SURFACE INSTABILITY OF THE POSITIVE COLUMN WITH AN APPLIED MAGNETIC FIELD

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(Received October 31, 1961)

The ambipolar diffusion theory of the positive column in an applied longitudinal magnetic field appears to breakdown at a critical magnetic field as it is evidenced by the fact that the longitudinal electric field increases with increasing magnetic field. In the earlier work, we treated the positive column with the magnetic field in which we pointed out that the radial distribution of the longitudinal electric field has a tendency of the decrease with the increasing radius due to the circumferential electric current. Although the joule heat due to the circumferential electric current is very small near the axis of positive column, it is increased with the increasing radius and becomes so large that the energy losses of electrons is well able to compensate depending on the strength of magnetic field B. Near the surface of plasma column defined in Reference 1, therefore, the longitudinal electric field, E_z, is smaller than E_z at $r=0$. From $\mathrm{rot}\cdot\vec{\mathrm{E}}=0$ (where $\vec{\mathrm{E}}$ is the strength of electric field), we obtain

$$\frac{\partial E_z}{\partial r} = \frac{\partial E_r}{\partial E}, \tag{1}$$

adopting the cylindrical coordinates. The equation means that $\partial E_r/\partial z$ is equal to zero only when $\partial E_z/\partial r=0$, whereas when E_z is not uniform in the radial distribution, E_r is dependent on z. Thus, the positive column in the magnetic field loses the uniformity along to z due to the presence of circumferential current, especially it is considerable near the surface of plasma column depending on B.

Let $E_r(r)$ be the radial component of electric field for the positive column of cylindrical symmetry. Let $E_r(r, z)$ be the radial component of electric field for the actual positive column. Then Eq. (1) is rewritten as follows, namely

$$\Delta E_r = E_r(r, z) - E_r(r) = \int \frac{\partial E_z}{\partial r} dz. \tag{2}$$

From the equation we can see that the increase of radial electric field due to the non-uniform radial distribution of longitudinal electric field, ΔE_r, acts on the electrons to accelerate towards a wall because $\partial E_z/\partial r$ is negative.

From the theory of the positive column with the magnetic field, the longitudinal electric field is given by

$$E_z = f_1\left\{1 - f_2\frac{J_1^2(\alpha r)}{J_0^2(\alpha r)}\right\}^{1/2}, \tag{3}$$

where f_1, f_2 are the functions of T_e, T_i and B denoting the electron- and ion-temperatures by T_e and T_i respectively, J_0, J_1 are the Bessel functions of the zero and the first orders and $\alpha^2 = Z/D_{AH}$. Here Z is the number of ionizations per sec and electron and D_{AH} the coefficient of the ambipolar diffusion across the line of magnetic field. If we assume f_1 and f_2 are spatially constant, then $\partial E_z/\partial r$ increases remarkably near the surface of plasma column depending on B in f and becomes infinite under the condition of $J_1^2/J_0^2 = f_2^{-1}$. The condition is expressed by the condition of $G_2 = 1$ for the surface of plasma column, where G_2 was defined in Reference 1. If the invariance of ion temperature is assumed, then G_2 is given by the following equation, namely

$$G_2 = f_0 (P_a R)^{2\gamma} \left\{ b_0 + a_0 \left(\frac{B}{p_a} \right)^2 \right\} \left\{ 1 + a_0 \left(\frac{B}{p_a} \right)^2 \right\}^{\gamma} I^{2/3}, \tag{4}$$

where f_0 is the coefficient depending on the nature of atom, R the radius of a discharge tube, p_a the pressure of atoms, γ is nearly equal to $0.7 \sim 0.8$ for most gases, I the discharge current and a_0 and b_0 are respectively given by

$$a_0 = \frac{\lambda_{e0} \lambda_{i0} e}{(m_i m_e)^{1/2}} \cdot \left(\frac{e}{k T_e} \right)^{1/2} \cdot \left(\frac{e}{k T_i} \right)^{1/2}, \tag{5}$$

$$b_0 = \frac{\lambda_{i0}}{2 \lambda_{e0}} \left(\frac{m_e}{m_i} \right)^{1/2}. \tag{6}$$

Here λ_{e0}, λ_{i0} are the mean free paths of electrons and ions respectively, e the electric charge of monovalent ion, k the Boltzmann constant and m_e and m_i are respectively the masses of electron and ion. From Eq. (4) the condition $G_2 = 1$ becomes approximately as follows, namely

$$B_c/p_a = a_0^{-1/2} \{ f_0^{-1} I^{-1/3} (p_a R)^{-2\gamma} - b_0 \}^{1/2}, \qquad \text{for } a_0 (B_c/p_a)^2 \ll 1, \tag{7}$$

$$B_c/p_a \propto (p_a R)^{-\gamma/1+\gamma} I^{-1/3(1+\gamma)}, \qquad \text{for } a_0 (B_c/p_a)^2 \gg 1. \tag{8}$$

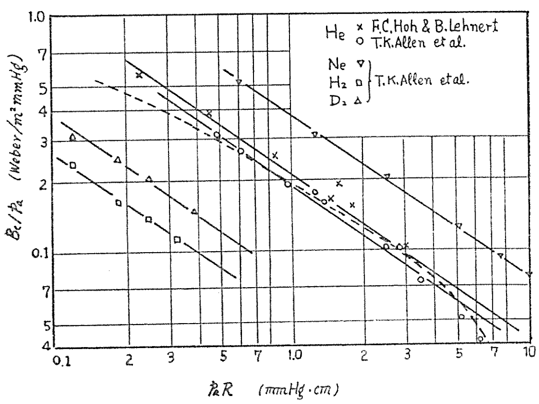

FIG. 1. The graph B_c/p_a vs. $p_a R$. The slopes of the straight-lines are nearly equal to 0.7 for the gases. A dotted line is the theoretical curve for He.

We, now, connect (B_c/p_a) given here to the critical magnetic field where the unusual behavior of the positive column with the magnetic field appears. The graph B_c/p_a vs. p_aR is written through various experiments in Fig. 1, in which the slopes of the straight lines are nearly equal to 0.7 for the gases used in the experiments. The theory is in good agreement with the experimental results for Ne. For He, however, the theoretical curve shown by the dotted line in Fig. 1 is found to be disconnected with the experimental results for the small values of p_aR. The discrepancy may be caused by the assumption that T_e, T_i are independent of the radius r.

Conclusion

In the positive column with the magnetic field, the longitudinal electric field has a tendency of the decrease with the increasing radius due to the circumferential electric current. The radially-distributed longitudinal electric field brings forth the non-uniform axial distribution of radial electric field, which is turned in the increase of particle losses. Near the surface of plasma column, the joule heat of circumferential electric current becomes so large as it compensates the energy losses of electrons depending on the magnetic field. In such a case the longitudinal electric field is equal to zero at the surface, so the particle losses to the wall are infinite. The condition is given by Eq. (4) and is connected with the critical magnetic field where the longitudinal electric field begin to increase. The theory is in good agreement with the experimental results. The surface instability predicted here differs from the hydrodynamic instability of the positive column described by B. B. Kadomtsev and A. V. Nedospasov.

Reference

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