RESEARCH ON SHEET DRAWING

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CONTENTS

Introduction

There are three important conditions to be satisfied for completion of the sheet drawing operation, *i.e.* 1) prevention of wrinkling, 2) diminution of working resistance of, so to say, the "deformation part" of drawn material where main contracting work is performed, 3) preservation of strength of, so to say, the "traction part" where tensile stresses govern the drawn material and the material transmits the tractive force necessary for drawing. In this research, the matters partaining to the second condition are chiefly referred to on account of its extensive influence on the result of the working.

Usually, in the plastic working of metals, we endeavour to reduce the external friction as much as possible to diminish the working resistance.

In the case of sheet drawing, the blank generally has so large surface area compared with its volume that effects of the external friction on the drawing resistance are of consequence especially. So, they cannot be disregarded in considering minimization of the resistance.

Though in many researches[1][2][3] on sheet drawing ever carried out the drawing resistance is almost always referred to either theoretically or experimentally, or in both ways, the character of the frictional resistance involved in it scarcely seems to be dealt with. Consequently, there remains the question to what extent the results of the theoretical calculation of drawing resistance will be reliable or how much the drawing resistance may be decreased by proper adoption of lubricant, or more essentially, what method is the most efficient for diminution of it.

The principal purpose of this research is to ascertain aspects of the external friction in sheet drawing so as to understand more fully the essential character of drawing resistance and the possibility of diminution of it, and eventually to contribute to improve the methods in leveling up the drawing limit.

Chapter I. Preliminaries

1. Forces involved in the drawing

Naturally, in a sheet drawing operation, the drawing force and the holding force are necessary for the working. Make P denote the drawing force and H the holding force, the punch and the blank holder are subject to the reactions of P and H respectively and the die stands under $P+H$. Provided that as in this research a flat circular die is used, a circular blank being drawn is subject to P around its centre portion in the axial direction. This causes tension in the blank which is equal to the drawing resistance. Part of the blank drawn partially into the die forms a conical wall of a cup. Hence, suppose the inclination of the generating line of the cone to the die plane is α, $0 \leqq \alpha \leqq \pi/2$, the tension in the material which is caused by P and balances with it must be writen as $T=P/\sin\alpha$.

The drawing resistance is governed chiefly by the worked condition of the "deformation part" of the material, and it may be said that, in fact that condition determines the drawing force P conversely.

Recently, it is reported by S. Y. Chung and H. W. Swift[4] that the actual drawing resistance can be represented without any appreciable irrelevancy by a resultant stress which may be expressed by simple algebraic sum of the elemental stresses corresponding to pure radial drawing, friction due to the holder, die profile friction and bending and unbending resistances over die profile. The author, too, has had long a view that the drawing resistance, which is equal to the tension T in the drawn material, is approximately expressed by the sum of pure drawing resistance and additional resistances due to the frictions of holder and die surfaces and to the bending over die profile. This view almost wholly agrees with that of Chung and Swift.

Now, if those were admitted, it will be desirable to determine the amounts of each elemental resistance or stress for given drawing condition either experimentally or theoretically. Swift gave an account of these elemental stresses rather theoretically in the same report, but experimental confirmation will be said to be still unsatisfactory because of lack in each separate measurement of elemental resistances.

In order to analize the drawing resistance into elemental ones we must understand first the character of friction in the working. So, the author attempted in the first place to make sure the amount and properties of external friction in sheet drawing and succeeded in measuring an element of the frictional resistance during working. It followed that the experimental analysis of the drawing resistance became possible with a consequent probability to consider more reasonably the significance of the drawing resistance.

2. Experimental apparatus

The experimental apparatus in which drawing force, holding force and the frictional resistance of the holder surface can be simultaneously recorded during working was built by the author according to the following principles:

Provided that a flat circular die is adopted, drawing force on punch and holding force on blank holder act in the axial direction and are relatively easy to be measured, so, the method applied for their measurements will be not mentioned in detail. On the other hand, the frictional force on the holder surface of which components are distributed in a circle and act along the surface centripetally as shown in Fig. 1 is a self-balanced force having no effect on the outside.

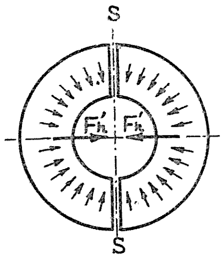


FIG. 1

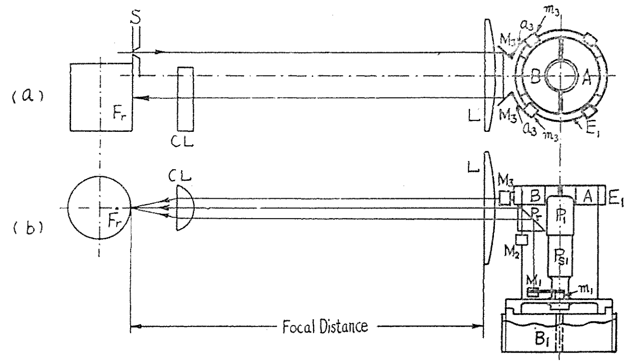


FIG. 2

Supposing the holder surface is divided into two sectors by a diameter, the resultant forces F_h' of the elemental frictional forces distributed in semi-circle on each sector act reciprocally and balance as shown in the figure. So, if the two sector blocks of a blank holder were connected with an elastic body, for instance, with the steel ring E_1 as shown in the right side of Fig. 2 (a), the ring will be deformed slightly by the resultant frictional forces F_h'. Provided that the deformation of the ring is elastic, amount of it must be approximately proportional to F_h'. Hence, the force F_h' and conformably the frictional resistance F_h on the holding surface will be known by measuring the deformation precisely. F_h is an additional frictional tension in the holding part of the blank, and approximately relates to F'_h with the expression $\pi F'_h = F_h$ if the holder surface is divided into half.

Now, in order to measure the deformation of the ring, a peculiar optical method was applied, the principle of which was based on the pressure measuring method employed by high speed optical indicator devised some 25 years ago by Dr. F. Nakanishi.[5)] The method is as follows: If the elastic ring E_1 combining the sector blocks, A and B, of the holder in Fig. 2 is subject to F'_h it slightly deforms itself to change the original circular form into somewhat elliptical shape.

In four symmetrical positions on the ring situated almost diagonally to the line of action of F'_h, the inclination of the tangential lines of the ring changes most extensively due to the deformation. In these positions of the ring four seat blocks were cut out integral with the ring as shown in Fig. 2 (a), where two of them are indicated by m_3. An arm was furnished on each m_3, on the free end of which the small mirror M_3 was glued so that the surfaces of the couple of mirrors M_3 have a relative inclination of about 90°. If F'_h acts and the ring E_1 deforms, the relative inclination between both of M_3 will decrease accordingly.

Light rays from the slit S which is placed at the focal distance apart from the lens L pass L and are reflected by the mirrors M_3 twice and pass L again returning and focus on rotating film F_r placed at the focal distance apart from L. Therefore, the image of the slit S is always focused on the film F_r and moves on the surface in the direction of axis of the film drum as E_1 deforms itself. To secure the displacement of image on the film as large as possible, the focal distance of the lens L was determined to 1 m. The cylindrical lens CL converges the image of the slit to a point on the film surface so that the change of F'_h during working can be autographed in a curve on the film.

Now, there are problems of how to transmit the holding force on the sector holding blocks excluding the friction undesirable to be measured and how to avoid the tendency of the holding surface of sector blocks to be inclined unduly during working. These problems were almost completely solved by the principle illustrated in Fig. 3. In the figure, a holder ring having the two holding surfaces, h_1-h_1 and h_2-h_2, which are perpendicular to a diameter of the ring and located symmetrically on the opposite side of the ring is once cut out from a solid steel bar. Then, the ring is divided into half by a plane of symmetry perpendicularly to its axis, and again recomposed by combining the separated parts with the elastic rings E_1 and E_2. The holder ring exerts holding force when it is inserted and compressed between two similar dies oppositely situated against the holding surfaces.

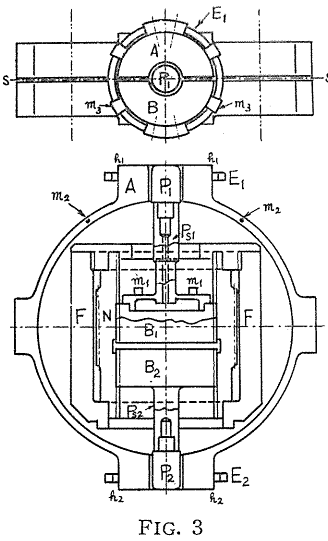


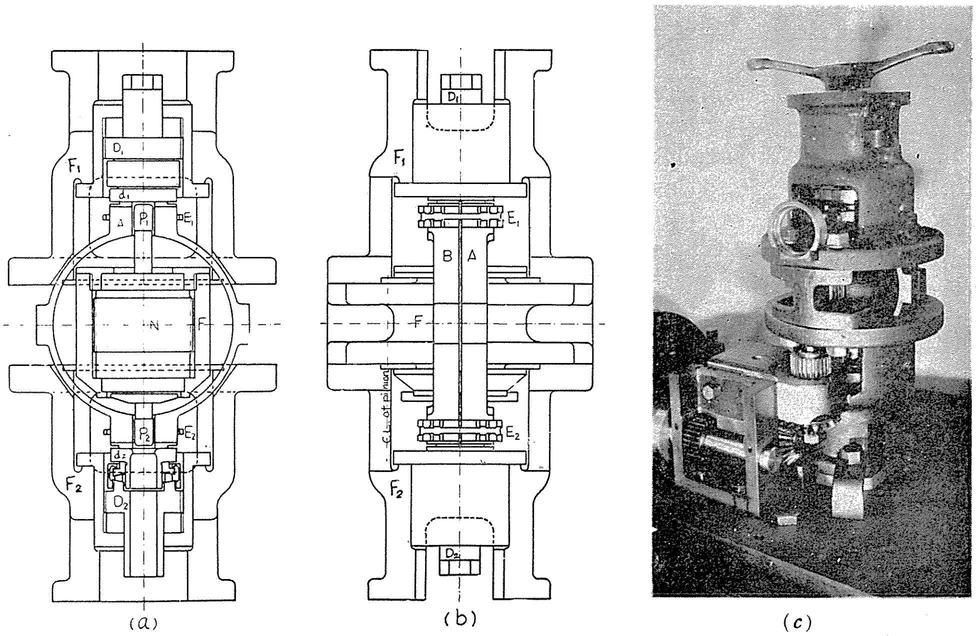
FIG. 3

In experiment, the punches, P_1 and P_2, proceed simultaneously from inside to outside of the ring towards corresponding dies, not shown in the figure, and in both dies workings are done on the similar blanks simultaneously and in the same working rate. Any force acting on the holder blocks during working has no undue effect on the measurement of F_h'.

In Fig. 2 (b), the method of measuring P and H is shown as well. The principle is essentially the same as that of F_h' except that a part of the light rays from S reflected by the right angle prism, P_r, so that the direction of light may be bent for the couple of the mirrors, M_1 and M_2. In the case of measuring P for instance, referring to Fig. 3, a circular diaphragm slightly deformable with P is fixed in its periphery to the moving piece, B_1, which has a male screw in its circumference and is driven by a female screw of the nut N. The center position of the diaphragm is integral with the punch stem P_{s1} which holds the punch head P_1. As the working goes on, the punch head P_1 moves in the axial direction, but this movement of punch has no effect on the record of the force owing to the autocollimation principle applied here.

The elastic member utilized for measuring H is, of course, the holder ring A or B itself.

Fig. 4 gives sectional drawings and outside view of the experimental drawing device in which, A and B: holder ring, d_1 and d_2: drawing die, P_1 and P_2: drawing punch, D_1 and D_2: die holder, F_1, F, F_2: frame, E_1 and E_2: elastic ring for measuring F_h', N: driving nut.



(a) (b) (c)

FIG. 4

3. Condition of experiment

Fig. 5 shows the standard size and shape of experimental tools; material of them is semi-hard steel not hardened, their surface roughness is about 7 μ.

Blanks to be drawn were at first of tin plate 0.28~0.29 mm thick and 29.0 mm in diameter, but the material was afterwards extended to copper, 60/40 brass and aluminium. In the standard blank size, the mean drawing ratio is 1.87. The blanks were all sheared with an alternative subpress die so carefully that the sheared edge exerted no harmful influence on the results.

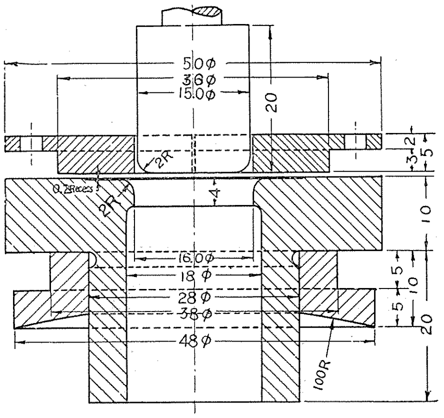


Fig. 5. Dimensions of tools.

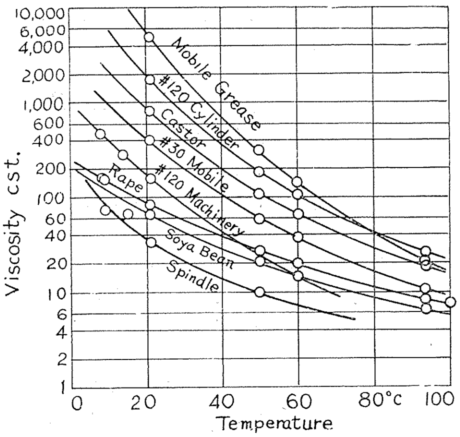


Fig. 6. Viscosity of lubricants.

The lubricant used was at first a kind of aluminium soap grease, the mobile grease as it is called,[6] of which viscosity is given in Fig. 6.

Lubricants other than that were also used as occasion demands.

Drawing speed, or travelling speed of punch, was ordinarily about 0.6 mm/sec.

Holding force was adjustable from about 50 to 350 kg and able to be kept almost constant throughout a process. It was adjusted by screwing the die holder, D_1, in the frame, F_1, so as to change the amount of compression of the holder ring.

In the above, as Chung and Swift[7] ever pointed out for some former experiments, there rises a question whether with the experimental size adopted here the reliable results widely adaptable for practical cases can be obtained or not. In this respect, the author has an opinion that if there exists such a size effect as cannot be disregarded it should be caused chiefly from the external friction, so we may be able to predict the case taking account of the effect when the properties of the friction are fully clarified.

Seeing from the consequence, it may be said that in spite of the small size of this experiment the results obtained do not include almost any discordance with those of large size, for instance, with the experimental results by Chung and Swift where blanks of 8″ (203 mm) and punches of 4″ (102 mm) in diameter are used.

Taking these into account, it seems that the results of this experiment are applicable with little anxiety that they may give only particular examples which are confined to the specific size.

Chapter II. Friction of the Holder Surface

1. Record of experiment

Fig. 7 gives some typical examples of the experimental records, in which standard size blanks are drawn under various holding force, initial values of them being 71, 169, 229 and 308 kg, the drawing speed is 0.28 mm/sec and viscosity of the lubricant is 2,300 cst. at the room temperature.

In any record in the figure, the holding force is kept almost constant during working until the blank is wholly drawn into the die. In the case of (*a*), the sheet was wrinkled to such a extent that the holding force increases towards the final stage of working, since the ridges formed in the material compress the holder ring additionally. The punch force in this case, is also relatively high in the final stage of drawing where the ridged portion of the cup passes through the die clearance.

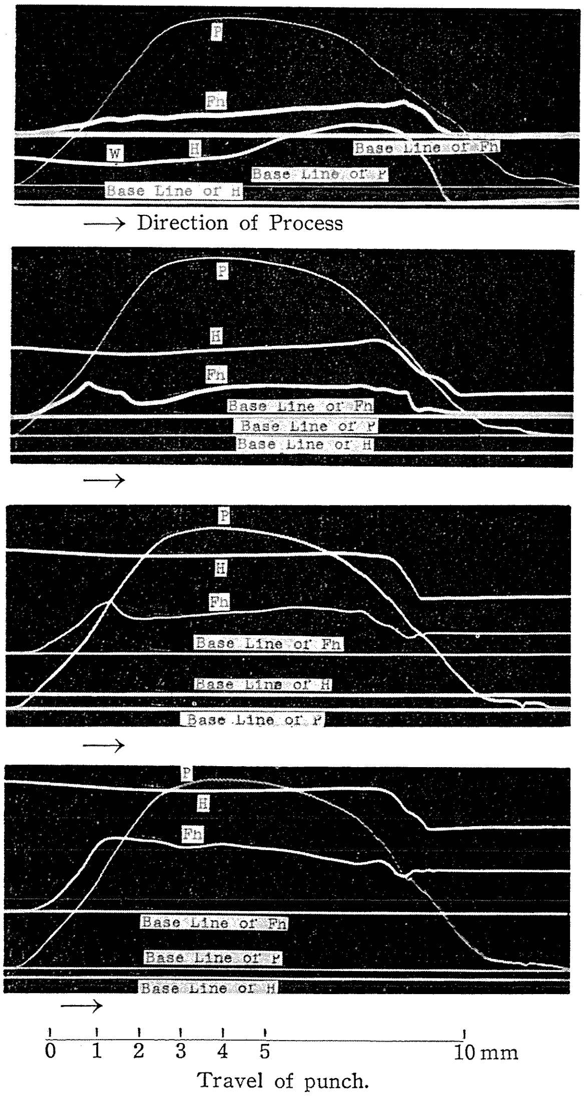
It will be seen in the figure that the mode of the frictional force acted on the holder surface changes according to the holding force. In the early stage, the

(*a*)
Initial holding force, $H_0 = 71$ kg
Maximum drawing force, $P_x = 510$ kg

(*b*)
$H_0 = 169$ kg
$P_x = 537$ kg

(*c*)
$H_0 = 229$ kg
$P_x = 548$ kg

(*d*)
$H_0 = 308$ kg
$P_x = 577$ kg



$D_0 = 29.0$ mm, $t_0 = 0.29$ mm, $m = 0.535$.

FIG. 7

force seems to increase gradually to a certain limiting value. But, considering the measuring method applied here, that portion of the frictional curves has little meaning on account of complexity of the relative movement of blank and the blank holder in the initial stage of the working. So, in the following sections the significance of that portion of the friction curve will be left out of consideration.

2. Significance of the frictional resistance curve

The frictional resistances experimentally recorded and shown in Fig. 7 should be regarded originally as F'_h, but they are easily convertible by simple computation into additional tensile forces, F_h, actuated in the blank due to the friction. So, regarding them as F_h hereafter, we will call them "frictional resistance of holder surface."

Now, in order to clarify the significance of F_h curve, the ratio of F_h and H was plotted against the punch travel. As the ratio, $\mu_h = F_h/H = \pi F'_h/H$, is regarded as a coefficient of friction, μ_h will be called here "frictional coefficient of holder surface." The average tendency of μ_h obtainable from the records similar to the above experiments is shown in Fig. 8 (a), where three kinds of holding forces in the region of wrinkleless working are taken for comparison. The coefficient of friction changes its value not only during working but also with holding force as is the case in the frictional resistance.

In the early stage, where F_h first reaches the maximum value, the coefficient is around 0.2 as indicated by "i" in the figure. As the working goes on beyond the stage of the maximum value, any μ_h decreases once with some difference in rate and amount of the decrease according to H. And, after passing the minimum value which is denoted by "m" in the figure, μ_h again increases to some extent and in the final stage of a process, it becomes to coinside with each other almost independently of H. At the ultimate points "u" in the figure which correspond to the stage, when the blank just wholly leaves from the holder surface, any μ_h curve converges to around 0.13.

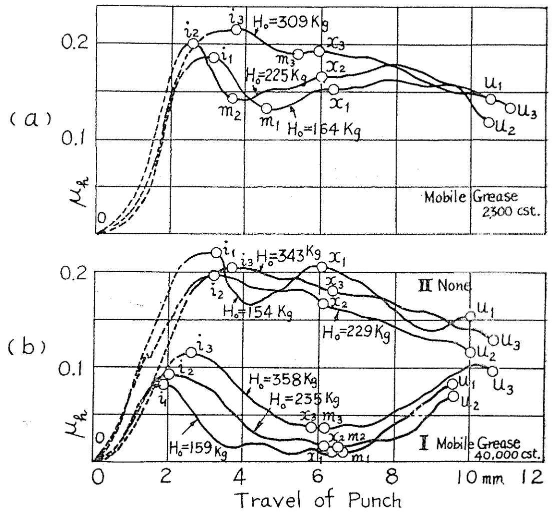


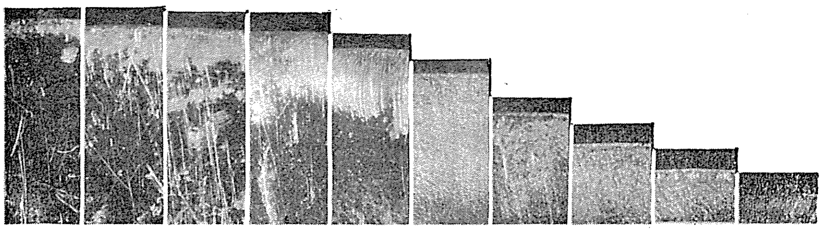
FIG. 8

The similar blanks were drawn with the same kind of lubricant as above in which the viscosity alone was exceedingly high (around 40,000 cst.). The drawing speed was risen to 0.6 mm/sec this time. The coefficients of friction in this case are shown by the curves I in Fig. 8 (b). As the working goes on, μ_h decreases considerably from "i" and in the middle stage of the process it becomes respectably small, and then again increases towards the end of the process. In this case also, the higher the holding force is the higher is the average value of μ_h throughout a process. In order to compare this with the case of non-lubrication, the similar blanks were drawn without any lubricant, keeping the holding forces nearly equal to the above. The result is shown by the curves II in Fig. 8 (*b*). Though the curves seem to be somewhat in disorder the average tendency of them is almost the same as that in the case of lubrication where the viscosity is 2,300 cst. and holding force is relatively high. From these, it is clear that μ_h depends on the viscosity of lubricant and the holding force. A friction of such a character is understood to be that under the boundary lubrication governed partially by fluid lubrication,[8] and the lubrication under such conditions is often called mixed lubrication.

In the early stage, attainable sliding speed of blank on the blank holder surface is so low, and in the final stage of a process, on account of extremely small contact area, contact pressure of blank and the blank holder becomes so high that in both stages a film lubrication is scarcely attainable.

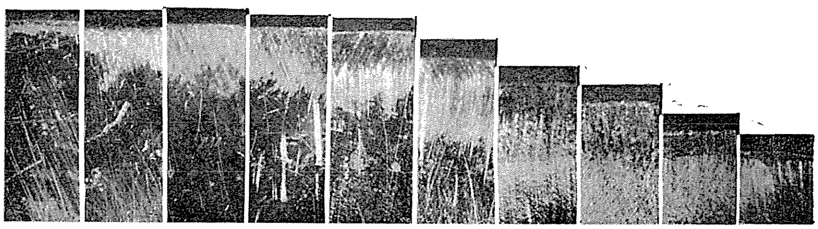
On the other hand, in the middle stage of a process, the sliding speed becomes sufficiently high and yet the effective area of the blank bearing holding force does not become so small. So, it will be said that an optimum condition for fluid lubrication is attainable in this stage. The mode of μ_h curves in the case applying the extremely viscous mobile grease proves clearly these facts. μ_h is considerably low around middle stage of the process and in both initial and final stages it bears relatively high values.

Lubricating Condition I



Punch travel (mm)	1	2	3	4	5	6	7	8	9	10
μ_h	—	0.09	0.10	0 06	0.04	0.03	0.03	0.05	0.07_5	0.10

Lubricating Condition II



Punch travel (mm)	1	2	3	4	5	6	7	8	9	10
μ_h	—	—	0.20	0.21	0.19	0.19	0.18	0.17	0.15	0.13

FIG. 9. The upper border between dark and lighter domain represents the rim of blank. Each longitudinal direction coincides with the radial direction of drawn material. ×8.

In order to ascertain that whether in the middle stage of a process a viscous lubricating condition governs evidently or not, or to what degree that condition is attainable, blanks were drawn and the punch was stopped at various stages of processes, then the blanks were took out carefully from the die and the surface condition of them was micro-photographed. Some examples of the photographs are shown in Fig. 9. The notations, I and II, correspond to the case applied the mobile grease in 40,000 cst. and the case of non-lubrication respectively. In the case I, the drawing depth of 6 mm corresponds to about the middle stage of the process. It will be seen that on the surface in this case no scratch is observable proving the existence of viscous lubrication evidently.

In the case of the less viscous lubricant, *i.e.* of 2,300 cst., mode of μ_h curve is not so simple but fluctuates during a process. The initial decrease of μ_h, (see Fig. 8 (a)), seems to correspond to a partial fluid friction not so influential.

Subsequent increase of μ_h seems to correspond to break down of the lubricating film due to increase in pressure and the second slight decrease to improvement in solid friction due to abrasion of tin coating.

The similar but somewhat modified mode of μ_h curves is obtainable for far less viscous lubricants. For example, a spindle oil of 90 cst.[9] gives the result as shown in Fig. 10.

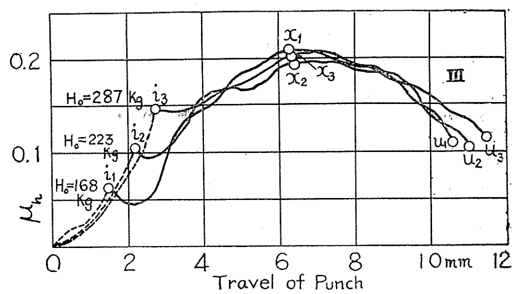


FIG. 10

3. *Maximum drawing resistance and the corresponding coefficient of friction*

The points indicated by "x" in Fig. 8 and 10 correspond to the drawing stage when the punch force reaches its maximum value in each process. The points "i" correspond to the initial stage of working under consideration here, and those "u" correspond to the final stage of effective working process.

Fig. 11, shows the relation between F_h and H for each stage of drawing and for the lubricating conditions, I and II. Significance of the notations is the same as in the preceding section. The inclination of the line indicates approximately the corresponding μ_h except in the stage "x" of the condition I.

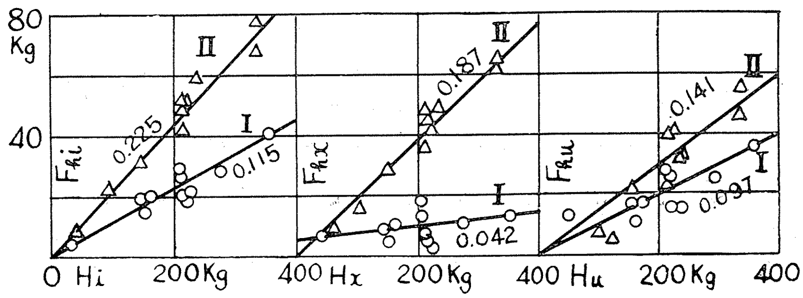


FIG. 11

Difference in μ_h according to lubricating conditions at each working stage is as follows: Ratio of μ_h in the condition II to in the condition I is at the stage "i" approximately 2/1, at "x" 4/1, at "u" 3/2.

We can also specify how much the maximum drawing resistance in an operation is affected by this difference in μ_h. In order to clarify that the values under consideration are in regard to the stage "x", the suffix x will be hereafter added to the notations. Fig. 12 shows the relations between P_x and H_x for these different lubricating conditions. The conditions, I and II, are the same as in the precedent, and the condition III corresponds to the case lubricated with spindle oil.

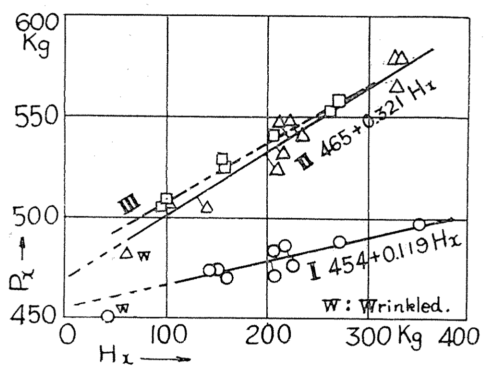


FIG. 12

Seeing these differences in P_x with lubricating condition, it is said that P_x reflects the effects of μ_{hx} considerably.

Hence, after all, as to the friction of the holder surface, the following statements can be made: The value of the frictional coefficient which has effects on the maximum drawing resistance in a process is determined in accordance with the mode of change of the coefficient and of the drawing resistance itself during working, and as regards constant tools and meterial, the mode of change of the coefficient is determined largely by the lubricating condition.

So, it is also to be said that, for the sake of a lubricant to be efficient for diminution of the maximum drawing force, P_x, the value of μ_{hx} available with the lubricant should be as low as possible, and according to the above experiments the viscosity of lubricants must at least be seriously high.

The question whether the viscosity alone is the most important factor for an effective lubricant or not, will be discussed later.

4. Effects of tangential contraction of material due to working

The material of blank under blank holder is subject to squeezing work in circumferential direction. This contraction of material may have effects on the friction of the holder surface. In order to ascertain this, the blanks were notched in their peripheries so as to draw them excluding the tangential compressive stress and thus the tangential contraction of material. An example of the notched blanks[10] is shown in Fig. 13, and some records on these blanks are shown in Fig. 14. The conditions, I and II, are the same as in the preceding sections. The μ_h curves in this event for various holding forces are given by Fig. 15. As shown in Fig. 15, the mode of change in μ_h in this case are quite similar to that of the blanks without notch (Fig. 8 (b)) except the effective travel of punch being shortened due to lack of the radial flow of the material, and the values on the whole being somewhat higher than the normal unnotched blanks. Considering the change of blank itself in accordance with notching, none of these differences are the matter not to be expected. So, it seems that the existence of tangential contraction of material has no significant effect on the friction of the holder surface.

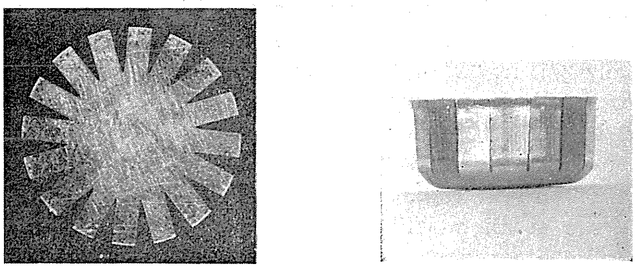
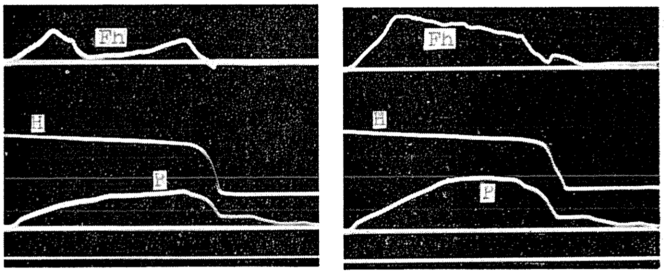


FIG. 13. Notched blank and the drawn shell.



⟶ Direction of process
Lubricating condition I.

⟶ Direction of process
Lubricating condition II.

FIG. 14

5. Effects of contact condition

As the working goes on, the rim of the blank slides radially inwards decreasing the area or the radial length of the blank lying under the holder surface. This radial sliding of the rim must cause a considerable change in contact condition. In this section, thats effects will be mentioned.

To ascertain that to what extent the change in contact condition has effects on the friction, especially in the case of non-lubrication, a beaded die as shown in Fig. 16 was prepared, and the

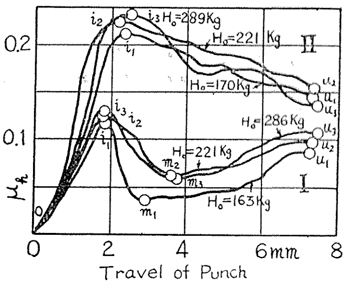


FIG. 15

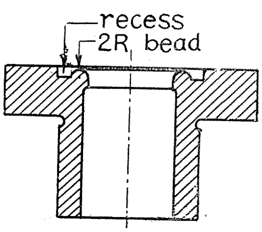
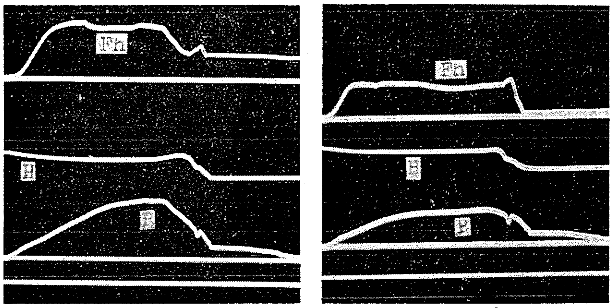


FIG. 16. Beaded die.



⟶ Direction of process
Condition I.

⟶ Direction of process
Condition II.

FIG. 17

frictional resistance was compared with that in the normal die. If a notched blank is drawn with the beaded die, the contact condition of blank and the blank holder will not appreciably change during working as the contact of them will coccur materially at the top of the bead, so that the frictional resistance also will be kept fairly invariable.

Examples of records for the experiments with beaded die are shown in Fig. 17, and μ_h in these cases is given by Fig. 18. As shown in the curves II, F_h or μ_h does not appreciablly change during the effective travel of punch except in the final stage, where they decrease abruptly from around 0.22 to 0.15.

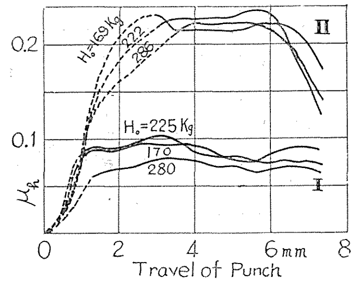


FIG. 18

These drops in μ_h coinside with that from "*i*" to "*u*" of the curves II in Fig. 15, and give an account of the effects of the rim of blank. Therefore, the gradual decrease of μ_h from the stage "*i*" to "*u*" in the case of non-lubrication shown in Fig. 8 (*b*) and 15 will be both said to be originated from the contact condition of blank and blank holder especially in the neighbourhood of the rim of the blank. It is already well known that a blank which is subject to a drawing work under blank holder usually contacts with the holder surface only in the neighbourhood of the rim. So, if there does not exist any lubricant between them, this contact condition will respectably affect the friction. The above result is not incompatible with this inference.

6. Effects of blank diameter

With tools of the standard size, blanks of various diameter were drawn under constant holding forces. Examples of the records are shown in Fig. 19, and μ_h curves for different diameters are compared in Fig. 20. The relations between μ_{hx} and the blank diameter D_0 are given by Fig. 21.

As shown in the figures, the mode of μ_h and P curves under a constant holding force varies according to D_0. μ_h is on the whole, smaller in the case of larger D_0 when the lubricant is applied. The point "*x*", where P reaches its maximum, shifts to a relatively late stage of the effective punch travel as the blank diameter decreases, so that the value of μ_{hx} increases in the case of I, and decreases in the case of II. With the drawing apparatus applied here the blanks of D_0 above 29 mm cannot be operated due to the constructive limits. But, it is infered that even with the larger blanks above 29 mm in diameter μ_{hx} shows almost no more decrease than in the case of 29 mm.

In practice, if the diameter of the blank varies, the minimum allowable holding force will be changed. So, the frictional resistance F_h which will be expected at least for each diameter is not easily estimated merely from the above results. The problems of size effect[11] accompanied with this can not be generally dealt with here.

7. Effects of die profile radius

With the dies having various profile radii from 1 mm to 3 mm, blanks of the standard size were drawn. Some examples of the records in this case are shown in Fig. 22, and μ_h curves in Fig. 23. As will be seen in the figures, the mode of

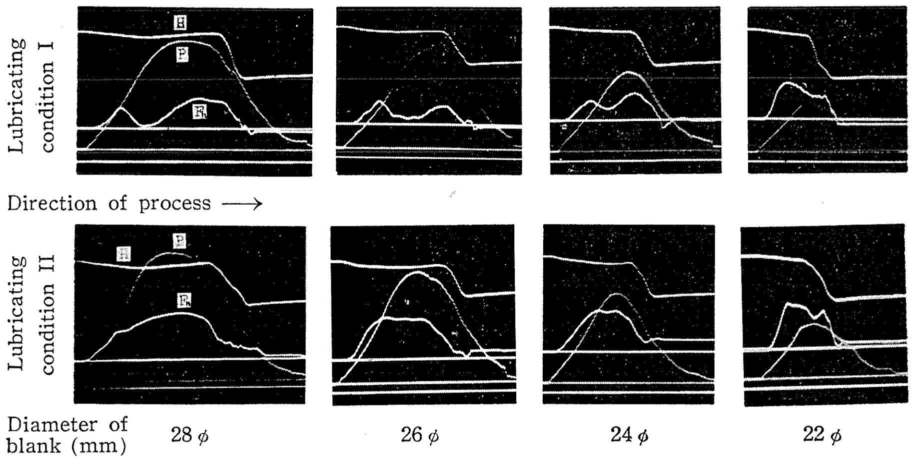


FIG. 19 Diameter of punch: 15 mm, *do.* of die hole: 16 mm, each profile radius: 2 mm.

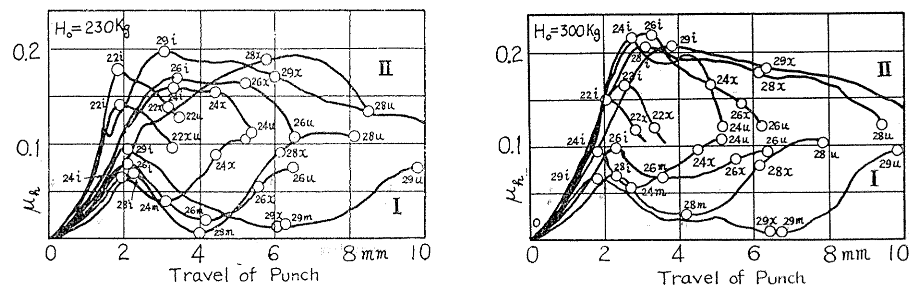


FIG. 20

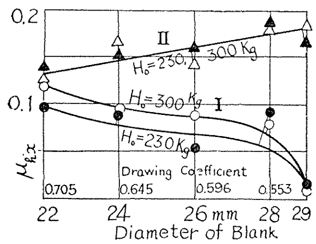
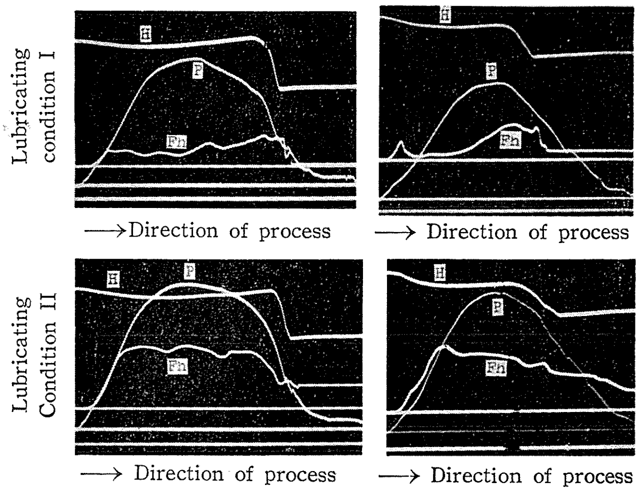


FIG. 21



$r_a = 1$ mm. $r_a = 3$ mm.

$D_0 = 29.0$ mm, $t_0 = 0.29$ mm, $m = 0.535$.

FIG. 22

changes in P and F_h or μ_h varies according to the radius, but, to what extent the mode of μ_h has effects on P_x cannot be generally clarified by these results. So, as in the preceding section, general consideration of the minimum expectable F_h for various profile radii will not be made here.

However, it is thought with confidence that the lubricating condition under the holder surface is considerably affected by both blank diameter and die profile radius as they govern the effective bearing area of holding force provided that the diameter of the die hole is constant.

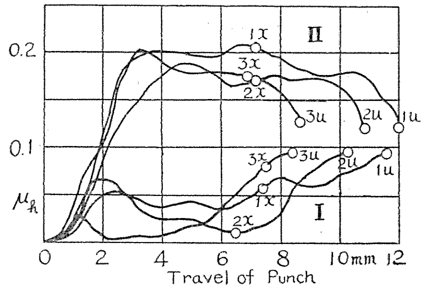


FIG. 23

Chapter III. Friction of the Die Surface

1. Experimental confirmation of frictional resistance of the die surface

Recently, H. W. Swift[12] has induced an expression for the additional stress due to die profile friction. Accordingly, the theoretical calculation of the resistance may be practicable, but, experimental confirmation for it is still not so satisfactory because the resultant drawing stress alone is ascertained experimentally.

In this research, as the forces, P, H and F_h are directly measured, the frictional resistance of the die surface, F_d, is relatively easy to be considered, although indirectly.

In the first place, effects of the difference in the die friction due to the change in the lubricating condition were ascertained. Fig. 24 shows the relation between P_x and F_{hx} taken from records of the experiments in which the lubricating condition and holding force are varied. The conditions, I and II, are the same as before, and I′ and II′ indicate the lubrication with the viscous mobile grease on the holder surface alone and on the die profile alone respectively. The difference of P_x for the same value of F_{hx} according to the lubricating conditions should be caused chiefly from the difference in the frictional resistance of the die surface, F_{dx}, if the other factors do not exceedingly vary according to the condition. Therefore, from the difference between these curves efficiency of the lubricant on the die surface may be compared.

It seems that the effect of lubrication is less on the die surface than on the holder surface, and it is less affected by holding force.

So long as we are confined to the above, the absolute value of the frictional resistance, F_d, cannot be known. F_d may be determined depending on the normal pressure on the die surface. However, as the normal pressure is not so easy to estimate experimentally, the vertical component of forces acting on the die, $P+H$, is considered here instead. When a notched blank is drawn and the forces, P, H and F_h are measured, the sum of the bending resistance and the frictional resistance of the die surface will approximately be found as $P/\sin\alpha - F_h = B_T + F_d$. Fig. 25 gives the $P/\sin\alpha - F_h$ and $P+H$ relation at several stages of working process. As shown, even though there are some disorders of points, the average tendencies are able to be regarded as straight lines. Since B_T must be a force approximately independent of any force experimentally measured, so, considering

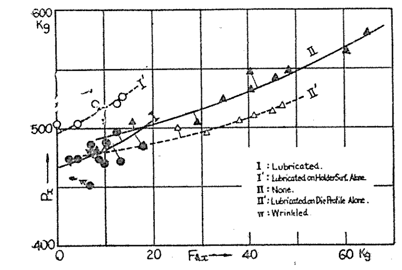


FIG. 24

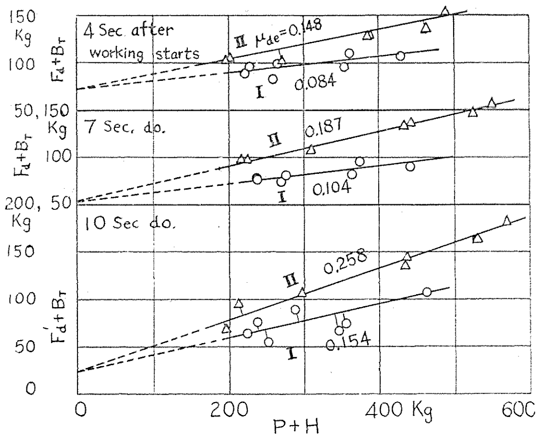


FIG. 25

that F_d is directly proportional to $P+H$, and if $P+H=0$, F_d will be also zero; B_T should be estimated by the ordinate at the intersection point of the lines with the ordinate axis. As will be seen, the lines for two lubricating conditions meet on the ordinate axis showing clearly that B_T is independent of the lubricating condition. The change of B_T during working thus estimated is given in Fig. 26 by a curve indicated by B_T. In the figure, curves indicated by μ_{de} are also shown. The value, μ_{de}, represents the inclination of the lines in Fig. 25, and is thought as a coefficient of friction or a value equivalent to coefficient of friction. Thus, by μ_{de} the condition of the die friction can be examined comparatively, but the value itself is somewhat ambiguous in physical significance and may be used merely for convinience. Therefore, a more detailed consideration of the die friction by μ_{de} will be avoided here.

The curves shown in Fig. 26 are obtained from the experiment on notched blanks. As mentioned previously, there is a possibility that the frictional resistance of the notched blanks is somewhat higher than that of normal blanks. Hence, μ_{de} also is supposed to be too high because of the serious difference in contact condition of blank and die surface. In order to ascertain this point the blanks were

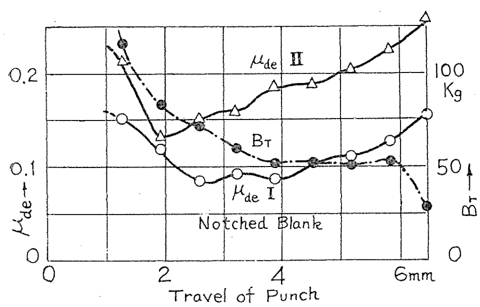


FIG. 26

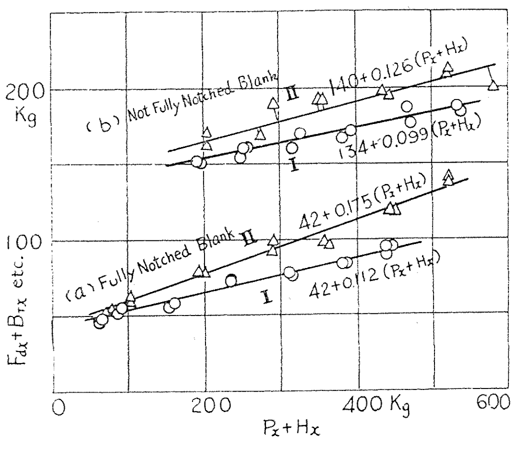


FIG. 27

prepared in which the notches are cut not so deep that, during working, the blank contacts with the die at least on die profile so intimately as in the case of the normal blanks. The experimental results on these shallowly notched blanks were compared with those of the fully notched ones. Fig. 27 gives the comparison, in which the lubricating conditions, I and II, are taken. The stage of working under consideration in this case is at 40 % of total effective travel of punch, which corresponds nearly to the stage "x". The lines (a) in the figure are for the fully notched blanks and (b) for the not sufficiently notched ones. As will clearly be seen, μ_{de} for (b) is less than that for (a). So, it is said that with notched blanks F_d is likely to be estimated too high as in the case of F_h.

2. Bending resistance

The bending resistance involved in the drawing resistance was experimentally examined formerly by M. Sommer[13] and C. L. Eksergian,[14] but this problem did not come into notice until recently when some analytical treatments were done by H. W. Swift[15] and R. Hill.[16] In this research, the component of drawing resistance which should be thought to be the bending resistance is experimentally found as mentioned in the preceding section. But, the experimental value of the bending resistance will be too small because of the effects of notching the blank. The notches decrease the amount of material to be bent and decrease the tendency of the material to be hardened by strain due to circumferential compressive working. The thickness of material must also change more in the case of normal blanks than in the notched ones. Therefore, the bending resistance estimated by notched blanks must anyway be too small.

In the case of this experiment, blanks are notched so that the petaloid part which may be left in each blank just forms a clearanceless cylindrical wall at the inner radius of die hole, R_1'. Supposing that the material is subject to bending work at the entrance of the die profile where the positional radius is R_2, the amount of the material lying at R_2 will be less by $\left(\frac{R_2}{R_1'}-1\right)\times 100\%$ than that of a full blank, at least. In this experiment, the shortage of bent material is about 25 %, and correction for this shortage in the bending resistance will be given easily.

The conditions on which Swift performed his calculation of the stress due to bending and unbending are very near to those in this experiment, and here, to compare both results briefly, the value of stresses obtained by him is temporarily applied in this case not modified. The bending resistances according to the stresses calculated by Swift are about 50 kg in the initial stage of working and 87 kg in the final stage. These values in general are not incompatible with those which were experimentally determined by the author.

3. Effects of die profile radius on drawing resistance

As to the effects of the die profile radius, r_d, researches have been made previously by G. Sachs and L. Herrmann,[17] S. Fukui,[18] S. Y. Chung and H. W. Swift.[19]

But, all of them treat the drawing resistance merely in gross except the theoretical treatment by Swift. In this research, the components of the drawing resistance, F_h, F_d, B_T, are experimentally determined, and so, the effects of r_d on F_d and B_T will be relatively easy to be ascertained.

The standard size blanks were drawn with dies of various r_d, the maximum drawing resistances during workings were taken from the records and plotted against r_d. The result is shown in Fig. 28. The notations I and II stand for those the same as before. According to the figure, it is clear that the effects of r_d on P_x is not so remarkable in the region of r_d larger than a certain limiting value, $r_d/t_0 = 6.7$ in this case (t_0 = blank thickness), and r_d is more influential in the case of I than in that of II.

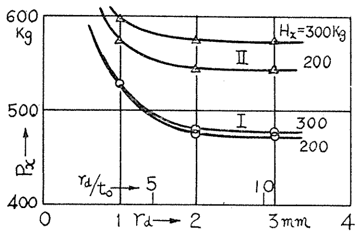


FIG. 28

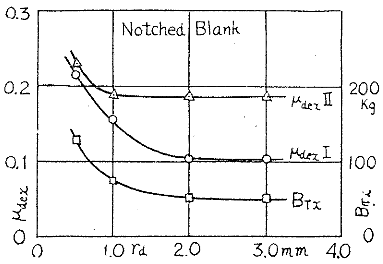


FIG. 29

Now, in order to ascertain the effects of r_d upon the bending resistance and the "equivalent coefficient of friction" of die surface at the stage "x" in processes, the notched blanks were drawn with dies of various profile radii. $F_{dx}+B_{Tx}$ was plotted against P_x+H_x as before and the inclinations of the lines and height of them on the ordinate axis for different r_d were examined. The results are gathered up and given in Fig. 29. As will be seen in the figure, μ_{dex} as well as B_{Tx} increases according to the diminution of r_d below a certain value. In the case of the condition I, μ_{dex} is more affected by r_d than in II where it resulted in more increase of P_x with diminution of r_d.

Anyway, it will be said that increase in the die profile radius brings about decrease in μ_{de} and B_T so that it contributes to the diminution of P, but, if the profile radius is larger than a certain limiting value, increase in the radius is not so effective to diminish P and application of an effective lubricant becomes far more important for it.

Chapter IV. Composition of Drawing Resistance

1. The pure drawing resistance

As already be seen, all elemental resistances supplemented to the resistance of pure radial drawing were extracted by experiment. So, we may be able to get the pure drawing resistance subtracting them from the actual drawing resistance. According to Fig. 27, by the curve (a) $B_{Tx}=53$ kg and by (b) $\mu_{dex}=0.0986$ for the case I and 0.1256 for the case II are able to be assumed. Using these, the pure drawing resistances S_x were calculated from 20 examples of experiment in both lubricating conditions. The result is plotted against H_x and shown in Fig. 30. As will be seen in the figure, the pure drawing resistance under both lubricating conditions and different holding forces are almost constant and around 347 kg.

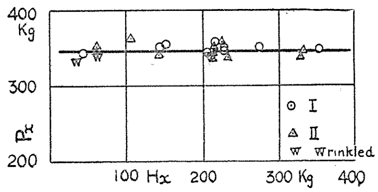


FIG. 30

Now, in many early researches, efforts were made to determine theoretically the drawing resistance or at least the pure drawing resistance, *i.e.* idealized force necessary for radial drawing alone, formerly by M. Sommer,[20] E. Siebel,[21] G. Sachs,[22] M. Asimow[23] and M. Masuda,[24] and recently by H. W. Swift[25] and R. Hill.[26]

The pure drawing resistance is rather simple to calculate since it should be determined principally by the mechanical properties of the material to be drawn. Here, as the force which is thought to be a pure drawing resistance is analized experimentally as above, it will be interesting to ascertain whether some simple methods of theoretical estimation of it will have validity or not. So, in the next place, some approximate estimations of the pure drawing resistance will be tried.

Taking cylindrical coodinates, r, θ and z, so as to make the axis z coincide with the axis of the punch, it was assumed that in the deformation part of the material being drawn, the following equation of stress holds for the first approximation,

$$\frac{d\sigma_r}{d\rho}+\frac{\sigma_r-\sigma_\theta}{\rho}=0 \tag{1}$$

where σ_r and σ_θ are the principal stresses of radial and circumferential direction respectively, aud $\rho=r/R_0$, $r=$ radial coodinate, $R_0=$ initial radius of blank. The stress in the direction of z is neglected.

The condition of plasticity is according to the shearing strain energy theory assumed as follows.

$$\sqrt{\sigma_r^2-\sigma_\theta\sigma_r+\sigma_\theta^2}=\sigma \tag{2}$$

The left hand side is often called the "stress intensity" and σ equals to a yield stress in pure tension.

$$\left.\begin{array}{ll}\text{The boundary conditions are at } \rho=R/R_0=v, & \sigma_r=0,\\ \text{and at } \rho=R_1/R_0=\rho_1, & \sigma_r=\sigma_{r_1}=S/2\pi R_1t_1,\end{array}\right\} \tag{3}$$

where R is the outside radius of drawn material and R_1 is the representative radius transmitting the tractive force necessary to draw material, and t_1 is the material thickness at R_1, and S is the pure drawing force.

The principal natural strains are $e_r=\ln\frac{dr}{dr_0}$, $e_z=\ln\frac{t}{t_0}$, $e_\theta=\ln\frac{r}{r_0}$, where r_0 is initial radial position of material which lies at r when deformed, and t_0 is the thickness of blank and t is the thickness at r when deformed.

The condition of constant volume will be writen as follows, provided that the blank deforms itself uniformly keeping axial symmetry.

$$\int_\rho^v \rho e^{e_z}d\rho=\frac{1}{2}(1-\rho^2e^{-2e_\theta}) \tag{4}$$

The stress strain relation is also approximately assumed as follows.

$$\frac{e_r-e_z}{\sigma_r-\sigma_z}=\frac{e_z-e_\theta}{\sigma_z-\sigma_\theta}=\frac{e_\theta-e_r}{\sigma_\theta-\sigma_r}=\frac{3}{2}\frac{e}{\sigma} \tag{5}$$

where $e=\frac{\sqrt{2}}{3}\sqrt{(e_r-e_z)^2+(e_z-e_\theta)^2+(e_\theta-e_r)^2}$ which is often called the "strain intensity."

Now, assuming for simplisity that a mean stress intensity $\bar{\sigma}$ governs throughout the deformation part of drawn material, the stress may be obtained by the mere stress equations, (1), (2) and the first condition of (3) as

$$\left.\begin{aligned}&\sigma_r=\frac{2}{\sqrt{3}}\sin\left(\varphi-\frac{\pi}{6}\right)\bar{\sigma}, \qquad \sigma_\theta=\frac{2}{\sqrt{3}}\sin\left(\varphi+\frac{\pi}{6}\right)\bar{\sigma},\\ &(\rho/v)^2=-\frac{\sqrt{3}}{2}\,\frac{e^{\sqrt{3}(\varphi-7\pi/6)}}{\cos\varphi}, \qquad \frac{5}{6}\pi\leqq\varphi\leqq\frac{7}{6}\pi\\ &\qquad\qquad\qquad \varphi=\text{parameter}\end{aligned}\right\} \qquad (6)^{27)}$$

In order to determine the value of the mean stress intensity, strains are approximately determined as follows.

First,

$$e_z/e_\theta=\frac{\sin\left(\varphi-\frac{\pi}{6}\right)+\sin\left(\varphi+\frac{\pi}{6}\right)}{\sin\left(\varphi-\frac{\pi}{6}\right)-2\sin\left(\varphi+\frac{\pi}{6}\right)} \qquad (7)$$

is obtained by putting (6) into the equation (5). Then, using (4), (7) and the third equation of (6) with the boundary condition, at $\rho=v$, $e_\theta=\ln v$, the principal strains at any point on the deformation part for any stage of drawing will be obtained by a graphical method. Fig. 31 represents the results put together. The thick lines in the figure indicate the strain intensity at various stages of drawing.

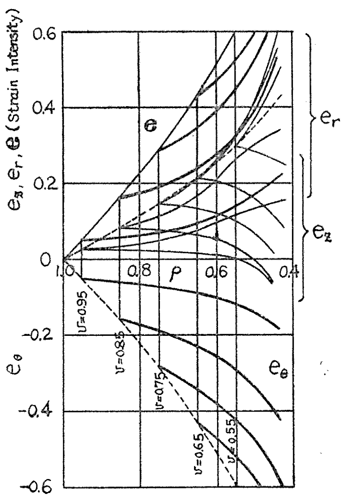


FIG. 31

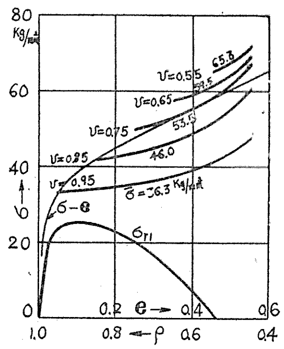


FIG. 32

The true stress and natural strain curve of the tin plate obtained by tension tests is given in Fig. 32, and distribution of the stress intensities corresponding to the strain intensities in Fig. 31 is also given in the figure. Supposing that the radial stress at R_1 determines the drawing resistance, the σ_r at $r=R_1$, σ_{r_1}, is calculated, and from $S=2\pi R_1 t_1\sigma_{r_1}=2\pi R_1 t_0\sigma_{r_1}\left(\frac{t_1}{t_0}\right)$ the pure drawing force, S, was eventually determined for each stage of working. The maximum value of S thus estimated is in this case 344 kg. This must be said to be a good agreement with the experimental result.

A more expedient but somewhat less strict method[28] for finding the maximum value of the pure drawing resistance in a process was applied by the author a few years ago. According to the method, however, the estimated value of the maximum resistance shows almost no difference from the previous result.

Hence, it will be said that the approximate method of calculation for pure drawing resistance have validity at least in the neighbourhood of the maximum value in a process, and conformably, it can be employed generally with fair confidence within the scope of that sense.

2. Composition of the drawing resistance and the drawing efficiency

Fig. 33 (*a*) and (*b*) give the conclusion of the results up to here showing composition of the drawing resistance, where the material is confined to 0.28~0.29 mm tin plate of the tensile strength $\sigma_B = 37.2$ kg/mm, and the diameter of blank is 29.0 mm, and the average diameter of the drawn cup is 15.5 mm.

In Fig. 33 (*a*), change of the elemental drawing resistances with the blank holding force, keeping the die profile radius 2 mm is given, and in Fig. 33 *b* that with the die profile radius keeping the holding force 300 kg is given. The curves in the figure show averages of many experimental results obtained hitherto. Among the elemental resistances, the pure drawing resistance S is to be regarded as an invariant throughout the experiment at least in this experimental region, and it should be considered as the required minimum drawing force also. If we consider all elemental resistances other than the pure drawing resistance to be the loss of the strength since they are inclinable to hinder the successful working, the ratio of the pure drawing resistance and the actual one will be able to be seen as a kind of efficiency of working.[29] Naturally, the efficiency referred to here is a mere ratio of forces and not correct in the strict sense of the word, but, the ratio of two forces partaining to the maximum drawing resistance in a process is far more important than the ratio of the ideal work and the actual work[30] of a process, especially in considering the limit of drawing.

Two curves of the efficiency for different lubricating conditions are shown in each figure. In this case, the attained maximum efficiency is around 73.5%.

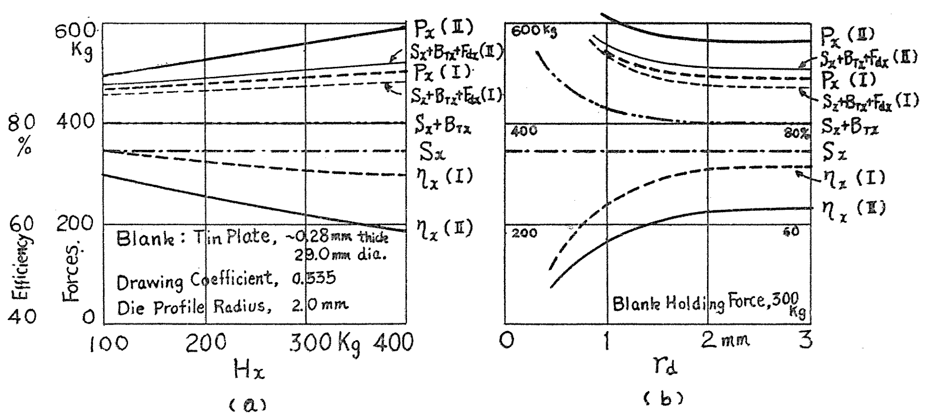


FIG. 33

Chapter V. Drawability and Mechanical Properties of Thin Sheets

In the preceding experiments, the material used was confined to tin plate. Now, materials other than that will naturally come into question. In this chapter, the drawability of copper, brass and aluminium sheets as well as of tin plate will be referred to by comparison taking account of thier frictional characteristics as well as the elemental resistances of drawing. The materials newly introduced are semi-hard copper sheets of purity 99.95%, 60/40 semi-hard brass sheets, and aluminium sheets of purity 99.6%, the thickness of each sheet being around 0.30 mm, and the diameter of the blanks, 29.0 mm. The aluminium sheets alone were not drawable as they were offered, and so they had to be annealed in the salt bath at the temperature, 300°C, and the time, 30′.

The experimental tools are, needless to say, of the standard size.

1. Mechanical properties of materials

The true stress and natural strain relations of the materials are shown in Fig. 34, and values of the tensile strength and corresponding true stress and strain are given in Table 1. In order to obtain the stress and strain relations, tension tests were made on small strips which were cut out from the standard market size sheets of each material and used as the test pieces. Taking account of the directional ununiformity of a rolled sheet, the strips were cut so that their longitudinal sides might coincide with the three directions, the direction of rolling, cross to the rolling and diagonal to the rolling. The thickness of them was about 0.30 mm, the breadth about 1.5 mm, and the effective length between the chucks, about 60 mm. The sectional area of the strip in the mid-portion of the effective length was somewhat smaller than in other sections so that the maximum tensile stress might govern in that section during the test. In order to get strains, the thickness and the breadth of the smallest section were measured with a micrometer of which measuring pressure was lightened as allowable and the spindle and the anvil were pointed to suitable radii so as to trace the deformation of that part. The loads were weights mounted on a pan connected to the lower chuck.

Along with the tensile tests, the Erichsen tests and the cupping tests analogous to the AEG process were made. The Erichsen tests were done with the standard tester, and the tools for the cupping tests were as follows: The die, 15.9 mmϕ, the punch, 14.9 mmϕ, each profile radius, 2 mm. The holding force in the latter tests was adjusted to the allowable minimum by oil pressure and kept almost constant during a process. The principal data obtained by above two tests are additionally given in the table.

Supposing that the limiting drawing ratios β_{max} obtained by the cupping tests represent the drawabilities of the materials, the representative values obtained by the tension tests and the Erichsen values were plotted against the limiting drawing ratio β_{max}. Fig. 35 gives the result. As will be seen, the values of tension test as well as the Erichsen value do not seem to be intimately correlated to the drawability of material, and particularly, the Erichsen value seems to indicate rather simply the elongation of material.

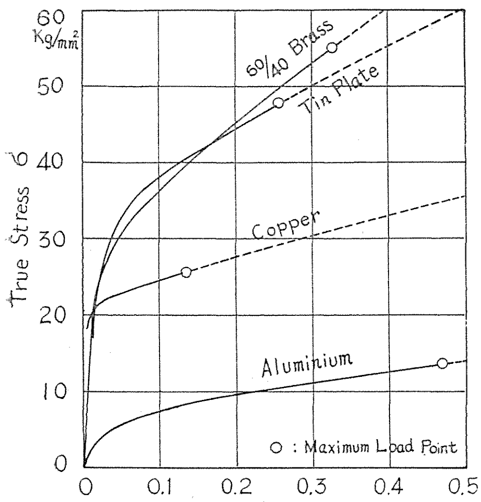


FIG. 34

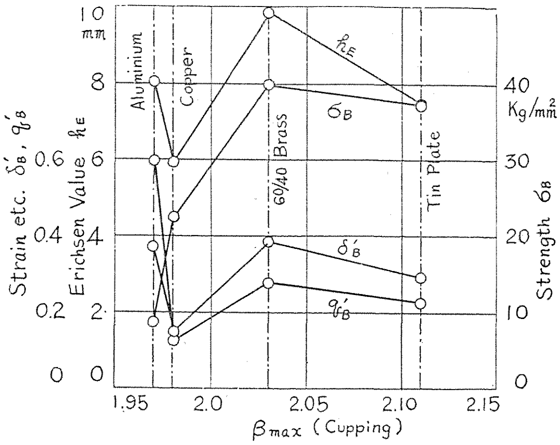


FIG. 35

TABLE 1. Values Pertaining to the Strength

	Tensile strength σ_B (kg/mm²) //	⊥	×	Mean value of the 4 directions σ_B (kg/mm²)	True stress correspond-ing to σ_B σ'_B (kg/mm²)	Contraction of area corresponding to σ_B q'_B //	⊥	×
60/40 Brass	41.5	39.5	39.3	39.9	55.0	0.265	0.243	0.289
Tin plate	37.4	38.0	36.6	37.2	48.0	0.211	0.250	0.219
Copper	22.2	23.3	21.9	22.3	25.5	0.105	0.123	0.109
Aluminium	9.0	8.7	8.5	8.6	13.7	0.316	0.419	0.347

	Mean value of the 4 directions q'_B	Elongation correspond-ing to σ_B δ'_B	Erichsen value h_E (mm)	Thickness of Erichsen test blank t_0 (mm)	Limiting drawing ratio $\beta_{max}=\frac{R_{0max}}{R_1}$	Limiting drawing Coefficient $m_{min}=\frac{1}{\beta_{max}}$
60/40 Brass	0.275	0.381	9.88	0.305	2.03	0.491
Tin plate	0.225	0.290	7.50	0.281	2.11	0.474
Copper	0.125	0.142	5.90	0.301	1.98	0.505
Aluminium	0.370	0.595	8.05	0.288	1.97	0.508

2. Limiting value of drawing based on stress-strain curve

In the earlier researches, it was often intended to determine the drawing limit by comparison of the drawing resistance and the limiting strength of material based on the stress-strain curve. In that event, since the true drawing resistance is essentially not so simple as to be estimated merely by the stress-strain curve, it is usual that the consideration is confined to the pure drawing resistance. The limiting strength of material during working also is not so simple as to be estimated by mere tensile strength. Therefore, many earlier trials[31] to presume the drawability do not seem successful. Here again, some trials of estimating drawing limit simply by the stress-strain curve will be made in order to give assurance of the possibility.

First, according to the approximate method for estimating the pure drawing resistance mentioned precedingly, the specific resistance of drawing s_{r_1} during a process was calculated for any drawing ratio as well as the radial stress σ_{r_1} at the representative radius of the traction part, R_1. Here, the specific drawing resistance is $s_{r_1}=\sigma_{r_1}e^{e_{z_1}}$ which is a quantity comparable with the tensile strength, provided that e_{z_1} is the natural thickness strain at R_1.

In this case, the most critical value for this purpose is the maximum value of s_{r_1} in each process. If the maximum, $s_{r_1\max}$, is plotted against the drawing coefficient, $m=\frac{1}{\beta}$, a curve expressing a theoretical relation between drawing resistance and drawing ratio will be obtained, and this curve is often called the "deep drawing curve." The deep drawing curves of the experimental materials are shown in Fig. 36 by dashed lines.

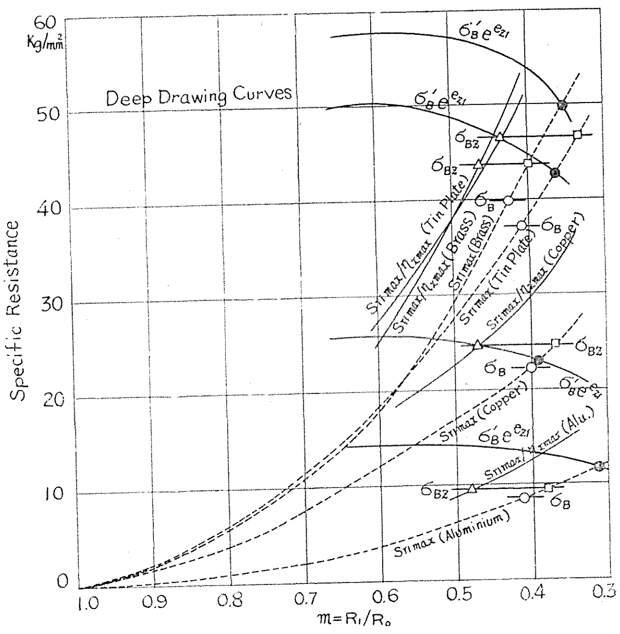


FIG. 36

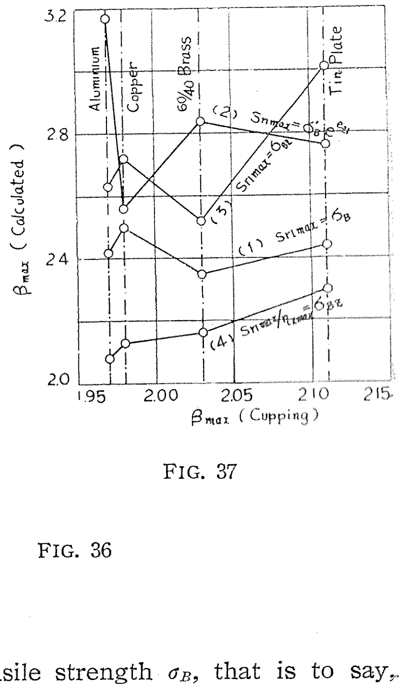


FIG. 37

Now, if the drawn material breaks at the tensile strength σ_B, that is to say, the material fractures when $s_{r_1\max}$ reaches σ_B, the abscissa of a deep drawing curve at $s_{r_1\max}=\sigma_B$ will give the limiting drawing coefficient. The coefficients and accordingly the drawing ratios thus obtained for the four kinds of materials were plotted against $\beta_{\max}$ by the cupping tests. The result is given in Fig. 37 by the line (1). As will be seen, it is clear that the calculated limits of drawing give almost nothing with regard to the drawing properties of the materials.

Now, for the second proposal, supposing that the material becomes fractured when $s_{r_1\max}$ reaches $\sigma'_B e^{e_{z_1}}$, where σ'_B is a true stress corresponding to the tensile strength of material and e_{z_1} is the natural thickness strain at R_1 when the specific drawing resistance there reaches $s_{r_1\max}$, the limit of drawing will obtained as shown in Fig. 37 by the line (2). This line also seems nonsense for a discussion of the drawability of material.

Hence, any simple theoretical method for estimating drawing limit may surrender.[32]

3. *Actual and theoretical drawing resistance*

Originally, values of S_{max} calculated from $s_{r_1 max}$ for different materials are merely theoretical ones and it is still not clear that to what extent they can be regarded as pure drawing resistance. In order to ascertain this, the experimental analyses of the drawing resistances into elemental ones were first attempted, and values of S_x were separated from the actual drawing resistances, T_x. The results are tabulated in Table 2. Now, there still remains a question whether S_x thus extracted from T_x can be regarded as a pure drawing resistance or not.

TABLE 2. Experimental Analysis of Drawing Resistance

Resistance → / Material ↓	Blank-holding force H_x (kg)	Maximum Drawing force P_x (kg)	Drawing depth at stage x h_x (mm)	Additional resistance: Bending B_{Tx} (kg)	Friction of the holder surface F_{hx} (kg)	Friction of the die surface F_{dx} (kg)
60/40 Brass	244	475	6.7	34.9	11.8	64.2
Tin plate	274	488	6.0	52.5	10.4	75.1
Copper	249	325	6.4	32.3	29.2	41.8
Aluminium	257	130	6.8	21.0	5.9	25.1

Resistance → / Material ↓	Experimental pure drawing force S_x (kg)	Theoretical maximum value of the pure drawing force S_{max} (kg)	Efficiency S_x/P_x η_x (%)	Allowable minimum holding force $H_{x min}$ (kg)	Drawing force applying $H_{x min}$ $P_{x min}$ (kg)	Maximum efficiency $\eta_{x max}$ (%)
60/40 Brass	364.1	365	76.7	75	454	80.2
Tin plate	349.5	342	71.6	150	473	73.5
Copper	221.7	216	68.2	75	293	75.6
Aluminium	78.0	77	60.0	50	110	71.0

Drawing condition, $t_0 \fallingdotseq 0.3$ mm, $D_0 = 29.0$ mm, $2R_1 = 15.5$ mm, $m = 0.535$, $r_d = r_p = 2$ mm (Die and punch profile radii) Lubricated with viscous mobile grease. $P_x \fallingdotseq T_x$ at stage "x."

TABLE 3. Drawing Strength of Material

Material	Cause of the fracture in experiment	Breaking punch force P_B (kg)	Breaking tension T_B (kg)	Thickness of blank t_0 (mm)	Referring sectional area $2\pi R_1 t_0$ (mm²)	Ultimate drawing strength σ_{Bz} (kg/mm²)	Ratio σ_{Bz}/σ_B
60/40 Brass	Too thin the blank thickness; Too high the blank-holding force	570	570	0.267	13.0	43.7	1.10
Tin plate	Too small profile radius of die; Too thin the blank thickness	605	610	0.270	13.1	46.6	1.25
Copper	Too high the holding force	419	422	0.310	15.1	24.8	1.11
Aluminium	*do.*	125	135	0.301	14.7	9.7	1.13

To ascertain this, photogrids were applied to the blanks, and strains during drawing were measured with a traveling microscope and a micrometer.

The distributions of the strains during the process are given in Fig. 38 for the brass. The corresponding distributions of the stress intensity during the process are given in Fig. 39. Thus the mean value of the stress intensity at the stage "x," $\bar{\sigma}_x$, was found for each material.

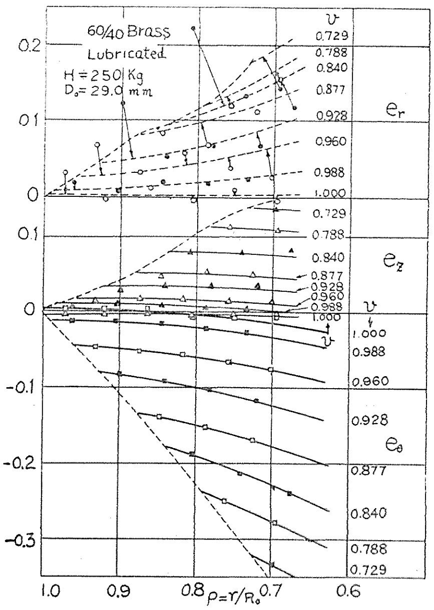


FIG. 38 (*a*)

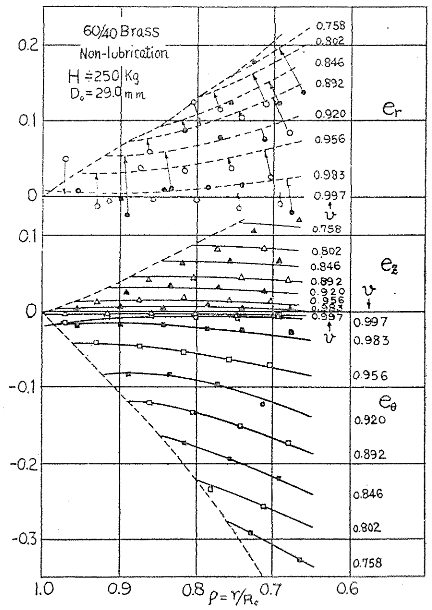


FIG. 38 (*b*)

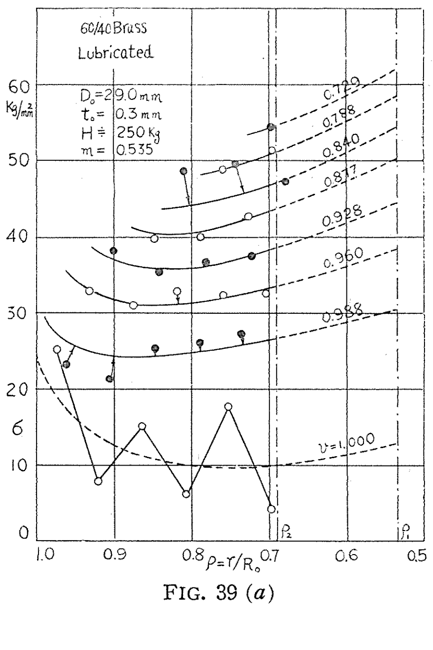


FIG. 39 (*a*)

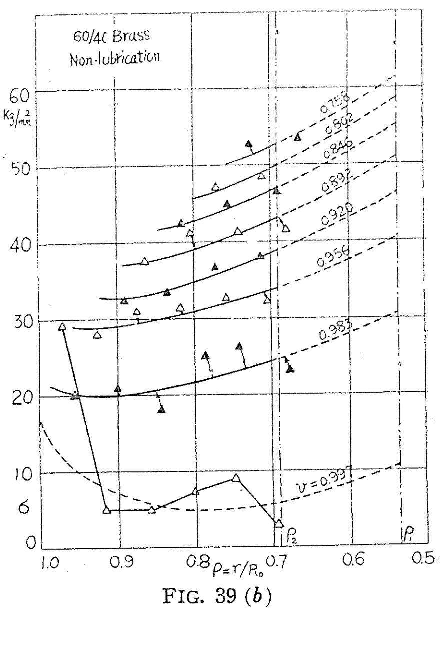


FIG. 39 (*b*)

The S_x which was experimentally extracted from T_x was plotted against $\bar{\sigma}_x$. The result is shown in Fig. 40 by a chain line. As shown, S_x is almost directly proportional to $\bar{\sigma}_x$ and clearly represents the pure mechanical properties of material. Therefore, the S_x extracted experimentally will be said to give surely an approximate value of the pure drawing resistance. The punch force P_x, on the other hand, is not proportional to $\bar{\sigma}_x$ but differs depending upon the lubricating condition as shown in the figure.

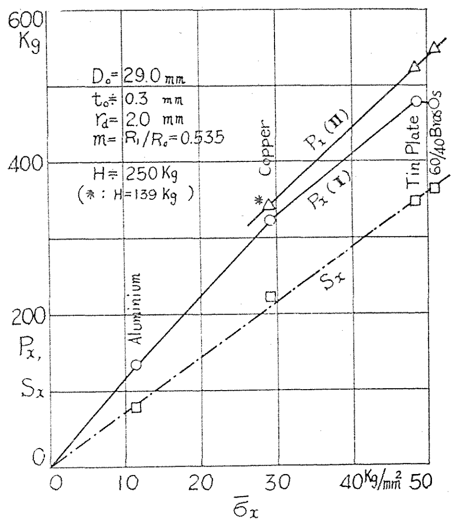


FIG. 40

Fig. 41 gives the comparison of the experimentally extracted S and the theoretically calculated S. As will be seen, the modes of change in both S's are considerably different, but, the maximum values of them happen to fairly agree with each other. Accordingly, the experimental S and the theoretical S are said to agree merely in the neighbourhood of their maximum points. Hence, S_{max} calculated from the stress-strain eurve will be said to be useful in such a sense that it fortunately gives the approximate maximum of the pure drawing resistance.

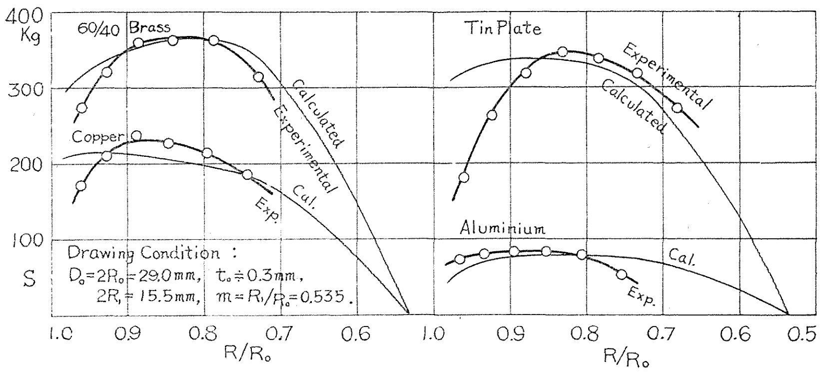


FIG. 41

4. *Actual drawing strength*

It has been pointed out previously by G. Sachs that the actual breaking strength of a drawn material is generally higher some 10 to 15% than the ultimate tensile strength by the tension test. The actual limiting drawing strength of materials (so called temporarily) in this experiment, σ_{Bz}, is tabulated in Table 3. It was calculated based on the data obtained from the actual drawing tests in which the materials fractured during working. As shown, the values of σ_{Bz}/σ_B are all larger than unity and differ according to the material.

TABLE 4. Strength Rotio and Number of Repetition of Bending

	Tensile strength σ_B(kg/mm²)	Drawing strength σ_{Bz}(kg/mm²)	Ratio σ_{Bz}/σ_B	Number of repetition N_B //	⊥	×	Mean value of 4 directions N_B
60/40 Brass	39.9	43.7	1.10	16	13	16	15
Tin plate	37.2	46.6	1.25	17	21	23	21
Copper	22.3	24.8	1.11	14	12	13	13
Aluminium	8.6	9.7	1.13	17	20	20	19

5. Estimation of drawing limit by semi-experimental methods

If it is intended to determine the limiting drawing coefficient using $s_{r_1\max}$ and σ_{Bz} as shown in Fig. 36, the limiting drawing ratios must be such as that shown in Fig. 37 by the line (3). According to the tendency of this line also drawing properties of the materials are not expressed. Since it has been shown in the earlier section that $s_{r_1\max}$ never represents the true specific drawing resistance, the immateriality of the line (3) will presumably owe to the difference between them. The theoretical $s_{r_1\max}$ is able to be corrected using the drawing efficiency $\eta_{x\max}$ given in Table 2. Supposing the true specific resistance be $s_{r_1\max}/\eta_{x\max}$ and comparing this with σ_{Bz}, the limiting drawing coefficient was determined. Going through the process shown in Fig. 36, the values are obtained and given by the line (4) in Fig. 37. In this event, the points are almost in a straight line and clearly express the approximate tendency of the drawing properties of the materials. The values of the limiting ratio themselves are however all somewhat higher than the cupping test results. This will be caused principally from the fact that a higher holding force is needed in the limit working than in the working, in which $\beta = 1.87$, and accordingly $\eta_{x\max}$ must be somewhat smaller in the case of actual limiting condition than in $\beta = 1.87$.

6. Summary

After all, it will be said that the drawability of a material, when it is drawn into a cylindrical shell, is neither presumable simply from the stress-strain curve nor comparable with that of others by the curve. The facts, $S_x/T_x \fallingdotseq 1$ and $\sigma_{Bz}/\sigma_B \fallingdotseq 1$, will probably be the chief cause of these unpracticabilities.

Hence, so long as we deal with the drawing limit, the additional elemental resistance and difference of the strength from the tensile strength should not be neglected.

As to the drawing efficiency or the ratio S_x/T_x, it is vaguely said that so-called high strength metals are generally higher in the efficiency than the lower ones as will be seen in Table 2, but, determination of the limiting efficiency for each material is not so simple.

With respect to the strength ratio σ_{Bz}/σ_z, the presumption of it for each material will still more troublesome unless actual breaking tests in the actual working is made.

Recently, C. Arbel[33] stated that for non-ferrous metals and alloys the power n which is used for representing the tendency of the stress-strain curve provided that it is expressed by $\sigma = Ke^n$ (σ = true stress, e = natural strain, K = a constant), specifies uniquely the limiting drawing ratio. But, S. Fukui and S. Yoshida[34] insisted that for ferrous alloys n can not specify the drawability methodically, and they proposed another method to predict generally the drawability.

In the author's experiments, the limiting drawing ratio of tin plate was exceedingly high, and will probably be not able to be predicted from the power *n*.

In considering the ratio σ_{Bz}/σ_B, we must take the fractured portion of the drawn material into account, for a fracture generally occurs in the portion of the material contact with the punch profile, where the material is subject to bending as well as tension. If there is a material particularly tough against bending, the ratio σ_{Bz}/σ_B must surely be high for such a material. In order to compare the bending strengths (so called temporarily) of the materials the repeated bending tests[35] were tentatively made on the sheets and the number of repetition before fracture occurred was taken. The result is tabulated in Table 4. As shown, the number of repetitions is seen as though it bears an intimate correlation to the strength ratio. So, it will at least be said that the actual drawing strength is essentially affected by the bending work at the punch profile.[36] Anyway, the bending work with which a drawing work is attended is never so immaterial as often considered in the earlier researches.

Chapter VI. Limit of Lubrication in Sheet Drawing

In the foregoing experiments, the lubricating conditions were confined to either the lubrication with the very viscous mobile grease or non-lubrication. This chapter gives an account of the lubricating properties of various lubricants other than that which are applied to copper, brass, and aluminium sheets. The limit of lubrication in sheet drawing will also be referred to.

1. Properties required for lubricants in sheet drawing

The properties of a lubricant required to lubricate efficiently in the drawing were specified by Evans and others[37] rather recently. But, what property is the most important for a lubricant to be effective is still not fully ascertained. In this research, consideration has been confined to the matter relating to decreasing friction, and in the earlier chapter, it was suggested that viscosity of lubricants must be high at least. Indeed, the viscosity seems to be an important factor for a drawing lubricant to be inhered in, but, it cannot be thought that the viscosity alone solves all the lubrication problems in sheet drawing.

First, a series of mineral oils were applied to the copper blanks in which the viscosities were widely different (see Fig. 6). The holding force was kept almost constant throughout the experiment and other conditions were in accordance with the standards the same as before. The frictional coefficient of the holder surface was calculated, and it is given comparatively in Fig. 42 (*a*). As shown in the figure, the coefficients for the so-called heavier stocks are smaller on the whole than those for the lighter ones.

Next, a series of the vegetable oils were tested. The result is given in Fig. 42 (*b*). As will be seen, the coefficients are all nearly equal except castor oil in which the coefficient shows a temporary drop about the middle stage of the working. Similar dropping of μ_h is observable also in the cylinder oil. The animal fats and other semi-solid lubricants were also tried and μ_h of them are shown in Fig. 42 (*c*) and (*d*).

Since μ_h in tallow is smaller than in lard, and tallow seems harder than lard judging from their consistency numbers,[38] if we confine our attention to Fig. 42 (*a*)

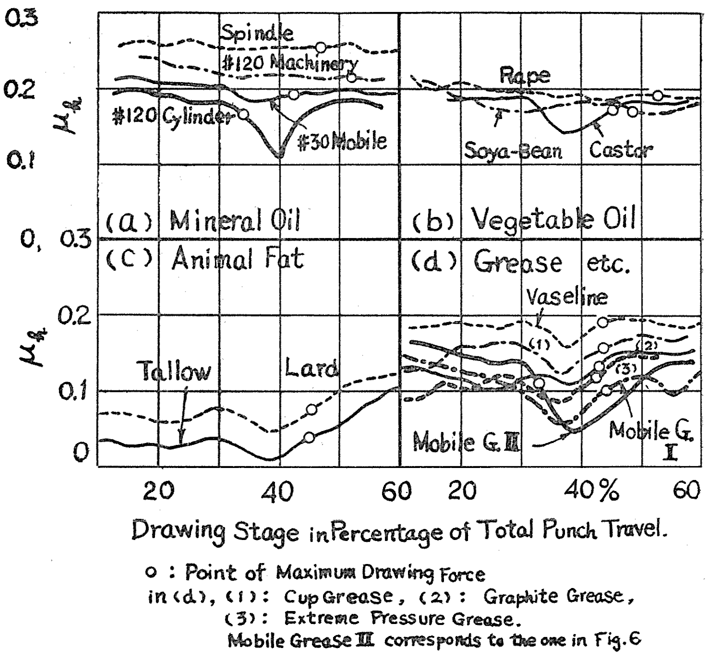


FIG. 42

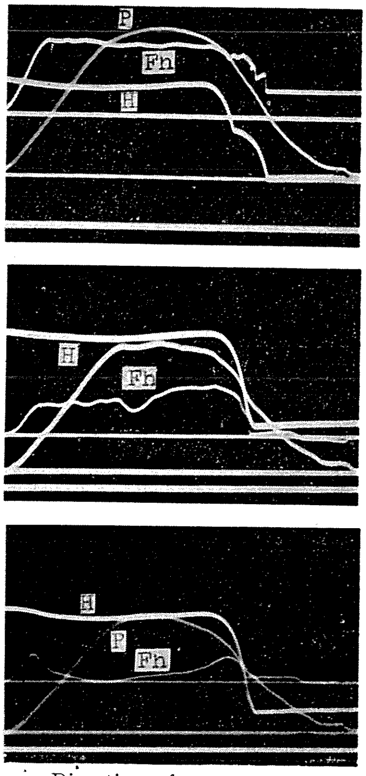
and (*c*), it will be thought that the viscosity or consistency of a lubricant is an important factor which dicides μ_h. But, as shown in Fig. 42 (*b*), in spite of that vaseline is rather harder than the other greases and the mobile greases are rather fluid to such degree that the viscosity is at any rate measurable, μ_h in the latters are considerably smaller. In addition, as to the vegetable oils, it is to be said similarly that even the viscosity of castor oil is much higher than that of soyabean oil (see Fig. 6), but the average heights of μ_h curves are not essentially different. On the other hand, it should be noticed that highly viscous oils or semi-solid lubricants to whatever series they may belong, without exception, show a temporaly drop of μ_h about the middle stage of the working. This dropping of μ_h must be the evidence that partial fluid lubrication governs the process.

From the above all, it will be said that viscosity or consistency of lubricants does not decide the average coefficient of friction on the whole but it gives them a faculty to play the rôle of fluid lubrication to some degree about the middle stage of the working. It seems also that the factor which determines the average value of the coefficient is so-called oiliness which is a character of lubricant displayed in cooperation with the lubricated metal. For example, the machinery oil and the machinery oil mixed with 3% stearic acid are almost the same in their viscosity, about 150 cst. at 21°C, but the frictional coefficient of the latter is kept less than a half of the former all over the process.

2. Types of friction of the holder surface

Some typical examples of experimental records for the copper blanks are given in Fig. 43.

In the figure, (*a*) is for a spindle oil in which F_h is at almost constant height



(*a*)
Copper with spindle oil.

(*b*)
Copper with E.P. grease.

(*c*)
Copper with graphite in tallow.

⟶ Direction of process

FIG. 43

throughout a process, and (*b*) is for an extreme pressure grease in which F_h increases gradually from the initial to the final stage of working, although a little dropping is recognized in the mid-stage. (*c*) is for a mixture of graphite in tallow, 1 : 3 by weight, in which F_h is generally very small except in the initial and final stages.

The author thinks that those given above are the three distinctive types of the frictional resistance of the holder surface, and proposes the following tentative designations: (*a*), "boundary lubrication type"; (*b*), "lubricant film wear type"; and (*c*), "fluid lubrication type."

The frictional character which will be exerted by a combination of any lubricant and metal can be expressed by a combination of the above three types.

There is of course a possibility that for the same combination of lubricant and metal the friction type differs according to the holding force. For example, if the extreme pressure grease is applied to aluminium blank under suitable temperature, the friction type is rather (*c*) in the case of low holding force, but as the force increases the type changes into (*a*).

3. Limit of lubrication

The problems of lubrication in deep drawing has arrested the researcher's attention from previous time. But, since the lubricating mechanism was not clarified, partaining to the limit of diminution of the drawing resistance by means of lubrication, or more generally, to the possibility of improvement of the operation by lubrication, hitherto almost nothing has been referred to. In this section, the possibilities and limitations of the lubrication in sheet drawing and the grading of performance of drawing lubricants will be explained.

Experiments were carried out with the standard size tools on copper, the brass (60/40) and aluminium sheets of the standard size. The lubricants applied are shown in Table 5.

TABLE 5. Lubricants Applied

Group	Name	Number referred to Fig. 46	Viscosity number cst. at 21°C or consistency number ASTM at 21°C
Mineral oil	Spindle oil		32 cst.
	♯120 Machinery oil	36	150 〃
	♯30 Mobile oil	35	400 〃
	♯120 Cylinder oil	33	1800 〃
Vegetable oil	Rape oil	18	85 〃
	Soyabean oil		68 〃
	Castor oil	30	810 〃
Animal fat	Lard		222 ASTM
	Beef Tallow	1	38 〃
Grease and petrolutum	(Vaseline)		191 〃
	Cup grease	20	280 〃
	♯190 Graphite grease	9	205 〃
	♯250 Extreme pressure grease	32	232 〃
	Mobile Grease I	24	—
	Mobile Grease II	23	5200 cst.

First, in order to find the limiting drawing resistance attainable by diminution of the external friction, frictional resistances, F_h and F_d, were subtracted from the actual drawing resistance. Fig. 44 shows the process of finding the limiting drawing resistance for the aluminium blanks lubricated with three kinds of lubricants, tallow, the extreme pressure grease and the Na-soap water of 60% soap in water. The figure shows $P/\sin\alpha - F_h$ in ordinate based on $P+H$ at various stages of working process. As shown, recognizing the approximate linear relations between them, height of the intersection points of the lines with the ordinate axis were taken and plotted against drawing stage. The results are given in Fig. 45 by the curves D. The cuves D give the drawing resistances attainable by an ideal lubrication, and are the limiting drawing resistances. The problem followed is that to what degree with actual lubricants is this ideal realizable.

The performance of a lubricant is not rationally specified till the actual drawing resistance, when the lubricant is applied, is compared with D. And with that, rational grading of lubricants will become possible.

In order to compare the performance of lubricants according to this principle, the values of P at the stage when $D\sin\alpha$ in Fig. 45 reaches its maximum were examined for various lubricants applied to the three kinds of sheets. The value of P at that stage, P_y, was graphed based on μ_{hy} which is the value of μ_h at the

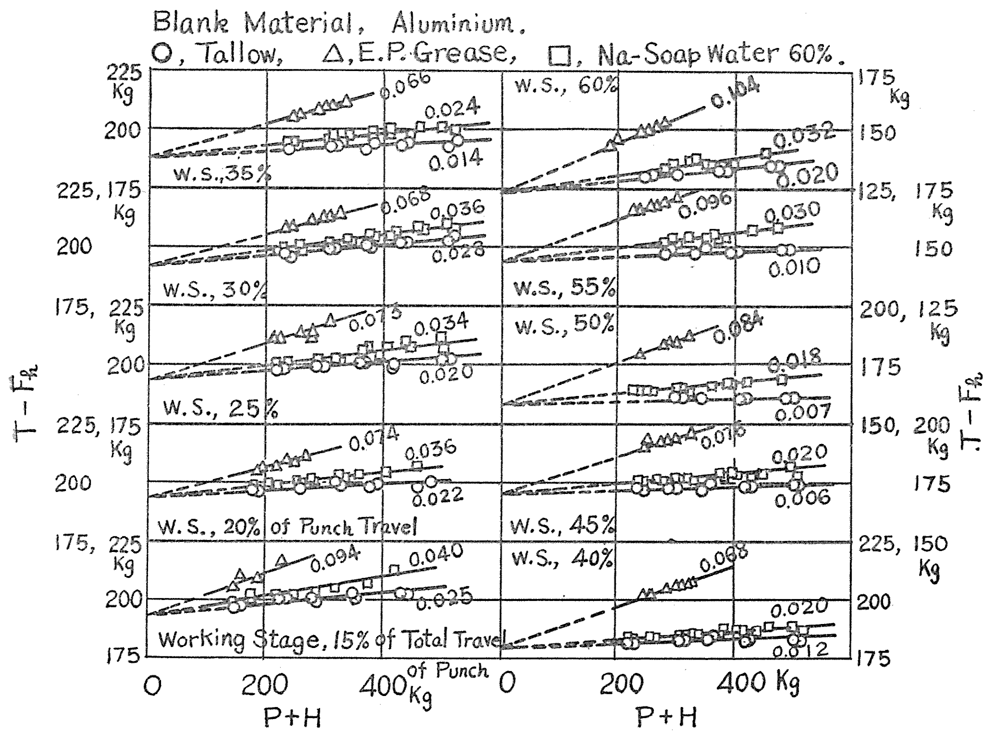
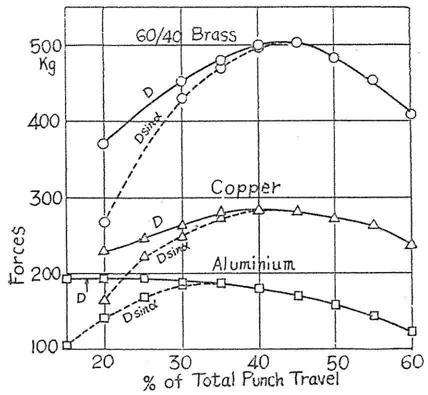


FIG. 44



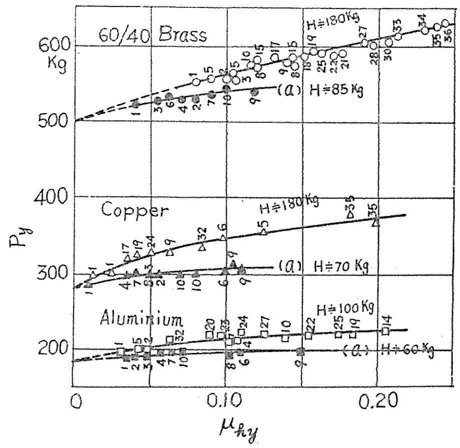


FIG. 45 (*left*)

FIG. 46 (*right*)

Blank, Diameter: 29.0mm, Thickness: 0.5mm
Die Hole, Diameter: 16.0 mm,
Mean Drawing Ratio, 1.87

same stage. The curves in Fig. 46 show the P_y-μ_{hy} relations for various metals. As shown, P_y's for any one metal and for a constant holding force line up almost in a curve and from the general tendency of the curve, P_y appears to approach a certain value, if μ_{hy} diminishes to zero. This ultimate value of P_y must be also

AUXILIARY TABLE FOR FIG. 46

Number indicated by in Fig. 46	Lubricant
1	Tallow (beef)
2	60% Na-Soap water
3	40% Na-Soap water
5	20% Na-Soap water
6	5% Na-Soap water
8	♯120 Machinery oil+3% Stearic acid (3% by weight)
9	♯190 Graphite grease
10	♯30 Mobile oil+Stearic acid (3% by weight)
14	♯120 Cylinder oil+Sulphur (1% by weight)
15	Spindle oil+Stearic acid (3% by weight)
17	♯120 Cylinder oil+Stearic acid (3% by weight)
18	Rape oil
19	♯120 Cylinder oil+Lead stearate (5% by weight)
20	Cup grease
21	♯30 Mobile oil+Sulphur (1% by weight)
22	♯30 Mobile oil+Lead stearate (5% by weight)
23	Mobile grease II
24	Mobile grease I
25	♯120 Machinery oil+Lead stearate (5% by weight)
27	Spindle oil+Lead stearate (5% by weight)
28	♯120 Machinery oil+Sulphur (1% by weight)
30	Castor oil
32	♯250 Extreme pressure grease
33	♯120 Cylinder oil
34	Spindle oil+Sulphur 1%
35	♯30 Mobile oil
36	♯120 Machinery oil
do. in Fig. 46 (*a*)	
1	Tallow (beef)+Stearic acid (5% by weight)
2	Sulphurized oil
3	Rape oil+SCl (10% in volume)
4	{Rape oil+SCl (8% in volume)}+Stearic acid (3% by weight)
6	60% Na-Soap water+Stearic acid (3% by weight)
7	Rape oil+SCl (8% in volume)
8	Tallow+Lead stearate (5% by weight)
9	Lard+Lead stearate (5% by weight)
10	Lard+Stearic acid (3% by weight)

specify the limiting drawing resistance. The three ultimate values determined by three points placed on the ordinate axis of Fig. 46 coincide rightly with the maximum values of $D \sin \alpha$ in Fig. 45 respectively.

As will be seen in the figure, tallow gives always the most satisfactory result for any material and it should be said to be an ideal lubricant in regard to diminution of friction. It is also clear that for copper and aluminium there is scarcely a possibility to make great stride in improvement of operation by any lubricant. For the brass, however, there still remains a little possibility of it. To prove this posibility, the holding foce is lowered as much as the condition allows and the lubricants made active with various additives are used for the experiment. The result is given by the curves (*a*) in the figure. The points obtaind in this case for the tallow with 5% addition of stearic acid are placed nearest to the ideal value, but compared with the case in the other materials, there still seems to exists a certain space to be occupied by some lubricants.

Hence, it should be said that in copper and aluminium, with the lubricants available in the present time an almost ideal working can be attained with little possibility of further improvement by lubrication, while, in the brass there remains a little possibility of it.

Résumé

The author clarified the mechanism of external friction in sheet drawing measuring directly the frictional resistance of the holder surface as well as the punch force and the holding force during working simultaneously, where as the measuring method, the principle applied in Dr. Nakanishi's high speed optical indicator was employed. Accordingly, an analytical consideration of the drawing resistance into elemental ones which were assumed to compose the resistance was made possible and the characteristics of the elemental resistances were specified in detail. Thus, the implication of the drawing resistance became explainable more satisfactorily than ever. A detailed consideration of the drawing limit became possible, and the limit of lubrication in the drawing and the performance of drawing lubricants were rationally specified, all based on the result of this experimental analysis of the resistance.

In concluding this paper, the author wishes to acknowledge the encouragement and guidance given in this research by Prof. Dr. S. Doi, the head of the laboratory of machine tools in Nagoya University, and the valuable advices and suggestions given by Prof. Dr. A. Wakuri and Prof. Dr. T. Ishibashi of Kyushu University. The author also wishes to thank Messrs. N. Kawai and M. Suzuki for their earnest assistance throughout this work.

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9) Viscosity of this spindle oil is given in Fig. 6.
10) Notched blanks like this were first applied by Dr. M. Sommer. cf. Sommer, M., Forsch.-Arb. V. d. I., H. **286** (1926).
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24) Masuda, M.: Trans. Jap. Soc. Mech. Engnrs., Vol. 8, No. 32 (1942), I-121.
25), 26) *loc. cit.*
27) cf. Nadai, A.: *Theory of Flow and Fracture of Solids* (1950), 472.
28) Kasuga, Y.: Trans. Jap. Soc. Mech. Engnrs., Vol. **14**, No. 47 (1948), I-60.
29) Oehler, G.: Werkst. Maschinenbau, Jg. **40** (1950), 225.
30) *loc. cit.*
31) For example, cf. M. Sommer. *loc. cit.* in (20).
32) Hofmann and Koelzer stated for automobile-body sheet iron that the product of several ratios partaining to tensile strength, yielding point and elongation represents the drawability fairly exactly. cf. Hofmann, W., Koelzer, H., Werkst. Maschinenbau, Jg. **42** (1952), 88.
33) Arbel, C.: Revue de Métallurgie, **47**, 5 (1950).
34) Fukui, S. and Kudo, H., Yoshida, S., Okawa, Y.: Mimeographed paper for the 549th lecture meeting in the plastic working section, J.S.M.E. Nov., (1952).
35) Bending radius is 1 mm in this case.
36) It is reported by Chung and Swift also that as to the drawability it is important to consider the order of merit which various materials shows with different punch profile. *loc. cit.*, 208, 209.
37) Evans, E. A., Silman, H., Swift, H. W.: Engineering, Vol. **164**, No. 4267, 4268 (1947), 454, 477
38) See Table 5.