# LOW FREQUENCY WAVE PROPAGATION IN A DRIFTING PLASMA

TAKAYOSHI OKUDA and KENZO YAMAMOTO

*Department of Electronics*

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ABSTRACT

In this paper, propagation of low frequency disturbance through a plasma having a drift velocity is dealt with the help of macroscopic approach. Existence of a drift velocity makes ion oscillation possible to propagate with constant amplitude under a certain condition.

The frequency of propagating wave is found to be higher than the ion plasma frequency. The frequency deviation depends on the magnitude of the drift velocity.

## I. Introduction

In gas discharge plasma, a number of types of oscillation may exist without any external excitation. For example, plasma electron oscillation has been observed in d.c. discharge under a special condition[1)-6)] and has been produced as a result of beam-plasma interaction.[7)-13)] Natural ion plasma oscillation has not been certified but several authors have understood low frequency oscillation observed in laboratory plasma as an ionic sound wave in which the frequency was determined by the dimension of the system used.[14)-16)]

Exsistence of low frequency oscillation has been verified by noise measurement, showing single or multiple peaks of noise amplitude in frequency spectrum.[17)-20)]

In this paper, our attention is directed towards propagation of low frequency oscillation in a drifting plasma. Hitherto, the drift velocity has been neglected in the analysis of the ion plasma oscillation.[3) 21) 22)]

The purpose of this paper is to clarify how the drift velocity or static electric field may affect the propagation of wave.

## II. Basic equations

In this paper, the macroscopic approach is used to describe velocity and density of charged particle in plasma, because under tne condition considered here an equilibrium in velocity distribution can be reached in a time shorter than the period of the oscillation.

Basic equations in the macroscopic approach are derived from Boltzmann's equation by assuming a displaced Maxwellian distribution of energy of charged particle. Those are refered as the equations of the continuity, the conservation of momentum and energy, respectively.[23) 24)] Those are expressed in one-dimensional case as follows;

$$\partial n/\partial t + \partial/\partial x(nu) = (\partial n/\partial t)_c \tag{1}$$

$$\partial/\partial t(nu) + \partial/\partial x\{n(u^2 + kT/m)\} \pm eEn/m = \sum P/m \tag{2}$$

$$\partial/\partial t\{n(u^2 + kT/m)\} + \partial/\partial x\{n(u^3 + 3\,ukT/m)\} \pm 2\,eEnu/m = \sum L/m \tag{3}$$

where $n$ is the density, $u$ the drift velocity, $(\partial n/\partial t)_c$ the changing rate of the density, $E$ the electric field, $m$ the mass, $e$ the unit charge, $T$ the temperature, $\sum P$ the transfered momentum, and $\sum L$ the transfered energy. Here, it is assumed that all the collision terms $(\partial n/\partial t)_c$, are negligibly small. Then, eqs. (1) to (3) are simplified as

$$\partial n/\partial t + \partial/\partial x(nu) = 0 \tag{4}$$

$$n\,\partial u/\partial t + nu\,\partial u/\partial x + \partial/\partial x(nkT/m) \pm eEn/m = 0 \tag{5}$$

$$n\,\partial/\partial t(u^2) + \partial/\partial t(nkT/m) + un\,\partial/\partial x(u^2) + \partial/\partial x(3\,nkTu/m) \pm 2eEnu/m = 0 \tag{6}$$

where the plus sign appearing in eqs. (5) and (6) refers to electron, while the minus to ion.

The last equation is Poisson's equation, which is expressed as

$$\partial E/\partial x = 4\,\pi e(n_2 - n_1) \tag{7}$$

where $n_1$ and $n_2$ are the density of electron and ion, respectively.

### III. Dispersion Relation for the Case with no Drift

Since we confine ourselves to the case of small amplitude, the follwing substituion method is applicable as far as propagation of small signal is concerned. The density and temperature are expressed as a sum of steady and perturbed parts as follows;

$$n_1 = n_{10}\{1 + \phi_1 \exp i(Kx - \omega t)\} \tag{8}$$

$$n_2 = n_{20}\{1 + \phi_2 \exp i(Kx - \omega t)\} \tag{9}$$

$$T_1 = T_{10}\{1 + \tau_1 \exp i(Kx - \omega t)\} \tag{10}$$

$$T_2 = T_{20}\{1 + \tau_2 \exp i(Kx - \omega t)\} \tag{11}$$

where $K$ is the wave number, and $\omega$ the frequency. In these expressions, it is assumed that $n_{10} = n_{20} = n_0$, $\phi \ll 1$ and $\tau \ll 1$.

As there is no steady drift velocity or electric field in this case, they are composed of only the perturbed part as

$$E = E \exp i(Kx - \omega t) \tag{12}$$

$$u_1 = u_1 \exp i(Kx - \omega t) \tag{13}$$

$$u_2 = u_2 \exp i(Kx - \omega t) \tag{14}$$

Substituting these into eq. (4) for electron and ion, we get

$$-i\omega\phi_1 + iKu_1 = 0 \tag{15}$$

$$-i\omega\phi_2 + iKu_2 = 0 \tag{16}$$

provided that the second or higher terms of $\exp i(Kx-\omega t)$ are neglected. Also, with the help of such linearization and the assumption that all the collision terms are neglected in the perturbed parts, we get the following equations from eqs. (5), (6) and (7);

$$-i\omega u_1 + i(kT_{10}/m_1)K\phi_1 + i(kT_{10}/m_1)K\tau_1 + eE/m_1 = 0 \tag{17}$$

$$-i\omega u_2 + i(kT_{20}/m_2)K\phi_2 + i(kT_{20}/m_2)K\tau_2 - eE/m_2 = 0 \tag{18}$$

$$-i\omega(\phi_1 + \tau_1) + 3\,iu_1K = 0 \tag{19}$$

$$-i\omega(\phi_2 + \tau_2) + 3\,iu_2K = 0 \tag{20}$$

$$iKE = 4\,\pi e n_0(\phi_2 - \phi_1) \tag{21}$$

Combining eqs. (15) to (21) and eliminating $u$, $\phi$, $\tau$ and $E$, we finally have the so-called dispersion relation as

$$1 = \omega_{p1}^2/\{\omega^2 - 3(kT_{10}/m_1)K^2\} + \omega_{p2}^2/\{\omega^2 - 3(kT_{20}/m_2)K^2\} \tag{22}$$

where
$$\omega_{p1}^2 = 4\,\pi e^2 n_0/m_1 \tag{23}$$
and
$$\omega_{p2}^2 = 4\,\pi e^2 n_0/m_2 \tag{24}$$

are the electron and ion plasma frequency, respectively. In addition to the dispersion relation, an adiabatic relation $\tau=2\,\phi$, or $p \sim n^{\gamma}$, where $\gamma=3$, can be deduced from eqs. (15), (16), (19) and (20). Now, the dispersion relation found here is identical with that obtained by several authors for the case of one kind of particle, by using Bolzmann's transfer equation.[25][26] If the temperature is assumed to be constant, *i.e.*, $\tau=0$, a similar dispersion relation results from combination of eqs. (15), (16), (17), (18) and (21). That is

$$1 = \omega_{p1}^2/\{\omega^2 - (kT_{10}/m_1)K^2\} + \omega_{p2}^2/\{\omega^2 - (kT_{20}/m_2)K^2\} \tag{25}$$

This expression has been given by J. J. Thomson and G. P. Thomson[21] and B. D. Fried and R. W. Gould[22] by means of the substitution method.

With any of these relations, one can not find a specific frequency and wave number of the wave propagating through a plasma with no drift.

## IV. Dispersion Relation for the Case with Drift

In this paragragh, we shall discuss the effect of drift on the dispersion relation.

For simplicity, we make an assumption that the temperature is not perturbed in time and space. Only difference from the case dealt in the preceding paragragh is that a steady drift velocity or electric field is involved in this case. With these respects and with use of a similar way in the preceding paragragh, the dispersion relation can be found.

In the present case, eqs. (12) to (14) are replaced by

$$E = E_0\{1 + \varepsilon \exp i(Kx - \omega t)\} \tag{26}$$

$$u_1 = u_{10}\{1 + \eta_1 \exp i(Kx - \omega t)\} \tag{27}$$

$$u_2 = u_{20}\{1 + \eta_2 \exp i(Kx - \omega t)\} \tag{28}$$

where $u_0$ is a constant drift velocity.

Introduction of eqs. (8), (9), (26), (27) and (28) into eqs. (5), (6) and (7) result in the following equations;

$$(-i\omega + iu_{10}K)\phi_1 + iKu_{10}\eta_1 = 0 \tag{29}$$

$$(-i\omega + iu_{20}K)\phi_2 + iKu_{20}\eta_2 = 0 \tag{30}$$

$$\{i(kT_{10}/m_1)K + eE_0/m_1 + i\omega_{p1}^2/K\}\phi_1 - i(\omega_{p1}^2/K)\phi_2 + (-iu_{10}\omega + iu_{10}^2K)\eta_1 = 0 \tag{31}$$

$$-i(\omega_{2p}^2/K)\phi_1 + \{i(kT_{20}/m_2)K - eE_0/m_2 + i\omega_{p2}^2/K\}\phi_2 + (-iu_{20}\omega + iu_{20}K)\eta_2 = 0 \tag{32}$$

where the following equation is used instead of eq. (20);

$$i\varepsilon E_0 K = 4\pi e n_0(\phi_2 - \phi_1) \tag{33}$$

Combining eqs. (29) and (30) and separating them into real and imaginary parts, two equations are obtained. They are

$$\begin{aligned} &K^4(m_1u_{10}^2 - kT_{10})(m_2u_{20}^2 - kT_{20})/m_1m_2 + 2K^3\omega\{m_2u_{20}(kT_{10} - m_1u_{10}^2) \\ &\quad + m_1u_{10}(kT_{20} - m_2u_{20}^2)\} + K^2\{-\omega^2(kT_{20}/m_2 + kT_{10}/m_1) + \omega_{p1}^2(kT_{20}/m_2 - u_{20}^2) \\ &\quad + \omega_{p2}^2(kT_{10}/m_1 - u_{10}^2) + \omega^2(4u_{10}u_{20} + u_{10}^2 + u_{20}^2) + e^2E_0^2/m_1m_2\} \\ &\quad + 2K\{\omega(u_{10}\omega_{p2}^2 + u_{20}\omega_{p1}^2) - \omega^3(u_{10} + u_{20})\} - \omega^2(\omega_{p1}^2 + \omega_{p2}^2) + \omega^4 = 0 \end{aligned} \tag{34}$$

$$K^2(kT_{10} + m_1u_{10}^2 - kT_{20} - m_2u_{20}^2) - 2K\omega(m_1u_{10} - m_2u_{20}) + \omega^2(m_1 - m_2) = 0 \tag{35}$$

Solutions of eqs. (34) and (35) allow us to find a dispersion relation, by which the frequency and wave number are given as a function of plasma parameters, such as the drift velocity and temperature of electron and ion. However, the resulting expressions for $K$ and $\omega$ have too complicate a form to understand. Since we have interest in the case usually encountered in laboratory, the problem is simplified by assuming that $kT_{10} \gg kT_{20}$, and $kT_{10} \gg m_1u_{10}^2$.

As a consequence, eq. (35) becomes

$$K \cong \pm\omega(m_2/kT_{10})^{1/2} \tag{36}$$

Providing that the condition $mu_0 < kT_0$ is also fulfilled for ion, combination of eqs. (34) and (36) yields

$$\begin{aligned} \omega^2 &= [(T_{20}/T_{10})\omega_{p1}^2 + \omega_{p1}^2 + e^2E_0^2/kT_{10}m_1 \pm 2(m_2/kT_{10})^{1/2}u_{10}\omega_{p2}^2 \pm 2(m_2/kT_{10})^{1/2}u_{20}\omega_{p1}^2] \\ &\times [m_2/m_1 - m_2T_{20}/m_1T_{10} \mp 2u_{20}(m_2/kT_{10})^{1/2}(m_2/m_1 + T_{20}u_{10}/T_{10}u_{20}) \pm 2u_{10}(m_2/kT_{10})^{1/2}]^{-1} \\ &\simeq \{\omega_{p1}^2 + e^2E_0^2/kT_{10}m_1 \pm 2(m_2/kT_{10})^{1/2}(u_{10}\omega_{p2}^2 + u_{20}\omega_{p1}^2)\} \\ &\times \{m_2/m_1 \pm 2(m_2/kT_{10})^{1/2}(u_{20}m_2/m_1 + u_{10}T_{20}/T_{10} - 2u_{10})\}^{-1} \end{aligned} \tag{37}$$

For example, we take the following numerical values in order to verify the present approach; $T_{10} = 3.10^4$ °K, $T_{20} = 300$ °K, $m_2 = 6.8.10^{-24}$ g, $E_0 = 1$ V/cm, $u_{10} = 7.10^5$ cm/sec., $u_{20} = 8.10^3$ cm/sec., $n_0 = 10^9$/cm$^3$, $\omega_{p1}^2 = 3.10^{18}$/sec$^2$., $\omega_{p2}^2 = 4.10^{14}$/sec$^2$. With these values, the third term in the numerator and the second term in the denominator of eq. (37) are negligibly small.

In such a case, the dispersion relation is given as

$$\omega^2 \simeq \omega_{p2}^2 + e^2E_0^2/kT_{10}m_2 \tag{38}$$

and $$K^2 \simeq (m_2/kT_{10})\omega_{p2}^2 + e^2E_0^2/k^2T_{10}^2 \tag{39}$$

The calculated values of the angular frequency and the wave length are 2.10⁷/sec. and 0.29 cm, respectively. Thus, we found the definite values of frequency and wave length of the progressive wave propagating through a drifting plasma. From this result, it is seen that progressive plasma wave may take place and that the frequency is shifted from the ion plasma frequency by an amount depending on the drift. The frequency shift in the example taken above is quite small, but it may become large as the steady electric field is increased. For example, if we take 100 V/cm as the value of $E_0$, leaving other values as before, the frequency and wave length are found to be $3.10^7$/sec. and 0.19 cm, respectively.

Next, we examine the problem with another assumption that neutrality of plasma is maintained even when a disturbance is applied. The existence of the neutrality allow us to ignore Poisson's equation and to set $\phi_1 = \phi_2$ in eqs. (29) and (30). Eliminating $\phi$, $\eta_1$, $\eta_2$ and $\varepsilon$ from eqs. (29) to (31) and considering the above condition, we finally find that the neutrality does not lead to propagation of progressive wave.

Secondly, we deal with the other case where $kT_{10} \ll m_1u_{10}^2$ and $kT_{20} \ll m_2u_{20}^2$ are assumed. In this case, eqs. (34) and (35) become

$$\begin{aligned} &K^4u_{10}^2u_{20}^2 - 2K^3\omega m_1m_2u_{10}u_{20}(u_{10}+u_{20}) + K^2\{\omega^2(4u_{10}u_{20}+u_{10}^2+u_{20}^2) \\ &\quad + e^2E_0^2/m_1m_2 - u_{20}\omega_{p1}^2 - u_{10}\omega_{p2}^2\} + 2K\{\omega(u_{10}\omega_{p2}^2 + u_{20}\omega_{p1}^2) - \omega^3(u_{10}+u_{20})\} \\ &\quad - \omega^2(\omega_{p1}^2+\omega_{p2}^2) + \omega^4 = 0 \end{aligned} \tag{36}$$

$$K^2(m_1u_{10}^2 - m_2u_{20}^2) - 2K\omega(m_1u_{10} - m_2u_{20}) + \omega^2(m_1 - m_2) = 0 \tag{37}$$

From eq. (37), we have

$$K = \omega\{m_1u_{10} - m_2u_{20} \pm (m_1m_2)^{1/2}(u_{10}-u_{20})/(m_1u_{10}^2 - m_2u_{20}^2)\} \tag{38}$$

From eqs. (36) and (38), we obtain an expression for $\omega$ or $K$ with function of the plasma parameters. As an example, we choose the following numerical values; $u_{10} = 7.10^8$ cm/sec, $u_{20} = 8.10^6$ cm/sec, $m_2 = 6.8.10^{-24}$ g, $E_0 = 1$ V/cm, $n_0 = 10^9/\text{cm}^3$, $\omega_{p1}^2 = 3.10^{18}/\text{sec}^2$ and $\omega_{p2}^2 = 4.10^{14}/\text{sec}^2$.

However, the result shows that no real value of $\omega$ or $K$ is attainable or no steady wave propagation is possible. This is connected with the so-called two stream instability.[27) 28)]

## V. Conclusion

In this paper, we emphasized the effect of drift on propagation of low frequency wave. Experimentally, oscillation or noise generated in plasma is observed by means of a probe or antenna which is located apart from the source of oscillation or noise. Thus, the observed signal may be controlled by the dispersive property of the plasma between the source and the receiving point, if there is no boundary responsible for generation of sound wave and no source adjacent to the receiving point. Our theory provided an explanation for the result of noise measurement that noise level was dominant in a frequency range near the plasma ion frequency.

We also found that the propagation with constant amplitude was possible only under the condition where $kT \gg mu_0^2$ was satisfied.

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