EQUILIBRIUM TEMPERATURE OF PARTICLES IN PARTIALLY IONIZED PLASMA

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ABSTRACT

The relation among electron, ion and gas temperature in equilibrium state is formulated. In particular, ion temperature is found as a function of various collision frequencies.

I. Introduction

The knowledge of ion temperature has been quite poor compared with that of electron. The principal reason for the fact is the difficulty in measuring the ion temperature. In fact, in the plasma usually seen in laboratory the degree of ionization is so small that the interaction between electron and ion is neglibly small, resulting in an ion temperature not different from the gas temperature.

Recently, attention has been paid towards hot plasma or highly ionized plasma in order to make use of high temperature. In such plasma, the ion temperature could become comparable to that of electron.[1]

In this paper, the formula of the ion temperature will be expressed in terms of the collision frequency characterizing interaction between the plasma constituents.

II. Basic Equations and Ion Temperature

We start with the macroscopic energy equation in order to get the relation among the temperatures of plasma constituents.[2][3] These are

$$\partial/\partial t\{n_1(m_1u_1^2+kT_1)\}+\partial/\partial x\{n_1u_1(m_1u_1^2+3\,kT_1)\}+2\,eEn_1u_1$$
$$=-(2\,m_1/m_2)\nu_{12}n_1(kT_1-kT_2)-(2\,m_1/m_3)\nu_{13}n_1(kT_1-kT_3)-\nu_i eV_in_1-\sum\nu_{ex}eV_{ex}n_1 \quad (1)$$

$$\partial/\partial t\{n_2(m_2u_2^2+kT_2)\}+\partial/\partial x\{n_2u_2(m_2u_2^2+3\,kT_2)\}-2\,eEn_2u_2$$
$$=-2\,n_2\nu_{21}(kT_2-kT_1)-n_2\nu_{23}(kT_2-kT_3) \quad (2)$$

where n is the density, u the drift velocity, m the mass, e the elementary charge, k the Boltzmann constant, T the temperature, E the electric field, ν_{mn} the collision frequency between the species m and n, ν_i the ionizing collision frequency, ν_{ex} the exciting collision frequency, V_i the ionization potential, V_{ex} the excitation potential. The subscripts 1, 2 and 3 refer to electron, ion and atom, respectively.

Since we are concerned with the equilibrium temperature, eqs. (1) and (2) are simplified by setting $\partial/\partial t$ and $\partial/\partial x$ to be zero as

$$2\,eu_1E = -(2\,m_1/m_2)\nu_{12}(kT_1-kT_2)-(2\,m_1/m_3)\nu_{13}(kT_1-kT_3)$$
$$-\nu_i eV_i - \sum \nu_{ex}eV_{ex}, \tag{3}$$
$$-2\,eu_2E = -2\,\nu_{21}(kT_2-kT_1)-\nu_{23}(kT_2-kT_3). \tag{4}$$

Eliminating E from eqs. (3) and (4), we find the expession for kT_2 as

$$kT_2=\left[\frac{m_1}{m_2u_1}(\nu_{12}+\nu_{23})kT_1-\left(\frac{m_1}{m_2}\frac{\nu_{13}}{u_1}+\frac{\nu_{23}}{2u_2}\right)kT_3+\frac{1}{2\,u_1}(\nu_ieV_i+\sum\nu_{ex}eV_{ex})\right]$$
$$\times\left[\frac{m_1}{m_2}\nu_{12}\left(\frac{1}{u_1}-\frac{1}{u_2}\right)-\frac{\nu_{23}}{2\,u_2}\right]^{-1} \tag{5}$$

where $m_2=m_3$ is used.

Now, we have to find the expression for u appearing in eq. (5) in order to complete formulation. This can be done with the equation of momemtum conservation, which is written by

$$\partial/\partial t(n_1u_1)+\partial/\partial x\{n_1(u_1^2+kT_1/m_1)\}+eEn_1/m_1=-\nu_{12}n_1(u_1-u_2)-\nu_{13}n_1(u_1-u_3) \tag{6}$$
$$\partial/\partial t(n_2u_2)+\partial/\partial x\{n_2(u_2^2+kT_2/m_2)\}-eEn_2/m_2=-\nu_{21}(u_2-u_1)-\nu_{23}(u_2-u_3) \tag{7}$$

In equilibrium state, these become

$$eE/m_1=-\nu_{12}(u_1-u_2)-\nu_{13}(u_1-u_3) \tag{8}$$
$$-eE/m_2=-\nu_{21}(u_2-u_1)-\nu_{23}(u_2-u_3) \tag{9}$$

Using relation $\nu_{21}=(m_1/m_2)\nu_{12}$, and assuming that $u_3=0$, we find the expression for u_1 and u_2 as a function of various collision frequencies

$$u_1=-eE(m_2/m_1)\frac{\nu_{23}}{m_1\nu_{12}\nu_{13}+m_2(\nu_{12}\nu_{23}+\nu_{13}\nu_{23})} \tag{10}$$

$$u_2=eE\frac{\nu_{13}}{m_1\nu_{12}\nu_{13}+m_2(\nu_{12}\nu_{23}+\nu_{13}\nu_{23})} \tag{11}$$

Finally, the expression for T_2 is obtained by making use of eqs. (5), (10) and (11) as

$$T_2=[T_1(m_1/m_2)^2(\nu_{12}+\nu_{23})/\nu_{23}+T_3\{\nu_{23}/2\,\nu_{13}-(m_1/m_2)^2(\nu_{13}/\nu_{23})\}$$
$$+(\nu_{23}/2\,k)(m_1/m_2)(eV_i\nu_i+\sum eV_{ex}\nu_{ex})][\nu_{23}/2\,\nu_{13}+(m_1/m_2)\nu_{12}(m_1/m_2\nu_{23}-1/\nu_{13})]^{-1} \tag{13}$$

It should be noted that the above formula was obtained assuming constant gas temperature.

It is known that the interaction between electron and ion becomes appreciable as the electron density increases or the electron temperature is lowered. The latter situation was realized in decaying plasma.[4] It seems that the contribution of the e-i interaction is not important in the ordinary discharge plasma.

Numerical examination for *He* plasma shows that at 2.10^{-2} mm Hg, $T_3=3.10^2$ °K and $T_1=3.10^4$ °K the ion temperature exceeds the gas temperature when the plasma density is higher than $2.10^{13}/\mathrm{cm}^3$ and reachs 4,900 °K at $2.10^{15}/\mathrm{cm}^3$, even if the inelastic collisions are ignored.

The excess from the gas temperature in the ordinary partially ionized plasma may come from the inelastic collision terms. The numerical value of several thousands as the ion temperature is not unreasonable from this point of view.

References

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