A STUDY ON THE ZIEGLER AND NICHOLS' RULE FOR AUTOMATIC CONTROLLER SETTINGS

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§ Introduction

Although there have been developed several theoretically advanced methods for automatic process controller settings, Ziegler and Nichols' rule[1] is still of great value and most prevailing in the industrial world because of its practicability. But the effectiveness of this rule is anticipated to vary with the change in the characteristics of the controlled plant. In this paper an attempt is made to find the extent to which this rule is successfully applicable and to call attentions of those who put this rule into practice in their plants.

§ Controlled plant

Throughout this paper the controlled plant is assumed to be a single capacity element with dead time, and its transfer function is shown as follows:

$$G=\frac{Me^{-Ls}}{1+sT}=\frac{Re^{-Ls}}{s+R/M} \tag{1}$$

Where,

M; process sensitivity.
T; time constant of the element.
L; dead time.
s; parameter of Laplace transformation.
R; unit process reaction rate, $R=\frac{M}{T}$.

It must be noted that by varying the ratio T/L, we can let the function (1) cover a wide range of the plant characteristics.

§ Proportional action rule by ultimate sensitivity method

Ziegler and Nichols' "Ultimate sensitivity method" for proportional action controller setting seems to be a very reliable rule from the stability point of view, as in this method the controlled plant is once brought to the critical state of stability, and thence the sensitivity dial of the controller is turned in the direction toward which the stability is improved.

But here remain some questions as to the direction and amount of the dial turning.

The fundamental principles which govern their rule are as follows;

(1) The stability of the control process is always improved with the decreasing sensitivity K.

(2) The ratio of the optimum sensitivity K_4 which gives a control process with 25% damping, to the ultimate sensitivity K_0 remains unchanged irrespective of varied characteristics of the controlled plant. Here the ultimate sensitivity K_0 is the sensitivity with which an undamped oscillation of cycling occurs in the proportional control circuit.

There may be some exceptions to these statements, but the first of the two, which specifies the direction of dial turning, is well accepted in the majority of process control problems. The following portion of this paper will be devoted to the study of the second statement which concerns the amount of dial turning.

In the proportional action rule by the ultimate sensitivity method, the optimum sensitivity K_4 is given by the following formula;

$$K_4 = K_0/2 \tag{2}$$

The analytical expressions of K_0 and K_4 in terms of M, T, L and R must now be derived.

When the controlled plant (1) is controlled by a proportional action controller of which sensitivity is K, the closed loop transfer function is of the form;

$$J = \frac{KMe^{-Ls}}{1+sT+KMe^{-Ls}} \tag{3}$$

The condition for obtaining an undamped oscillation is that the denominator of (3) goes to zero when

$$s = i\omega_0 \tag{4}$$

then

$$1 + i\omega_0 T + K_0 M e^{-i\omega_0 L} = 0$$

Separation of this equation into real and imaginary parts gives the two following equations.

$$\left.\begin{aligned} MK_0 &= \sqrt{1+(T\omega_0)^2} \\ \tan\omega_0 L &= -T\omega_0 = -\frac{T}{L}(L\omega_0) \end{aligned}\right\} \tag{5}$$

where ω_0 is the angular frequency of the undamped oscillation.

In a similar manner the conditions for obtaining a control process with 25% damping are derived by putting the following value for s instead of Eq. (4)

$$\left.\begin{aligned} s &= (-\sigma_4 + i)\omega_4 \\ \sigma_4 &= \frac{\log_e 4}{2\pi} \end{aligned}\right\} \tag{6}$$

hence

$$
\left.
\begin{aligned}
MK_4 e^{\sigma_4 \omega_4 L} &= \frac{\omega_4 T}{\sin \omega_4 L} \\
[1 - \sigma_4 \omega_4 L(T/L)] \tan \omega_4 L + \omega_4 L(T/L) &= 0
\end{aligned}
\right\} \quad (7)
$$

where ω_4 is the angular frequency of the damped oscillation.

By assigning values to T/L between 0 and ∞ in Eqs. (5) and (7) and calculating the corresponding values of MK_0 and MK_4, we can finally obtain the ratios K_0/K_4. These values are plotted against T/L in Fig. 1 curve (a). It is seen from this curve that the formula (2) gives 25% damping only when $L \gg T$, and with increasing T/L it gives a control process more than 25% damping.

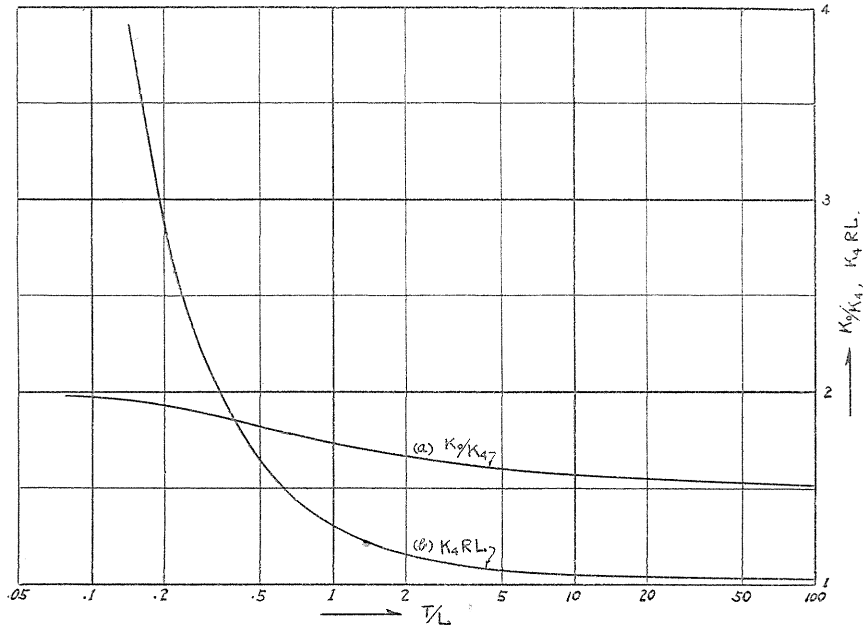

FIG. 1. K_0/K_4 and K_0RL vs. T/L

But the deviation from desired value is not only tolerable in its amount but always in the safty side in its direction.

From this fact Ziegler and Nichols' P-action rule by ultimate sensitivity method might be said to be a quite reliable one.

§ Relation between the ultimate sensitivity method and indicial response method

The ultimate sensitivity method and the indicial response method for automatic controller settings by Ziegler and Nichols may be looked upon as two different expressions of the same principle. In the latter, lag time L and unit reaction rate R, defined by Ziegler and Nichols, represent the plant characteristic.

Brief investigation of the two rules reveals the existence of the following relations between R, L and K_0, τ_0;

$$\left.\begin{array}{lll} K_0/2=1/RL & \text{or} & K_0RL=2 \\ \tau_0/4=L & \text{or} & \tau_0/L=4 \end{array}\right\} \qquad (8)$$

where τ_0 is the undamped cycling period, $\tau_0=2\pi/\omega_0$.

By definition $R=M/T$, so Eqs. (5) offer the method to evaluate K_0RL end τ_0/L corresponding to given values of T/L. These values are plotted in Fig. 2. It must be noted that the right parts of the curves asymptotically approach the following values which are the cases with an astatic controlled plant of the kind $G=Re^{-Ls}/s$.

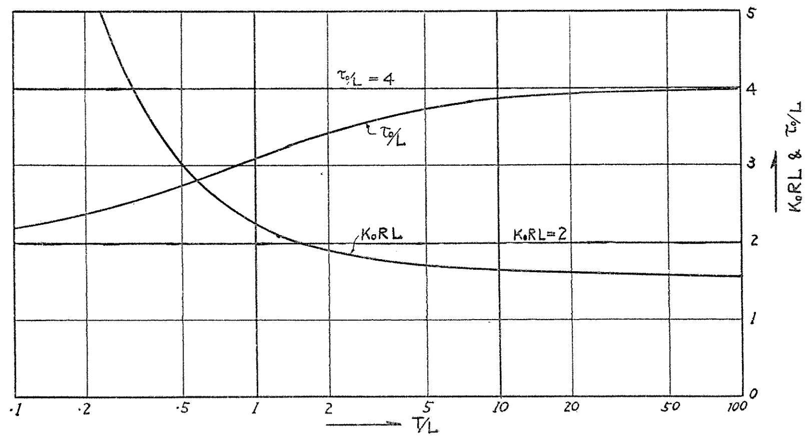

FIG. 2. K_0RL and τ_0/L vs. T/L

$$\left.\begin{array}{l} K_0RL=\pi/2=1.5708 \\ \tau_0/L=4 \end{array}\right\} \qquad (8')$$

Eqs. (8) claim to substitute two horizontal lines for these curves as shown in Fig. 2. Where $T\gg L$, approximation is fairly well, but with decreasing value of T/L, Eqs. (8) are not acceptable.

§ Indicial response method for proportional action

In the indicial response method the optimum value K_4 for the proportional action controller sensitivity is given by the following equation,

$$K_4=1/RL \qquad \text{or} \qquad \frac{K_4}{1/RL}=K_4RL=K_4\frac{M}{T}L=(K_4M)(L/T)=1 \qquad (9)$$

By the use of Eqs. (7) values of K_4ML/T are calculated and plotted against T/L in Fig. 1, curve (b).

As was foreseen from the trend of curves in Fig. 2, the rule represented by Eq. (9) is correct when $T>L$, but once L becomes greater than T, the rule gives too small sensitivity and poor control results.

§ Proportional and integral action rule

For the discussion of the PI-action rule by Ziegler and Nichols, it is convenient to chose R, L system for ease of analytical handling. Since we have discussed the relation between the two systems, R, L and τ_0, K_0, argument about the one enables us to estimate the other.

This time the transfer function of the controller is

$$H = K(1 + 1/T_I s) \tag{10}$$

where T_I is the integral action time.

According to the Ziegler and Nichols' rule for the PI-action controller

$$K = \frac{0.9}{RL} = \frac{0.9}{M}(T/L) \tag{11}$$

Combining Eq. (1), (6), (10), (11) and equalizing the denominator of the closed loop transfer function to zero, we get the following equations for the control process of 25% damping;

$$\left.\begin{aligned} \frac{T}{L} &= \frac{\sigma_4 \sin L\omega - \cos L\omega}{0.9e^{\sigma_4 L\omega} - L\omega\,[(1-\sigma_4^2)\sin \omega L + 2\,\sigma_4 \cos \omega L]} \\ \frac{T_{I4}}{L} &= \frac{0.9e^{\sigma_4 L\omega}(\sigma_4 \sin \omega L - \cos L\omega)}{L\omega(1+\sigma_4^2)(0.9\,e^{\sigma_4 L\omega}\sin L\omega - L\omega)} \end{aligned}\right\} \tag{12}$$

where T_{I4} is the integral action (reset) time at which 25% damping control process is gotten.

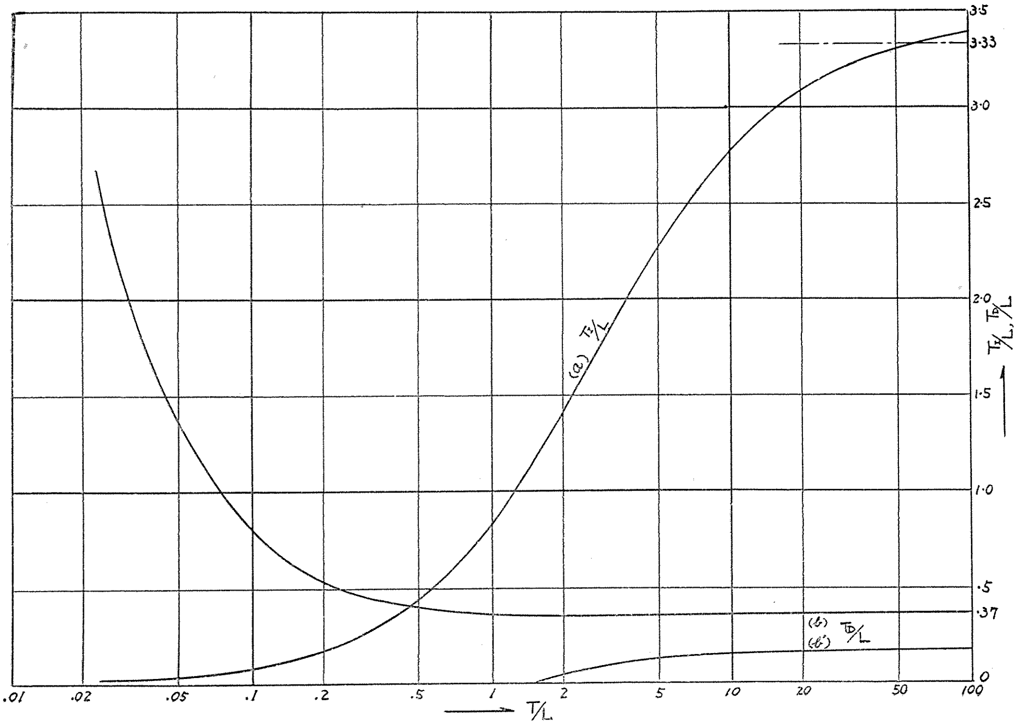

FIG. 3. T_I/L and T_D/L vs. T/L

These equations give us a curve T_{I4}/L v.s. T/L as shown in Fig. 3 curve (a). On the other hand what the rule by Ziegler and Nichols specifies is,

$$\frac{T_I}{L} = 3.3333 = \text{constant.}$$

This horizontal line is also shown in Fig. 3, and it reveals the fact that the integral action time by the Ziegler and Nichols' rule is greater than adequate for the majority of plants, namely for the plant of $(T/L)<60$.

Since the smaller integral action time tends to cause unstability, their rule is said to be in the safty side from the stability point of view. But where quick reset is required, modification of T_I would become necessary.

As to the ultimate sensitivity method for *PI*-action, it is clear from Fig. 2 that the yardstick by which the integral action time is measured is smaller in this case than that of the indicial response method. This fact, having a trend to specify a smaller value for T_I, gives a favourable testimony for the ultimate sensitivity method.

§ On the derivative action

In the Ziegler and Nichols' rule *D*-action is used only together with *P* and *I* actions. But it is quite laborious to carry out on the *PID*-action rule the same procedure as before.

Here focusing our attention only to the effect of *D*-action on the stability, we shall examine the *PD*-action instead of the *PID*-action.

Even though no formula for the *PD*-action is given by Ziegler and Nichols, we can assume the formula by extrapolation from the others as follows;

$$\left.\begin{aligned} K &= \lambda/RL \\ T_{D4} &= \mu L \end{aligned}\right\} \tag{13}$$

Where T_{D4} is the optimum derivative action time, and λ and μ are certain constants.

The ideal transfer function of controller is

$$H = K(1+T_D s) \tag{14}$$

Again combining Eqs. (1), (6), (14) and the first of Eq. (13), and equalizing the denominator of the closed loop transfer function to zero, we get

$$\left.\begin{aligned} \frac{T}{L} &= \frac{\sigma_4 \sin L\omega + \cos L\omega}{(1+\sigma_4^2)L\omega \sin L\omega - \lambda e^{\sigma_4 L\omega}} \\ \frac{T_{D4}}{L} &= \frac{\lambda e^{\sigma_4 L\omega} \sin L\omega - L\omega}{\lambda L\omega e^{\sigma_4 L\omega}(\sigma_4 \sin L\omega + \cos L\omega)} \end{aligned}\right\} \tag{15}$$

As a numerical example, letting λ be 1.2, which is identical to the corresponding factor in *PID*-action rule, we get curves (b) and (b′) for T_{D4}/L v.s. T/L in Fig. 3.

When $T/L>1.43$ two values for T_D are consistent with the conditions (15), but the upper curve (b) is, if possible, preferable since ω on this line is larger than that on the curve (b′).

The curve (b) remains fairly constant for the wide range of T/L, and this fact justifies the way in which Ziegler and Nichols give the optimum derivative action time: namely to use L as the yardstick in measuring T_D, and to set the ratio T_D/L to a constant.

Adopting only the right part of the curve (b), it would also not be so far wrong to establish a new rule for the *PD*-action as follows;

$$\left.\begin{aligned} K&=1.2/RL \\ T_D&=(0.36\sim0.37)L \end{aligned}\right\} \qquad (16)$$

Where $T/L<0.5$ the value of $T_{D4}/L=\mu$ increases rapidly. This means that in this range the rule of the type Eqs. (16) gives smaller value for T_D than adequate. This point can not be compensated by using the τ_0, K_0 system, as the τ_0/L curve in Fig. 2 declines with decreasing value of T/L.

§ Conclusion

When the Ziegler and Nichols' rule for automatic controller settings are used, the following points must be borne in mind.

1) The *P*-action rule by the ultimate sensitivity method gives fairly good control point for all values of T/L. But with the increasing T/L, damping of the control process becomes more than 25%.

2) The difference between the ultimate sensitivity method and the indicial response method is not serious if T/L is not less than all around unity.

3) Smaller value of T_I than what the Ziegler and Nichols' *PI*-action rule specifies is permissible where T/L is less than 60.

4) The way in which Ziegler and Nichols give the optimum value for T_D seems to be successful provided that T/L is greater than 0.5.

5) When T/L approaches zero, $1/RL$, the measure of the sensitivity in the indicial response method, does so too. This fact prevents the indicial response method from being used in the range of smaller T/L. On the other hand, K_0, the measure of sensitivity in the ultimate sensitivity method, does not go to zero even when T/L tends to zero. This makes the latter give better control points than the former when T/L is small.

The whole discussion in this paper is based on the assumption that the controlled plant is of the kind shown by Eq. (1). The deviations of dynamic characteristics of actual plants from that of Eq. (1) may be appreciable, and must be the subject of further study.

1) Ziegler, J. G. and Nichols, N. B.: Optimum Settings for Automatic Controllers, Trans. A.S.M.E., **64**, 1942, p 759.