ELECTROACOUSTIC WAVE IN QUASI-STEADY APPROACH

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ABSTRACT

An attempt is made to discuss an electroacoustic wave in plasma with the help of the quasi-steady approach and the substitution method.

The result shows that the attenuation of the wave is a function of the steady electric field existing in the plasma or the diffusion coefficient.

I. Introduction

A number of types of oscillation or fluctuation may exist in gaseous discharge plasma.[1)-4)] Recently, many people have paid their attentions to low frequency oscillation in connection with the problem on plasma instability.

The plasma ion oscillation may take place in plasma if a density disturbance occurs. Under a certain condition, such local oscillation may propagate through the plasma as an electroacoustic wave.

In a previous paper,[5)] we discussed the problem as to whether the ion oscillation could be transmitted or not with the macroscopic description and the substitution method and found that under a certain condition the propagation of a steady low frequency oscillation was possible if the thermal velocity exceeds the steady drift.

Our purpose of the present paper is to solve the same problem by means of the quasi-steay approach.

II. Basic Equation and Dispersion Relation

Many authors have used the quasi-steady approach to treat low frequency phenomena such as moving striation[6)-9)] and screw instability.[10)]

In this approach, the velocity expression is given by a sum of the drift velocity due to the electric field, the diffusion and the thermal conduction. Thus, the velocities of electron and ion are expressed respectively by

$$u_1 = -b_1E - (D_1/n_1)\partial n_1/\partial x - (b_1k/e)\partial T_1/\partial x \tag{1}$$

$$u_2 = b_2E - (D_2/n_2)\partial n_2/\partial x - (b_2k/e)\partial T_2/\partial x \tag{2}$$

where b is the mobility and D the diffusion coefficient. Here, for simplicity, the second and the third terms are disregarded.

Then the velocities are

$$u_1 = -b_1E \tag{3}$$

$$u_2 = b_2E \tag{4}$$

The basic equations in the quasi-steady approach are the continuity and the Poisson's equations. These are written as

$$\partial n_1/\partial t+\partial/\partial x(-b_1En_1)=n_1\nu_i-n_1/\tau=0 \tag{5}$$

$$\partial n_2/\partial t+\partial/\partial x(b_2En_2)=n_1\nu_i-n_1/\tau=0 \tag{6}$$

$$\partial E/\partial x=4\pi e(n_2-n_1) \tag{7}$$

where ν_i is the production rate of electron and τ the life time of electron.

The electric field and the densities of electron and ion consist of two parts, steady and perturbed, as follows;

$$E=E_0\{1+\varepsilon\exp i(Kx-\omega t)\} \tag{8}$$

$$n_1=n_0\{1+\phi_1\exp i(Kx-\omega t)\} \tag{9}$$

$$n_2=n_0\{1+\phi_2\exp i(Kx-\omega t)\} \tag{10}$$

where K is the propagation constant and ω the angular frequency. If the amplitude of the perturbed part is assumed to be much smaller than those of the steady part, *i.e.*, $1\gg\varepsilon$, $1\gg\phi_1$ and $1\gg\phi_2$, eqs. (5), (6) and (7) are combined to yield the following linearized equations;

$$\phi_1(-i\omega+4\pi en_0b_1-iKb_1E_0)-4\pi en_0b_1\phi_2=0 \tag{11}$$

$$-4\pi en_0b_2\phi_1+\phi_2(-i\omega+4\pi en_0b_2+iKb_2E_0)=0 \tag{12}$$

Combining eqs. (11) and (12), we have

$$K^2b_1b_2E_0^2-\omega KE_0(b_1-b_2)-\omega^2+i4\pi en_0(b_1+b_2)\omega=0 \tag{13}$$

Since we are concerned with the attenuation of wave, K is divided into two parts, real and imaginary, as

$$K=K_r+iK_i \tag{14}$$

where K_r is the wave number and K_i the attenuation constant. Substituting eq. (14) into eq. (13) and separating them into the real and imaginary parts, we get

$$(K_r^2-K_i^2)b_1b_2E_0^2-K_r\omega E_0(b_1-b_2)-\omega^2=0, \tag{15}$$

$$2K_rK_ib_1b_2E_0^2-K_i\omega E_0(b_1-b_2)-4\pi en_0\omega(b_1+b_2)=0. \tag{16}$$

Further simplification is made assuming that $|K_i|\ll|K_r|$. Therefore, the validity of the result obtained from this simplification is justified only when the attenuation is small.

Eq. (16) results in

$$K_i=4\pi en_0(b_1+b_2)\omega/[2K_rb_1b_2E_0^2-\omega E_0(b_1-b_2)]. \tag{17}$$

Under the assumption made above, eq. (15) leads to

$$K_r=\omega/b_2E_0 \quad \text{or} \quad -\omega/b_1E_0. \tag{18}$$

From eq. (15) the corresponding K_i is found to be

$$K_i = \pm 4\pi e n_0 / E_0. \tag{19}$$

According to the final expression for K_i, it is obvious that the attenuation or growing of wave toward the direction x is proportional to the plasma density and inversely to the stedy electric field.

Further analysis will be given by making another assumption that the steady electric field considered above does not exist, but a diffusion takes place instead.

With this assumption, the velocities are prescribed as

$$u_1 = -b_1 E - (D_1/n_1)\partial n_1/\partial x \tag{20}$$

$$u_2 = b_2 E - (D_2/n_2)\partial n_2/\partial x \tag{21}$$

instead of eqs. (3) and (4). Here, it is noted that E comes from the perturbation, which is expressed as

$$E = E \exp i(Kx - \omega t) \tag{22}$$

The continuity equations are

$$\partial n_1/\partial t - b_1 \partial/\partial x(E n_1) - D_1 \partial^2 n_1/\partial x^2 = 0 \tag{23}$$

$$\partial n_2/\partial t + b_2 \partial/\partial x(E n_2) - D_2 \partial^2 n_2/\partial x^2 = 0. \tag{24}$$

Adopting the same procedure as used above, we get the following relations from eqs. (7), (9), (10), (20) to (24);

$$(-i\omega + D_1 K^2 + 4\pi e n_0 b_1)\phi_1 - 4\pi e n_0 b_1 \phi_2 = 0, \tag{25}$$

$$-4\pi e n_0 b_2 \phi_1 + (i\omega + D_2 K^2 + 4\pi e n_0 b_2)\phi_2 = 0. \tag{26}$$

The combination of these relations yields

$$D_1 D_2 K^4 + \{4\pi e n_0 (b_2 D_1 + b_1 D_2) - i\omega(D_1 + D_2)\} K^2 - \omega^2 - i 4\pi e n_0 (b_1 + b_2)\omega = 0. \tag{27}$$

Using eq. (14) and neglecting the second order of K or more, eq. (27) is re-written by a set of the following relations, one being the real part and another the imaginary part;

$$D_1 D_2 K_r^4 + 4\pi e n_0 (b_2 D_1 + b_1 D_2) K_r^2 + 2\omega K_r K_i (D_1 + D_2) - \omega^2 = 0, \tag{28}$$

$$4 D_1 D_2 K_r^3 K_i - \omega K_r^2 (D_1 + D_2) + 8\pi e n_0 (b_2 D_1 + b_1 D_2) K_r K_i - 4\pi e n_0 (b_1 + b_2)\omega = 0. \tag{29}$$

Combination of eqs. (28) and (29) leads to the following equation, from which the wave number K_r can be found as a function of various plasma parameters;

$$4 D_1^2 D_2^2 K_r^6 + 24\pi e n_0 D_1 D_2 (b_2 D_1 + b_1 D_2) K_r^4 + \{32\pi^2 e^2 n_0^2 (b_2 D_1 + b_1 D_2)^2 + 2(D_1 + D_2)^2 \omega^2\} K_r^2 + 8\pi e n_0 (b_1 + b_2)(D_1 + D_2)\omega^2 = 0. \tag{30}$$

This equation may have a real solution if the following condition is satisfied;

$$r = \frac{1}{D_1^6 D_2^6}\left[\frac{(D_1 + D_2)^2}{216}\omega^6 + \pi^2 e^2 n_0^2 \left\{\frac{(b_2 D_1 - b_1 D_2)^2 (D_1^2 - D_2^2)^2}{4} - \frac{(b_2 D_1 + b_1 D_2)^2 (D_1 + D_2)^4}{9}\right\}\omega^4 + \frac{8}{9}\pi^4 e^4 n_0^4 (b_2 D_1 + b_1 D_2)^4 (D_1 + D_2)^2 \omega^2 - \frac{64}{27}\pi^6 e^6 n_0^6 (b_2 D_1 + b_1 D_2)^6\right] = 0 \tag{31}$$

In this case, K_r is given by

$$K_r^2 = 2\sqrt[3]{-e/2} - a/3 \tag{32}$$

where

$$e = \frac{-\pi e n_0 \omega^2}{D_1^3 D_2^3}(b_2 D_1 - b_1 D_2)(D_1^2 - D_2^2) \tag{33}$$

$$a = \frac{6\pi e n_0 (b_2 D_1 + b_1 D_2)}{D_1 D_2} \tag{34}$$

From eq. (32), we obtain

$$K_r^2 = \frac{2}{D_1 D_2}[\{\pi e n_0 \omega^2 (b_2 D_1 - b_1 D_2)(D_1^2 - D_2^2)/2\}^{1/3} - \pi e n_0 (b_2 D_1 + b_1 D_2)] \tag{35}$$

The attenuation constant K_i is found from eq. (29) as

$$K_i = \frac{\omega\{K_r^2(D_1 + D_2) + 4\pi e n_0 (b_1 + b_2)\}}{K_r\{4 D_1 D_2 K_r^2 + 8\pi e n_0 (b_2 D_1 + b_1 D_2)\}} \tag{36}$$

General solution can be obtained without any restriction for r. The solution for K_r is

$$K_r^2 = \sqrt[3]{-\frac{e}{2} + \sqrt{r}} + \sqrt[3]{-\frac{e}{2} - \sqrt{r}} - a/3 \tag{37}$$

III. Conclusion

We derived the dispersion relation for two extreme cases; one being the drifting plasma with zero temperature, while the other the diffusion plasma with a finite temperature. The form of the dispersion relation is quite different from each other. Further study will be done in future.

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