RESEARCH REPORTS

ON THE THERMAL TRANSFER IN SUPERSONIC AND HYPERSONIC FIELDS

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I. Preliminaries

In the ultrasonic fields of comparatively low frequency, it is considered that the thermal process may occur under the *adiabatic* condition. While, at the region of extremely high frequency, it has been considered that liquids behave like an elastic body, and the thermal process may become *nearly adiabatic*. Thermal transfer, however, is rather complicated and can be hardly said so determinately. Accordingly, many authors have had their discussions about the thermal process really occuring in liquids under the supersonic and the hypersonic regions of mechanical oscillation. From the initial stage of the ultrasonic investigation, where the sound becomes inaudible and is faded away from human hearing sensation, its threshold being nearly at 2×10^4 Hz., the ambiguity of the thermal process in fluids has been alive intensely in several researchers' mind.

In their early paper, Herzfeld and Rice[1] considered that the *adiabatic* state is best guaranteed for low frequencies, while for higher frequencies the influence of *heat conduction* becomes larger, thus resulting in the decrease of velocity with increasing frequency. According to Rocard's investigation,[2] the absorption due to the thermal conduction is consisted of two terms, the first proportional to the square of frequency and the second proportional to the fourth power of frequency. The former gives the ordinary absorption originated from the heat conduction, and the latter can be neglected at low frequencies. While at higher frequencies, this term becomes larger and larger untill comparable with the first one at about 5×10^8 Hz. At the higher frequencies over 5×10^8 Hz., the sound amplitude absorption due to the thermal conduction reaches a limitting value independent of frequency, *e.g.* being 6×10^4 *c.g.s.* for air. This corresponds to such a damping that the amplitude decreases to the half of its initial value after the wave propagated 1,800 Å in the distance. The sound velocity also has a limiting value equal to the one in the isothermal state. At extremely high frequencies, it is considered by several authors[3][5] that the sound velocity again becomes to a value corresponding to the isothermal state. Thus they might consider that the nearly adiabatic process is maintained at a certain intermediate region of frequency lying between these two limitting isothermal regions.

Kneser's kinetic theory[4] of ultrasonics, almost neglecting the thermal conduction, leads to the operator corresponding to a *dynamic specific heat*. And one can hardly say anything if the thermal state really occuring in liquids is adiabatic or isothermal. Hiedemann[5] considered that in some liquids the isothermal process may be guaranteed for the region of hypersonic frequencies such as over 10^{10} Hz. Frenkel,[6] Oshida,[7] and the present author[8] took the ratio of thermal conduction (devided by heat capacity) multipled by frequency to the square of the sound velocity in the adiabatic condition, and compared the ratio with unity, and then they considered that for most kinds of liquid the adiabatic process is still realizable even in the hypersonic region under 10^{12} Hz.

Their criterion as well as that of Herzfeld-Rice is equivalent to merely take into consideration the comparison of thermal conduction with mechanical work done by the fluid in the infinitely extended medium. Moreover, they have treated only one wave corresponding to the density change. In reality, however, there exist *two waves*, one due to the density change and the other due to the temperature fluctuation.[8] Thus, these treatments cited above, may be said unsound, if their theory of sound should be re-examined in the bounded space as Sakadi[9] considered. Because of the existence of one more extra wave due to the thermal conduction and fluctuation, and of the nonexistence of the infinitely extended liquids, we should take two waves into account and consider the effect of the boundary of the medium. The boundary conditions bound the independent choice of two independent variables, which may be chosen freely in the infinitely extended medium. Thus we can descriminate strictly whether the process occuring in the medium is isothermal or adiabatic.

In this paper the thermal process occuring in liquids under the supersonic and the hypersonic regions of frequency shall be discussed in detail. The result obtained shows that the adiabatic condition maintains fairly well for most kinds of liquid at low frequencies. And in liquids far from the boundary, the process takes place in the course of almost adiabatic state for the high frequency regions under 10^{12} Hz. It is to be emphasized that most liquids undergo *adiabatic change* even for such extremely high frequencies. In the vicinity of the boundary, however, it is shown that the thermal conduction can not be neglected for such a region of high frequency. Accordingly the thermal conduction is of some importance inside the so-called *boundary layer*, and this means that the process occuring in this layer is more shifted towards the isothermal state rather than the adiabatic, being compared with the space region inside the medium.

II. Notations and Fundamental Equations

Notations

x_i : rectangular coordinates, $(i=1, 2, 3)$

ξ_i : components of displacement,

ϑ : temperature deviation from initial temperature,

p_0 : initial static pressure,

ρ_0 : density in static state,

k_0 : satatic bulk modulus,

$\lambda_r,\ \mu_r$: partial compressional and shearing rigidities,

$\lambda_r'=\lambda_r\tau_r$: partial volume viscosities,

$\mu_r'=\mu_r\tau_r^{(1)}$: partial shearing viscosities,

α_r/k_0 : partial cubic thermal expansion,

κ : thermal conductivity for external degree of freedom,
C_0: static specific heat at constant volume,
ε_0, C_r, and $\varepsilon_{r,ij}$: material constants,
τ_r, $\tau_r^{(1)}$, $\tau_r^{(2)}$, $\tau_r^{(3)}$, and $\tau_r^{(4)}$: relaxation times,
$D=\dfrac{d}{dt}=\dfrac{\partial}{\partial t}$: partial differential operator with respect to time t,
$\sigma_{ij}=\dfrac{1}{2}\left(\dfrac{\partial\xi_j}{\partial x_i}+\dfrac{\partial\xi_i}{\partial x_j}\right)$: components of strain tensor, $(i,\ j=1,\ 2,\ 3)$

$A_{ij}=p_0\cdot\delta_{ij}-\left(k_0+\sum_{r=1}^{l}\dfrac{\lambda_r' D}{1+\tau_r D}-\dfrac{2}{3}\sum_{r=1}^{m}\dfrac{\mu_r' D}{1+\tau_r^{(1)}D}\right)\sigma_{kk}\cdot\delta_{ij}-2\sum_{r=1}^{m}\dfrac{\mu_r' D}{1+\tau_r^{(1)}D}\sigma_{ij}$
$+\sum_{r=1}^{n}\dfrac{\alpha_r\tau_r^{(2)}D}{1+\tau_r^{(2)}D}\vartheta\cdot\delta_{ij}$: components of strass tensor,

$dU=\left(C_0+\sum_{r=1}^{s}\dfrac{C_r\tau_r^{(3)}D}{1+\tau_r^{(3)}D}\right)\cdot d\vartheta+\left(\varepsilon_0\cdot\delta_{ij}+\sum_{r=1}^{q}\dfrac{\varepsilon_{r,ij}\tau_r^{(4)}D}{1+\tau_r^{(4)}D}\right)\cdot d\sigma_{ij}$: increment of internal energy per unit mass.

Fundamental Equations

The fundamental equations for liquids at the rheological point of view, were obtained by the present author[8] in a generalized form, taking into account the thermal expansion and thermal conduction as well as the mechanical work done by the fluids.[9][10] The medium, through which the ultrasonics propagate, shall be either liquids or gases as well as solids, *mutatis mutandis*. To the present case of the supersonic and the hypersonic waves, these equations are conveniently applied.

The linear equations of motion and of conservation of energy are respectively as follows:

$$\rho_0\frac{\partial^2\xi_i}{\partial t^2}=-\frac{\partial A_{ij}}{\partial x_j},\tag{1}$$

$$\rho_0\frac{\partial U}{\partial t}=-A_{ij}\frac{\partial\sigma_{ij}}{\partial t}+\kappa\Delta\vartheta.\tag{2}$$

In one-dimensional case: $x=x_1,\ \xi=\xi_1,\ \xi_2=\xi_3=0$, and $\dfrac{\partial}{\partial x_2}=\dfrac{\partial}{\partial x_3}=0$, (1) and (2) are written in the forms:

$$\frac{\partial^2\xi}{\partial t^2}=-a^*\frac{\partial\vartheta}{\partial x}+b\frac{\partial^2\xi}{\partial x^2}+(l^*+m^*)\frac{\partial^2\xi}{\partial x^2},\tag{3}$$

$$(1+c^*)\frac{\partial\vartheta}{\partial t}=h\frac{\partial^2\vartheta}{\partial x^2}-(g+f^*)\frac{\partial^2\xi}{\partial t\partial x},\tag{4}$$

with

$$\left.\begin{aligned}
&b=\frac{k_0}{\rho_0},\qquad h=\frac{\kappa}{\rho_0 C_0},\qquad g=\frac{p_0}{\rho_0 C_0}+\frac{\varepsilon_0}{C_0},\\
&a^*=\frac{1}{\rho_0}\sum_{r=1}^{n}\frac{\alpha_r\tau_r^{(2)}D}{1+\tau_r^{(2)}D},\qquad l^*=\frac{1}{\rho_0}\sum_{r=1}^{l}\frac{\lambda_r' D}{1+\tau_r D},\\
&m^*=\frac{1}{\rho_0}\,\frac{4}{3}\sum_{r=1}^{m}\frac{\mu_r' D}{1+\tau_r^{(1)}D},\qquad c^*=\frac{1}{C_0}\sum_{r=1}^{s}\frac{C_r\tau_r^{(3)}D}{1+\tau_r^{(3)}D},\\
&f^*=\frac{1}{C_0}\sum_{r=1}^{q}\frac{\varepsilon_{r,11}\tau_r^{(4)}D}{1+\tau_r^{(4)}D}.
\end{aligned}\right\}\tag{5}$$

The quantities with an asterisk * appeared in (5) show that they are integro-differential operators having hereditary characteristics of Maxwellian relaxational process.

III. Plane Sound Wave

As a representative case, we shall consider a plane sound wave in the space region: $0 \leqq x < +\infty$, where $\xi_2 = \xi_3 = 0$, and $\frac{\partial}{\partial x_2} = \frac{\partial}{\partial x_3} = 0$.

For the sake of simplicity, we shall take in (5),

$$l = n = s = q = 1, \qquad \text{and} \qquad m = 2.$$

The choice of $m = 2$ means that we take ordinary shearing viscosity μ_1' and one more extra shearing viscosity μ_2' subjected to the relaxational process.

Let

$$\left.\begin{aligned} &\xi = \Re \xi \cdot e^{i\omega t + \beta x}, \\ &\vartheta = \Re \vartheta \cdot e^{i\omega t + \beta x}. \qquad \Im m(\beta) < 0, \qquad \Re(\beta) < 0. \end{aligned}\right\} \tag{6}$$

Inserting (6) into (3) and (4), we obtain:

$$\left.\begin{aligned} &-\{\omega^2 + (b + l^* + m^*)\beta^2\}\xi + a^*\beta\vartheta = 0, \\ &-i\omega\beta(g + f^*)\xi + \{-i\omega(1 + c^*) + h\beta^2\}\vartheta = 0, \end{aligned}\right\} \tag{7}$$

where D included in the operators marked with *, is replaced by $i\omega$.

Eliminating ξ and ϑ from (7), we have a quadratic equation to determine β^2. This equation can be written in the non-dimensional form:

$$(1+\varepsilon)\gamma^2 + \{\delta - i(1+c^*)(1+\varepsilon) - i\varphi\}\gamma - i\delta(1+c^*) = 0, \tag{8}$$

with

$$\beta = -\sqrt{\frac{\omega}{h}}\sqrt{\gamma},$$

and

$$\left.\delta = \frac{h\omega}{b}, \qquad \varepsilon = \frac{l^* + m^*}{b}, \qquad \varphi = \frac{a^*(g + f^*)}{b}.\right\} \tag{9}$$

From (8), γ is obtained as follows:

$$\gamma = \frac{1}{2(1+\varepsilon)}\{-\delta + i(1+c^*)(1+\varepsilon) + i\varphi \pm \sqrt{\prime\prime}\}, \tag{10}$$

with

$$(\sqrt{\prime\prime})^2 = \{\delta - i(1+c^*)(1+\varepsilon) - i\varphi\}^2 + 4i\delta(1+c^*)(1+\varepsilon). \tag{11}$$

The numerical values in *c.g.s.* unit for some kinds of liquid at 20°C and 1 atm. pressure, are given in Table 1, with the data for air under the same condition. The order of magnitude of ε_0, f^*, and relaxation times $\tau_1^{(j)}$ is merely estimated in Table 2.

The two roots of (10) shall be denoted as γ_1 and γ_2. Then we have two waves, one due to the *density change* and the other due to the *thermal conduction.*

TABLE 1. Numerical Values (20°C, 1 atm. press.)
(c.g.s. unit)

	ρ_0	$\dfrac{\text{Ordinary thermal expansion}}{\rho_0}=a$	b	$\frac{1}{\rho_0}\frac{4}{3}\mu_1'$	h	$g=\frac{p_0}{\rho_0 C_0}$ $(\varepsilon_0=0)$	$\frac{ag}{b}$
Water H_2O	1.00	$10^7\times 0.31$	$10^{11}\times 0.237$	$10^{-2}\times 1.35$	$10^{-3}\times 1.40$	$10^{-1}\times 0.24$	$10^{-5}\times 0.31$
Acetic acid CH_3COOH	1.05	$10^8\times 0.12$	$10^{11}\times 0.118$	$10^{-2}\times 1.56$	$10^{-3}\times 0.81$	$10^{-1}\times 0.47$	$10^{-4}\times 0.48$
Benzene C_6H_6	0.879	$10^8\times 0.18$	$10^{11}\times 0.143$	$10^{-2}\times 0.982$	$10^{-3}\times 1.11$	$10^{-1}\times 0.65$	$10^{-4}\times 0.83$
Mercury Hg	13.54	$10^7\times 0.34$	$10^{11}\times 0.185$	$10^{-3}\times 1.53$	$10^{-1}\times 0.45$	$10^{-1}\times 0.54$	$10^{-5}\times 0.98$
Air	1.21×10^{-3}	$10^7\times 0.31$	$10^9\times 0.839$	0.200	0.270	$10^3\times 0.12$	0.43

Accordingly, putting again

$$\left.\begin{aligned}\xi &= \Re(A_1\cdot e^{i\omega t+\beta_1 x}+A_2\cdot e^{i\omega t+\beta_2 x}),\\ \vartheta &= \Re(B_1\cdot e^{i\omega t+\beta_1 x}+B_2\cdot e^{i\omega t+\beta_2 x}),\end{aligned}\right\} \tag{12}$$

with $\beta_{\frac{1}{2}}=-\sqrt{\frac{\omega}{h}}\sqrt{\gamma_{\frac{1}{2}}}$ as in (9) and referring to equation (7), we obtain

$$\sigma_1\equiv\frac{B_1}{A_1}=\omega^2\cdot\frac{\delta+(1+\varepsilon)\gamma_1}{\beta_1 a^*\delta}, \tag{13}$$

$$\sigma_2\equiv\frac{B_2}{A_2}=\omega^2\cdot\frac{\delta+(1+\varepsilon)\gamma_2}{\beta_2 a^*\delta}. \tag{14}$$

Let the medium occupy the region: $0\leqq x<+\infty$, and the boundary conditions at $x=0$, shall be taken as follows:

$$\left.\begin{aligned}&(\xi)_{x=0}=C\cdot\cos\omega t,\\ &\left(\frac{\partial\vartheta}{\partial x}\right)_{x=0}=0,\end{aligned}\right\} \tag{15}$$

with C real positive.
These relations result in:

$$\left.\begin{aligned}A_1&=C\frac{\beta_2\sigma_2}{\beta_2\sigma_2-\beta_1\sigma_1},\\ A_2&=C\frac{-\beta_1\sigma_1}{\beta_2\sigma_2-\beta_1\sigma_1}.\end{aligned}\right\} \tag{16}$$

Let

$$\begin{aligned}\kappa\frac{\partial^2\vartheta}{\partial x^2}&\equiv Q\cdot\cos(\omega t+\Omega(x))\\ &=\Re\kappa(\beta_1^2\sigma_1 A_1\cdot e^{i\omega t+\beta_1 x}+\beta_2^2\sigma_2 A_2\cdot e^{i\omega t+\beta_2 x}),\end{aligned} \tag{17}$$

and

$$\begin{aligned}-p_0\frac{\partial^2\xi}{\partial t\partial x}&\equiv Q'\cdot\cos(\omega t+\Omega'(x))\\ &=\Re(-i\omega p_0)(\beta_1 A_1\cdot e^{i\omega t+\beta_1 x}+\beta_2 A_2\cdot e^{i\omega t+\beta_2 x}),\end{aligned} \tag{18}$$

with

$$Q \quad\text{and}\quad Q'>0.$$

We finally obtain

$$\frac{Q}{Q'} = \delta \cdot \zeta \cdot \frac{|\sqrt{\gamma_2}\, e^{\beta_2 x} - \sqrt{\gamma_1}\, e^{\beta_1 x}|}{|\sqrt{\gamma_2}\,(\delta + (1+\varepsilon)\gamma_1) e^{\beta_2 x} - \sqrt{\gamma_1}\,(\delta + (1+\varepsilon)\gamma_2) e^{\beta_1 x}|}. \tag{19}$$

The explicit expression for ζ is given in the following two paragraphs for the supersonic and the hypersonic regions of frequency, respectively. The ratio (19) can be a measure of the thermal state occuring in the medium. From the definitions (17) and (18), if the value (19) is far smaller than unity, the process is adiabatic, while, if not, the thermal change may be called isothermal rather than adiabatic. The numerical value of (19) shall be discussed in the following.

IV. Supersonic Waves

In the supersonic region: $10^4 < \omega < 10^8$ Hz., the quantities δ, ε, φ, and c^* are all far smaller than unity. Accordingly, by expanding (11) in the powers of δ, ε, φ and c^*, and neglecting the higher order terms of these small quantities, we have from (11),

$$\sqrt{\gamma} = i(1+c^*)(1+\varepsilon) + \delta + i\varphi - \frac{2\varphi\delta}{(1+c^*)(1+\varepsilon)} + O(\delta^3). \tag{20}$$

From (10) and (20), we obtain:

$$\left.\begin{aligned} \gamma_1 &= i(1+c^*) + \frac{i\varphi}{1+\varepsilon} + \ldots, \\ \gamma_2 &= \frac{-\delta}{1+\varepsilon} \cdot \left\{1 - \frac{\varphi}{(1+c^*)(1+\varepsilon)} + \ldots\right\}. \end{aligned}\right\} \tag{21}$$

Thus we have two waves of density change and of temperature origin:

$$\left.\begin{aligned} \sqrt{\gamma_1} &= \frac{1+i}{\sqrt{2}} \sqrt{1+c^*} \cdot \left\{1 + \frac{\varphi}{2(1+c^*)(1+\varepsilon)} + \ldots\right\}, \\ \sqrt{\gamma_2} &= i\sqrt{\frac{\delta}{1+\varepsilon}} \cdot \left\{1 - \frac{\varphi}{2(1+c^*)(1+\varepsilon)} + \ldots\right\}. \end{aligned}\right\} \tag{22}$$

Taking a linear combination of these two waves as in (12), and imposing the boundary conditions (15) at $x=0$, we see, after some calculations, that (19) is given by:

$$\begin{aligned} \frac{Q}{Q'} &\fallingdotseq \delta \cdot \frac{\left|1 + \dfrac{f^*}{g}\right|}{\left|1 - \dfrac{\varepsilon_0}{g_0 C_0}\right|} \cdot |1 + \psi - i\psi'| \times \\ &\times \frac{|\sqrt{\gamma_2}\, e^{\beta_2 x} - \sqrt{\gamma_1}\, e^{\beta_1 x}|}{|\sqrt{\gamma_2}\,(\delta + (1+\varepsilon)\gamma_1)\, e^{\beta_2 x} - \sqrt{\gamma_1}\,(\delta + (1+\varepsilon)\gamma_2) e^{\beta_1 x}|}, \end{aligned} \tag{23}$$

with

$$\psi = \frac{\varphi}{(1+c^*)(1+\varepsilon)},$$

and

$$\psi' = \frac{\delta}{(1+c^*)(1+\varepsilon)}.$$

For the ordinary shearing viscosity, μ_1' is of the order 10^{-2} c.g.s. (poises), and $\tau_1^{(1)}$ is 10^{-11} sec. Taking the ordinary thermal expansion coefficient, the formula (23) leads to the one obtained by Sakadi,[9] considering that l^*, ε_0, c^*, and f^* are all reduced to zero.

In the region: $x>0$, (23) takes the values as shown in Table 2 for the frequencies $\omega<10^8$ Hz. By using these figures,

TABLE 2. Rheological Constants (20°C, 1 atm. press.)
(c.g.s. unit)

	$\delta=\frac{h}{b}\omega$	ε	$\varphi \doteqdot \frac{ag}{b}\times 10^2$	c^*	f^*	(23)	(24)
Water	$0.58\times10^{-13}\,\omega$	10^{-2}	0.3×10^{-3}	10^{-1}	10^{-3}	δ	$\sqrt{\delta}$
Acetic acid	$0.69\times10^{-13}\,\omega$	10^{-2}	0.5×10^{-2}	10^{-1}	10^{-3}	δ	$\sqrt{\delta}$
Penzene	$0.88\times10^{-13}\,\omega$	10^{-2}	0.8×10^{-2}	10^{-1}	10^{-3}	δ	$\sqrt{\delta}$
Mercury	$0.24\times10^{-11}\,\omega$	10^{-5}	1.0×10^{-3}	—	—	δ	$\sqrt{\delta}$
Air	$0.32\times10^{-9}\,\omega$	10^{-3}	$ag/b=0.43$	—	—	δ	$\sqrt{\delta}$

we see that (23) is almost equal to

$$\frac{Q}{Q'}\doteqdot\delta(1+\varphi)/(1+\varphi+\varepsilon+c^*)\doteqdot\delta, \tag{23'}$$

considering that

$$\frac{|\sqrt{\gamma_2}\,e^{\beta_2 x}-\sqrt{\gamma_1}\,e^{\beta_1 x}|}{|\sqrt{\gamma_2}\,(\delta+(1+\varepsilon)\gamma_1)e^{\beta_2 x}-\sqrt{\gamma_1}\,(\delta+(1+\varepsilon)\gamma_2)e^{\beta_1 x}|}\doteqdot\frac{1}{|\delta+(1+\varepsilon)\gamma_1|},$$

for this region of frequency and for the space region: $x>0$.

In the vicinity of the boundary $x=0$, where

$$x\ll\sqrt{\frac{h}{\omega}},\quad \text{(inside the so-called boundary layer)}$$

(23) leads to

$$\frac{Q}{Q'}\doteqdot\sqrt{\delta}. \tag{24}$$

From Table 2, we can definitely conclude that the ratio Q/Q' is far smaller than unity for most kinds of liquid in this region of frequency under 10^8 Hz. In another word, the process occuring in liquids is *almost adiabatic* at the *supersonic region* of frequency.

V. Hypersonic Waves

For sufficiently high frequencies such as $10^{10}<\omega<10^{12}$ Hz., we can still treat the liquids as continua in a fairly good approximation.[7][8] From Table 2, we can see that in most liquids the condition $\delta<1$ is best guaranteed for the frequency region $\omega < 10^{12}$ Hz. Thus, the conclusion presented in the preceding paragraph is *still well available* for most kinds of *liquid* even in such a region of extremely high frequency, often called *hypersonic region.*

It happens sometimes, that in some fluids such as gases the value δ in Table 2 is formally larger than unity for $\omega > 10^{10}$ Hz. In reality, however, when the frequency of mechanical oscillation reaches greater than 5×10^9 Hz., the wave-length of air at 20°C, 1 atm. pressure, for example, becomes smaller than 7×10^2 Å. While, the mean free path of air at the same condition is known to be 6×10^2 Å. Accordingly, it is obvious that the existence of such a wave with extremely short wave-length is quite impossible for air. In this case of high frequency wave, the treatment of gases as continua, becomes utterly inadequate.

On the contrary, in most liquids, there exist the hypersonic waves, whose frequency lies in the region smaller than 5×10^{13} Hz. In crystals the most upper limit of the frequency of wave which can exist, reaches 10^{14} Hz. Thus the mathematical treatment as in (23) can be still valid for liquids and solids even in such high frequencies as 10^{12} Hz. The values of δ in Table 2, still remain to be smaller than unity for most liquids.

Mercury is not the case. The most upper limit of the frequency of wave being able to exist in mercury, is approximately 5×10^{12} Hz. In mercury, for $\omega < 10^{11}$ Hz., δ becomes also smaller than unity.

Accordingly, we shall treat the hypersonics in most liquids, whose frequency lies in the region: $10^9 < \omega < 10^{12}$ Hz. In this case, ε, φ, f^*, and c^* are all far smaller than unity.

Considering that

$$\begin{aligned}
&\tau_1^{(1)} \fallingdotseq 10^{-11}\,\text{sec.},\\
&\tau_1^{(2)} \fallingdotseq 10^{-7} \sim 10^{-8}\,\text{sec.},\\
&\tau_1 \fallingdotseq 10^{-8}\,\text{sec.},\\
&\tau^{(2)},\ \tau^{(3)},\quad \text{and} \quad \tau^{(4)} \fallingdotseq 10^{-8} \sim 10^{-9}\,\text{sec.},
\end{aligned}$$

for most kinds of liquid, we calculate (11), by expanding in the powers of δ, ε, φ and c^*. Neglecting the higher order terms of these small quantities, for $\sqrt{\eta}$ we obtain the same expression as (20). This results in the same expressions as (21), (22), (23), and (23′).

Accordingly, the discussion presented in the preceding paragraph for $\omega < 10^8$ Hz., can be also valid even in the region of extremely high frequency $10^9 < \omega < 10^{12}$ Hz., with slight modification of numerical values of δ. In other words, for the hypersonic region, we have several thousandfold or hundred thourandfold numerical values of δ at the supersonic frequency. It is to be emphasized, however, that values of δ at this hypersonic region are still smaller than unity. Table 2 can be also useful in this case. And the conclusion drawn in paragraph IV is also hold in the hypersonics:

$$\frac{Q}{Q'} \fallingdotseq \delta(1+\varphi)/(1+\varphi+\varepsilon+c^*).$$

Thus, we conclude that for most *liquids* the *adiabatic* process is still maintained fairly well even in the *hypersonic region,* whose frequency lies under 10^{12} Hz.

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