HYDRAULIC PRESSURE DISTRIBUTION IN FILTER-CAKES UNDER CONSTANT PRESSURE FILTRATION

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Introduction

Since P. C. Carman[1] and B. F. Ruth[6] introduced compression permeability technique for theoretical analysis of filtration, most of the filtration studies have been focused on obtaining the theoretical and experimental correlation between the data obtained from compression permeability experiment and the data resulted from actual filtration test. Many theoretical and experimental studies for this approach have been published by H. P. Grace[2], F. M. Tiller[12-14], M. Shirato[7-11], S. Okamura[5], Kotwitz and Boylan[4], and many others. One of the most important assumptions for predicting filtration characteristics from the result obtained by compression-permeability experiment, is that the local value of porosity and the local specific filtration resistance of filter cake, in dynamic state, are equivalent to the values of compressed cake, in statical equilibrium state, in a consolidometer.

In this paper, the conventional expression for hydraulic pressure distribution is revised in view of the so-called modern filtration theory of the variation of flow rate with respect to distance through a filter cake. In order to prove the plausibility of the assumption mentioned above, the newly derived expression is compared with the experimental measurements of hydraulic pressure variation during actual constant pressure filtration runs.

Using eight kinds of slurry materials, constant pressure filtrations are executed under several combinations of pressures and slurry concentrations. During the filtration run, the hydraulic pressure variations in cake in dynamic state are directly measured by introducing several probes into cake. Observed values of the hydraulic pressure distributions are compared with the new theoretical values predicted from the compression-permeability data, and with an analytical approximate expression which is derived in this paper.

Variation of flow rate within a filter cake

Under a filtration of constant pressure p and slurry concentration s, the volume of filtrate per unit area, v, is expressed from material balance as

$$v=\frac{(1-ms)w}{\rho s} \tag{1}$$

where ρ is the density of filtrate, w the mass of dry solid per unit area, and m the ratio of wet to dry cake mass defined by

$$m = 1 + \frac{\rho \varepsilon_{av}}{\rho_s(1-\varepsilon_{av})}$$

In the above equation, ρ_s is the true density of solid and ε_{av} denotes the average porosity of entire cake. Eq. (1) is differentiated with respect to the time θ. There results

$$q_1 = \frac{dv}{d\theta} = \frac{(1-ms)}{\rho s}\frac{dw}{d\theta} - \frac{w}{\rho}\frac{dm}{d\theta} \tag{2}$$

$(dw/d\theta)$ in Eq. (2) means the accumulation rate of cake solid. Denoting the flow rate of liquid reached the cake surface accompanied with the dw of solid mass by q_0, it is expressed as

$$q_0 = \frac{1-s}{\rho s} \cdot \frac{dw}{d\theta} \tag{3}$$

If the porosity of cake surface is ε_i and dL is the increase of cake thickness in time $d\theta$, a part of liquid volume reached the cake surface, which equals $(\varepsilon_i dL)$, stays in the surface layer, and the excess liquid flows into the cake, then

$$q_i = q_0 - \varepsilon_i \frac{dL}{d\theta} \tag{4}$$

where q_i is the rate of flow through the surface into the cake. From material balance for solids,

$$dw = \rho_s(1-\varepsilon_{av})dL \tag{5}$$

Substituting Eq. (3) and (5) into Eq. (4),

$$q_i = \left\{\frac{1-s}{\rho s} - \frac{\varepsilon_i}{(1-\varepsilon_{av})\rho_s}\right\}\frac{dw}{d\theta} \tag{6}$$

$(dm/d\theta)$ in Eq. (2) is negligibly small when the filter medium resistance can be ignored. Ratio of filtrate flow into the cake surface and out from the cake bottom (q_i/q_1) is obtained by dividing Eq. (6) by Eq. (2).

$$\frac{q_i}{q_1} = \frac{\varepsilon_{av}(1-s) - \varepsilon_i(m-1)s}{\varepsilon_{av}(1-ms)} \tag{7}$$

A general relationship between rate of porosity change and internal flow rate variation is expressed by the following equation in view of the continuity of flow.

$$\frac{\partial q_x}{\partial x} = \frac{\partial \varepsilon_x}{\partial \theta} \tag{8}$$

where x is the distance measured from the cake bottom. When it is assumed, in accordance with the experimental facts[5], that ε_x is solely a function of x/L and that the cake thickness L is a function of θ, Eq. (8) can be modified as follows.

$$\frac{\partial q_x}{\partial x}=\frac{d\varepsilon_x}{d(x/L)}\cdot\frac{d(x/L)}{dL}\cdot\frac{dL}{d\theta}=-\frac{d\varepsilon_x}{d(x/L)}\cdot\frac{x}{L^2}\cdot\frac{dL}{d\theta}$$

$$\text{at } x=0\,;\quad \left(\frac{\partial q_x}{\partial x}\right)_{x=0}=0$$

$$\text{at } x=L\,;\quad \left(\frac{\partial q_x}{\partial x}\right)_{x=L}=-\left[\frac{d\varepsilon_x}{d(x/L)}\right]_{x=L}\cdot\frac{1}{L}\cdot\frac{dL}{d\theta}$$

Then Eq. (9) is obtained and it becomes clear that the q_x-variation is solely a function of x/L.

$$\frac{\left[\dfrac{dq_x}{d(x/L)}\right]}{\left[\dfrac{dq_x}{d(x/L)}\right]_{x=L}}=\frac{x}{L}\frac{\left[\dfrac{d\varepsilon_x}{d(x/L)}\right]}{\left[\dfrac{d\varepsilon_x}{d(x/L)}\right]_{x=L}}\equiv f(x/L) \tag{9}$$

Rearranging Eq. (9), it results

$$dq_x=\left(\frac{dq_x}{d\varepsilon_x}\right)_{x=L}\cdot\left(\frac{x}{L}\right)d\varepsilon_x \tag{10}$$

Integrating Eq. (10) from the cake bottom to a point in the cake x yields

$$q_1-q_x=-\left(\frac{dq_x}{d\varepsilon_x}\right)_{x=L}\int_{\varepsilon_1}^{\varepsilon_x}(x/L)\,d\varepsilon_x$$

Integrating Eq. (10) through the entire cake results

$$q_1-q_i=-\left(\frac{dq_x}{d\varepsilon_x}\right)_{x=L}\int_{\varepsilon_1}^{\varepsilon_i}(x/L)\,d\varepsilon_x$$

where ε_1 is the porosity at $x=0$, *i.e.* the bottom of cake. From the two expressions derived above, $(q_1-q_x)/(q_1-q_i)$ is expressed as

$$\frac{q_1-q_x}{q_1-q_i}=\frac{1-q_x/q_1}{1-q_i/q_1}=\frac{\displaystyle\int_{\varepsilon_1}^{\varepsilon_x}(x/L)\,d\varepsilon_x}{\displaystyle\int_{\varepsilon_1}^{\varepsilon_i}(x/L)\,d\varepsilon_x} \tag{11}$$

$$\int_{\varepsilon_1}^{\varepsilon_x}(x/L)\,d\varepsilon_x=\varepsilon_x(x/L)-\int_0^{x/L}\varepsilon_x d(x/L)=(\varepsilon_x-\varepsilon_{avx})(x/L)$$

$$\int_{\varepsilon_1}^{\varepsilon_i}(x/L)\,d\varepsilon_x=(\varepsilon_i-\varepsilon_{av})$$

Substituting Eq. (7) into Eq. (11), and solving for (q_x/q_1) leads to

$$\frac{q_x}{q_1}=1-\frac{(\varepsilon_x-\varepsilon_{avx})(m-1)s}{\varepsilon_{av}(1-ms)}\cdot\frac{x}{L} \tag{12}$$

where ε_{avx} is the average porosity for cake lying between medium and distance x. The variation of filtrate flow rate is now expressed by Eq. (12). It has become clear that (q_x/q_1) is a function of the normalized distance x/L and the slurry concentration s even under a constant pressure condition.

Theoretical equations of hydraulic pressure distribution

The basic differential equation for relating flow rate q_x at a point x in a constant pressure filtration cake to the pressure gradient is

$$q_x = \frac{g_c}{\alpha_x \mu} \cdot \frac{dp_x}{dw_x} \tag{13}$$

where α_x is a local value of specific filtration resistance at x, and w_x is a mass of solid per unit area in distance x from the medium and is related to distance x by the expression

$$dw_x = \rho_s(1-\varepsilon_x)dx \tag{14}$$

In accordance with the force balance equation, the cake compressive pressure p_s, caused by accumulation of drag force to cake particles, may be calculated by means of

$$p_x + p_s = p$$

Then the following expression,

$$dp_x = -dp_s \tag{15}$$

is concluded. Substituting Eqs. (14) and (15) into Eq. (13) yields the basic differential equation relating the flow rate q_x to the pressure gradient

$$g_c\frac{dp_x}{dx} = -g_c\frac{dp_s}{dx} = \mu\rho_s(1-\varepsilon_x)\alpha_x q_x \tag{16}$$

Solving for p_x and integrating through entire cake thickness L and through cake thickness x under the condition of negligible filter medium resistance, results in the expression for p_x-distribution

$$\begin{aligned}\frac{p_x}{p} &= \frac{\int_0^{x/L}(1-\varepsilon_x)\alpha_x q_x d(x/L)}{\int_0^1 (1-\varepsilon_x)\alpha_x q_x d(x/L)} \\ &= \frac{\int_0^{x/L}\left[\varepsilon_{av}(1-ms)-(\varepsilon_x-\varepsilon_{avx})(m-1)s\frac{x}{L}\right](1-\varepsilon_x)\alpha_x d\left(\frac{x}{L}\right)}{\int_0^1\left[\varepsilon_{av}(1-ms)-(\varepsilon_x-\varepsilon_{avx})(m-1)s\frac{x}{L}\right](1-\varepsilon_x)\alpha_x d\left(\frac{x}{L}\right)}\end{aligned} \tag{17}$$

Local porosity ε_x and local specific resistance α_x in Eq. (17) are obtained as functions of cake compressive pressure p_s from compression-permeability data. So it is convenient to rearrange Eq. (17) as follows,

$$\frac{\int_{p_s}^{p}\frac{dp_s}{\alpha_x(1-\varepsilon_x)}}{\int_0^{p}\frac{dp_s}{\alpha_x(1-\varepsilon_x)}} = \frac{\int_0^{x/L}(q_x/q_1)d(x/L)}{\int_0^1(q_x/q_1)d(x/L)} = \frac{\int_0^{x/L}\left[\varepsilon_{av}(1-ms)-(\varepsilon_x-\varepsilon_{avx})(m-1)s\left(\frac{x}{L}\right)\right]d\left(\frac{x}{L}\right)}{\int_0^1\left[\varepsilon_{ax}(1-ms)-(\varepsilon_x-\varepsilon_{avx})(m-1)s\left(\frac{x}{L}\right)\right]d\left(\frac{x}{L}\right)} \tag{18}$$

By solving Eq. (18) with a numerical processes, it is possible to predict p_x-distribution using the data of compression-permeability experiment.

When the slurry concentration s is so small that the flow rate of filtrate q_x can be regarded as constant through the entire cake, (q_x/q_1) is unity independent of (x/L). Therefore Eq. (18) simply becomes

$$\frac{\displaystyle\int_{p_s}^{p} \frac{dp_s}{\alpha_x(1-\varepsilon_x)}}{\displaystyle\int_{0}^{p} \frac{dp_s}{\alpha_x(1-\varepsilon_x)}} = \frac{x}{L} \tag{19}$$

By calculating the integral values of the left hand side of Eq. (19), p_x-distribution is obtained without any tedious trial and error method.

Analytical approximate equations of hydraulic pressure distribution

As an example of the result of compression-permeability experiment, the data obtained from Hong Kong kaolin "pink (1)" is shown in Fig. 1-1. It is clear that the relation of equilibrium porosity ε vs. cake compressive pressure p_s may be expressed as following approximate equations.

$$\begin{aligned} \varepsilon &= \varepsilon_0 p_s^{-\lambda} & p_s &> p_i \\ \varepsilon &= \varepsilon_i = \varepsilon_0 p_i^{-\lambda} & p_s &\leqq p_i \end{aligned} \tag{20}$$

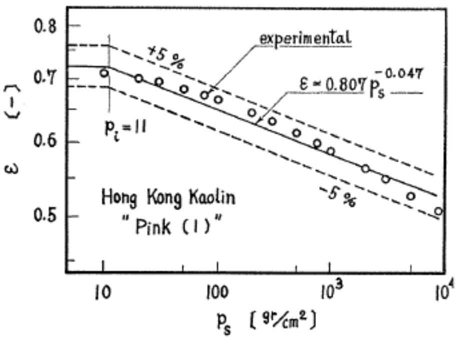

FIG. 1-1. Result of compression experiment, log ε vs. log p_s; Hong Kong kaolin "pink (1)".

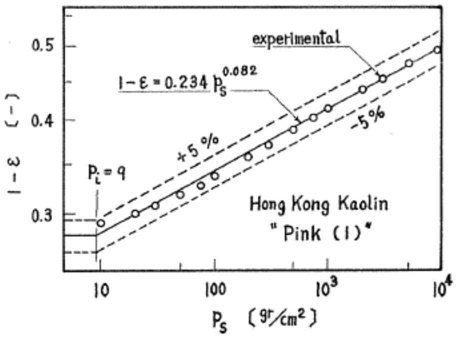

FIG. 1-2. Result of compression experiment, log $(1-\varepsilon)$ vs. log p_s; Hong Kong kaolin "pink (1)".

In Eq. (20) p_i is a constant for a given material and takes different values for different slurries. However, the value p_i is as small as 1 to 10 gr/cm². In the same manner $(1-\varepsilon)$ vs. p_s are plotted in Fig. 1-2, and their relation may be expressed as follows, but they are incompatible with Eq. (20) simultaneously.

$$\begin{aligned} 1-\varepsilon &= Bp_s^{\beta} & p_s &> p_i \\ 1-\varepsilon &= 1-\varepsilon_i = Bp_i^{\beta} & p_s &\leqq p_i \end{aligned} \tag{21}$$

Kozeny's equation for a dx layer within cake can be written as

$$\frac{g_c}{\mu} \cdot \frac{dp_x}{dx} = \frac{kS_0^2(1-\varepsilon_x)^2}{\varepsilon_x^3} q_x$$

Comparing the above equation with Eq. (16), the local specific filtration resistance α_x is expressed as

$$\alpha_x = \frac{kS_0^2(1-\varepsilon_x)}{\rho_s \varepsilon_x^3} \tag{22}$$

From experimental results[5], it has been known that the value of kS_0^2 in Eq. (22) varies with the porosity ε_x. In many cases, however, it can be assumed to be constant for approximate calculations.

Neglecting p_i and substituting Eqs. (20), (21) and (22) into Eq. (19) yields

$$\frac{\displaystyle\int_{p_s}^{p} \frac{\varepsilon_0^3 p_s^{-3\lambda}}{kS_0^2 B^2 p_s^{2\beta}} dp_s}{\displaystyle\int_0^{p} \frac{\varepsilon_0^3 p_s^{-3\lambda}}{kS_0^2 B^2 p_s^{2\beta}} dp_s} = \frac{x}{L}$$

Caluculation of the above expression results in

$$1 - \left(\frac{p_s}{p}\right)^{1-2\beta-3\lambda} = \frac{x}{L}$$

or

$$\left(1 - \frac{p_x}{p}\right)^{1-2\beta-3\lambda} = \left(1 - \frac{x}{L}\right) \tag{23}$$

Eq. (23) is an approximate analytical equation of the p_x-distribution based on the validity of power function relationship between ε_x and $(1-\varepsilon_x)$ vs. p_s, under the condition of neglecting the variation of flow rate and kS_0^2 through cake. There is no necessity to make permeability experiment in order to calculate p_x-distribution by Eq. (23).

Tiller and Cooper[12] also presented an approximate expression for p_x-distribution. As an example, the data of permeability test of Hong Kong kaolin "pink (1)" are illustrated in Fig. 1–3. It is possible to approximate the relation between specific filtration resistance α_x and cake compressive pressure p_s to the next empirical equations.

$$\begin{aligned} \alpha_x &= \alpha_0 p_s^n & p_s &> p_i \\ \alpha_x &= \alpha_i = \alpha_0 p_i^n & p_s &\leqq p_i \end{aligned} \tag{24}$$

Neglecting p_i and substituting Eqs. (21) and (24) into Eq. (19) yields

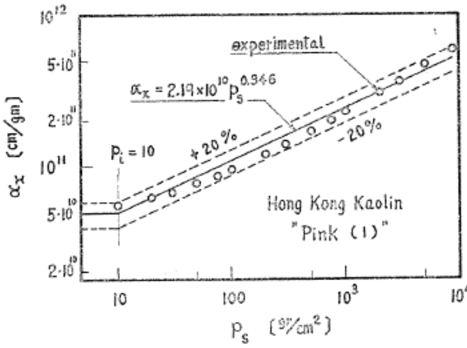

FIG. 1–3. Result of permeability experiment, $\log \alpha_x$ vs. $\log p_s$; Hong Kong kaolin "pink (1)".

$$\frac{\int_{p_s}^{p} \frac{dp_s}{\alpha_0 B p_s^{n+\beta}}}{\int_0^{p} \frac{dp_s}{\alpha_0 B p_s^{n+\beta}}} = \frac{x}{L}$$

and, therefore

$$1 - \left(\frac{p_s}{p}\right)^{1-n-\beta} = \frac{x}{L}$$

or

$$\left(1 - \frac{p_x}{p}\right)^{1-n-\beta} = \left(1 - \frac{x}{L}\right) \tag{25}$$

Eq. (25) is also an approximate analytical equation of p_x-distribution under the condition of neglecting the variation of flow rate through cake. Eq. (25) requires the data of both compression and permeability experiments, whereas Eq. (23) requires only the compression data.

In Table 1 the data of compression-permeability experiments and the approximate analytical equations for eight kinds of examined slurries are shown.

TABLE 1. The results of compression-permeability experiments

Approximate Equation→	$\varepsilon=\varepsilon_0 p_s^{-\lambda}$ ···(20)				$1-\varepsilon=Bp_s^{\beta}$ ···(21)				$\alpha_x=\alpha_0 p_s^{n}$ ···(24)				Eq. (23)	Eq. (25)
Slurry ↓	ε_0	λ	p_i	range dev.	B	β	p_i	range dev.	α_0	n	p_i	range dev.	$1-2\beta-3\lambda$	$1-n-\beta$
Cement Material (1)	0.670	0.073	2.3	~1000 ±2%	0.384	0.065	0.6	~1000 ±3%	5.93×10^9	0.361	1.6	~1000 ±10%	0.651	0.574
〃 (2)	0.627	0.058	1.0	~9000 ±0%	0.415	0.048	0.1	~9000 ±5%	8.43×10^9	0.307	0.5	~9000 ±10%	0.731	0.645
〃 (3)	0.588	0.052	0.8	~9000 ±0%	0.440	0.041	0.1	~9000 ±3%	1.20×10^{10}	0.255	0.2	~9000 ±10%	0.762	0.704
Filter Cel	0.890	0.011	5.0	~1000 ±0.5%	0.116	0.059	3.7	~1000 ±2%	1.09×10^{10}	0.210	4.5	~1000 ±10%	0.850	0.732
Hara-Gairome Clay (1)	0.924	0.055	13.7	~5000 ±5%	0.146	0.132	11.0	~9000 ±5%	9.92×10^{10}	0.515	12.6	~5000 ±20%	0.570	0.353
Hong Kong Kaolin "Pink (1)"	0.807	0.047	11.2	~9000 ±4%	0.234	0.082	8.8	~9000 ±3%	2.19×10^{10}	0.346	10.3	~9000 ±20%	0.694	0.572
Ignition Plug	0.888	0.054	10.8	~5000 ±5%	0.175	0.113	7.5	~9000 ±4%	1.27×10^{10}	0.449	9.5	~5000 ±20%	0.611	0.438
Kieselguhr	0.795	0.017	13.9	~1000 ±2%	0.213	0.047	13.2	~1000 ±3%	1.21×10^9	0.216	13.6	~1000 ±20%	0.855	0.747

Measurement of hydraulic pressure variation

In Fig. 2 a schematic view of experimental apparatus is shown. A bomb-filter is employed as test-filter, and air-sealed manometers[5] and strainmeter pressure-heads are used as pressure measuring devices. These pressure measuring devices are connected to several probes which are approximately 1 to 3 mm. consecutively shorter than the longest and nearest probe to the medium. After setting the pressure heads to the bomb filter, slurry is poured and filtered at constant pressure. The filtration time, the volume and the temparature of filtrate and the reading of strain-meters are recorded.

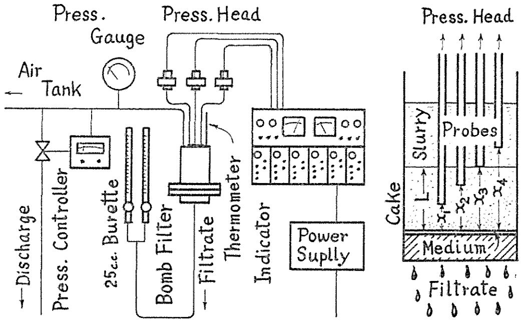

FIG. 2. Schematic view of experimental apparatus.

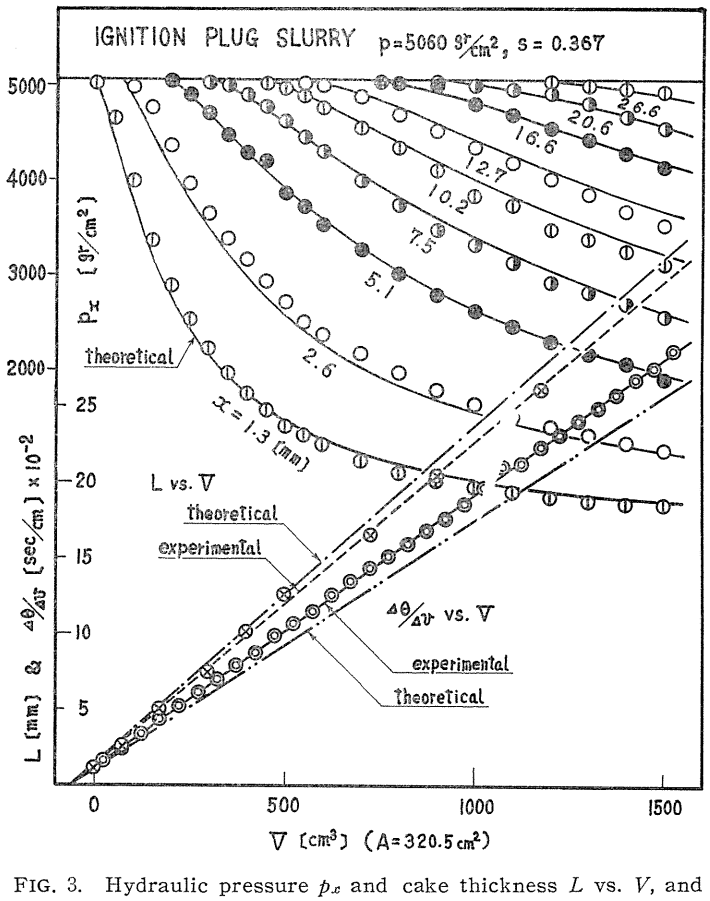

FIG. 3. Hydraulic pressure p_x and cake thickness L vs. V, and $\Delta\theta/\Delta v$ vs. V; Ignition plug slurry, p=5060 g-force/cm², s=0.367.

Experimental results

The results obtained with the setup shown in Fig. 2 are presented in Fig. 3, where the pressure at different probe heights and the cake thickness are plotted against filtrate volume for Ignition plug slurry filtered at a constant pressure of 5060 g-force/cm². As long as a probe is in the slurry, the pressure remaines constant; as soon as the cake forms around the probe, the pressure begins to fall as shown in Fig. 3.

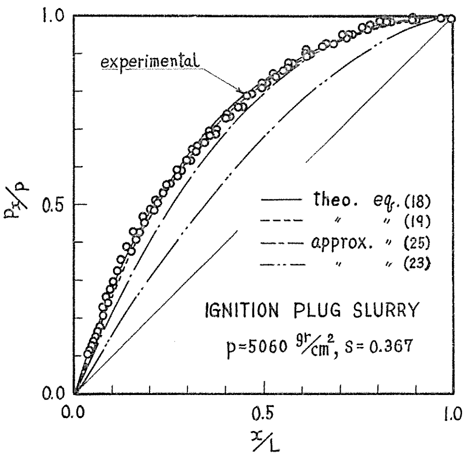

FIG. 4. Hydraulic pressure distribution in a cake; Ignition plug slurry, $p=5060$ g-force/cm², $s=0.367$.

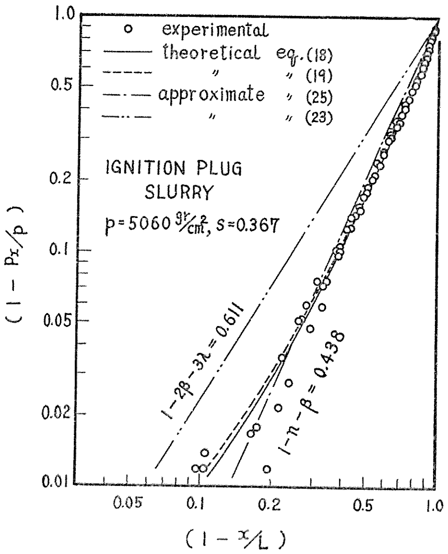

FIG. 5. Logarithmic plot of the hydraulic pressure distribution in a cake; Ignition plug slurry, $p=5060$ g-force/cm², $s=0.367$.

On the basis of data obtained as illustrated in Fig. 3, p_x-distributions are plotted in Figs. 4 and 5, in which pure theoretical and approximate analytical values are plotted. The value of $(1-2\beta-3\lambda)$ in Eq. (23) is 0.611 and the value of $(1-n-\beta)$ in Eq. (25) is 0.438. They are approximately equal to the value in the following experimental equation for Ignition plug slurry reported by S. Okamura and M. Shirato[5].

$$(1-x/L) \doteqdot (1-p_x/p)^{\nu} \qquad (26)$$

$$\nu: \ 0.40\text{-}0.56$$

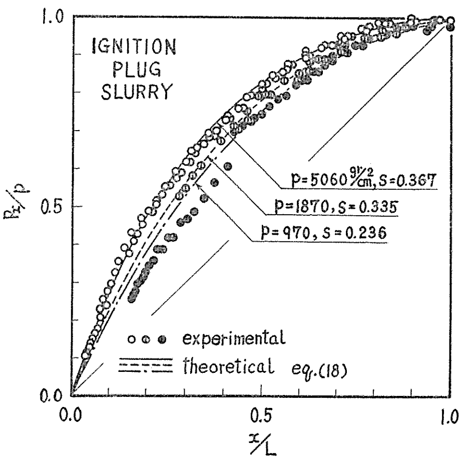

FIG. 6. Effect of filtration pressure on the hydraulic pressure distribution in a cake; Ignition plug slurry.

In Fig. 3, the theoretical vlues of p_x vs. V, $\Delta\theta/\Delta v$ vs. V and L vs. V, which are calculated from Eq. (18) with an electronic computer, are also plotted.

Fig. 6 shows the effect of filtration pressure on p_x-distribution using the date of Ignition plug slurry. Similar results are obtained about other kinds of slurries except Cement material slurry. The authors are inclind to believe that the exception of Cement material slurry is affected by the fact that the wall friction of Cement material during the compression test is rather larger than that of cakes of other slurries. However, further investigations are necessary to solve the problem.

As may be seen in Fig. 5, it can generally be said that Eq. (25) is nearer to the newly derived theoretical value rather than Eq. (23). Experimental values are nearly close to the derived Eq. (17) and lie between the approximate analytical Eqs. (23) and (25).

The distribution of the flow rate of filtrate within a cake is greatly affected by slurry concentration s[8], therefore the p_x-distribution may be varied by slurry concentration; but this fact cannot be shown by experimental measurements. It will be considered to be difficult to obtain more accurate data using the probes inserted in filter cake as attempted in this paper, and the data will scatter to the extent of the expected small p_x variations caused by large changes in slurry concentration.

Conclusion

1) In view of the new theory[14] of the variation of flow rate with respect to distance through a filter cake, the theoretical p_x-distribution Eq. (18) for a constant pressure cake is derived and solved by using a numerical process with a digital computer.

2) Eq. (23) is derived as an approximate analytical equation of the p_x-distribution, assuming an approximate power relation of ε vs. p_s. While it is a simple equation obtainable only from the compression data, it is still useful for predicting the p_x-distribution in cakes.

3) The higher the filtration pressure and the larger the compressibility of cake, the p_x-distribution curve is the more further from a linear relation between p_x and x/L.

4) The theoretical p_x-distribution should be varied with the slurry concentration s, but its change is too small to detect by actual measurements used in this paper.

5) The theoretical values of filtration characteristics predicted from compression-permeability data differ from their experimental values in 5 to 10%.

6) To develop the more exact equations for predicting the p_x-distribution and the filtration characterstic values, the variation of both the flow rate of liquid and the migration of solids through a cake and the effect of the wall friction in a consolidometer[9] should be taken into account for mathematical analysis correlating filtration and permeation.

Nomenclature

A : cross sectional area, cm^2
B : constant defined in Eq. (21)
g_c : conversion factor, dyne/g-force
k : Kozeny's constant, dimensionless

L : cake thickness, cm
m : ratio of wet to dry cake mass, dimensionless
n : compressibility coefficient, exponent in Eq. (24)
p : applied filtration pressure, g-force/cm^2
p_i : low pressure below which ε_x and α_x are constant, g-force/cm^2
p_s : cake compressive pressure at distance x from the medium, g-force/cm^2
p_x : hydraulic pressure at distance x from the medium, g-force/cm^2
p_1 : pressure at the interface of medium and cake, g-force/cm^2
q_i : value of q_x at surface layer of cake, cm^3/(cm^2)(sec)
q_x : rate of flow of liquid in cake at distance x from the medium, cm^3/(cm^2)(sec)
q_0 : rate of flow of liquid approaching cake surface cm^3/(cm^2)(sec)
q_1 : value of q_x at the medium, cm^3/(cm^2)(sec)
s : mass fraction of solids in slurry, dimensionless
S_0 : effective specific surface area of cake solids, cm^2/cm^3
v : volume of filtrate per unit area, cm^3/cm^2
V : volume of filtrate, cm^3
w : total mass of dry solids per unit area, g-mass/cm^2
w_x : mass of solids per unit area in distance x from the medium, g-mass/cm^2
x : distance from the medium, cm

Greek Letters

α_x : local value of specific filtration resistance at cake pressure p_s, cm/g-mass
α_i : value of α_x when $p_s \leqq p_i$, cm/g-mass
α_0 : constant defined in Eq. (24)
β : exponent in Eq. (21)
ε : equilibrium porosity at cake pressure p_s, dimensionless
ε_{av} : average porosity of entire cake, dimensionless
ε_{avx} : average porosity for cake lying between medium and distance x, dimensionless
ε_i : porosity infinitesimal surface layer of cake, dimensionless
ε_x : local value of porosity at distance x from the medium, dimensionless
ε_0 : constant defined in Eq. (20)
ε_1 : porosity in layer adjacent to medium, dimensionless
θ : time, sec
λ : exponent in Eq. (20)
μ : viscosity, g-mass/(cm)(sec)
ν : exponent in experimental equation, Eq. (26)
ρ : density of liquid, g-mass/cm^3
ρ_s : true density of solids, g-mass/cm^3

Literatures Cited

1) Carman, P. C.: Trans. Instn. Chem. Engrs. (London), **16**, 168 (1938).
2) Grace, H. P.: Chem. Eng. Progr., **49**, 303, 367 (1953).
3) Hoffing, E. H., and F. J. Lockhart: Chem. Eng. Progr., **47**, 3 (1951).
4) Kottwitz, F. A., and D. R. Boylan: A. I. Ch. E. Journal, **3**, 433 (1957).

5) Okamura, S., and M. Shirata: Chem. Eng. (Japan), **19**, 104, 111 (1955).
6) Ruth, B. F.: Ind. Eng. Chem., **38**, 564 (1946).
7) Shirato, M., T. Murase, and H. Kato: Preprint of 28th annual meeting of the Society of Chem. Engrs., Japan, 187 (1963).
8) Shirato, M., and M. Sambuichi: Chem. Eng. (Japan), **27**, 470 (1963).
9) Shirato, M., M. Sambuichi, and H. Kato: Preprint of 2nd general symposium of the Society of Chem. Engrs., Japan, 110 (1963).
10) Shirato, M., M. Sambuichi, T. Murase, M. Hamajima, and Y. Miyata: Preprint of Yokkaichi meeting of the Society of Chem. Engrs., Japan, 57 (1964).
11) Shirato, M., M. Sambuichi, and S. Okamura: A.I.Ch.E. Journal, **9**, 599 (1963).
12) Tiller, F. M., and H. Cooper: A.I.Ch.E. Journal, **6**, 595 (1960); **8**, 455 (1962).
13) Tiller, F. M., and M. Shirato: Chem. Eng. (Japan), **26**, 925 (1962).
14) Tiller, F. M., and M. Shirato: A.I.Ch.E. Journal, **10**, 61 (1964).