SOME CHARTS FOR OBTAINING DAMPED FREQUENCY RESPONSE DIAGRAMS IN PROCESS CONTROL

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Introduction

Wherever quantitative approach to the best controller settings is desired in the process control problem, frequency response diagrams of both the controller and controlled plant are indispensable.

But even if either the transfer functions or the frequency response curves for undamped oscillations are given, they would be half as useful without correction for damping. This correction is often quite laborious if analytically carried out. To cope with this, graphical method is of great value. So several charts and tables for this end are introduced in this paper.

Controller

The performances of actual automatic controllers deviate largely from that of the orthodox or theoretical ones because of the interaction and other factors which unavoidably result from the mechanism in which they take part. But since the performance differs from controller to controller and does not allow generally to be treated, only the theoretical controller is discussed here.

TABLE 1. Frequency Response of the Theoretical Controller

Symbols used:
- s.r.: Subsidence ratio of damped oscillation
- F: Total gain of controller
- K: Sensitivity of controller
- τ : Period of oscillation
- ψ : Phase advance of controller.
- T_I: Integral action time of controller
- T_D: Derivative action time of controller

a) PI-action

s.r. $= e$

τ/T_I	F/K	ψ deg
0	1.00	0
0.224	0.995	−2
0.556	0.991	−5
1.00	0.9876	$-9°\sim03'$
2.22	1.004	−20
3.40	1.058	−30
4.77	1.141	−40
6.46	1.307	−50
8.75	1.567	−60
12.3	2.03	−70
19.2	3.00	−80
26.2	4.07	−85
40.5	6.28(2)	−90

s.r. $= 4$

τ/T_I	F/K	ψ deg
0	1	0
0.228	0.992	−2
0.565	0.985	−5
1.000	0.9783	$-8°\sim55'$
2.22	0.986	−20
3.36	1.024	−30
4.66	1.101	−40
6.21	1.232	−50
8.24	1.45	−60
11.26	1.82	−70
16.6	2.56	−80
21.4	3.25	−85
29.9	4.50	−90

b) PD-action

s.r. = e

τ/T_D	F/K	ψ deg
∞	1.000	0
36.6	0.989	10
18.26	1.006	20
11.90	1.059	30
8.49	1.152	40
6.27	1.310	50
4.625	1.568	60
3.285	2.03	70
2.108	3.03	80
1.549	4.08	85
1.000	6.28(2)	90

s.r. = 4

τ/T_D	F/K	ψ deg
∞	1.000	0
36.986	0.977	10
18.646	0.986	20
12.286	1.025	30
8.876	1.102	40
6.656	1.233	50
5.011	1.45	60
3.671	1.82	70
2.494	2.56	80
1.935	3.26	85
1.386	4.54	90

c) PID-action ($T_I=\tau$)

s.r. = e

τ/T_D	F/K	ψ deg
40.5	0.951	0
19.72	0.940	10
13.01	0.956	20
9.56	1.005	30
7.32	1.096	40
5.67	1.244	50
4.34	1.488	60
3.19	1.93	70
2.10	2.875	80
1.565	3.88	85
1.025	5.97	90

s.r. = 4

τ/T_D	F/K	ψ deg
41.3	0.933	0
20.20	0.913	10
13.45	0.919	20
9.99	0.956	30
7.74	1.027	40
6.09	1.15	50
4.75	1.35	60
3.59	1.70	70
2.51	2.39	80
1.98	3.03	85
1.43	4.23	90

As far as the theoretical controller is concerned, no much complexity is involved in calculating the frequency response for damped oscillations.

In practice the subsidence ratio of the recovery curve between e and 4 is important, and these two cases are shown in Table 1 for the PI, PD, and PID-actions. In the PID-action the integral action time T_I is assumed to be equal to the period of the damped oscillation.

This table is helpful in drawing the frequency response diagram of the controller.

Controlled Plant

Mathematical theory of correction from the frequency response curve for undamped oscillation to that for damped one was originally suggested by N. Ream B. A.[1] As an application of this theory, Geraldne A. Coon[2] proposed a practical chart arranged for the graphical correction to the frequency response curve of which variables are phase lag ϕ, gain G, and frequency f, and the subsidence ratio of 4 is desired.

In the process control field, however, the following combinations of the variables are more commonly used in the frequency response diagrams;

Abscissa	Ordinate	
Angular frequency ω	Gain in decibel D	Phase lag ϕ
Period τ	Attenuation A	Phase lag ϕ

ω, τ, D, and A are on log-scale and ϕ is on normal scale as is usually the case.

Fundamental equations of the correction based on the Ream's theory are derived and listed up in Table 2 for each combination of variables.

TABLE 2. Formulae for Correction

Variables used	Subsidence ratio	Correction for ϕ	Correction for D, A, and G
$\omega \sim D,\ \phi$	e	$\Delta\phi^\circ = -0.452(\partial D/\partial \log \omega)$	$\Delta D = 0.0104(\partial\phi^\circ/\partial \log \omega)$
	4	$\Delta\phi^\circ = -0.622(\partial D/\partial \log \omega)$	$\Delta D = 0.0143(\partial\phi^\circ/\partial \log \omega)$
$\tau \sim A,\ \phi$	e	$\Delta\phi^\circ = -9.043(\partial \log A/\partial \log \tau)$	$\Delta \log A = 5.196\times 10^{-4}(\partial\phi^\circ/\partial \log \tau)$
	4	$\Delta\phi^\circ = -12.44(\partial \log A/\partial \log \tau)$	$\Delta \log A = 7.148\times 10^{-4}(\partial\phi^\circ/\partial \log \tau)$
$f \sim G,\ \phi$	e	$\Delta\phi^\circ = -9.043(\partial \log G/\partial \log f)$	$\Delta \log G = 5.196\times 10^{-4}(\partial\phi^\circ/\partial \log f)$
	4*	$\Delta\phi^\circ = -12.44(\partial \log G/\partial \log f)$	$\Delta \log G = 7.148\times 10^{-4}(\partial\phi^\circ/\partial \log f)$

* The chart proposed by Coon is a graphical expression of these two equations.

A graphical procedure for obtaining the corrections to these process frequency response curves may be shown with the help of Fig. 1, Fig. 2, and Fig. 3.

Let Fig. 4 be the frequency response curves of a controlled plant, and the scales used are the same as those of Fig. 2.

In order to correct the attenuation curve the following steps are needed:

1. To draw the tangent to the phase curve at the period when the correction is required.
2. The increment of ϕ in the tangent over one decade is measured with a pair of dividers and laid off on the Y-scale of Fig. 2.
3. To measure the distance y in Fig. 2 with a pair of dividers corresponding to Y.
4. To lay this amount of distance y off above the attenuation curve at the period when the tangent is drawn. This gives a point on the corrected attenuation curve.

In a similar manner other points are to be found so as to yield a correction curve over the required range of period.

The procedure for obtaining the phase correction curve is mostly similar to the above.

A tangent is drawn to the attenuation curve instead of the phase curve, X and x are found on Fig. 2 instead of Y and y, and the distance x is laid off above the phase curve, which gives a point on the corrected phase curve.

In this instance of Fig. 4, the subsidence ratio 4 is assumed. Attention must be also paid to the fact that Y, X and y are negative in sign and x is positive, consequently y is measured downward from the attenuation curve and x is upward from the phase curve.

The way in which Fig. 1 and Fig. 3 are applied is much the same as the above and no explanation would be necessary.

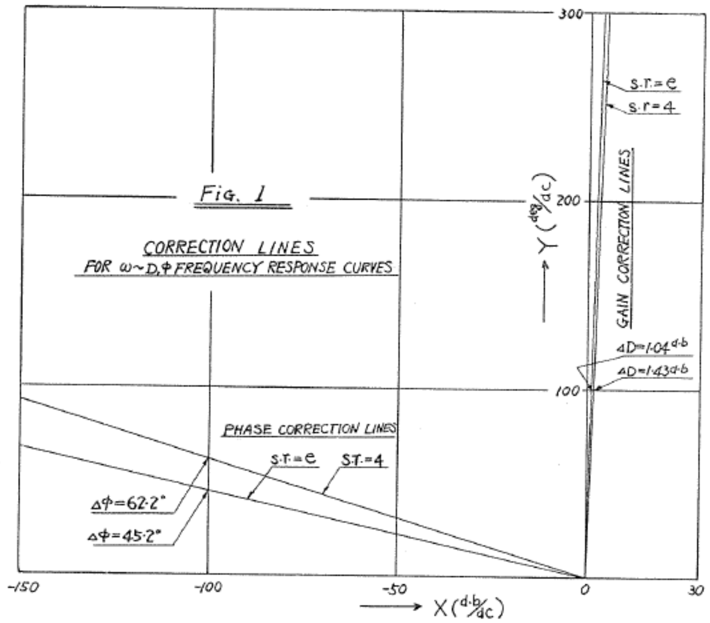
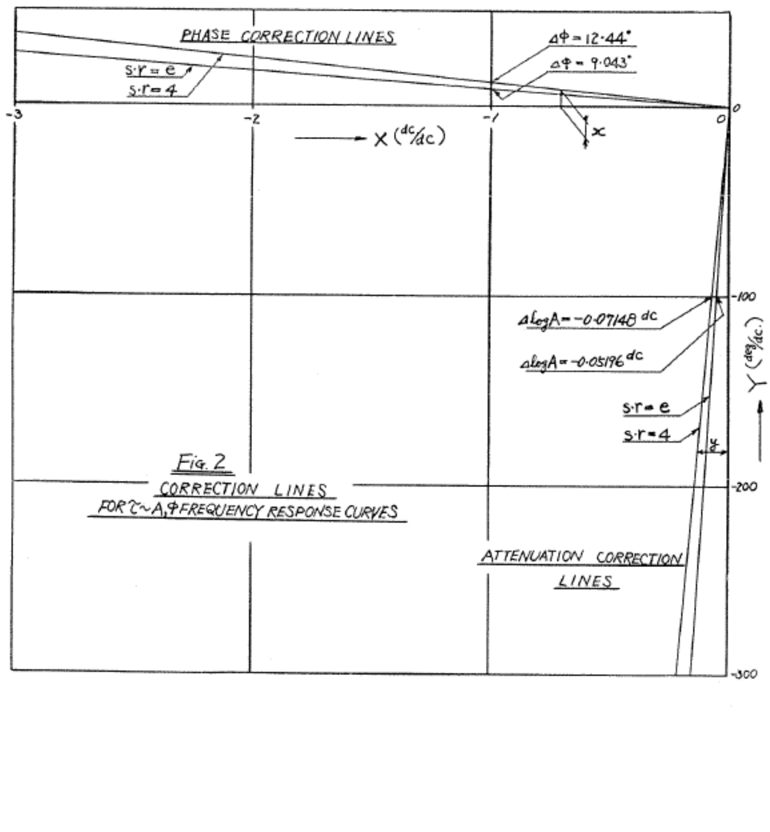
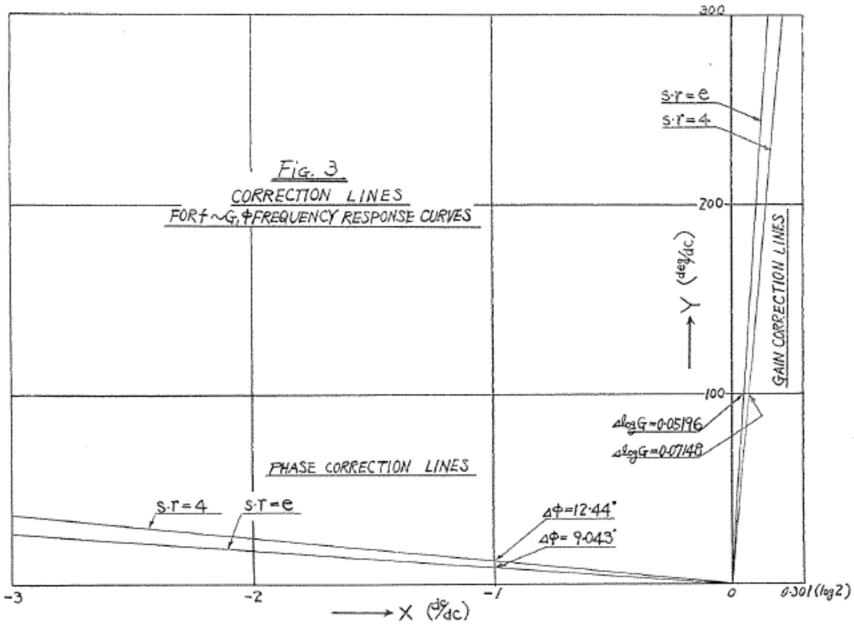


Fig. 1
CORRECTION LINES
FOR ω~D,φ FREQUENCY RESPONSE CURVES
GAIN CORRECTION LINES
s.r.=e
s.r=4
ΔD=1.04 d.b
ΔD=1.43 d.b
PHASE CORRECTION LINES
s.r.=e
s.r.=4
Δφ=62.2°
Δφ=45.2°
Y (deg/dc)
X (d.b/dc)
300
200
100
-150
-100
-50
0
30


PHASE CORRECTION LINES
s.r=e
s.r=4
Δφ=12.44°
Δφ=9.043°
X (dc/dc)
x
-3
-2
-1
0
Δlog A=-0.07148 dc
Δlog A=-0.05196 dc
s.r=e
s.r=4
Y (deg/dc.)
Fig. 2
CORRECTION LINES
FOR τ~A,φ FREQUENCY RESPONSE CURVES
ATTENUATION CORRECTION LINES
-100
-200
-300



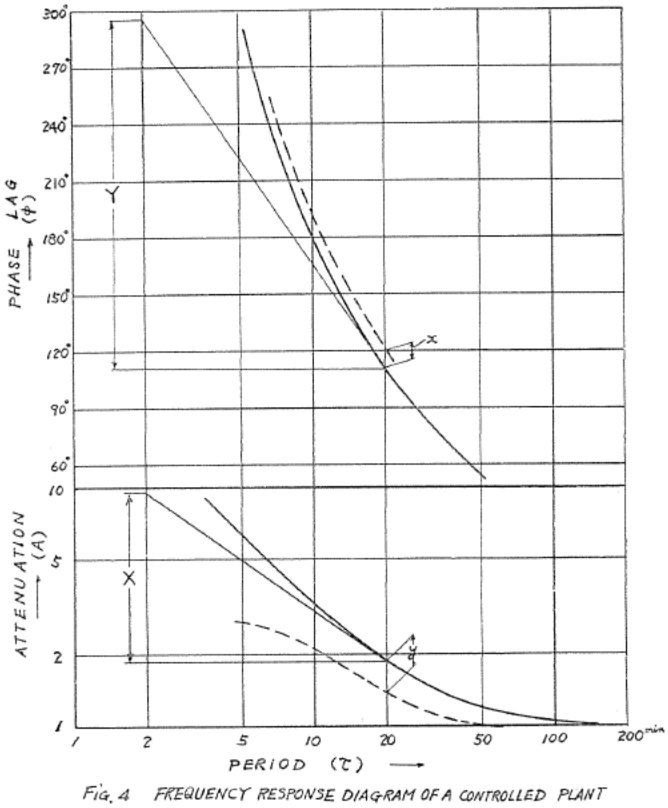


FIG. 4 FREQUENCY RESPONSE DIAGRAM OF A CONTROLLED PLANT

Summary

Three workable charts were introduced in order to facilitate the correction procedures to the process plant frequency response curves, taking the subsidence ratios of e and 4 into consideration. The frequency response of the theoretical controller for the same damping as the above was also considered.

References

1) N. Ream, B. A., "The Calculation of Process Control Settings from Frequency Characteristics," Trans. S.I.T., **6**, 1, March, 1954, p. 19.

2) G. A. Coon, "How to Find Controller Settings from Process Characteristics," Control Engineering, May, 1956, p. 66.