THEORY OF SPACE CHARGE IN VACUUM DIODE TUBE

PART II. THE TEMPERATURE OF ELECTRON GAS

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I. Introduction

In the preceding paper,[1] we developed a space charge theory on the basis of hydrodynamics. Under the two extreme conditions that (1) the temperature is constant in the system and (2) the temperature changes adiabatically, we solved the space charge problem. With the constant temperature approximation (hereafter referred to as C.T.A.), the results were in good agreement with those of Langmuir theory. With the adiabatic approximation (A.A.), however, the results were poor for the current smaller than the saturated current.

In this paper, we describe the equation of thermal energy and discuss the applicable conditions for the above two approximations. As it will be seen later, the applicable conditions depend on the thermal conductivity of electron gas or the mean free path. We estimate these quantities using the results of the calculation by Chapman and Cowling.[2]

II. The Equation of Thermal Energy

On the macroscopic point of view, the equation of thermal energy is written as follows,

$$n\frac{du}{dt}+p\,\mathrm{div}\,v=-\mathrm{div}\,J_w, \tag{1}$$

where n is the number density, u the thermal energy per particle, p the pressure, v the macroscopic velocity and J_w the flow of heat. The flow of heat is approximately given by

$$J_w=-\kappa\,\mathrm{grad}\,T, \tag{2}$$

where κ is the thermal conductivity and T is the temperature.

From the equation (1) we can conclude that the temperature changes adiabatically when the characteristic time of the phenomenon is smaller than the collision time of electron gas. When it is not the case, the divergence of J_w is important.

We shall consider the situation that the system is in the steady state: The equation (1) then becomes

$$n(v\cdot\mathrm{grad})u+p\,\mathrm{div}\,v=\mathrm{div}\,(\kappa\,\mathrm{grad}\,T) \tag{3}$$

using Eq. (2). Since the order of the magnitude of nu or p is nkT, where k is the Boltzmann constant, the temperature changes adiabatically when

$$nv \gg \frac{\kappa}{Lk}, \tag{4}$$

where L is the characteristic length of the phenomenon. When

$$nv \ll \frac{\kappa}{Lk}, \tag{5}$$

the temperature is approximately constant in the system.

From the theory of gas kinetics, κ is

$$\kappa \simeq nv_0 lk, \tag{6}$$

where v_0 is the mean thermal velocity and l is the mean free path. Using Eq. (6), Eqs. (4) and (5) are rewritten as follows,

$$v \underset{\ll}{\gg} v_0 \frac{l}{L}, \tag{7}$$

where the inequality of the above side in Eq. (7) is to be taken for A.A. and that of the below side is for C.T.A.

III. The Spatial Variation of Temperature in Vacuum Diode Tube

Under the conditions that the system is steady and one-dimensional, Eq. (3) becomes

$$\frac{d^2 T}{dx^2} - \frac{kj}{2e\kappa} \cdot \frac{dT}{dx} - \frac{kj}{e\kappa} \cdot \frac{1}{v} \frac{dv}{dx} \cdot T = 0, \tag{8}$$

where

$$j = env. \tag{9}$$

From Eq. (8) and the equations in Part I we can get the quantities, n, v, T and E or V as a function of x, where E and V are the strength of electric field and the electric potential respectively. Since the differential equations are very complicate and the boundary conditions at the anode surface are uncertain, we shall discuss applicable conditions of the two extreme approximations in part I.

For the space charge problem, the applicable conditions are deduced from the preceding section. Using Eq. (9), Eqs. (4) and (5) are altogether written as follows,

$$j \underset{\ll}{\gg} \frac{e\kappa}{k(x+x_0)}, \tag{10}$$

where $x_0 = (kT_0/8\pi n_0 e^2)^{1/2}$ and n_0 is the number density at the cathode and T_0 is the temperature of electron gas at the cathode. It is noted that in Eq. (10) the sign of the inequality of the above side is to be taken for A.A. and that the below side is for C.T.A. We use $x+x_0$ instead of x in Eq. (10) because, from the Part I, the quantities just as the velocity, the density etc. are affected by $x+x_0$ instead of x. From Eq. (7), the alternative form of Eq. (10) is get, namely

$$\frac{v}{v_0} \gtrless \frac{l}{x+x_0}. \tag{11}$$

In the cathodic side of a point of minimum electric potential, $v/v_0<1$, so that the temperature is nearly constant in the region of x satisfying the condition

$$\frac{l}{x+x_0} \gg 1. \tag{12}$$

In the anodic side $v/v_0>1$, the temperature is adiabatic in the region of x satisfying the condition

$$\frac{l}{x+x_0} \ll 1. \tag{13}$$

IV. The Thermal Conductivity and the Mean Free Path

The scattering of the electron-electron interaction is given by Rutherford's law. According to the law, the cross-section of scattering is infinity when the scattering angle is equal to zero. Hence, the determination of the lower limit of scattering angle is very important.

From the calculations by Chapman and Cowling, in which the lower limit of scattering angle is taken to correspond to the mean distance of particles, the mean free path and the thermal conductivity of electron gas are respectively given by

$$l=(nQ)^{-1} \tag{14}$$

and

$$\kappa=\frac{75\pi}{128}\cdot\left(\frac{nkT}{\pi}\right)^{1/2}Q^{-1}, \tag{15}$$

where

$$Q=\frac{\pi}{4}\left(\frac{e^2}{kT}\right)^2 \ln\frac{4kT}{e^2 n^{1/3}}. \tag{16}$$

For the interesting range of the temperature we plot Eq. (14) and Eq. (15) in Fig. 1 and Fig. 2 respectively.

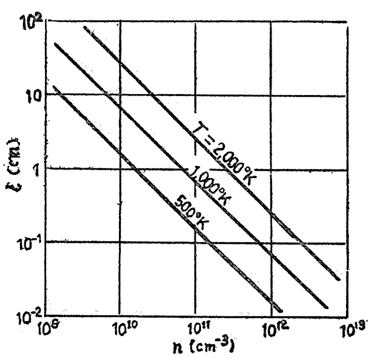

FIG. 1. Mean free path, l, versus number density, n, for various temperature.

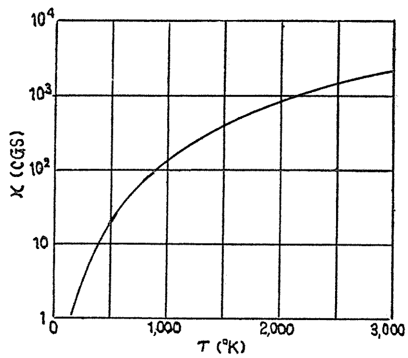

FIG. 2. The dependence of thermal conductivity on temperature.

V. Conclusion

On the basis of the thermodynamics, we discuss the spatial variation of electron temperature in vacuum diode tube and deduced the applicable conditions of constant temperature approximation and of adiabatic approximation. The applicable condition is given by Eq. (10).

References

1) S. Miyajima and K. Yamamoto: Memo. Fac. Engi., Nagoya Univ., Vol. **12**, No. 1, 1960.
2) Chapman and Cowling: The Mathematical Theory of Non-Uniform Gases, Camb. Univ. Press, Cambridge, 1953, p. 177.