PENETRATION OF ION SHEATH INTO PLASMA

TAKAYOSHI OKUDA and KENZO YAMAMOTO

Department of Electronics

(Received October 31, 1962)

ABSTRACT

An estimation of penetrating voltage is made for negative probe in high pressure plasma with aid of the sheath criterion. The penetration may be a major source of errors appearing in the probe measurement.

The surrounding space of a space charge sheath formed in plasma is disturbed by penetration of applied electric field. The authors has investigated the disturbance phenomena in probe measurement and found that the discrepancies of the theoretical space charge thickness and voltage-current characteristics from these measured became appreciable as the plasma density was decreased.[1]

The penetration of ion sheath into plasma may be resposible for this kind of disturbance. As a matter of fact, the penetration may result in that a certain part of voltage is applied across a penetrating sheath developing just outside the space charge sheath.

We shall estimate the voltage difference across the penetrating sheath with aid of the sheath criterion.

The existence of the penetrating sheath has been theoretically suggested from the point of view of sheath stability by Bohm[2] and Boyd,[3] in the cases of low and high pressure, respectively. The expression for the ion velocity V_b, the density n_b and the potential V_b at the boundary between the penetrating sheath and the space charge sheath is given for the case of low pressure as

$$v_b = (kT_e/M)^{1/2} \tag{1}$$

$$n_b = n_0 e^{1/2} \tag{2}$$

$$V_b = kT_e/2e \tag{3}$$

where k is the Boltzmann constant, T_e the electron temperature, M the mass of ion, n_0 the density of the undisturbed plasma and e the elementary charge. On the other hand, for the case of high pressure the electric field or the velocity is given as

$$v_b = bE_b = bkT_e/e\lambda \tag{4}$$

where b is the mobility of ion and λ the mean free path of ion.

The density and the potential at the boundary for the case of high pressure will be calculated in this paper.

Assuming neutrality in the penetrating sheath, Poisson's equation in cylindrical coordinate is described as

$$(1/r)\partial/\partial r(rE) = 0 \tag{5}$$

It follows that

$$rE = \text{const.} \tag{6}$$

Making use of the sheath criterion postulated by Boyd, *i.e.*, the expression for the electric field at the boundary $r = r_b$, $E_b = kT_e/e\lambda$, the electric field at a radius r is expressed as

$$E = (r_b/r)(kT_e/e\lambda) \tag{7}$$

We also assume that continuity of the current density is held in the penetrating sheath. As a consequence, we get the expression for the current density as

$$i = 2\pi rebEn_i = 2\pi br_b kT_e n_i/\lambda \tag{8}$$

On the other hand, the electron density is determined by the Boltzmann relation. Thus, we have

$$n_e = n_0 \exp\{-eV/kT_e\} = n_i \tag{9}$$

Then, eq. (8) is rewritten as

$$i = (2\pi br_b kT_e n_0/\lambda)\exp\{-eV/kT_e\} \tag{10}$$

Let us assume that the electric field at the boundary between the undisturbed plasma and the penetrating sheath is prescribed as

$$E_0 = kT_i/e\lambda \tag{11}$$

From eq. (7), the outer radius of the penetrating sheath is

$$r_0 = r_b(T_e/T_i) \tag{12}$$

The potential difference across the penetrating sheath and the density at $r = r_b$ are given as follows;

$$V_b = \int_{r_b}^{r_0} Edr = (r_b kT_e/e\lambda)\ln(T_e/T_i) \tag{13}$$

$$n_b = n_0(T_i/T_e)\exp\{\lambda/r_b\} \tag{14}$$

Then, eq. (10) is

$$i = (2\pi br_b kT_i n_0/\lambda)\exp\{\lambda/r_b\} \tag{15}$$

According to eq. (13), the potential difference across the penetrating sheath depends on the mean free path of ion, the electron temperature and the outer radius of the space charge sheath. The outer radius of space charge sheath is decreased with increasing plasma density and consequently the penetration becomes pronounced, according to eq. (13). The experimental result presented in the previous paper confirms this statement. Eq. (13) can also account for the enhanced penetration with increasing the applied voltage across the space charge sheath.

As the magnitude of V_b may be of the order of ten volts even at 1 mm Hg, ignorance of the penetration effect will lead to errornous estimation of plasma density. Correct way to prevent this error is to consider the real voltage difference across the space charge sheath which is smaller than the applied voltage by an amount of the voltage difference across the penetrating sheath.

References

1) T. Okuda and K. Yamamoto, J. Phys. Soc. of Japan, **13**, 1212, 1958.
2) D. Bohm, Characteristics of Electrical Discharge in Magnetic Field, McGraw Hill, 1949.
3) R. L. F. Boyd, Proc. Roy. Soc., **A 201**, 329, 1950.