# AN EXAMPLE OF THE HIGHER GRADE CONTROL SYSTEM

## CONTROL OF CONTROL SYSTEM

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One year ago we proposed a method for synthesizing new control systems. The function of control elements designed according to our method is divided into several actions, i.e.; primary controller, secondary controller, etc. The secondary one changes the character of the primary controller. Consequently, our control elements are of non-linear character. We can quickly obtain the final output with small error and slight overshoot.

The present paper describes examples of our new "Method of Control of Control System." First, we select the control-element-constants so that the entire system has oscillatory character. When the ratio of error to signal arrives at a constant value $\alpha$ ($\alpha<1$), some of the controller-constants automatically change making the system over-damped. Thus, our system arrives at a stable state much faster than with the usual linear system.

### 1. Introduction

As is well known, the customary automatic control system design is to control the input into the controlled object according to the difference between the output and the desired objective value. In such cases, the requirements for the system are: 1) to get the final value with as little error as possible: 2) to have the minimum overshoot: 3) to converge at the steady state value in minimum time. It is, however, very difficult to meet all these reqirements under the following conditions: A) when the objective value varies over a wide range, and B) when any large disturbance comes to the system. In 1954, we offered a proposed design* for a higher order automatic control system. In that proposal, we classified the function of the control action into several orders, for instance, functions of the first order, of the second order and of the third order, etc. The order is determined by the number of methods for output-estimation. When the system has a method of output-estimation, we call it the "first order control system." When the system has two methods of output-estimation, we term it the "second order control system," etc. For example, the conventional control systems having proportional, rate and reset actions, may well be called the first order control system, because they have only one way for output-estimation. They have three control actions, but the three actions are derived from only one estimation of output, i.e., the difference between the output and the objective values. In such a system, the rate of each action is determined by the desigher based upon the information obtained on the characters of controlled systems, on the objective values, on disturbances and on requirements.

* T. Koga: Prepaper of the 13th annual meeting of the Society of Applied Physics, Japan, 1954, 113-115.

The second order control system has two methods for output-estimation: one is the estimation described above: the other an estimation for the first order control system. One method is for detecting the difference between the output and the objective value: the other method uses the human consideration and operation for determining the rate of each action. The second order control system controls the controller constants of the first order control system according to an estimation for the output, in order to obtain the most desirable control effect.

While the foregoing is a simple example only, in general, the higher order control system changes not only the controller constants over a wide range, but also the mode of transfer function of the lower order controller, and the transfer function has the property of a functional. Consequently, the control system as a whole has non-linearity in even the most simple cases. It is not important to make the system non-linear, but it is important to select the best non-linearity. To emphasize the complexity of the contents of the system, we have called it the "Functional Controlled System." The essential problem cannot be solved by thinking that we can get a better control effect by applying non-linearity to the control system instead of using the usual linear system, as J. B. Lewis had done. This problem will be discussed in a later chapter.

## 2. Control of Controller Constants

To illustrate our method of second order estimation mentioned above, let us consider the absolute value of error. Our aim is to produce a system having the following characteristics: 1) to form the stand-up character immediately, and 2) to make the overshoot as low as possible. Assume that the system consists of the controlled object and the controller, both having the first order delay term. Then the system equation becomes a second order differential equation. To improve the stand-up character, we must, on one hand, reduce the damping factor of the characteristic equation, and in order to lower the overshoot, we must select the large damping factor of the characteristic equation. To satisfy these requirements, the second order controller should decrease the damping factor immediately after the objective value changes, and then to increase it so as not to overshoot. Because the damping factor is changed by the value of the error, the system equation becomes a non-linear differential equation. Solving the non-linear differential equation is laborious and consequently we use the two position control. Fig. 1 gives a block diagram of the original control system. In this system, the second order controller controls the first order controller constant, $C_0$, by means of a two position control. That is, when the error $\varepsilon(t)$ arrives at a certain value, the second order controller changes $C_0$ to a new constant $C_1$. When the error $\varepsilon(t)$ does not arrive at a certain value, the system equation can be written as follows:

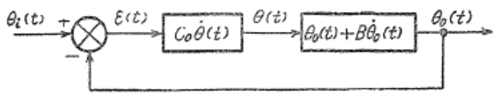


FIG. 1. Conventional feedback system.

$$\varepsilon(t) = C_0\dot{\theta}(t), \tag{1}$$

$$\theta(t) = \theta_0(t) + B\dot{\theta}_0(t), \tag{2}$$

$$\varepsilon(t) = \theta_i(t) - \theta_0(t). \tag{3}$$

Solving the above equation with $\varepsilon(t)$ and $\theta_i(t)$,

$$\ddot{\varepsilon}(t) + \frac{1}{B}\dot{\varepsilon}(t) + \frac{1}{C_0B}\varepsilon(t) = \ddot{\theta}_i(t) + \frac{1}{B}\dot{\theta}_i(t), \tag{4}$$

and by using

$$\zeta = \frac{1}{2}\sqrt{\frac{C_0}{B}}, \qquad \omega_n = \sqrt{\frac{1}{C_0B}}, \tag{5}$$

we replace

$$\ddot{\varepsilon}(t) + 2\zeta\omega_n\dot{\varepsilon}(t) + \omega_n^2\varepsilon(t) = \ddot{\theta}_i(t) + 2\zeta\omega_n\dot{\theta}_i(t). \tag{6}$$

Giving the initial conditions for the input changes as a unit step,

$$\left.\begin{aligned} t<0: \quad & \theta_i(0_-) = 0, \\ t=0: \quad & \theta_i(0_+) = 1, \\ & \dot{\theta}_i(0_+) = 0, \\ & \ddot{\theta}_i(0_+) = 0, \\ & \varepsilon(0_+) = 1, \\ & \dot{\varepsilon}(0_+) = 0, \end{aligned}\right\} \tag{7}$$

then, we have

$$\left.\begin{aligned} \varepsilon(t) &= \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,\omega_n t + \phi), \\ \phi &= \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}. \end{aligned}\right\} \tag{8}$$

where

The steady-state solution for Equation (6) for a constant input is

$$E_{ss} = 0. \tag{9}$$

We begin with this condition, and change $C_0$ to $C_1$ at time $t_1$ when the error arrives at $\varepsilon(t_1) = \alpha$ $(\alpha < 1)$. We may solve the new equation

$$\ddot{\varepsilon}(t-t_1) + 2\zeta_1\omega_{n_1}\dot{\varepsilon}(t-t_1) + \omega_{n_1}^2(t-t_1) = 0 \tag{10}$$

with the next initial conditions:

$$\left.\begin{aligned} &\varepsilon(t_1) = \alpha, \\ &\theta_i(t_1) = 1, \\ &\dot{\theta}_i(t_1) = 0, \\ &\ddot{\theta}_i(t_1) = 0, \\ &\dot{\varepsilon}(t_1) = \frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_1}\cos(\sqrt{1-\zeta^2}\,\omega_n t_1 + 90^\circ), \end{aligned}\right\} \tag{11}$$

where

$$\zeta_1 = \frac{1}{2}\sqrt{\frac{C_1}{B}} \text{ and } \omega_{n_1} = \sqrt{\frac{1}{C_1 B}}. \tag{12}$$

Fig. 2 shows the indicial response for various values of $\zeta$, $\omega_n$, $\alpha$, $\zeta_1$ and $\omega_{n_1}$, in which Curve 1 is the response for $\zeta = 1$ (critical damping): Curve 5 is the response for $\zeta = 0.2$: Curves 2, 3 and 4 the responses for our new control method. Curves 2, 3 and 4 are more suitable for our requirements than the linear response curves 1 or 5. Fig. 3 shows the conventional block diagram of our control system which is a complex control system. It is easier to check the stability of this system than to check that of the usual control system, because our system is overdamped at the stational state.

The following is the description of a model of this new control system. Fig. 4 is the actual network controlled by the second order controller. Let us illustrate that network. The amplifier 6AU6 and the 3-stage C-R network form a phase-shift oscillator. If the gain in the amplifier is high enough, then the circuit becomes an oscillator. But, if the amplifier gain is too low, then the circuit becomes an amplifier with selective feedback. We shall control the amplifier. To simplify the mechanism and to compare the effect of the higher order control with the effect of the conventional linear control, we made the system so that it would work with a higher order controller when the input changed to the positive side. When the input changed to the negative side, we used only the first order controller. The

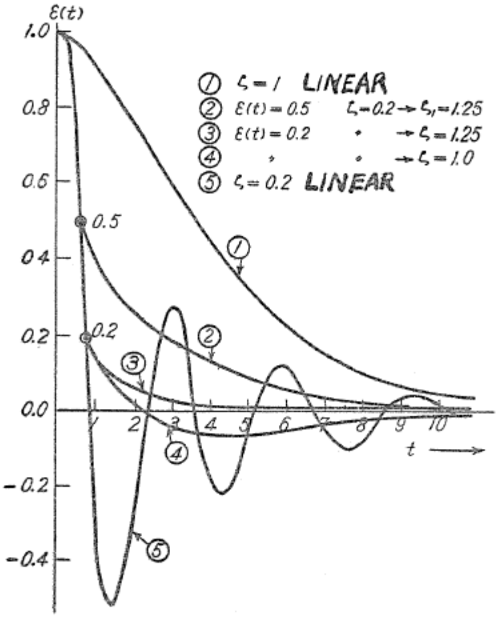


FIG. 2. Indicial responses of linear system and higher order control systems.

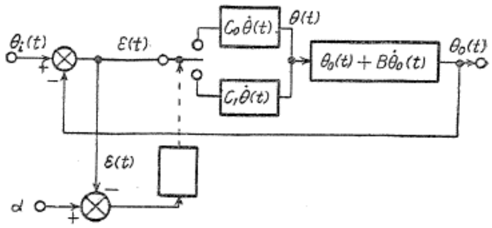


FIG. 3. Block diagram of higher order control system.

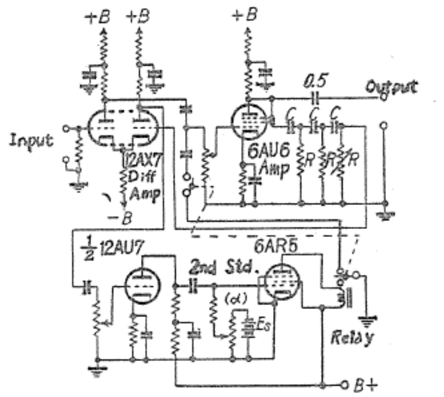


FIG. 4. An example of higher order control system.

rectangular wave was used for the input. This system can be changed as shown by the conventional block diagram in Fig. 5.

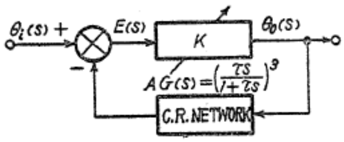


FIG. 5. Block diagram of the network shown in Fig. 4.

The system equation in $S$ function is

$$E(S) = \theta_i(S) - AG(S)\theta_0(S). \tag{13}$$

Assume that $K$ is the total gain of both the amplifier and the differential amplifier, and that $AG(S)$ is the transfer function of the $C$-$R$ network, then

$$\frac{\theta_0(S)}{E(S)} = K, \tag{14}$$

$$\therefore \quad \frac{\theta_0(S)}{\theta_i(S)} = \frac{K}{1+KAG(S)} = \frac{K}{1+KA\left(\frac{\tau S}{1+\tau S}\right)^3}, \tag{15}$$

where

$$\tau = CR.$$

The characteristic equation is

$$1+KAG(S) = 1+KA\left(\frac{\tau S}{1+\tau S}\right)^3 = 0. \tag{16}$$

Equation (16) becomes

$$(1+KA)\tau^3 S^3 + 3\tau^2 S^2 + 3\tau S + 1 = 0. \tag{17}$$

According to Hurwitz, the criteria of stability in this system is

$$\left.\begin{aligned} &H_1 = 3\tau^2 > 0 \\ &H_2 = \begin{vmatrix} 3\tau^2 & 1 \\ (1+KA)\tau^2 & 3\tau \end{vmatrix} > 0 \end{aligned}\right\}. \tag{18}$$

In the usual physical system, the first inequality is always satisfied. Calculated from the second inequality, $KA < 8$ is the condition for stability. Consequently, the system starts with the condition $KA > 8$, and as soon as the error $\varepsilon(t)$ arrives at 0.2, the system comes under the condition $KA \ll 8$. This control has taken place by grounding the capacitor at the grid of 6AU6, and the apparent gain in the amplifier has been decreased. Thus the system arrives at the final state without oscillation or overshoot. Fig. 6 illustrates the oscillogram of the output of this system: (a) is the wave-form of input signal, (b) is the under-damped output, (c) is the over-damped output made by grounding the capacitor at the grid of 6AU6, and (d) is the output of this higher order control system. The reason why the wave-form of the positive side slopes downward slightly to the right, is that this system lacks the proportional action because it has a coupling capacitor. The capacitor at the grid of 6AU6 serves not only to decrease the gain in the amplifier but also to by-pass the higher frequency component of noises such as the chattering noise of the relay, ambient noise, etc. It also serves to control the transfer function of the controller. The delay time of the relay may be neglected by setting the value of $\alpha$ slightly higher than the usual value of $\alpha$ without delay time.

This model satisfies our requirements but it is still unsuitable for practical application, showing only the possibilities of our proporsal. Here the input has been regarded as a unitstep. This system cannot work satisfactorily for any given value of the input. For a large input, the output wave-form is as shown in (b), Fig. 6. For the small input, the output is as shown in (c), Fig. 6.

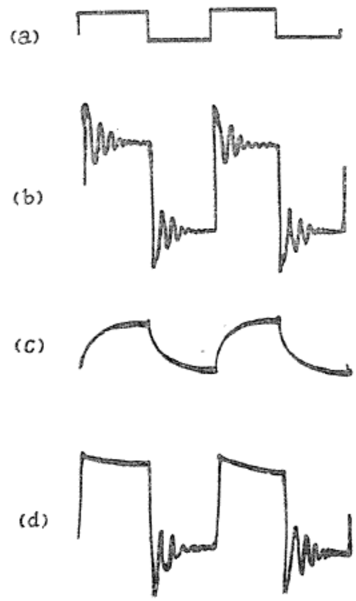

FIG. 6. Responses of the network shown in Fig. 4 for the rectangular-wave.

### 3. Standardization of Given Value Step-Iuput

We should make our control system so that it is able to work for any given input. To do this, we must solve Equation (6) not for the unitstep input but for any step input $\Theta_i$. Then Equation (8) becomes,

$$\varepsilon(t) = \Theta_i \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2}\,\omega_n t + \phi), \tag{19}$$

dividing both term by $\Theta_i$, we have

$$\frac{\varepsilon(t)}{\Theta_i} = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2}\,\omega_n t + \phi). \tag{20}$$

Let us take the ratio, $\alpha$, of the error $\varepsilon(t)$ to input $\Theta_i$ as the input of the second order controller. In this way the second order controller is always worked with unit-step. By so doing, the input of the second order controller is standardized. As shown in Fig. 7, a dividing network was added to the model previously described. There are two important reasons for using the dividing network. One is that a standardization of the input of the second order controller permits the controller to work independently of the input value. The other important reason is to give it all the functions of estimation of which the human brain is capable.

The human sensational magnitude, as shown by Weber-Fechner's law,* is nearly proportional to the logarithm of physical stimulus. Thus, to compare the sen-

* See any psychological text book.

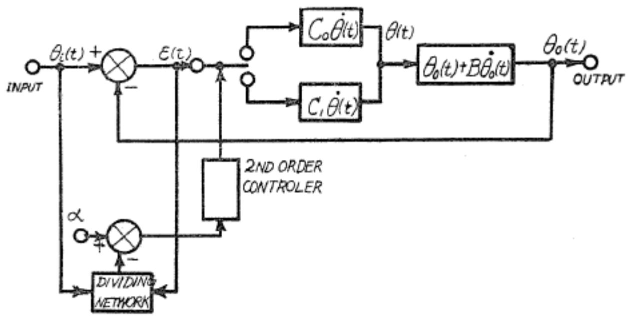


FIG. 7. A new proposed control system with the dividing network as the second order control device.

sational magnitudes in the human brain is analogous to dividing calculations among the physical magnitudes. A human being can handle physical magnitudes over a wide range. For this reason we used the dividing network for the second order controller. Estimation of output is the same as calculating the effect of control. The first order control system has a calculator—differential calculator—as the means for estimation of output.

In our study we characterized the second order controller as a dividing calculator. But the dividing network is only one means for estimating the control effect. There may be many ways to estimate the control effect.

It is important to note that the use of a calculator can contribute a very great deal to the study of higher order control systems.

### 4. About the Non-Linearity

Up to this time, non-linearity has been discussed only from a passive point of view except in the sampling control or the on-off control. We were obliged to solve the non-linear equation when the solution of the linear eqation did not agree with our experiment. There are few reports which apply non-linearity to the control system in the active sense. The proposal of Dr. J. B. Lewis* is a most typical and interesting example. We outline it briefly. Fig. 8 is the block diagram of Lewis's method. The non-linearity was produced from differentiator $S$ and the multiplier network. The system equation is

$$J\ddot{x} + A\dot{x} - B(v-x)\dot{x} + x = v$$

This result is the same as that reported by us† in 1954, showing that we can get a better indicial response by means of the non-linear control. It was considered an epoch-making idea, but our proposal is that making a system non-linear is not important. The important thing is to estimate the output and to control the lower order control system. As a result, our system has non-linearity, not as a required "convenience" but from "necessity."

* J. B. Lewis: A.I.E.E., Jan., 1953, 449–453.

† K. Tsuchiya, T. Yamamoto, T. Koga: Prepaper of the 13th annual meeting of the Society of Applied Physics, Japan, 1954, 115–117.

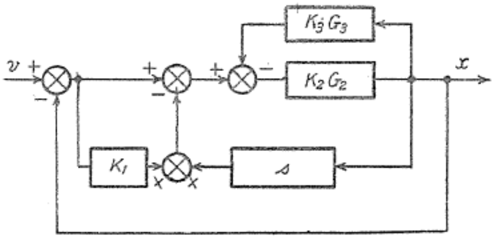


FIG. 8. Black diagram of Lewis's nonlinear control system.

Nevertheless, an estimation is necessary for the higher quality control system and the system should be so prepared that the control action is varied. By using this method, we can design a system which satisfies all requirements and in which we can completely meet the necessary control conditions by programming.

## 5. Conclusion

We proposed a design for a higher order control system, and we have shown experimentally that we can, by this method, design a system having a far better response than the usual control system. Moreover, we supplied the system with a calculator in the form of a dividing network, as a second order estimation of output. A report will be published on this in the future.

In general, the estimations or the criteria for the controlling effect are of many types. The important and interesting problem is how to actualize our proposal in time to come.

This is a tentative proposal for picking up the "ratio of error to input" as a measure of a criterion. We hope for your comments on this report.

## 6. Acknowledgement

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