TIME CHANGE OF PERMEABILITY AND HYSTERESIS LOOP DUE TO DIFFUSION AFTER-EFFECT IN FERROMAGNETICS

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Introduction

There is a disaccomodation phenomenon of permeability in some ferromagnetic materials which reveal the time lag effect of magnetic induction. This time effect was explained by Snoek as due to diffusion of impurity atoms among the interstitial positions of crystal lattice and therefore named *diffusion after-effect* or sometimes called *reversible after-effect*[1]. Another important kind of magnetic after effect is the so-called *fluctuation after-effect*[1], because it is affected by the thermal fluctuation of spontaneous magnetization. This latter one is also called *irreversible after-effect.*

Of these two after-effects, the present paper is concerned exclusively with the first one. The magnetic measurement in the weak field of *Rayleigh region* was made of the time change of the permeability and the hysteresis loop of silicon iron, in the cases where 1) measuring field was applied with various time intervals after demagnetization, and 2) measuring field was applied immediately after demagnetization.

The observed results of the experiments are interpreted using Preisach diagram, by which Preisach[2] made a explanation of *Rayleigh law* and afterward Néel[3] extended it, and in terms of Néel's theory[4] on the diffusion after-effect.

Apparatus and Specimen

The main part of measuring apparatus, that is a variation of the Maxwell bridge, is shown in Fig. 1 (I), where L_S is a variable standard inductance, L_X unknown inductance with a specimen core (ring-shaped). In this circuit R is chosen as $R \gg 2\pi f L$ so that the *constant-current characteristic* is given to the specimen inductance L_X. When the core specimen has a hysteresis characteristics shown in Fig. 1 (II), and the bridge is balanced to the L_X corresponding to $\mu = B_m/H_m$, the out-put voltage at the terminal T is proportional to $d/dt(\Delta B)$. This voltage, after being integrated and amplified, is connected to the vertical terminal of the cathode ray tube and thus hysteresis characteristics can be visualized as a hysteresis loop of ΔB on the screen. This method makes the more precise observation possible of hysteresis characteristics, especially in the weak field, than the usual visualizing method. In the following the hysteresis loops of ΔB will be shown instead of the more general ones of B-H.

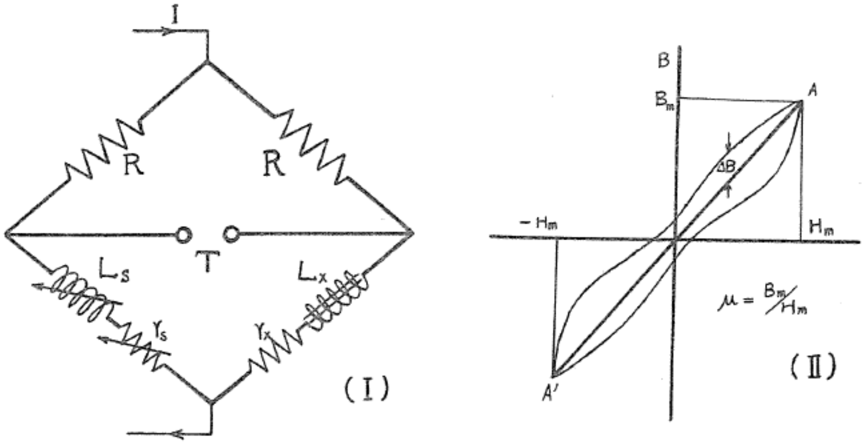


FIG. 1. (I) Maxwell bridge, (II) Hysteresis loop of the specimen.

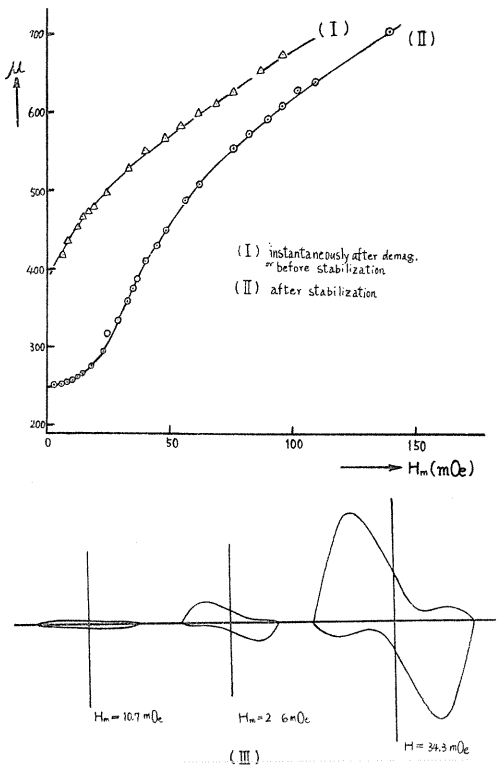
Measurement was made on various kinds of specimen, 1∼4.16% silicon-irons and commercial pure iron. But only the representative results of 4.16% silicon-iron are presented here, because the phenomenon is almost the same to the others: only the difference is in the magnitude of physical constants.

Experimental results

As an initial state of ferromagnetics, the writer chooses hereafter a demagnetized state by the *A.C.* magnetic field of 25 Oe. In Fig. 2 (II) is shown the relation between the permeability and the strength of measuring field (100 cycle/sec., sinusoidal) applied with an interval of 50 hours after the initial state. In other words the permeability was measured in the completely stabilized state. The corresponding hysteresis loops to the curve (II) are shown in Fig. 2 (III). It is to be noted that the loop when magnetized in the field below the value of about 20.6 mOe is *one-humped*, while the other above 34.3 mOe is *two-humped*, which if transformed into the usual hysteresis of *B-H* is somewhat like the so-called *wasp-waist* loop observed in Perminvar. At a certain value between 20.6 mOe and 34.3 mOe there should be a transition from the *one-humped* to the *two-humped.*

In the second place, the writer observed the change of permeability and hysteresis loop at every step during stabilization starting from the unstabilized state. The permeability change is shown in Fig. 3 (I) for 13.3 mOe, and in Fig. 4 (I) for 29.5 mOe. The change of hysteresis loop is shown in Fig. 5.

Next, the writer will show the hysteresis and permeability change with time, when the specimen is kept magnetized in a constant *A.C.* field. In this case, the permeability decreases as well as those above mentioned, the difference being only in the magnitude of decrease (Fig. 3 (II) and Fig. 4 (II)). Fig. 6 (II) shows the time change of hysteresis loop, in which one sees the completely different way of change from those in Fig. 5. Not so apparent deformation is observed but the maximum value of ΔB gradually declines to the right as well as the decrease of its absolute value and the tangent at the tip of left side of the loop. At $t=90$ min. after the initial state, if one increases the strength of measuring field, he



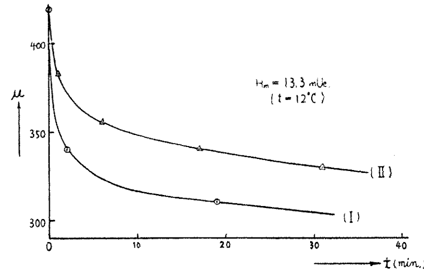


FIG. 3

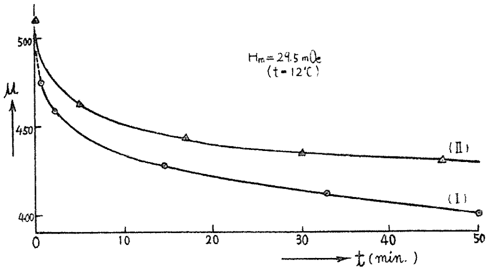


FIG. 4

FIG. 2. (I) Permeability variation with the field strength (before stabilization).
(II) Permeability variation with the field strength (after complete stabilization).
(III) Hysteresis loop corresponding to the curve (II).

FIG. 3 and 4. (I) Time change of permeability after demagnetization (stabilized without no applied field).
(II) Time change of permeability after demagnetization (stabilized in a constant amplitude of *A.C.* field).

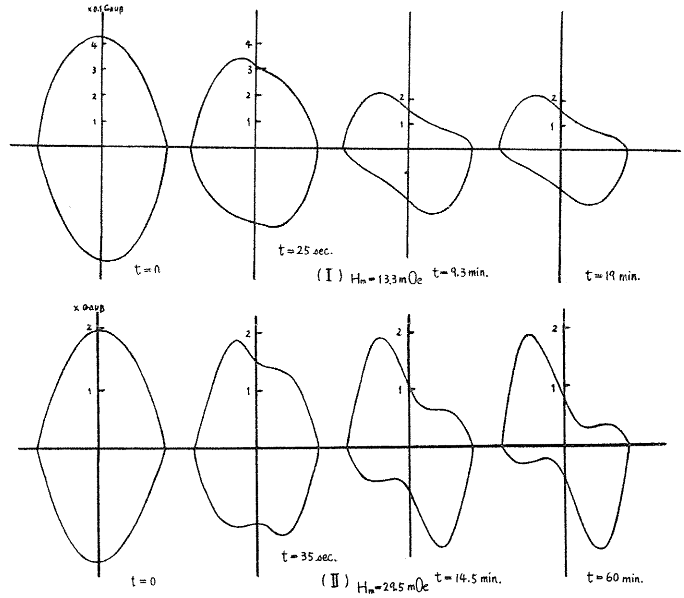


FIG. 5. (I) Hysteresis change with time corresponding to Fig. 3 (I).
(II) Hysteresis change with time corresponding to Fig. 4 (I).

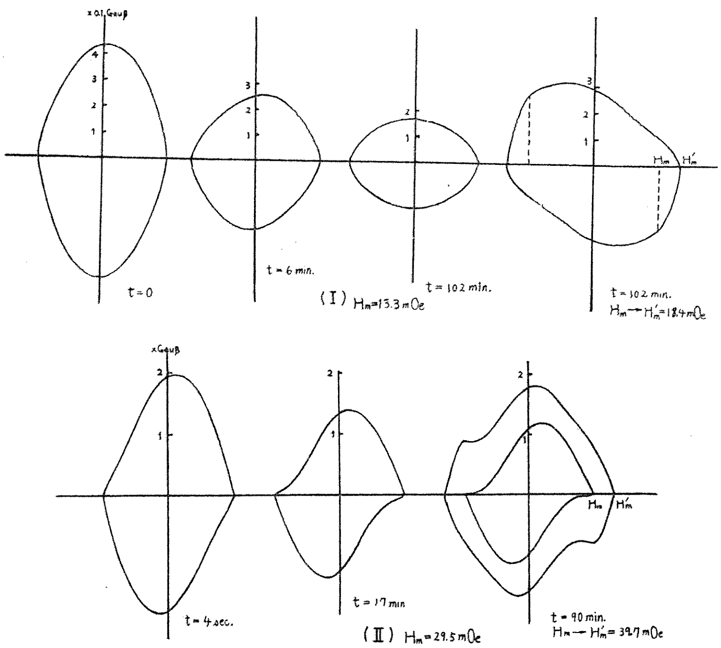


FIG. 6. (I) Hysteresis change with time corresponding to Fig. 3 (II).
(II) Hysteresis change with time corresponding to Fig. 4 (II).

will see the unusual form of hysteresis loop in which the steep transition points can be seen at the points corresponding to $H = 29.5$ mOe where the specimen has been magnetized by now. But this is merely a transient phenomenon and gradually changes into the same form as at $t = 17$ min., in 30 to 60 minutes. In the comparatively small field, the mode of deformation of hysteresis is a little differnt. The result is shown in Fig. 6 (I). The marked decrease of tangent at the left side tip is not the case here.

Discussion

When the magnetic field applied is increased from a certain value H_1 to H, then the corresponding value of B also increases from B_1 to B according to the following equation.

$$B - B_1 = \alpha(H - H_1) + 1/2\,\beta(H - H_1)^2 \qquad H > H_1 \qquad (1)$$

If the field is decreased from H_2, then the corresponding decrease of B_2 can be expressed as follows:

$$B - B_2 = \alpha(H - H_2) - 1/2\,\beta(H - H_2)^2 \qquad H < H_2 \qquad (2)$$

This law was presented in 1887 by *Lord Rayleigh*, which is called *Rayleigh law* after his name. If the magnetic field alternates sinusoidally between $-H_m$ and $+H_m$, the hysteresis loop of ΔB and permeability change with H_m can be written as follows:

$$\Delta B = \pm 1/2\, \beta (H_m - H)^2 \tag{3}$$

$$\mu = B_m/H_m = \alpha + \beta H_m \tag{4}$$

These results are to be compared with the experimental, *i.e.* with those in Fig. 5 (I), (II) $t=0$ or Fig. 6 (I) $t=0$, and Fig. 2 (I). Comparatively good accordances can be seen between them. In other words, the ferromagnetics with magnetic after-effect follow the *Rayleigh law* only momentarily just after demagnetization or just before stabilization begins to occur. In this case the values of α and β are about 400 and 5 $(\mathrm{mOe})^{-1}$ respectively for the specimen.

According to Néel's theory [4] on diffusion after-effect, when the domain wall is rested at a certain position A in the potential hole shown in Fig. 7, it is stabilized after a long time compared with the time-constant of diffusion of impurity atoms. After stabilization, the wall is more hard than before to displace from the position and requires an additional energy for it. This supplement of the additional energy means that the wall displacement requires the additional field besides the usual field for magnetization. This additional field can be written as a function of displacement of wall U from the stabilized position A, and if $U \leqq 3d$ (thickness of 90°-wall),

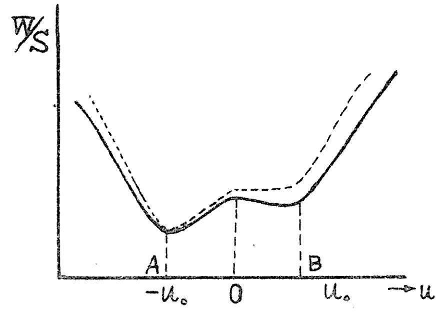


FIG. 7. Potential valley into which the domain wall is fallen.

$$\Delta H = -\frac{W}{J_s}\frac{2}{3}\left(\frac{U}{d}\right)$$

W; stabilization energy
J; saturation magnetization

If $2u_0 \leqq 3d$ is assumed, the corresponding variation on the elementary hysteresis is shown in Fig. 8 (I)

$$\left.\begin{aligned} \Delta a &\approx \frac{W}{J_s}\frac{2}{3}\left(\frac{c_1 u_0}{d}\right) \\ \Delta b &\approx \frac{W}{J_s}\frac{2}{3}\left(\frac{c_2 u_0}{d}\right) \end{aligned}\right\} \tag{6}$$

where c_1 and c_2 have average values 1/2 and 3/4 respectively. This variation as a whole appears in Preisach diagram [2) 3)] as a parallel displacement of $\Delta\gamma O\delta$ to $\Delta\gamma' O'\delta'$ as well as $\Delta\alpha O\delta$ to $\Delta\alpha' O''\delta''$. According to the modified diagram, one can calculate the hysteresis loop of ΔB, which is shown in Fig. 9, as a function of parameter n defined by $n = H_m/H_0$.

When the specimen is magnetized in a constant amplitude of *A.C.* field, stabilization occurs also. While the applied field alterates sinusoidally, the domain wall fallen into the potential vally shown in Fig. 10 (II) is skipping from A to B or *vice versa*. When the strength of the applied field reaches the value a, the wall

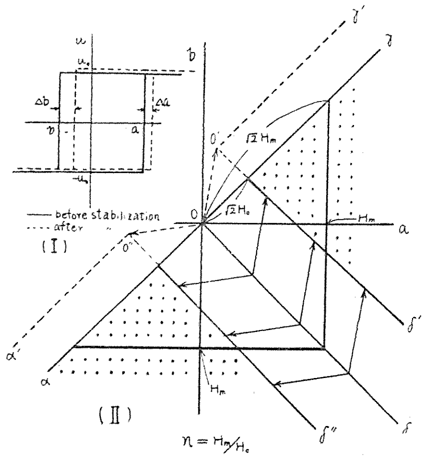


FIG. 8. (I) Elementary hysteresis loop corresponding to the potential valley in Fig. 7. (II) Preisach diagram modified by stabilization.

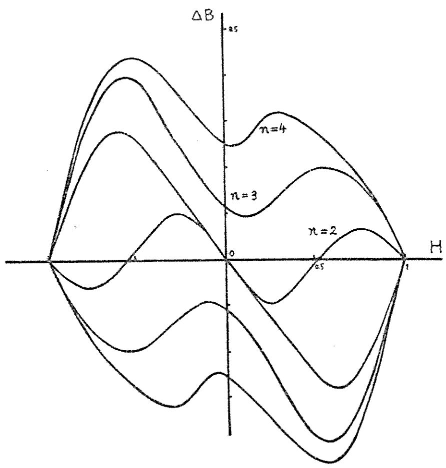


FIG. 9. Hysteresis loop of ΔB calculated with Fig. 8 (II).

skips to B from A, or *vice versa* when the field strength is b, so that the sojourn time at each position of A and B is t_1 and t_2 in one cycle as shown in Fig. 10 (I). When the stabilization is achieved after numbers of this skipping process, the effective field strength required to ride over the slope a or b varies by Δa or by Δb. This variation also can be calculated employing Néel's theory.

Considering above process, the wall displacement can be defined as a function of time as follows: (Fig. 10 (I))

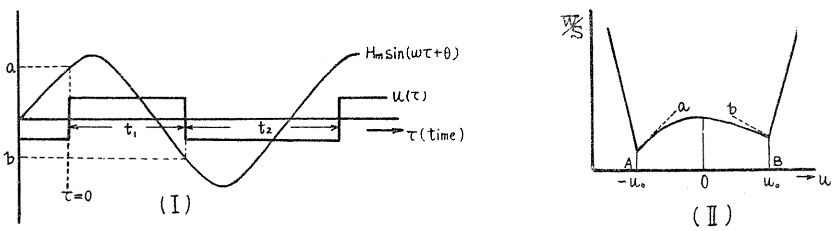


FIG. 10. (I) Movement of domain wall fallen into the potential valley (II), under the action of the field $H(\tau) = H_m \sin(w\tau + \theta)$.

$$\left.\begin{array}{lcccr} \text{from} & \tau = 0 & \text{to} & t_1 & u = u_0 \\ \text{from} & t_1 & \text{to} & t_1 + t_2 & -u_0 \\ & \vdots & & \vdots & \\ & 2n(t_1+t_2) & \text{to} & 2n(t_1+t_2)+t_1 & u_0 \\ & 2n(t_1+t_2)+t_1 & \text{to} & (2n+1)(t_1+t_2) & -u_0 \end{array}\right\} \quad (7)$$

Néel's equation from which the fictive field of difussion after-effect after complete stabilization can be deduced is,

$$\left.\begin{aligned} \varDelta H &= -\frac{W}{cJ_s}\int_0^{t=\infty} f(U)g(t-\tau)d\tau \\ c=1 &\text{ for } 90^\circ\text{-wall} \qquad c=2 \text{ for } 180^\circ\text{-wall} \end{aligned}\right\} \tag{8}$$

where U is defined by $U=u(\tau)-u(t)$ and if $U \lessdot 3d$ ($3d$ is the order of thickness of 90°-wall), then to the first approximation $-f(U)=c\cdot 2/3\cdot U/d$ in the case of only one time-constant of after-effect, $g(t-\tau)=\exp(-(t-\tau)/\theta)$ and W is stabilization energy. Then the equation (8) becomes,

$$\varDelta H = -\frac{W}{cJ_s}\int_0^{t=\infty} \frac{2}{3}\frac{cU}{d} e^{-(t-\tau)/\theta} dt. \tag{9}$$

If one regards the after-effect as a small perturbation and supposes that in the expression of U the difference between the real wall position and the wall position which should be occupied in the absence of after-effect is sufficiently small to be neglected, then the $u(\tau)$ and $u(t)$, which appear in U, are the positions which the wall should occupy at the time τ and t, in the absence of after-effect, under the action of the field $H(\tau)$ and $H(t)$, and therefore taking accout of the relation (7),

$$\begin{aligned} \varDelta H &= -\frac{2W}{3J_s d}\int_0^{t=\infty} [u(\tau)-u(t)]e^{-(t-\tau)/\theta} d\tau, \\ &= -\frac{2W}{3J_s d}\,\frac{2(1-e^{t_1/\theta})}{1-e^{(t_1+t_2)/\theta}} \qquad (\text{for } u=u_0) \\ \text{or} \quad &= -\frac{2W}{3J_s d}\,\frac{-2(1-e^{t_2/\theta})}{1-e^{(t_1+t_2)/\theta}} \qquad (\text{for } u=-u_0) \end{aligned}$$

generally because $\theta \gg t_1,\ t_2$ (θ: time-constant of after-effect) then if $2u_0 \lessdot 3d$

$$\left.\begin{aligned} -\varDelta H_B &= \varDelta b \approx \frac{2W}{3J_s d}\left(\frac{2t_1}{t_1+t_2}\right) \qquad (\text{for } u=u_0) \\ -\varDelta H_A &= \varDelta a \approx -\frac{2W}{3J_s d}\left(\frac{2t_2}{t_1+t_2}\right) \qquad (\text{for } u=-u_0) \end{aligned}\right\} \tag{10}$$

Thus the stabilization results in a form variation of each elementary hysteresis as shown in Fig. 11 (I) and as a whole this is to appear on Preisach diagram as a corresponding group shift of elementary hysteresis. Outside of triangle $\alpha\delta\gamma$, the wall sojourns at A or B without skipping to the other and the parallel shift of elementary hysteresis is the case here, just like that of Fig. 8 (II). As a result of these two kinds of shift in opposite direction, there appears a vacancy along the lines $\overline{\alpha\delta}$ and $\overline{\delta\gamma}$ in which exists no elementary hysteresis.

Now one must remark that the result is effective only in a case where $|\varDelta H| < |a|$ or $|b|$, because when it was derived an assumption was made that the after effect was a small perturbation. Thus the application can be only to the part indicated by full line in Fig. 11 (II), but the shift indicated by dotted line is most likely to be the case for the part where the above mentioned result cannot be applied.

The writer simplified this modified Preisach diagram further into that shown in Fig. 12 (I) and thereby calculated the hysteresis loop of $\varDelta B$, which is shown in

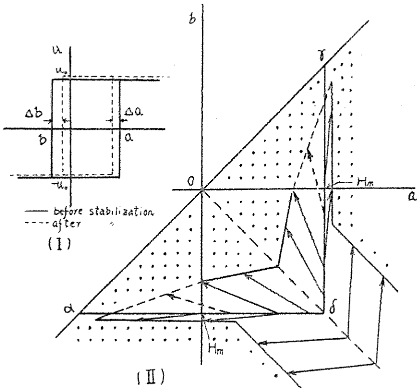


FIG. 11. (I) Elementary hysteresis loop corresponding to Fig. 10 (II).
(II) Redistribution of elementary hysteresis after stabilization.

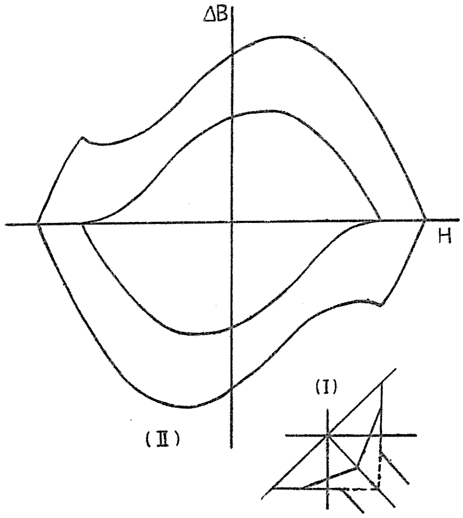


FIG. 12. (II) Hysteresis loop of ΔB calculated with the simplified modified Preisach diagram after stabilization (I).

Fig. 12 (II). This result is to be compared with that of Fig. 6 (II). If a proper form and magnitude of vacancy is fixed, a qualitatively good accordance will be seen. But this simple theory is unsuccessful for the phenomena in the comparatively small magnetic field (Fig. 6 (I)). If, however, the redistribution of elementary hysteresis after stabilization is studied more precisely, the phenomena will be explained.

Acknowledgements

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