FORMULATION OF DISSIPATION MECHANISMS IN A PARTIALLY IONIZED PLASMA

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(Received October 31, 1961)

1. Introduction

It is shown that a quiescent partially ionized plasma possesses two important mechanisms for heat dissipation. One of them is the joule heat and the other is the ambipolar heat. The latter may be connected with Cowling's prediction[1] that partially ionized plasma possesses a mechanism for the heat dissipation of electric current which is absent in a fully ionized plasma. His treatment, however, is discernible in mathematical expression. We formulate, here, the dissipation mechanisms.

2. The heat dissipations as a whole

We consider a system constituted with electrons, ions and atoms and assume that the ion is singly ionized ion. From thermodynamics[2], the internal energy per unit mass of the system, u, is given by the following equation, namely

$$\rho \frac{du}{dt} + p \cdot \operatorname{div} \mathbf{v} = \sum_s \mathbf{J}_s \mathbf{F}_s - \operatorname{div} \mathbf{J}_w, \qquad (s = e,\ i,\ a), \tag{1}$$

where ρ the mass density as a whole, p the pressure, $\mathbf{v}$ the velocity of the center of mass, $\mathbf{J}_s$ the mass flow of component s relative to $\mathbf{v}$, $\mathbf{F}_s$ the body force per unit mass on the component s, $\mathbf{J}_w$ the flow vector of heat, and the subscripts e, i and a mean the quantities regard with the electrons, ions and atoms respectively. The quantities ρ, $\mathbf{v}$ and $\mathbf{J}_s$ are respectively defined by

$$\rho = \sum \rho_s, \tag{2}$$

$$\mathbf{v} = \sum \rho_s \mathbf{v}_s / \sum \rho_s, \tag{3}$$

$$\mathbf{J}_s = \rho_s(\mathbf{v}_s - \mathbf{v}). \tag{4}$$

Here ρ_s the mass density of component s and $\mathbf{v}_s$ the flow speed of component s.

The work to be done on the system as a whole, namely $\sum \mathbf{J}_s \cdot \mathbf{F}_s$ in Eq. (1), is calculated as follows: At the first stage we express $\mathbf{J}_s$ by the density of electric current $\mathbf{j}$ and by the ambipolar velocity $\mathbf{v}_A$. The latter is the velocity of charged particles relative to uncharged particles. They are defined by

$$\mathbf{j} = en_e(\mathbf{v}_i - \mathbf{v}_e), \tag{5}$$

$$\mathbf{v}_A = \mathbf{v}_p - \mathbf{v}_a, \tag{6}$$

respectively. Here e the unit of electric charge, n_e the density of electrons which is nearly equal to the density of ions and $\mathbf{v}_p$ the flow speed of charged particles which is defined as follows, namely

$$\mathbf{v}_p = (\rho_e \mathbf{v}_e + \rho_i \mathbf{v}_i)/(\rho_e + \rho_i). \tag{7}$$

Using $\mathbf{j}$ and $\mathbf{v}_A$, the mass flow relative to the center of mass $\mathbf{J}_e$, $\mathbf{J}_i$ and $\mathbf{J}_a$ are expressed as follows,

$$\mathbf{J}_e = -\frac{m_e}{e}\left\{\frac{m_i}{m_i+m_e}\mathbf{j} - (1-x)\mathbf{j}_A\right\}. \tag{8}$$

$$\mathbf{J}_i = \frac{m_i}{e}\left\{\frac{m_e}{m_i+m_e}\mathbf{j} + (1-x)\mathbf{j}_A\right\}, \tag{9}$$

$$\mathbf{J}_a = -\left(\frac{m_i+m_e}{e}\right)\cdot(1-x)\mathbf{j}_A, \tag{10}$$

where m_e is the mass of an electron, m_i the mass of an ion and $\mathbf{j}_A$ and x are defined by

$$\mathbf{j}_A = en_e\mathbf{v}_A, \tag{11}$$

$$x = (\rho_e + \rho_i)/(\rho_e + \rho_i + \rho_a), \tag{12}$$

in which the latter is the degree of ionizations.

The electron current relative to $\mathbf{v}$, $-e\mathbf{J}_e/m_e$, is composed of the electronic part of $\mathbf{j}$ and of the contribution of $-(1-x)\mathbf{j}_A$. On the other hand the ionic current relative to $\mathbf{v}$, $e\mathbf{J}_i/m_i$, is constructed with the ionic part of $\mathbf{j}$ and with the contribution of $(1-x)\mathbf{j}_A$. Consequently, we can find that the contribution of $(1-x)\mathbf{j}_A$ to the flow of charged particles is equal to zero, whereas the contribution of that to the mass flow of charged particles is finite and it is equal to the mass flow of uncharged particles although the directions are opposit each other.

In the electro-magnetic field, the forces $\mathbf{F}_e$, $\mathbf{F}_i$ and $\mathbf{F}_a$ are given by

$$\mathbf{F}_e = -\frac{e}{m_e}(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}), \tag{13}$$

$$\mathbf{F}_i = \frac{e}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \tag{14}$$

$$\mathbf{F}_a = 0, \tag{15}$$

where $\mathbf{E}$ is the intensity of electric field and $\mathbf{B}$ the intensity of magnetic field.

From Eqs. (8) ~ (15), $\sum \mathbf{J}_s \cdot \mathbf{F}_s$ is given by the following equations, nemely

$$\sum \mathbf{J}_s \cdot \mathbf{F}_s = \mathbf{j} \cdot (\mathbf{E} + \mathbf{v}_p \times \mathbf{B}) + (1-x)\mathbf{v}_A \cdot [\mathbf{j} \times \mathbf{B}], \tag{16}$$

$$= \mathbf{j} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{17}$$

In Eq. (16), first term of the right-hand side is equal to zero when the electric field relative to $\mathbf{v}_p$ is a right angle to the electric current and hence it is considered as the longitudinal heat. On the other hand, the second term is the transverse heat, since if the effective electric field of the Hall-effect $[\mathbf{j}\times\mathbf{B}]/en_e$ is perpendicular to the effective electric current $(1-x)\mathbf{j}_A$ then the second term is equal to zero.

The hydrodynamics of partially ionized plasmas shows that $\mathbf{v}$, $\mathbf{v}_A$ and $\mathbf{j}$ are respectively given by the following equations;[3)~5)]

$$\rho\frac{d\mathbf{v}}{dt}=\mathbf{j}\times\mathbf{B}-\operatorname{grad}p, \tag{18}$$

$$\rho_p\frac{d_p\mathbf{v}_p}{dt}=\mathbf{j}\times\mathbf{B}-\operatorname{grad}p_p-a(\mathbf{v}_A+b\mathbf{j}), \tag{19}$$

$$\frac{m_e}{e^2}\frac{\partial}{\partial t}\frac{\mathbf{j}}{n}+\eta\mathbf{j}=\mathbf{E}+\mathbf{v}_p\times\mathbf{B}+\frac{1}{en_e}\operatorname{grad}p_e-\frac{1}{en_e}[\mathbf{j}\times\mathbf{B}]-ab\,\mathbf{v}_A, \tag{20}$$

where $d_p/dt=\partial/\partial t+(\mathbf{v}_p\cdot\operatorname{grad})$, η is the electrical resistivity, p_e the partial pressure of electrons, p_p the pressure of charged particles, ρ_p the mass density of charged particles (which is defined by $\rho_p=\rho_e+\rho_i$) and a and b are both some coefficients (see Eqs. (34) and (35)). The last terms in Eqs. (19) and (20) relate to the collisions of charged particles with uncharged particles so that these terms are absent in fully ionized plasmas.

Using Eq. (20), the longitudinal heat becomes

$$\mathbf{j}\cdot(\mathbf{E}+\mathbf{v}_p\times\mathbf{B})=\\ \eta\mathbf{j}^2-\frac{1}{en_e}(\mathbf{j}\cdot\operatorname{grad}p_e)+\frac{ab}{en_e}\mathbf{j}\cdot\mathbf{j}_A+\frac{m_en_e}{e^2}\frac{\partial}{\partial t}\left(\frac{\mathbf{j}^2}{2n_e^2}\right). \tag{21}$$

In the right-hand side of Eq. (21), the first term is the joule heat and the second term is the heat dissipation due to the thermo-electric effect. The third term is the heat dissipation by $\mathbf{j}$ and $\mathbf{j}_A$.

On the other hand, the transverse heat becomes

$$(1-x)\mathbf{v}_A\cdot[\mathbf{j}\times\mathbf{B}]=\frac{a}{e^2n_e^2}\mathbf{j}_A^2+\frac{1}{en_e}\{\mathbf{j}_A\cdot(\operatorname{grad}p_p-x\operatorname{grad}p)\}\\ +\frac{ab}{en_e}\mathbf{j}\cdot\mathbf{j}_A+\frac{x}{1-x}\frac{d}{dt}\left\{\frac{(1-x)^2}{2}\mathbf{v}_A^2\right\}, \tag{22}$$

using Eqs. (18) and (19). Here we use the following approximate equation, namely

$$\frac{d_p\mathbf{v}_p}{dt}\simeq\frac{d\mathbf{v}}{dt}+\frac{d}{dt}\{(1-x)\mathbf{v}_A\}$$

In Eq. (22), the first term of the right-hand side is the heat dissipation of the effective electric current $\mathbf{j}_A$ and the second term is the contribution of the therm-electric effect. We call the first term, "The ambipolar heat". The transverse heat is also wrtten as follows;

$$(1-x)\mathbf{v}_A \cdot [\mathbf{j}\times\mathbf{B}] = \frac{(1-x)^2}{a}[\mathbf{j}\times\mathbf{B}]^2$$

$$-\frac{1-x}{a}[\mathbf{j}\times\mathbf{B}]\cdot(\mathrm{grad}\,p_p - x\,\mathrm{grad}\,p) - x\frac{1-x}{a}[\mathbf{j}\times\mathbf{B}]\cdot\frac{d}{dt}\{(1-x)\mathbf{v}_A\}, \quad (23)$$

in which the first term of the right-hand side relates to the heat dissipation predicted by T. G. Cowling[6].

3. The heat dissipation as the component

The temperature of the components of the partially ionized plasma just as the electron temperature etc. differ from the temperature of the plasma as a whole. If we denote the velocity of a particle of component s by $\mathbf{C}_s$, then it is plausible to be defined the kinetic temperature of component s, T_s, by

$$\frac{3}{2}kT_s = \overline{\frac{m_s(\mathbf{C}_s-\mathbf{v}_s)^2}{2}}, \quad (24)$$

where k is the Boltzmann constant and the bar on the right-hand side means the mean value over the velocity space. Using the above definition of the kinetic temperature, the equation of heat for the component s is given by

$$n_s\frac{d_s}{dt}\left(\frac{3}{2}kT_s\right) + p_s\cdot\mathrm{div}\,\mathbf{v}_s = \mathbf{J}_s\cdot\mathbf{R}_s - \mathrm{div}\,\mathbf{J}_{ws} - \sum_{r\neq s} n_s n_r \beta_{sr}(T_s - T_r), \quad (25)$$

where $d_s/dt = \partial/\partial t + (\mathbf{v}_s\cdot\mathrm{grad})$, $\mathbf{R}_s$ is the frictional force per unit mass on the component s due to the collisions with the other components, $\mathbf{J}_{ws}$ the heat flow of component s and β_{sr} the coefficient of the linear approximation for the collision term in Boltzmann's transport equation. The equation (25) is deduced from the equations in Case 3 of Chapt. 8 in the book, "The Mathematical Theory of Non-Uniform Gases", by Chapman and Cowling (Cambridge University Press, 1953), pp 135, using the equation of motion for the component s given by

$$\rho_s\frac{d_s\mathbf{v}_s}{dt} = \rho_s\mathbf{F}_s - \mathrm{grad}\,p_s - \rho_s\mathbf{R}_s. \quad (26)$$

In Eq. (25), the first term of the right-hand side is the frictional heat and only the term is different from the equations given by the earlier workers[7)~9)]. The third term is the direct energy losses of component s due to the collisions with the other components.

Let us now calculate $\mathbf{J}_s\cdot\mathbf{R}_s$. With the first approximation, $\mathbf{R}_s$ is[10)]

$$\rho_s\mathbf{R}_s = \sum_{r\neq s} n_s n_r \alpha_{sr}(\mathbf{v}_s - \mathbf{v}_r), \quad (27)$$

where α_{sr} is the coefficient of the linear approximation. Expressing with the terms of $\mathbf{j}$ and $\mathbf{j}_A$, $\mathbf{J}_e\cdot\mathbf{R}_e$ etc. can be written as follows;

$$\mathbf{J}_e\cdot\mathbf{R}_e = \eta_e\mathbf{j}^2 - \{\eta_{ea}+\eta_e(1-x)\}\,\mathbf{j}\cdot\mathbf{j}_A + (1-x)\eta_{ea}\mathbf{j}_A^2, \quad (28)$$

$$\mathbf{J}_i \cdot \mathbf{R}_i = \eta_i \left(\frac{m_e}{m_i} \mathbf{j} \right)^2 + \left(\frac{m_e}{m_i} \right) \{ \eta_{ia} + \eta_i (1-x) \} \mathbf{j} \cdot \mathbf{j}_A + (1-x) \eta_{ia} \mathbf{j}_A^2, \tag{29}$$

$$\mathbf{J}_a \cdot \mathbf{R}_a = \frac{a}{e^2 n_e^2} x \mathbf{j}_A^2 + \frac{ab}{en_e} x \mathbf{j} \cdot \mathbf{j}_A, \tag{30}$$

where,

$$\eta_e = \eta_{ei} + \eta_{ea}, \tag{31}$$

$$\eta_i = (m_i / m_e) \eta_{ei} + \eta_{ia}, \tag{32}$$

$$\eta_{ei} = \frac{m_e \nu_{ei}}{e^2 n_e}, \qquad \eta_{ea} = \frac{m_e \nu_{ea}}{e^2 n_e}, \qquad \eta_{ia} = \frac{m_i \nu_{ia}}{2 e^2 n_e}, \tag{33}$$

$$a = n_e (m_i \nu_{ia}/2 + m_e \nu_{ea}) \simeq n_e m_i \nu_{ia}/2, \tag{34}$$

$$b = \frac{m_e}{ea} \left(\frac{\nu_{ia}}{2} - \nu_{ea} \right) \simeq - \frac{2}{en_e} \cdot \frac{m_e}{m_i} \cdot \frac{\nu_{ea}}{\nu_{ia}}. \tag{35}$$

Here η_e and η_i are the electronic part and ionic part of electrical resistivity η, η_{sr} is the effective resistivity due to the collisions of component s with component r and ν_{sr} the frequency of $s-r$ collisions. In Eqs. (28) ~ (35), we assume that

$$1 \ll (\nu_{ea}/\nu_{ia}) \ll (m_i/m_e) \tag{36}$$

From the above equations, one can find that the joule heat $\eta \mathbf{j}^2$ is mainly supplied to the heating of the electrons and that the ambipolar heat $a \mathbf{v}_A^2$ is mainly supplied to the heating of the ions and atoms. Furthermore, even when the electric current is not present the components of plasma is heated by the ambipolar heat except by the adiabatic compression. It is noted that

$$\sum_s \mathbf{J}_s \cdot \mathbf{R}_s = \eta \mathbf{j}^2 + \frac{2ab}{en_e} \mathbf{j} \cdot \mathbf{j}_A + \frac{a}{e^2 n_e^2} \mathbf{j}_A^2. \tag{37}$$

4. Conclusion

We have formulated the dissipation mechanisms in the partially ionized plasmas. From the formulation, we find the ambipolar heat proportional to the ambipolar flow $\mathbf{j}_A$ (see Eq. (11)), just as the joule heat is proportional to the square of the electric current $\mathbf{j}$. Furthermore, it is shown that the heat dissipation proportional to $\mathbf{j} \cdot \mathbf{j}_A$ is present.

When we turn our attention to the components of plasma, we can find that the joule heat is mainly supplied to the heating of electrons and that the ambipolar heat is supplied to the heating of ions and atoms mainly. It is noted that even when the electric current is absent the contributions of ambipolar heat to the heating of plasma components are finite.

References

1) T. G. Cowling: Magnetohydrodynamics, Interscience Pub., 1957, pp 105.
2) R. S. De Groot: Thermodynamics of Irreversible Processes, Amsterdam, 1951.

3) H. Maecker and Th. Peters: Z. Physik, **144**, 586, 1956.
4) A. Schlüter: Z. Naturforsch., 6a, 73, 1951.
5) T. Kihara: J. Phys. Soc. of Japan, **15**, 473, 1958.
6) T. G. Cowling: see reference (1), pp 111, Eq. (6–28).
7) A. A. Dougal and L. Goldstein: Phys. Rev., **109**, 615, 1958.
8) L. Goldstein and T. Sekiguchi: Phys. Rev., **109**, 625, 1958.
9) see L. B. Loeb: Fundamental Processes in Gas Discharges, New York, 1947, Chapt. 4.
10) M. H. Johnson: Phys. Rev., **84**, 566, 1951.