + u_0 \frac{\partial \Delta u}{\partial x} + \frac{u_0}{T_0}\left(\nu_{i_0} \frac{eV_i}{kT_0^2} + \frac{\nu_{c_0}}{2}\right) \Delta T + \left(\frac{e}{m} + \frac{i\omega \nu_{c_0}}{4\pi e n_0}\right) \Delta E$$
$$= \frac{\nu_{c_0}}{en_0} \Delta J_e, \tag{18}$$

$$-4 u_0 \nu_{c_0} \Delta u + \frac{3kT_0}{m} \frac{\partial \Delta u}{\partial x} + \left\{\frac{i\omega k}{m} - \frac{u_0^2}{T_0}\left(\nu_{c_0} + \frac{\nu_{i_0} eV_i}{kT_0}\right) + \frac{3 \nu_{c_0} k}{M}\right.$$

$$+\frac{(eV_i)^2\nu_{i_0}}{mkT_0^2}\Big\}\Delta T=0, \tag{19}$$

$$\Delta J=-n_0 e\Delta u, \tag{20}$$

$$\Delta J_e=-n_0 e\Delta u+\frac{i\omega\Delta E}{4\pi}=\Delta J+\frac{i\omega\Delta E}{4\pi}. \tag{21}$$

As a consequence, the extenal or internal impedance can be calculated by eliminating Δu, $\partial\Delta u/\partial x$ and ΔT from these equations thus obtained.

In proceeding the evaluation, the following d.c. equation is employed:

$$u_0^2=\frac{kT_0}{M}+\frac{\nu_{i_0}eV_i}{2m\nu_{c_0}}. \tag{22}$$

which is obtained from eq. (5).

The resulting expression for the external impedance Z_e is given by

$$Z_e=\frac{\left(H-\frac{\omega^2 k}{m}\right)+i\omega\left(G+\frac{\nu_{c_0}k}{m}\right)}{\left\{\frac{(\omega_p^2-\omega^2)G}{4\pi}-\frac{\omega^2 k\nu_{c_0}}{4\pi m}\right\}+\frac{i\omega}{4\pi}\left\{H+\frac{k}{m}(\omega_p^2-\omega^2)\right\}}, \tag{23}$$

where

$$G=\frac{1}{T_0}\left\{\frac{5}{2}\frac{eV_i\nu_{i_0}}{m}+\frac{2kT_0}{M}\left(\nu_{c_0}-\frac{eV_i\nu_{i_0}}{2kT_0}\right)+\frac{\nu_{i_0}(eV_i)^2}{mkT_0}\left(1-\frac{\nu_{i_0}}{2\nu_{c_0}}\right)\right\}, \tag{24}$$

$$H=\frac{\nu_{c_0}}{T_0}\left\{\left(\frac{kT_0}{M}+\frac{eV_i\nu_{i_0}}{2m\nu_{c_0}}\right)\left(\frac{7eV_i\nu_{i_0}}{kT_0}+\nu_{c_0}\right)+\frac{eV_i\nu_{i_0}}{m}\left(3+\frac{eV_i}{kT_0}\right)+\frac{3\nu_{c_0}kT_0}{M}\right\}, \tag{25}$$

and $\omega_p^2=4\pi e^2 n_0/m$ is the electron plasma frequency.

The real component or the resistance R_e is thus obtained as follows:

$$R_e=4\pi\frac{\left(H-\frac{\omega^2 k}{m}\right)\left\{(\omega_p^2-\omega^2)G-\frac{\omega^2 k\nu_{c_0}}{m}\right\}+\omega^2\left(G+\frac{k\nu_{c_0}}{m}\right)\left\{H+\frac{k}{m}(\omega_p^2-\omega^2)\right\}}{\left\{(\omega_p^2-\omega^2)G-\frac{\omega^2 k\nu_{c_0}}{m}\right\}^2+\omega^2\left\{H+\frac{k}{m}(\omega_p^2-\omega^2)\right\}^2}. \tag{26}$$

On the other hand, the inductance L_e is defined as the imaginary part divided by ω and hence it is

$$L_e=4\pi\frac{\left(G+\frac{k\nu_{c_0}}{m}\right)\left\{(\omega_p^2-\omega^2)G-\frac{\omega^2 k\nu_{c_0}}{m}\right\}-\left(H-\frac{\omega^2 k}{m}\right)\left\{H+\frac{k}{m}(\omega_p^2-\omega^2)\right\}}{\left\{(\omega_p^2-\omega^2)G-\frac{\omega^2 k\nu_{c_0}}{m}\right\}^2+\omega^2\left\{H+\frac{k}{m}(\omega_p^2-\omega^2)\right\}^2}. \tag{27}$$

The concept of the internal impedance seems to be more convenient than the external impedance when the dynamical property of plasma itself is concerned. The internal impedance is evaluated in a similar way as that mentioned above with the exception that the term $i\omega\Delta E/4\pi$ is removed from eq. (21) or eq. (20) is used instad of eq. (21).

The resulting form of the internal impedance is given by

$$Z_i = 4\pi \frac{\left(H - \frac{\omega^2 k}{m}\right) + i\omega\left(G + \frac{k\nu_{c_0}}{m}\right)}{\omega_p^2\left(G + \frac{i\omega k}{m}\right)}. \tag{28}$$

The internal resistance R_i and the internal inductance L_i are respectively given by

$$R_i = 4\pi \frac{HG + \frac{\omega^2 \nu_{c_0} k^2}{m^2}}{\omega_p^2\left\{G^2 + \frac{\omega^2 k^2}{m^2}\right\}}, \tag{29}$$

$$L_i = 4\pi \frac{\frac{k}{m}\left(\frac{\omega^2 k}{m} - H\right) + G\left(G + \frac{k\nu_{c_0}}{m}\right)}{\omega_p^2\left\{G^2 + \frac{\omega^2 k^2}{m^2}\right\}}. \tag{30}$$

(b) The case $\partial\Delta n/\partial x = 0$, $\partial\Delta u/\partial x = 0$ and $\partial\Delta T/\partial x = 0$

In this section, all the derivatives with respect to x are assumed to be zero. With the same prodedure as in the foregoing section, eqs. (12) to (16) are simplified as follows:

$$i\omega\Delta n = \frac{n_0 \nu_{i_0} e V_i}{k T_0^2}\Delta T, \tag{31}$$

$$i\omega n_0 \Delta u + \left(\frac{n_0 u_0}{T_0}\right)\left(\nu_{i_0}\frac{eV_i}{kT_0} + \frac{\nu_{c_0}}{2}\right)\Delta T + \frac{eE_0}{m}\Delta n + \left(\frac{en_0}{m} + \frac{i\omega\nu_{c_0}}{4\pi e}\right)\Delta E = \frac{\nu_{c_0}}{e}\Delta J_e, \tag{32}$$

$$-4u_0 n_0 \nu_{c_0}\Delta u + \left\{-\frac{(n_0 u_0^2)}{T_0}\left(\nu_{c_0} + \frac{\nu_{i_0} e V_i}{kT_0}\right) + \frac{3kn_0\nu_{c_0}}{M} + \frac{(eV_i)^2 n_0 \nu_{i_0}}{mkT_0^2} + \frac{i\omega k n_0}{m}\right\}\Delta T$$
$$+\left\{-2u_0^2\nu_{c_0} + \frac{2k\nu_{c_0}T_0}{M} + \frac{eV_i\nu_{i_0}}{m} + \frac{i\omega k T_0}{m}\right\}\Delta n = 0, \tag{33}$$

$$\Delta J_e = -u_0 e\Delta n - n_0 e\Delta u + \frac{i\omega\Delta E}{4\pi} = \Delta J + \frac{i\omega\Delta E}{4\pi}. \tag{34}$$

Elimination of Δn, Δu and ΔT from the above equations leads to an expression containing ΔE and ΔJ_e as the variables, from which the impedance formula is derived. In the derivation, we use the d.c. equation (22) and

$$\frac{eE_0}{m} = -\nu_{c_0} u_0. \tag{35}$$

The external impedance is thus found to be

$$Z_e = \frac{(UX + \omega^2 VY) + i\omega(VX - UY)}{X^2 + \omega^2 Y^2}, \tag{36}$$

where

$$X = \frac{\omega_p^2 - \omega^2}{4\pi}\left\{-\left(a + \frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\nu_{c_0} + \nu_{i_0}\frac{eV_i}{kT_0}\right) + 3a\nu_{c_0} + \nu_{i_0}(b+c)\frac{eV_i}{kT_0}\right\} - \frac{c\nu_{c_0}}{4\pi}\omega^2,$$

$$Y=\frac{\omega_p^2-\omega^2}{4\pi}\left\{-4\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\frac{eV_i}{kT_0}\right)\nu_{i_0}\nu_{c_0}+\omega^2 c\right\}+\frac{\nu_{c_0}}{4\pi}\left\{-\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\right.$$

$$\left.\left(\nu_{i_0}\frac{eV_i}{kT_0}-\nu_{c_0}\right)+3\nu_{c_0}a+\nu_{i_0}(b+c)\frac{eV_i}{kT_0}\right\},$$

$$U=\nu_{c_0}\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\nu_{c_0}+3\nu_{i_0}\frac{eV_i}{kT_0}\right)\nu_{c_0}\nu_{i_0}(b+c)-c\left(\omega^2-3\frac{m\nu_{c_0}^2}{M}\right),$$

$$V=-\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\nu_{c_0}+\nu_{i_0}\frac{eV_i}{kT_0}\right)+\nu_{i_0}\frac{eV_i}{kT_0}(b+c)+\nu_{c_0}(c+3a),$$

$$a=\frac{kT_0}{M},\quad b=\frac{eV_i}{m}\quad \text{and}\quad c=\frac{kT_0}{m}. \tag{37}$$

The external resistance and inductance are respectively given by

$$R_e=\frac{UX+\omega^2VY}{X^2+\omega^2Y^2}, \tag{38}$$

$$L_e=\frac{VX-UY}{X^2+\omega^2Y^2}. \tag{39}$$

On the other hand, the internal impedance is expressed as

$$Z_i=\frac{(UX'+\omega^2VY')+i\omega(VX'-UY')}{X'^2+\omega^3Y'^2}, \tag{40}$$

where

$$X'=\frac{\omega_p^2}{4\pi}\left\{-\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\nu_{c_0}+\nu_{i_0}\frac{eV_i}{kT_0}\right)+3a\nu_{c_0}+\frac{eV_i}{kT_0}\nu_{i_0}(b+c)\right\},$$

$$Y'=\frac{\omega_p^2}{4\pi\omega^2}\left\{-4\left(a+\frac{b\nu_{i_0}}{2\nu_{c_0}}\right)\left(\frac{eV_i}{kT_0}\right)\nu_{i_0}\nu_{c_0}+\omega^2c\right\}. \tag{41}$$

The internal resistance and inductance are respectively given by

$$R_i=\frac{UX'+\omega^2VY'}{X'^2+\omega^2Y'^2}, \tag{42}$$

$$L_i=\frac{VX'-UY'}{X'^2+\omega^2Y'^2}. \tag{43}$$

(c) The case $\partial\Delta n/\partial x=0$, $\partial\Delta u/\partial x=0$ and $\partial\Delta T/\partial x=0$, but the quasi-steady approach

Here, we make an attempt to use a quasi-stady approach, in which the velocity of the electron is presumed to be in phase with the electric field, and is given by

$$u=-\mu E, \tag{44}$$

where μ is the mobility of the electron. In this approach, we neglected the diffusion and thermal conduction for simplicity. The validity of this approach is justified if many collisions take place during a period of the a.c. field. Eq. (44) is obtained from eq. (35) with the following substitution:

$$\mu=\frac{e}{m\nu_c}. \tag{45}$$

Concerning the temperature dependence of ν_i, W and μ, we employ eqs. (9), (11) and

$$\Delta\mu = -\frac{\mu_0}{2T_0}\Delta T, \tag{46}$$

which is obtained from eqs. (10) and (45).

With a similar way as in the case (a), the continuity and energy equations for the a.c. components can be found for the present case. They are

$$i\omega\Delta n = \frac{n_0\nu_{i_0}eV_i}{kT_0^2}\Delta T, \tag{47}$$

$$\frac{n_0 k}{m}\left\{i\omega + \frac{3m\nu_{c_0}}{M} + \left(\frac{eV_i}{kT_0}\right)^2\nu_{i_0} + \frac{e\mu_0E_0^2}{kT_0}\right\}\Delta T + \left\{\frac{eV_i\nu_{i_0}}{m} - 2\frac{eE_0^2\mu_0}{m} + 2\frac{\nu_{c_0}kT_0}{M}\right.$$
$$\left. + i\omega\mu_0^2E_0^2 + i\omega\frac{kT_0}{m}\right\}\Delta n + \left\{-4\frac{n_0\mu_0E_0}{m} + 2i\omega n_0\mu_0^2E_0\right\}\Delta E = 0. \tag{48}$$

The quation for the a.c. current density is

$$\Delta J_e = \mu_0E_0e\Delta n + en_0\mu_0\Delta E + i\omega\frac{\Delta E}{4\pi} = \Delta J + \frac{i\omega\Delta E}{4\pi}. \tag{49}$$

Again, we introduce the following d.c. energy equation in order to express E_0 in terms of the loss terms:

$$\mu_0^2E_0^2 = \frac{kT_0}{M} + \frac{\mu_0V_i\nu_{i_0}}{2}. \tag{50}$$

Combining the above equations to eliminate Δn, Δu and ΔT, we obtain the impedance formula for the present case. The resulting form of the external impedance is

$$Z_e = \frac{(IL+\omega^2KN) + i\omega(IN-KL)}{I^2+\omega^2K^2}, \tag{51}$$

where

$$I = 16\pi e^2n_0B - \frac{k^2T_0^2}{eV_i\nu_{i_0}}\omega^2A,$$

$$K = -\frac{k^2T_0^2}{eV_i\nu_{i_0}}\omega^2 + 4\pi en_0\mu_0m(2C-B),$$

$$L = -\frac{4\pi k^2T_0^2}{eV_i\nu_{i_0}}\omega^2,$$

$$N = 4\pi\frac{k^2T_0^2}{eV_i\nu_{i_0}}A + 4\pi mC. \tag{52}$$

The symbols A, B and C appearing in the above expressions are respectively

$$A = \frac{3m\nu_{c_0}}{M} + \left(\frac{eV_i}{kT_0}\right)^2\nu_{i_0} + \left(\frac{e}{kT_0\mu_0}\right)\left(\frac{kT_0}{M} + \frac{\mu_0V_i\nu_{i_0}}{2}\right),$$

$$B = \frac{kT_0}{M} + \frac{\mu_0V_i\nu_{i_0}}{2},$$

$$C = \frac{kT_0}{M} + \frac{\mu_0 V_i \nu_{i_0}}{2} + \frac{kT_0}{m} = B + \frac{kT_0}{m}. \tag{53}$$

From eq. (51), the external resistance and inductance turn out to be respectively

$$R_e = \frac{IL + \omega^2 KN}{I^2 + \omega^2 K^2}, \tag{54}$$

$$L_e = \frac{IN - KL}{I^2 + \omega^2 K^2}. \tag{55}$$

On the other hand, the internal impedance is written by

$$Z_i = \frac{(I'L + \omega^2 K'N) + i\omega(I'N - K'L)}{I'^2 + \omega^2 K'^2}, \tag{56}$$

where

$$I' = 16\,\pi e^2 n_0 B,$$
$$K' = 4\,\pi e n_0 \mu_0 m(C - 2\,B). \tag{57}$$

The internal resistance and inductance are found to be respeotively

$$R_i = \frac{J'L + \omega^2 K'N}{I'^2 + \omega^2 K'^2}, \tag{58}$$

$$L_i = \frac{I'N - K'L}{I'^2 + \omega^2 K'^2}. \tag{59}$$

(d) The case $\partial\Delta n/\partial x = 0$ and $\partial\Delta T/\partial x = 0$, but the quasi-steady approach

Here, we shall consider the effect of the term $\partial\Delta u/\partial x$ which was neglected in the previous sections. In the quasi-steady approch, the term $\partial\Delta u/\partial x$ is written by $\mu\Delta E/\partial x$. Now the quantity $\partial\Delta E/\partial x$ is connected with the space charge field arising from the charge separation. The relation between them is described by the Poisson's equation

$$\frac{\partial\Delta E}{\partial x} = -4\,\pi e\Delta n. \tag{60}$$

Substituting eq. (60) into eq. (12), we obtain

$$(4\,\pi e n_0 \mu_0 + i\omega)\,\Delta n = \frac{n_0 \nu_{i_0} e V_i}{kT_0^2}\,\Delta T, \tag{61}$$

which differs from eq. (44) by an additional term $4\,\pi e n_0 \mu_0$. Also, the following additional term due to charge separation appears in the bracket for Δn in eq. (48):

$$12\,\pi e n_0 \mu_0 \left(\mu_0^2 E_0^2 + \frac{kT_0}{m}\right)$$

Such additional terms lead to a different form of plasma impedance from that for the case (c). The expressions for the exernal impedance, resistance and inductance are formally given by eqs. (51), (54) and (55), respectively, but with different I, K, L and N, which are

$$I = 16\pi e^2 n_0 B + \frac{k^2 T_0^2}{eV_i \nu_{i_0}} \{(16\pi^2 e^2 n_0^2 \mu_0^2 - \omega^2)A - 8\pi e n_0 \mu_0 \omega^2\} + mC(48\pi^2 e^2 n_0^2 \mu_0^2 - \omega^2),$$

$$K = \frac{k^2 T_0^2}{eV_i \nu_{i_0}} (8\pi e n_0 \mu_0 A + 16\pi^2 e^2 n_0^2 \mu_0^2 - \omega^2) + 8\pi e n_0 \mu_0 m(2C - B),$$

$$L = 4\pi \cdot \frac{k^2 T_0^2}{eV_i \nu_{i_0}} (4\pi e n_0 \mu_0 A - \omega^2) + 48\pi^2 e n_0 \mu_0 mC,$$

$$N = 4\pi \frac{k^2 T_0^2}{eV_i \nu_{i_0}} (4\pi e n_0 \mu_0 + A) + 4\pi mC. \tag{62}$$

where the symbols A, B and C are just same as given by eq. (53).

On the other hand, the internal impedance, resistance and inductance are given respectively by eqs. (56), (58) and (59) but with different I', K', L' and N' which are

$$I' = 16\pi e^2 n_0 B + \frac{k^2 T_0^2}{eV_i \nu_{i_0}} 4\pi e n_0 \mu_0 (4\pi e n_0 \mu_0 - \omega^2) + 48\pi^2 e^2 n_0^2 \mu_0^2 mC,$$

$$K' = \frac{k^2 T_0^2}{eV_i \nu_{i_0}} 4\pi e n_0 \mu_0 (A + 4\pi e n_0 \mu_0) + 4\pi e n_0 \mu_0 m(C - 2B),$$

$$L' = L,$$

$$N' = N. \tag{63}$$

IV. Graphical Representation of the Impedance

In the foregoing section, the plasma impedance are evaluated for four cases. In the cases (a) and (b) the analysis was made with the unsteady treatment, but in the cases (c) and (d) the quasi-steady approach. Numerical examination of the impedance, especially of the frequency dependence, will be done for various cases, in the present section. As the formulae obtained in this paper are too complicated to understand the dependence, it may be better to examine numerically for typical examples.

Fig. 1 illustrates the frequency dependence of R_e, R_i, L_e and L_i for the case (a) when the following numerical values of parameters are taken as one of typical examples: $\nu_{i_0} = 3.10^4$/sec., $\nu_{c_0} = 2.10^9$/sec., $T_0 = 3.10^4$°K and $n_0 = 10^7$, 10^9 and 10^{11}/cm^3.

The curve of the internal resistance R_i at low and high frequency sides has a flattened portion and in the middle range there appears a declining portion. As regards L_i, the sign changes at a certain frequency from negative to positive with increasing frequency. The critical frequency is given by

$$\omega^2 = \left\{\frac{kH}{m} + G\left(G + \frac{k\nu_{c_0}}{m}\right)\right\}\left(\frac{m}{k}\right)^2, \tag{64}$$

which is obtained from eq. (45).

However, as shown in Fig. 2, which is plotted taking the numerical values corresponding to lower pressure, *i.e.*; $\nu_{i_0} = 3.10^6$/sec., $\nu_{c_0} = 2.10^7$/sec., and $T_0 =$ 3.10^4°K, the internal inductance L_i is always positive independently on the frequency. Concerning the external resistance, an abrupt decrease at a certain

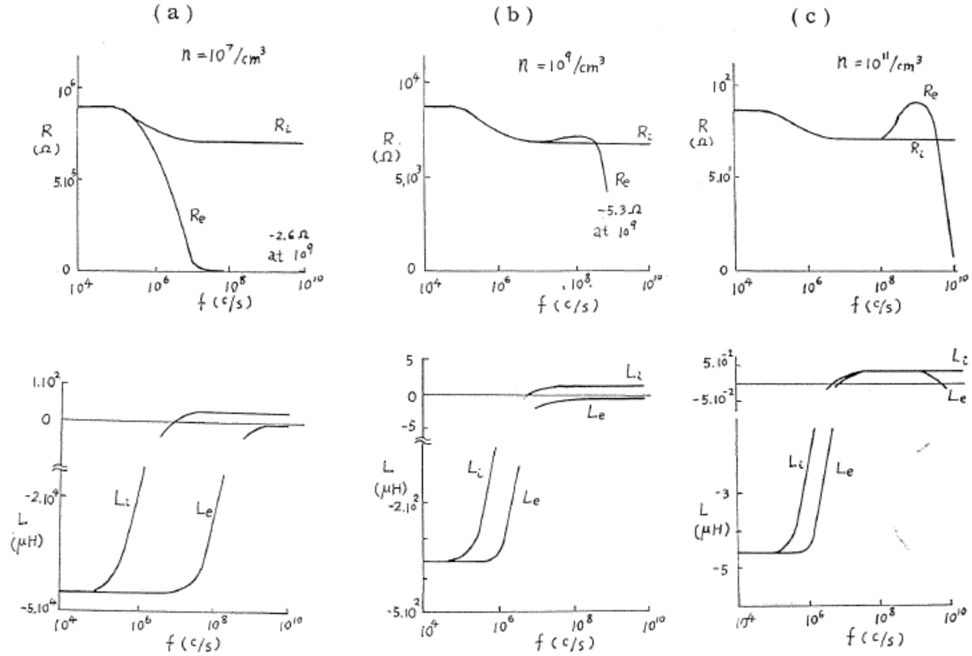

FIG. 1. The plasma impedance as a function of the frequency for the case (a). Higher pressure.

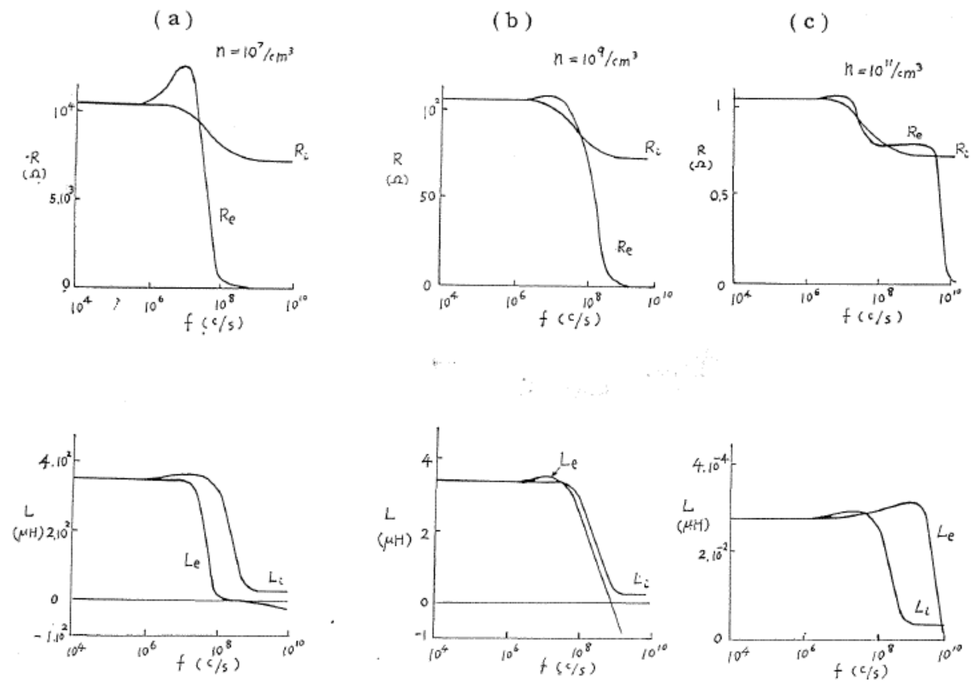

FIG. 2. The plasma impedance as a function of the frequency for the case (a). Lower pressure.

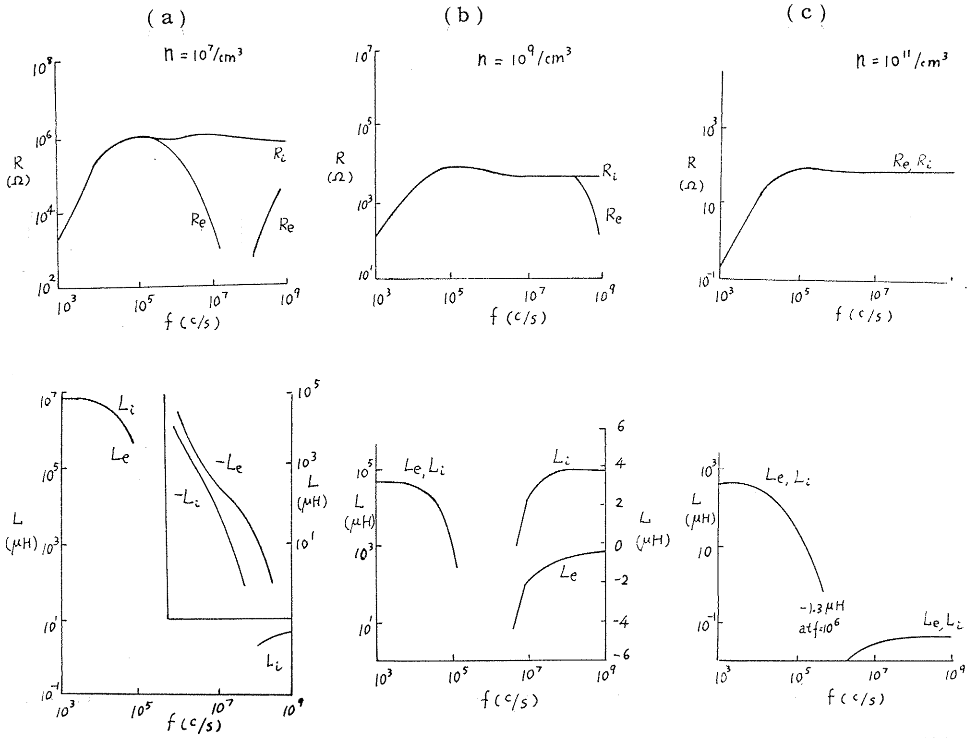

FIG. 3. The plasma impedance as a function of the frequency for the case (b). Higher pressure.

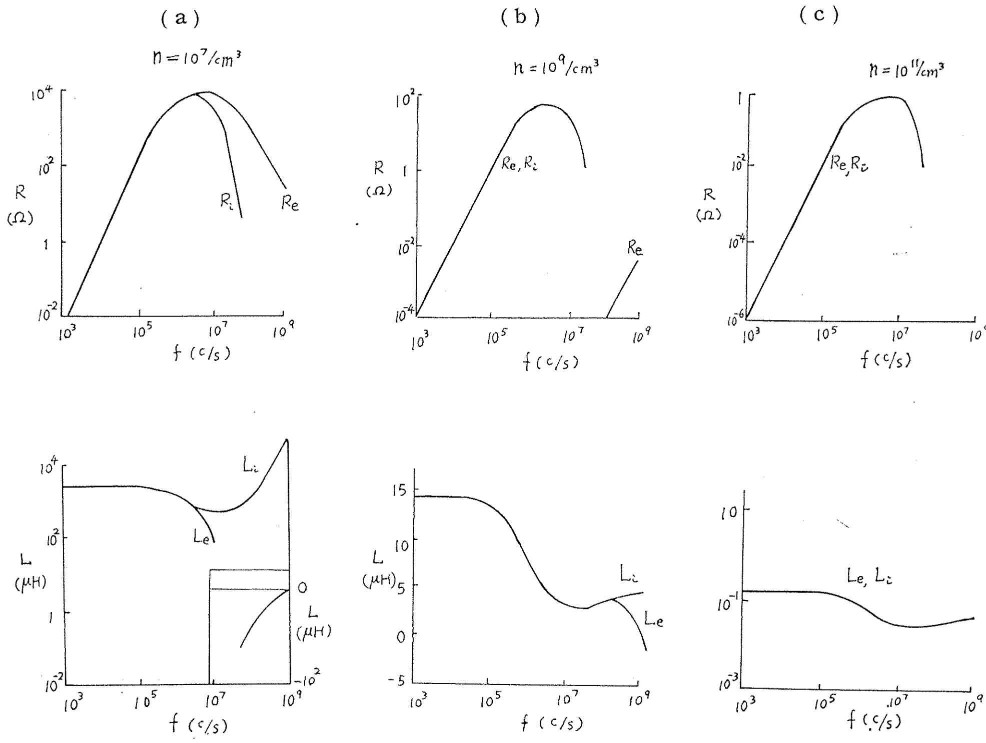

FIG. 4. The plasma impedance as a funbtion of the frequency for the case (b). Lower pressure.

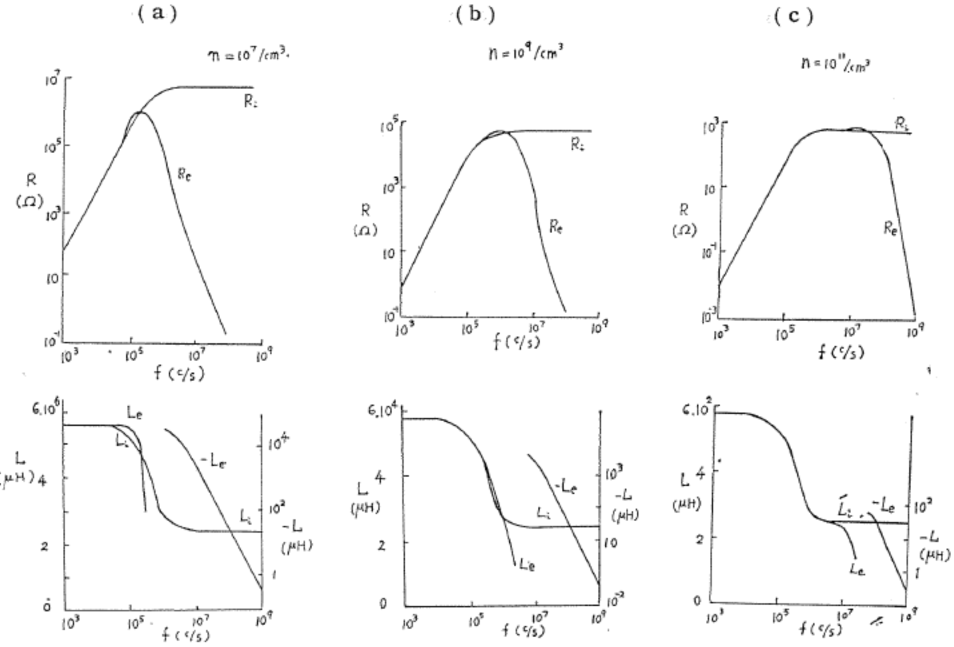

FIG. 5. The plasma impedance as a function of the frequency for the case (c). Higher pressure.

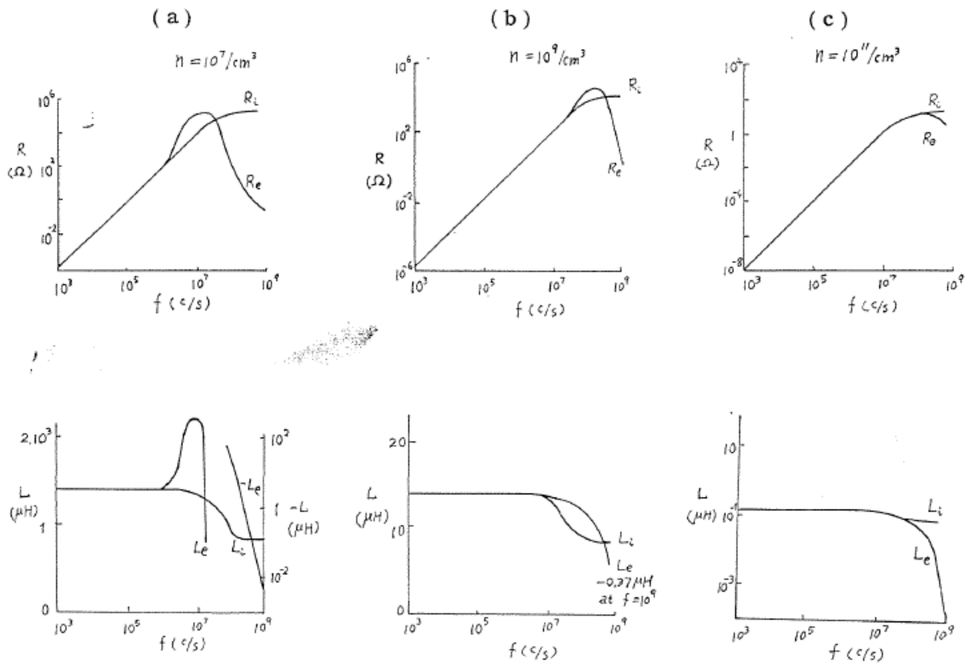

FIG. 6. The plasma impedance as a function of the frequency for the case (c). Lower pressure.

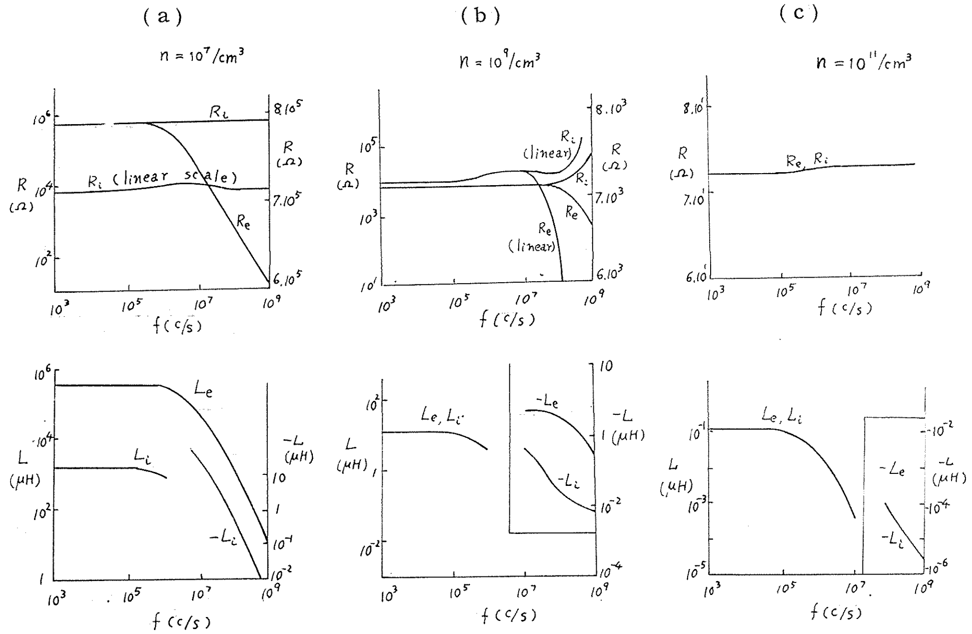

FIG. 7. The plasma impedance as a function of the frequency for the case (d). Higher pressure.

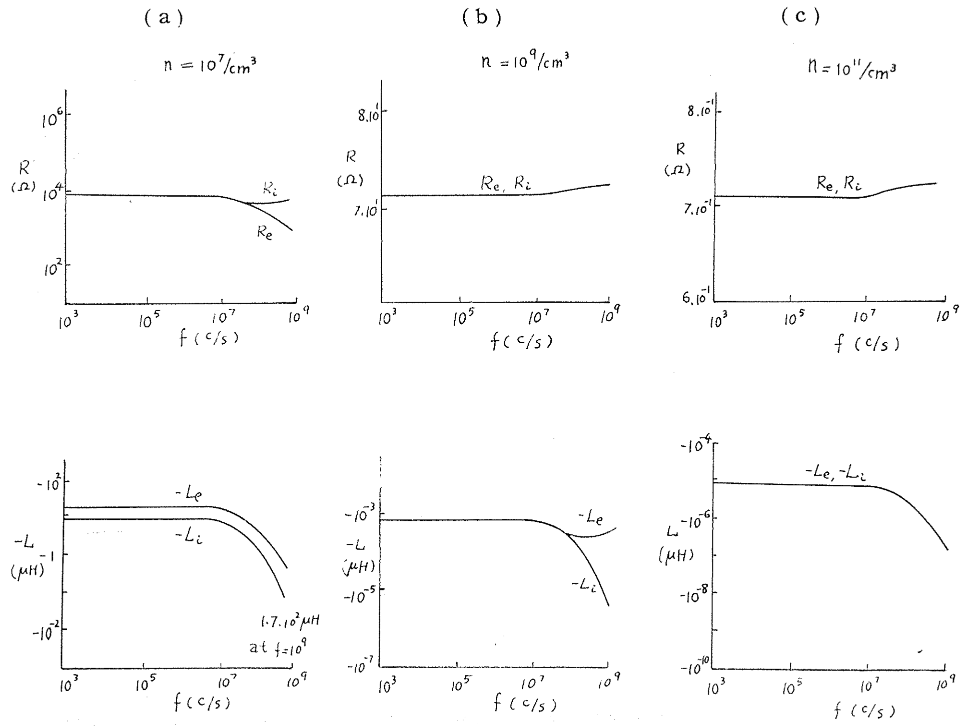

FIG. 8. The plasma impedance as a function of the frequency for the case (d). Lower pressure.

frequency depending on the plasma density is characteristic.

Fig. 3 shows the plots for the case (b) by use of the same numerical values as those in Fig. 1. Contrary to the result of Fig. 1, the sign of L_i in low frequency is positive, but with increasing frequency it changes into negative and then returns to positive. The frequency at which L_i becomes negative first is approximately given by

$$\omega^2 = 4\left\{\frac{m}{M} + \frac{eV_i\nu_{i_0}}{2kT_0\nu_{c_0}}\right\}\frac{eV_i}{kT_0}\nu_{i_0}\nu_{c_0}, \tag{65}$$

which is obtained from eq. (55) setting $Y'=0$.

As shown in Fig. 3, the inteunal resistance R_i increases steeply with increasing frequency in the low frequency range. This is understandable from that Y' in eq. (41) varies proportionally to ω^2 and the remaining terms are almost invariant under the condition considered here. In the high frequency range, the plot shows a rather flattened characteristic. However, a distinct decrease with increasing frequency appears for the low pressure case as shown in Fig. 4, in which the numerical values of parameters are same as thosein Fig. 2.

In Figs. 5 and 6, we plot the impedance curves for the case (c), indicating that both R_i and L_i always have positive value. Also, the plot is different from the corresponding plot for the case (b) in the magitude as well as the frequency dependence. In particular, the difference is remarkable in the high frequency range, where both R_i and L_i are almost constant for the case (c). This result implies that the quasi-steady approach is not valid in the high frequency range.

Figs. 7 and 8 represent the frequency dependence for the case (d), which are plotted by use of the same numerical values of parameters as those used for other cases. The variation of the internal resitance with frequency is not so much as in the other cases, whereas that of the internal inductance is unlike that for the foregoing cases in such a way that it is negative over the whole range of frequency when the low pressure case is considered.

V. Conclusion

So far as the authors know, the experimental data of plasma impedance over the whole range of frequency considered here has not been given enough for the pupose of convincing us. Therefore, at the present time it is not adequate to make a choice of an appropriate expression of plasma impedance from those presented by us in the present paper. Another reason for this statement comes from the fact that most of the experimental result of plasma impedance involve an influence of the electrode fall, because the measurement is made by applying a.c. voltage through the discharge tube.

However, it seems to be true that the measured impedance varies with frequency according to the earlier experiments. This fact can not be explained with the simple theory basing on the simple assumption that only the velocity perturbation contributes to the response of plasma. The theory considering all the perturbation, *i.e.*, the density, velocity and temperature, may provide an explanation for the frequency dependence of the plasma impedance.

According to the earlier experiments, it is likely that of these formulae presented by us the second, that is, the case (b) is most appropriate. The third one is not valid especially at high frequency, because of its smoothed characteristic.

Conclusively, it is said that the collision effect leads to the frequency dependent characteristic of plasma impedance.

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