PROPAGATION OF TEMPERATURE DISTURBANCE IN GASEOUS PLASMA

TAKAYOSHI OKUDA and KENZO YAMAMOTO

Department of Electronics

(Received October 31, 1962)

ABSTRACT

In this paper, we propose a mechanism for the propagation of temperature disturbance through a plasma having drift velocity. The analysis is based on the macroscopic approach and the substitution method. The result shows that the steady propagation of temperature disturbance is possible under a certain condition and the frequency is of the order of *Mc*/sec in a low pressure plasma. An attempt of examining the use of the quasi-steady approximation is also made, but no steady propagation of temperature disturbance is predicted with this treatment.

I. Introduction

Various types of low frequency oscillation or fluctuation have been observed in gaseous discharge plasma [1)-4)] Recently, many people have paid their attention to such low frequency phenomena in connection with electrostatic instability.

Natural ion plasma oscillation may take place in plasma if density disturbance occurs. However, whether the ion oscillation can be transmitted or not is determined by the nature of the surrounding plasma. A problem arises as to under what condition oscillation or disturbance would be transmitted without attenuation.

In a previous paper, we dealt with a similar problem in which density perturbation was considered with the help of the macroscopic approach and the substitution method, and found that under a certain condition the propagation of steady low frequency oscillation could be possible if the thermal velocity exceeded the steady drift velocity.[5)]

II. Basic Equation in Macroscopic Approach

In our analysis, the macroscopic approach is used to describe the density and the velocity of the charged particle in plasma. Validity of this approach is justified by the statement that under the condition considered here an equilibrium in velocity distribution can be reached in a time shorter than the period of the oscillation.

Basic equations in the macroscopic approach are derived from the Boltzmann equation by assuming a displaced Maxwellian distribution. These are refered as the equations of the continuity, the conservations of momentum and energy, respectively.[7)] Those in one-dimensional case are described as follows;

$$\partial n_1/\partial t + \partial/\partial x(n_1 u_1) = n_1 \nu_i - W \quad (1)$$

$$\partial n_2/\partial t + \partial/\partial x(n_2 u_2) = n_1 \nu_i - W \quad (2)$$

$$\partial/\partial t(n_1u_1)+\partial/\partial x\{n_1(u_1^2+kT_1/m_1)\}+eEn_1/m_1=-\nu_{c1}u_1n_1 \tag{3}$$

$$\partial/\partial t(n_2u_2)+\partial/\partial x\{n_2(u_2^2+kT_2/m_2)\}-eEn_2/m_2=-\nu_{c2}u_2n_2 \tag{4}$$

$$\partial/\partial t\{n_1(u_1^2+kT_1/m_1)\}+\partial/\partial x\{n_1(u_1^3+3\,u_1kT_1/m_1)\}+2\,eEn_1u_1/m_1$$
$$=-(2\,m_1\nu_{c1}n_1/m_2)(kT_1/m_1)-\nu_ieV_in_1/m_1 \tag{5}$$

$$\partial/\partial t\{n_2(u_2^2+kT_2/m_2)\}+\partial/\partial x\{n_2(u_2^3+3\,u_2kT_2/m_2)\}-2\,eEn_2u_2/m_2=-(\nu_{c2}n_2kT_2/m_2) \tag{6}$$

where n is the density, u the drift velocity, ν_i the ionization frequency, W the loss in number density, k the Bolzmann constant, T the temperature, e the unit charge, E the electric field, m the mass, ν_c the collision frequency with neutrals and V_i the ionization potential. The subscript 1 and 2 denote electron and ion, respectively.

Poisson's equation is expressed as

$$\partial E/\partial x=4\,\pi e(n_2-n_1) \tag{7}$$

III. Dispersion Relation

In the substitution method, quantities such as the density, the velocity, the temperature, and the electric field are presoribed as a sum of a steady and a small a.c. component. It allows us to linearize the basic equations in order to get the dispersion relation, which provides information of the relation between the frequency and the wave number.

The associated quantities are expressed as follows;

$$E=E_0\{1+\varepsilon\exp i(Kx-\omega t)\}, \tag{8}$$

$$n_1=n_0\{1+\phi_1\exp i(Kx-\omega t)\}, \tag{9}$$

$$n_2=n_0\{1+\phi_2\exp i(Kx-\omega t)\}, \tag{10}$$

$$u_1=u_{10}\{1+\eta_1\exp i(Kx-\omega t)\}, \tag{11}$$

$$u_2=u_{20}\{1+\eta_2\exp i(Kx-\omega t)\}, \tag{12}$$

$$T_1=T_{10}\{1+\tau_1\exp i(Kx-\omega t)\}, \tag{13}$$

$$T_2=T_{20}\{1+\tau_2\exp i(Kx-\omega t)\}, \tag{14}$$

where the relative amplitudes ε, ϕ_1, ϕ_2, η_1, η_2, τ_1 and τ_2 are much smaller than 1 and neutrality is assumed in the steady state, and K and ω are the wave number and the frequency of the wave.

Substitution of eqs. (8) to (14) into eqs. (1) to (7) leads to a relation between K and ω, by neglecting the second or higher order of exp $i(Kx-\omega t)$ and separating them into the real and imaginary parts.

Our special interest is in the propagation of wave through the temperature perturbation. For this purpose, ϕ_1 and ϕ_2 are set to be zero. With this assumption, Poisson's equation is excluded from the analysis.

The continuity equation for the perturbed part is obtained by means of the substitution method from eqs. (1) and (2). Combination of those for electron and ion yields:

$$iKu_{10}\eta_1 - iKu_{20}\eta_2 = 0. \tag{15}$$

independent of the functional form of ν_i and W.

The momentum quations for the perturbed part are derived from eqs. (3) and (4). They are given respectively by

$$\eta_1 u_{10}(-i\omega + 2iKu_{10} + \nu_{c1}) + iK\tau_1(kT_{10}/m_1) + (eE_0/m_1)\varepsilon = 0 \tag{16}$$

$$\eta_2 u_{20}(-i\omega + 2iKu_{20} + \nu_{c2}) + iK\tau_2(kT_{20}/m_2) - (eE_0/m_2)\varepsilon = 0 \tag{17}$$

The ionization frequency ν_i appearing in eq. (5) is a function of the electron temperature as

$$\nu_i = A(kT_1/e)^{1/2}\exp\{-eV_i/kT_1\}, \tag{18}$$

where A is a constant. From eq. (18), ν_i has the following form in the first approximation;

$$\nu_i = \nu_{i0}\{1 + \tau_1(eV_i/kT_{10} + 1/2)\} \tag{19}$$

Now, we get the energy equation for the perturbed part with eq. (19) as

$$\eta_1 u_{10}\{-2i\omega u_{10} + 3iK(u_{10}^2 + kT_{10}/m_1) + 2eE_0/m_1\} + \tau_1\{-i\omega(kT_{10}/m_1) + 3iKu_{10}(kT_{10}/m_1)$$
$$+ 2\nu_{c1}(kT_{10}/m_2) + (eV_i\nu_{i0}/m_1)(eV_i/kT_{10} + 1/2)\} + (2eE_0u_{10}/m_1)\varepsilon = 0 \tag{20}$$

$$\eta_2 u_{20}\{-2i\omega u_{20} + 3iK(u_{20}^2 + kT_{20}/m_2) - 2eE_0/m_2\} + \tau_2\{-i\omega(kT_{20}/m_2)$$
$$+ 3iKu_{20}(kT_{20}/m_2) + \nu_{c2}kT_{20}\} - (2eE_0u_{20}/m_2)\varepsilon = 0 \tag{21}$$

Here, for simplicity, we assume that $\tau_2 = 0$, *i.e.*, the ion temperature is constant. It turns out that the quantities η_1, η_2, τ_1 and ε can be eliminated from eqs. (15), (16), (17), (20) and (21). The combination of these equations gives a relation in which ω and K are connected with each other. Since this relation consists of a real and imaginary part, we separate it into the following two equations;

$$K^2\{2u_{10}u_{20}(u_{10}m_1 + u_{20}m_2) + 3u_{20}kT_{10} + 3u_{10}kT_{20}\}kT_{10}u_{10}$$
$$+ \omega K\{u_{10}m_1 + 2u_{20}m_2 - u_{10}m_2\}kT_{10}u_{10}u_{20} - \omega^2 kT_{10}u_{10}u_{20}(m_1 + m_2)$$
$$+ u_{10}u_{20}(m_1\nu_{c1} + m_2\nu_{c1})\{2m_1\nu_{c1}kT_{10}/m_2 + eV_i\nu_{i0}(eV_i/kT_{10} + 1/2)\} = 0 \tag{22}$$

$$\omega/K = [3kT_{10}u_{10}u_{20}(m_1\nu_{c1} + m_2\nu_{c2}) + 2u_{20}(u_{10}m_1 + u_{20}m_2)\{2m_1\nu_{c1}kT_{10}/m_2$$
$$+ eV_i\nu_{i0}(eV_i/kT_{10} + 1/2)\} - kT_{10}\{2eE_0(u_{20} - u_{10})\}][kT_{10}u_{20}(m_1\nu_{c1} + m_2\nu_{c2})$$
$$+ u_{20}(m_1 + m_2)\{2m_1\nu_{c1}kT_{10}/m_2 + eV_i\nu_{i0}(eV_i/kT_{10} + 1/2)\}]^{-1} \tag{23}$$

When eq. (23) is introduced into eq. (22), the final expression for K or ω can be deduced. However, the form is too complicate to understand the dependence on various plasma parameters. The only thing seen from the result without further numerical examination is that K and ω are connected through the interaction term such as ν_0 or ν_i. This seems to be a prominent character of this type of wave motion.

Owing to the complexity of the form of the dispersion relation seq. (22) and (23), we are forced only to examine numerically whether a real value of K or ω is attainable under the condition which is ordinary in laboratory plasma.

We take the following numerical values as an example; $V_i=24.5$ V, $\nu_{i0}=2.6.10^4$/sec., $E_0=1$ V/cm, $T_{10}=3.10^4$ °K, $T_{20}=3.10^2$ °K, $m_2=6.8.10^{-24}$ g, $u_{10}=7.10^5$ cm/sec., $u_{20}=8.10^3$ cm/sec., $\nu_{c1}=2.10^9$/sec. and $\nu_{c2}=2.6.10^7$/sec. They are typical for the values at 1 mm Hg of He. However, the result of numerical calculation shows that no real K and ω exist simultaneously. Then we repeat the same procedure for the case of 10, 0.1 and 0.01 mm Hg of He. For this purpose, u_{10} and u_{20} are regarded to be inversely proportional to the pressure, while ν_{c1} and ν_{c2} proportional to it.

The result shows that the propagation without attenuation is possible only for the case of 0.01 mm Hg. The numerical values of ω and K are $4.7.10^6$/sec. and 3.2/cm, respectively.

Leaving major terms in eqs. (22) and (23) alone, the following approximate expressions for this case can be obtained;

$$K^2=\nu_{c2}\nu_{i0}eV_i/u_{10}u_{20}m_2k^2T_{10}^2 \qquad (24)$$

$$\omega=3\,k^2T_{10}^2u_{10}\nu_{c2}K/e^2V_i^2\nu_{i0} \qquad (25)$$

IV. Quasi-Steady Approach

Many authors have used the quasi-steady approximation to analyze the dynamic behavior of plasma such as wave propagation. In this appraximation, velocity is expressed in terms of the drift due to electric field, the diffusion and the thermal conduction. Thus, the velocities of electron and ion are given as

$$u_1=-b_1E-(D_1/n_1)\partial n_1/\partial x-(b_1k/e)\partial T_1/\partial x \qquad (26)$$

$$u_2=b_2E-(D_2/n_2)\partial n_2/\partial x-(b_2k/e)\partial T_2/\partial x \qquad (27)$$

where b is the mobility and D the diffusion coefficient.

Making the same assumption, $\phi_1=\phi_2=\tau_2=0$, and taking the same procedure as in the preceding paragragh, we obtain two equations instead of eqs. (15), (16), (17), (20) and (21). They are

$$-iK\varepsilon E_0(b_1+b_2)+(b_1k/e)T_{10}K^2\tau_1=0 \qquad (28)$$

$$\begin{aligned}&\varepsilon\{-2\,i\omega b_1E_0-3\,iK(b_1^2E_0^2+kT_{10}/m_1)-4\,eE_0(1/m_1)\}b_1E_0\\&+\tau_1\{-i\omega kT_{10}/m_1+(2\,b_1^2kT_{10}E_0/e)\omega K+(3\,kT_{10}b_1/e)(b_1^2E_0^2+kT_{10}/m_1)K^2\\&-iK(5/m_1)b_1E_0kT_{10}+2\,ekT_{10}/m_1m_2b_1+(\nu_{i0}eV_i/m_1)(eV_i/kT_{10}+1/2)\}=0 \qquad (29)\end{aligned}$$

Combining eqs. (28) and (29) and separating them into the real and imaginary parts, two relations between K and ω are obtained as

$$\omega=-b_1E_0K \qquad (30)$$

$$\begin{aligned}&3\,b_1b_2kT_{10}/e(b_1^2E_0^2-kT_{10}/m_1)K^2+(2\,b_1^2b_2kT_{10}E_0/e)\omega K\\&+(b_1+b_2)\{2\,ekT_{10}/m_1m_2b_1+(\nu_{i0}eV_i/m_1)(eV_i/kT_{10}+1/2)=0 \qquad (31)\end{aligned}$$

Then, from these equations we finally find the expression for K^2 as

$$\begin{aligned}K^2=&-e(b_1+b_2)\{2\,ekT_{10}/m_1m_2b_2+(\nu_{i0}eV_i/m_1)(eV_i/kT_{10}+1/2)\}\\&\times\{b_1b_2kT_{10}(b_1^2E_0^2+3\,kT_{10}/m_1)\}^{-1} \qquad (32)\end{aligned}$$

It is evident from eq. (32) that K is always imaginary, showing no possibility of progressive wave propagation with no attenuation or growing.

Now, we shall deal with the attenuation or growing with this approach. To do this, K in eqs. (28) and (29) is replaced by K_r+iK_i and the resulting equation is divided into the real and imaginary parts. The former is written by

$$K_i = (eb_1E_0K_r/2\,b_2m_1)\left\{\omega E_0b_1 + 3\left(b_1^2E_0^2 + \frac{kT_{10}}{m_1}\right)K_r\right\}^{-1} \tag{33}$$

With an assumption that $|K_r| \gg |K_i|$, the latter results in

$$K_r^3 + AK_r^2 + BK_r + C = 0 \tag{34}$$

where $A = \dfrac{4}{3}\dfrac{\omega E_0b_1}{(b_1^2E_0^2 + kT_{10}/m_1)}$

$$B = \frac{(eb_1E_0)^2}{m_1} + 4(\omega E_0b_1b_2)^2m_1 + 6\,b_2e(b_1^2E_0^2 + kT_{10}/m_1)$$

$$\times\left\{\frac{2\,e}{Mb_1} + \frac{\nu_{i0}eV_i}{kT_{10}}\left(\frac{eV_i}{kT_{10}} + \frac{1}{2}\right)\right\}\{18\,b_2^2m_1(b_1^2E_0^2 + kT_{10}/m_1)^2\}^{-1}$$

$$C = 2\,b_1b_2E_0e\omega\left\{\frac{2\,e}{Mb_1} + \frac{\nu_{i0}eV_i}{kT_{10}}\left(\frac{eV_i}{kT_{10}} + \frac{1}{2}\right)\right\}\{18\,b_2^2m_1(b_1^2E_0^2 + kT_{10}/m_1)^2\}^{-1} \tag{35}$$

Under a certain condition, eq. (34) may have a real root. This special condition for existence of real root is written by

$$\frac{1}{4}\left\{\frac{A}{3}\left(\frac{2\,A^2}{9} - B\right) + C\right\}^2 + \frac{1}{27}\left(B - \frac{A^2}{3}\right)^3 = 0 \tag{36}$$

In this case, K_r is given by

$$K_r = 2\sqrt[3]{-e/2} - A/3 \tag{37}$$

where

$$e = A(2\,A^2/27 - B/3) + C \tag{38}$$

Then eq. (37) becomes

$$K_r = 2\sqrt[3]{C/2 + AB/6 - A\,3/27} - A/3 \tag{39}$$

On the other hand, the attenuation constant can be found from eq. (33).

It is seen from those results that on the contrary to the case when only density perturbation is concerned as discussed in the previous paper,[5] the propagation constant does not depend on the plasma density but on the ratio of the directed energy to the thermal energy and also on the energy loss.

Furthermore, if desired, the following d.c. equation is employed to rewrite E_0 as a function of the loss terms:

$$b_1^2E_0^2 = kT_{10}/M + \nu_{i0}V_ib_1/2 \tag{40}$$

V. Conclusion

A proposed mechanism for transmission of a disturbance through a temperature perturbation was described. According to the analysis basing on the unsteady treatment, a progressive wave with a definite frequency and wave number can be transmitted if a certain condition is fulfilled. However, this treatment gives us only information as to whether the progressive wave with constant amplitude is possible or not.

Another treatment basing on the quasi-steady approach allows us to study the attenuation or growing with time or space. However, only a simplified case is dealt because of anlytical easiness. Further analysis will be done in future.

References

1) J. D. Cobine and C. J. Gallagher, J. Appl. Phys. **18**, 110, 1947 and J. Franklin Inst., **243**, 41, 1947.

2) N. R. Labrum and E. K. Bigg, Proc. Phys. Soc., **B 65**, 350, 1952.

3) H. Martin and H. A. Woods, Proc. Phys. Soc., **B 65**, 281, 1951.

4) F. W. Crawford, J. Appl. Phys., **33**, 15, 1962.

5) T. Okuda and K. Yamamoto, "Electroacoustic Wave in Quasi-steady Approach" in this issue.

6) G. Ecker, Phys. of Fluid. **4**, 127, 1961.