NEUTRAL POTENTIAL TRACER AND ITS APPRICATIONS TO POWER ENGINEERING

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Contents

1. Introduction

More extensive range and much accuracy have been brought in the measuring techniques of recent electric power engineering by taking advantage of vacuun tube and Braun tube. Some of new measuring device have been established so that valuable field data might be collected as readily as in the laboratory researches. The operating device as well as watching equipment assume a new aspect and the possibility in the past turns out a great contribution to the future development which serves as the familiar fundamental facts. The author has lately had a much interest in the application of high frequency techniques to the analysis of power systems, especially to the researches in the abnormal voltage phenomena in transmission systems. Besides the instantaneous phenomena which are concerned with time duration of one millionth of a second, durable quasi-static transient ones also come into question. The former requires the high-speed Braun tubes, but vector operation is often effective for the latter to be understood better. Thus, the anthor's strenuous efforts in recent years have brought his research to the extent that he can accomplish such an apparatus that A.C. terminal voltage is directly to indicate its vector quantity. In order that the alternating electrical quantities are indicated with those of vector, basic frequency must be chosen properly, and a steady vector must have a constant amplitude of that frequency in its oscillographic representation, but a transient vector a constant or a variable amplitude with regularly variable phase angles or frequency.

A.C. power network, in its steady state, will be explained by means of steady vectors, but in many a case transient state requires to be shown with transient vectors. For example, at the recovery of one-line-grounded fault in an off-tuned reactor compensated transmission system, the frequency of the damped oscillation in the zero phase sequence circuit is entirely different from that of system network. The variation of phase angles is of much significance when any power surging is discussed within the stability limits of the system.

In the ordinary three-phase A.C. transmission system, the phase-to-phase voltages and their phase angles are, except the case of directly grounded network, given from the beginning, but the phase-to-ground voltages vary in a wide range according to the circumstances. The insulation of lines is carried out, without exception, against ground conductors, and the line-to-ground voltages are the decisive elements to be considered especially from a standpoint of insulation coordination. In a case that the vector operation is applicable, the voltage vector between the ground and the so called neutral point, which is the geometrical centre of the triangle given by the normal source voltages, naturally determines the line potentials against the ground. The direct measurement of floating neutral potential has, for this reason, been strongly claimed for the explanation of abnormal overvoltage accompanied with some durable faults. This is better understood by conceiving to the contrary how the ground potential varies against the neutral point of the source triangle, assuming no surge of the faults disturbs the power source.

Figures which emerge upon the fluorescent surfaces of televisions or radars are nothing but the loci of the quick movements of a mathematically coordinated bright point. The same devices will denote easily the successive displacements of the neutral potential in polar coordinates.

This treatise relates the detailed explanation of the apparatus for the direct measurement of vector voltages, which the author has recently accomplished after his several years' research, and illustrates the facts in regard to the intuitive comprehension of the voltages and currents as well as the phase angles which enabled us to extend the field of A.C. measurements and to understand more easily the results of the network phenomena. In other words, it serves as the neutral potential tracer for the experimental analysis of the arc-suppressing reactor system, the states of the steady and transient overvoltages being quite simply explained for each phase as the displacements of the neutral potential against the ground. Hence the successive developments of the overvoltages can easily be traced without distinction as to the case that the system is set up actually or as a model, and the result turns out fundamental data for the insulation coordination and the arrester design. It is of great use, too, for the determination of the effectively grounded conditions in the fundamentally resonant transmission system. The author is confident of the utility of the apparatus for the conventional calculation in transmision systems, which not only serves as a new measuring device of the non-directly grounded network, but as a measuring instrument such as the vector indicator of the A.C. network analyser.

In the following articles, he first tries to recognize some properties of the A.C. voltage vectors, and then explains briefly the meanings of the neutral potential which is the research object of the abnormal voltages in power systems. Next, he refers to the neutral potential tracer proper, and its development, construction, action and characteristics reviews in short. Theoretical and experimental results are then discussed extensively when the tracer is applied to the analysis of the transient phenomena in the arc-suppressing reactor system that is the most valuable test-case, and its another example of application reveals the convenient method for the pursuit of the current locus, or the circle diagram of a loaded induction motor in which sufficient accuracy is achieved. Last, some fundamental items in relation to the extensive application other than above examples, which the author could not enumerate here, are introduced with its characteristics and then his future expectation is expressed.

2. A.C. Voltage Vector and Neutral Potentials in Power Systems

Meaning of the A.C. Voltage Vector. The vector denotation of the A.C. quantity has been adopted since 1893 by A. E. Kennelly and C. Steinmetz. More or less constancy and time duration are necessary for the A.C. quantity to be denoted with vector, but a transient phenomenon refered to a steady state can be denoted as a vector quantity as well. Steady vectors should be accompanied by their own reference frequency, but others, slightly different from that frequency, are understood as moving vectors. Harmonic components of higher orders may be different vectors, but generally these are not put in practice because of their complicated denoting operation. Oscillatory phase angles are denoted with oscillatory vectors, and those which change in one direction with constant rate make another constant frequency and are denoted with rotating vectors. The former may be treated as a variable frequency. Damped free oscillations of approximately the same frequency as that of the coordinate surface have vector loci of logarithmic spirals, which are often the case with the transient phenomena in power transmission systems. Originally as the projections of a vector upon some given axis denotes the instantaneous values of the A.C. quantity, the amplitude of the vector must be the same as the maximum value of the sinusoidal A.C. quantity. But afterwards, as the vector operation became of vulgar use, now we make it a rule to denote the amplitude by the effective value of the A.C. for convenience' sake. This is also the same in this paper. In other words, the transient vectors applied to quick phenomena show no more than $1/\sqrt{2}$ of the equivalent A.C. maximum.

Neutral Potentials in Power Systems. The neutral point in the windings of Y-connection generator or transformer may be naturally defined, but generally no more than three sorts of phase-to-phase voltages are given from the electrical point of view, and the potential of the mechanical neutral point against the ground, which is of great importance for the insulation problems, varies for a wide range. In Δ-connection system in which, however, the mechanical neutral point cannot be found, the imaginary neutral may be properly chosen. Phase voltages of given phase angles stress between the neutral and the lines, so the magnitudes and the phase angles of the neutral potential against the ground only determine the line-to-ground potentials, with which the line insulation can easily be discussed. Hence the abnormal voltages of the lines are directly connected with the neutral potential, and this again results in the shifting problem of the ground potential against the source triangle or its neutral. The neutral shift in a system may be recognized as the shift of the ground potential against the system, which is more comprehensive and coincides with the method of analysis adopted hereafter by the author. Transient displacements of the neutral potential caused by the faults are the adequate objects to the analysis of the neutral potential tracer.

In a far transmission system, there exist many sorts of natural oscillation, which are chiefly determined with the whole line-to-ground and line-to-line capacities and the leakage inductances of the generators and transformers involved.

Especially it must be born in mind that the transient inductances of the generators vary with the lapse of time. Intense resonant overvoltages will occur when the natural frequencies of the system transiently organized during the faults or the switching performances occasionally coincide with the source frequencies. Negative phase sequence component of the fault current especially in a generator without damper windings, sometimes causes harmonic oscillations in the network, odd in steady and even in transient.[1)]

The resonance frequency of the zero phase sequence circuit in an arc-suppressing reactor system is variable generally within $\pm 10\%$ of that of the system, and the abnormal overvoltages suddenly come out across the reactor terminals when one-line-grounded fault is recovering in a perfect compensation system, one-line-broken occurring in an under compensation, and also one-line-grounded in one of the systems which are drawn parallel with each other in an over compensation, all of them taking place as a result of intense series resonance phenomena.

These resonances, in their fundamental frequency band, backed up by the powerful sources give rise to severe and durable overvoltages and the consequences not only threaten the insulation of the system but have significant effects on the arrester design. By the method of symmetrical components, the calculation of transient abnormal voltages[2)] becomes possible by the aid of operational impedances, however, it is not so easy but complicated. Many of the resonance phenomena in an arc-suppressing reactor system are found in the zero phase sequence circuit and have little influences upon phase-to-phase voltages, and again, in case of resonances of the transmission system in general, it is fully recognized that, on the conditions which make the fault current or the non-fault current maximum, the overvoltages are decreased with the lowering neutral impedances.[3)] The neutral potential against ground, therefore, is of importance part in both cases and the abnormal resonance overvoltages in transmission network and the observation of the neutral potential are decisively co-related with each other. The neutral potential in a steady balanced network may be regarded as zero, that is equal to the ground, but the vector locus, in its free oscillation state, should be explained with a logarithmic spiral as mentioned above. For this purpose conventional equations are introduced showing the neutral vector displacement and hence the transient abnormal voltages of lines against the ground. But when, for example, such inductance consisting the resonance circuit as the fault neutralizer varies in its value accoring to the passing current, the rotating angular velocity of the spiral becomes irregular. This saturation effect of the iron core can hardly be recognized theoretically, but the vector denotation greatly assists us in our comprehension and is still accurate. The quasi-static transient phenomena as shown above become secured substantially by measuring directly the displacement of the neutral potential.

3. Neutral Potential Tracer

General Description. This is the main chapter of this paper. The so called "Neutral Potential Tracer" named after the property in its practical application is rather properly called the apparatus indicating the vector quantity. The equipment has been studied since 1948 and is accomplished lately by the author, and gives a very convenient analytic method in the reactor compensated system as well as transmission network in general. In Fig. 1, suppose one of the selsyn machines, which are directly

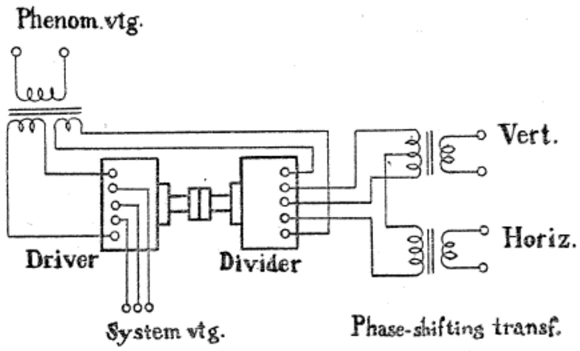

Fig. 1. Voltage divider by selsyn machines.

shaft-coupled, is the driver and the other the voltage divider, the rotor position will be determined and hence the phenomenal voltage divided into two rectangular components in that position through the phase-shifting transformers in accordance with the phase angles of the system and phenomenal voltages. These are led to a couple of deflecting plates, vertical and horizontal, of the Braun tube respectively, so that the vector value of the phenomenal voltage will be visible with its radius. This method of early days failed to denote the accurate values because of the mechanical frictions and momentum or the electrical interferences between the input voltages both steady and transient. Afterwards the method has been developed by using the vacuum tubes in which the symmetric amplification method is adopted in place of the compensation method as follows. The principle is that, upon each of the rectangular two phase voltages which are previously prepared from the three phase system, the projections of the phenomenal voltage are obtained. The projections are the D.C. voltages if the phenomenal frequency is the same as that of the system and the slowly changing voltages with difference frequency if the both frequencies are not the same, and they are led to the deflection plates of the Braun tube through D.C. amplifiers respectively. Thus the bright spot indicates the corresponding vector topography without radius. Let the neutral potential be introduced as the phenomenal voltage, then the displacement of the neutral potential will intuitively be observed or photographed at a glance, however the faults may be complicated in connection with the three coordinate points shown as the invariable phase-to-neutral potentials.

The main part of the tracer consists of dividing amplifiers, to which the input voltages are admitted, a Braun tube indicator and a photographic equipment, and is also attended with the power source and the timing device.

Constructions and Actions of the Tracer. The circuit diagrams of the main part are shown in Fig. 2, *a* and *b*. The three phase input voltages do not appear directly

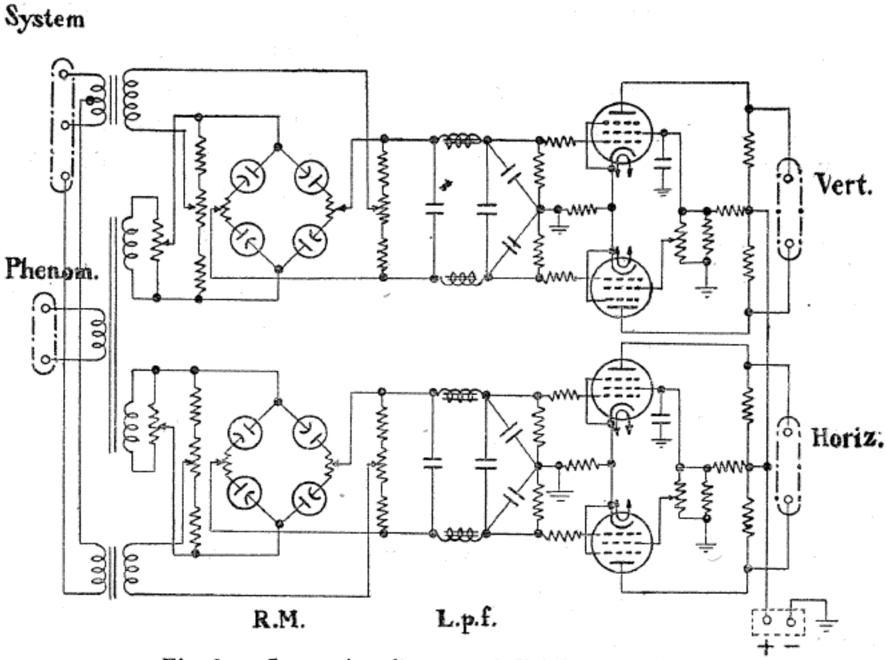

Fig. 2*a*. Connection diagram of dividing amplifiers.

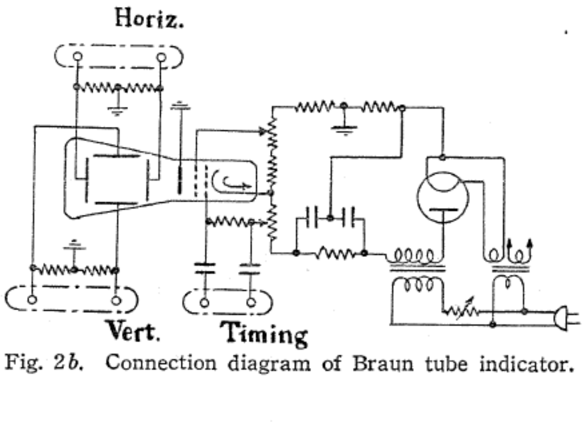

Fig. 2*b*. Connection diagram of Braun tube indicator.

in the results of measurement at all, but they make the coordinate voltages of the phenomenal vectors, and their frequency is accordingly the standard of the vector surface. So, they must be symmetrical three phase voltages of sinusoidal wave forms, constant in their frequency and amplitudes during the measurements. Moreover, their phase sequence must be properly selected so that the phase voltages which are applied with given polarities to the phenomenal terminals in regular periodical order should denote the clockwise rotating vertices of the source triangle, the each stretching angle of which is 120° with one another in accordance with our common sense.* The inversed polarity of the supplied A.C. phenomenal voltage, of course, results in the opposite phase for the vector observed. To obtain the projection voltages, a set of ring modulators which operates with the approximately equal frequencies and low pass filters are used. As the output voltages are rather insufficient for the deflection purpose, with necessary D.C. amplification are those voltages amplified without distortion to the extent required. The projections can never be obtained when either of the phenomenal or the coordinate voltages is not afforded, so that the output voltages do not appear across the deflection terminals.

The Braun tube indicator is no more complicated than the ordinary one, and the vertical or horizontal displacement of the bright spot is brought into effect by the deflection voltages. It has, in addition, a pair of brightness modulation terminals, through which supplied negative impulses, occur every one cycle of the coordinate frequency, will extinguish the bright transient vector locus for a moment.

Only a single phase A.C. 100 V is required as the auxiliary power source for the apparatus, the total input being less than 100 W. It must be taken into consideration that any static or electromagnetic induction should be annihilated, as the apparatus needs a little power and deals with the different frequencies. The type of the camera employed is characterized by 6×6 cm and F 3.5 with an attachment for proximity photography, but this may be replaced by a 35 mm camera of F 1.5 if necessary.

The phase-shifting transformers in Fig. 1 do not make, strictly speaking, any phase shifting circuit, as they are operated by single phase source. In Fig. 2*a*, on the contrary, the input three phase voltages are divided into two phases' potentials of 90° phase-difference. This is a special application of the constant phase difference circuit independent of frequencies, and the frequency characterized phase shifters are of no use because variable frequencies are dealt with in this apparatus. The ring modulator is the main part of the dividing amplifier. In ordinary high frequency circuits, it bears only modulated frequencies from carrier and modulation ones, and, in this case, the voltages of sum and difference frequencies, that is nearly the twice of the frequency and approximately equal to zero, are tentatively obtained from the coordinate and the phenomenal voltages, those frequencies being approximately equal. The former, being devised so as to be absorbed with a low pass filter, the latter

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* The clockwise rotation of the vector triangle is taken for the normal sequence.

alone is led to the amplifier. So far as the phenomenal voltage is at the maximum no more than 1/10 of the coordinate, the difference voltage, in practically ample accuracy, is to denote the vector projection of that upon this as shown in Appendix I. Those projections on the rectangular two-phases are again combined on the surface of the Braun tube and denote the vector value of the phenomenal voltage. Thus the method of experimental vector composition becomes possible by means of the phase shifting transformers and the ring modulators.

By putting proper time marks upon the transient vector loci, the phenomena might be conceived clearly along the lapse of time. The negative impulses which occur every one cycle of the coordinate frequency are introduced to the brightness modulation terminals of the indicator. The circuit is shown in Fig. 3, in which the sinu-

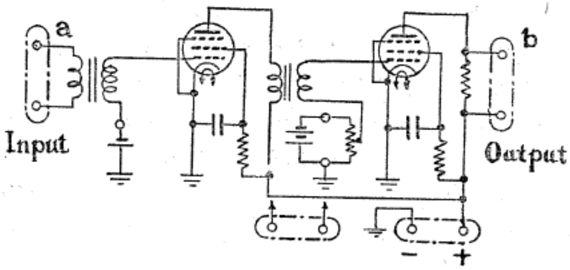


Fig. 3. Connection diagram of timing device.

soidal A.C. voltage applied at terminal "*a*" produces cyclic negative impulses at "*b*" about the zero points of the voltage. If the phase control of the phenomenon is possible in a model test, the timing is correctly counted from the outbreak of the fault, but is otherwise easily calculated by an exterpolation method. The three dimensional denotation of the vector locus is thus adopted.

Variable resisters shown in various parts of Fig. 2*a* are the compensating equipments for giving perfectly balanced conditions of the dividing amplifiers.

The three phase voltages of 100 V impressed upon the coordinate potential terminals turn out a couple of rectangular voltages each of which is 40 V. The phenomenal input is linearly amplified up to A.C. 5 V including amplifier characteristics. The vertical and the horizontal displacement of the spot, then, is 2.5 cm for the former and 3.5 cm for the latter in the same direction. As the voltages supplied to the modulators are about 1/3 of the phenomenal voltage, the voltage ratio becomes $\alpha = 0.042$, and thoroughly fulfils the condition of Appendix I. The frequency which changes from 55 to 65 c/s of the phenomenal voltage, the amplitude being kept constant, never affects the diameter of its circle locus where the coordinate frequency is 60 c/s. Within the circle of 50 to 60 mm diameter about the centre of the Braun tube whose diameter is 120 mm, sufficient accuracies are expected for photographing vector loci.

4. Applications to the Arc-Suppressing Reactor Systems

General Description. Most of the faults found in the super-high-tension transmission systems below 77 KV are one-line-grounded phenomena. To minimize the fault current and extinguish the arc spontaneously, an ingenious method of grounding the system neutral with the specially chosen reactor dependent on the line capacities was invented by W. Petersen in 1919. Under such devices the source can supply the power even during the faults to the load without interruption, but rating current is passing through the reactor during that time continuously. Moreover the electro-magnetic energies stored in the reactor discharge through line capacites in the re-covering periods, and then arouse damped free oscillations that are taking place in the same order of frequency as the system sometimes lasting for scores of cycles. The value of the reactance which minimizes the grounded fault current is called that of the perfect compensation, and those greater or smaller are called the under compensation for the greater and the over compensation for the smaller from the view-point of the compensation current. In an under compensation system, a violent series resonance will take place with the system frequency if one of the lines is broken, and the grounded fault of the other lines which are drawn parallel along the same towers also causes series resonance in an over compensation system. Both of these are the causes of the abnormal rise of the reactor voltages or the neutral potentials against the ground, and the overvoltages of the lines and their time du-ration are determined by the reactor terminal voltages. For those extraordinary phenomena in the arc-suppressing reactor system, only the neutral potential tracer is the most adequate device, and herein lies the reason why it comes into existence.

When faults occur in a reactor system including the zero phase sequence circuit, the transient terminal voltages of the reactor are generally accounted for by the sum of two sorts of voltages ; one the final steady value itself of the system frequency and the other the damped oscillation of natural frequency decided by the last circuit conditions. These may be calculated by means of operational impedances, but in their vector denotation, the logarithmic spiral which damps rotating around the fixed topography of the final vector designates the neutral potential, or the ground potential, at every instant. When the one-line-ground fault is recovering the locus of the neutral potential is indicated by a logarithmic spiral which rotates around the centre of the source triangle, because the steady value may be generally zero. In case of the one-line-broken fault, on the contrary, the centre of the rotation of the spiral frequently lies outside of the source triangle. In both cases, the direct distances between the point on the locus corresponding to some given instant and the points which indicate the topography of the vertices denote the vector values of the phase-to-ground voltages of that period, those may be, under some certain circumstances, unlimitedly grown up to dangerous values in the latter case if the saturation effect of the reactor is disregarded. The author introduced the equations of vector loci in place of the calculation formulae of the symmetrical components. The vector loci are determined by the compensation factors, loss components of the reactor system, and the phase angles, when the phenomena take place, but the detailed explanation of the theoretical operation will be omitted here. The frequency of free oscillations incident to the one-line-broken faults was about 55 to 72 c/s after the practical field data of 77 KV, 60 cycle system, and the less deviations are experienced by the one-line-grounded system. The conditions of the occurred abnormal voltages analysed from the electro-magnetic oscillograms and vector loci are shown in Table 1.

Table 1*a*. Abnormal voltages in the restoring period of one-line-grounded fault.

Compensation	Maximum potential exposed against ground
Over	Phase just succeeding to the grounded
Perfect	——
Under	Phase just preceding the grounded

Table 1*b*. Fault which produces the series resonance.

Compensation	Occasion by which the extraordinary resonant overvoltages are caused
Over	Ground of the other system drawn parallel
Perfect	Ordinary
Under	One-line-broken

Table 1*c*. Abnormal voltages in case of one-line-broken fault.

Compensation (for one-line-grounded)	Line-to-ground potential (source side)		
	Max.	Min.	Note
Over	Preced. & succeed.	Broken	Minimum is the smallest
Perfect	〃	〃	A little greater for the phase preceding
Under (resonance for one-line-broken)	Preced.	Succeed.	Factor is of the same order for each phase and the largest
Under	Broken	〃	——

If any saturation is taken into account, the increase of the terminal voltages does not keep up with that of the current and is so restricted as not to break out any excessive overvoltages. The terminal voltages are generally of sinusoidal wave form and can be indicated by vector loci, while those of the passing current are not so. Dependent incidentally on the conditions of the broken fault, abrupt rise of the reactor terminal voltage may occur during the transient change of the resonance current, which is designated as the ferro-resonance. It is generally no more than 2.5 times the phase voltage in magnitude, but its phase angles are so greatly changed simultaneously that any one of the phases may suddenly suffer from dangerous overvoltages, and this phenomenon is the case sometimes called "the inversion phenomenon of the neutral potential." The leap of the terminal voltage, near at the critical limit, is also influenced by the phase angles when the fault begins.

By making use of the model network and the neutral potential tracer, the quasi-static transient abnormal voltages can systematically be surveyed in details in regard to the effect of the compensation factors, the loss components, the initial phase angles and the saturation rate, etc. which are caused incidentally by the one-line-grounded and restoring, one-line-broken or the transient switching phenomena. The practical field tests may be carried out as easily as those in the laboratory, when the coordinate voltages are led from the generator and the phenomenon is from the reactor terminals both through potential transformers.

Detailed Argument. In a reactor compensated transmission system, as soon as the one-line-grounded fault may occur, the ground potential coincides with that of the phase concerned, but it does not come back to the normal neutral position at once in its restoring stage as is always the case with the reactor systems. This is explained by the following vector equation of logarithmic spiral:

$$\dot{E}_{ng}=EK\epsilon^{-\alpha t}\cdot\epsilon^{j[(\beta-\omega)t+\chi]} \qquad \ldots\ldots\ldots (4.1)$$

where

$K=\dfrac{\sin\psi}{\sin\chi}=\sqrt{\sin^2\psi+A^2}$ ...... multiplying factor of vector overvoltage,

$\chi=\tan^{-1}\dfrac{\sin\psi}{A}$,

$A=\dfrac{\sqrt{\rho}}{\sqrt{1-\rho}}\left(1-\dfrac{2\varepsilon}{1+4\rho\varepsilon}\right)\sin\psi+\dfrac{1}{\sqrt{1-\rho}}\dfrac{\sqrt{\varepsilon}}{1+4\rho\varepsilon}\cos\psi$,

$\varepsilon=\dfrac{1}{\omega^2\cdot 3LC_0}$ .... compensation factor,

$\rho=\dfrac{3R^2C_0}{4L}$ .... loss factor,

$\alpha=\omega\sqrt{\varepsilon\rho}=\omega\eta$ .... damping coefficient,

$\beta=\omega\sqrt{\varepsilon(1-\rho)}=\omega\zeta$ ..... angular frequency of natural oscillation,

and

E : magnitude of normal phase voltage,
L : inductance of the reactor,
C_0: line capacities to ground per phase,
ω : angular frequency of the source,
ψ : initial phase angles.

In ordinary systems we may assume $\varepsilon=1.2\sim0.8$ and $\rho=\overset{-3}{1.0}\sim\overset{-4}{1.0}$.* Eq. (4.1) prescribes the vector loci rotate anticlockwise for the over compensation systems or $\beta>\omega$, and clockwise for the under compensation or $\beta<\omega$.[4] The condition that $\beta \gtreqless \omega$ generaly corresponds to $\varepsilon \gtreqless 1$, but as the values of ρ draw near to 1, K becomes rapidly greater and the vector locus comes to rotate clockwise under the condition of $\beta<\omega$ even in case of $\varepsilon>1$. These instances are few examples in practical network. Many sets of vector locus diagrams are arranged by changing the values of ε, ρ

* The notation means, for example, the equation $\overset{-3}{3.78}=3.78\times10^{-3}$.

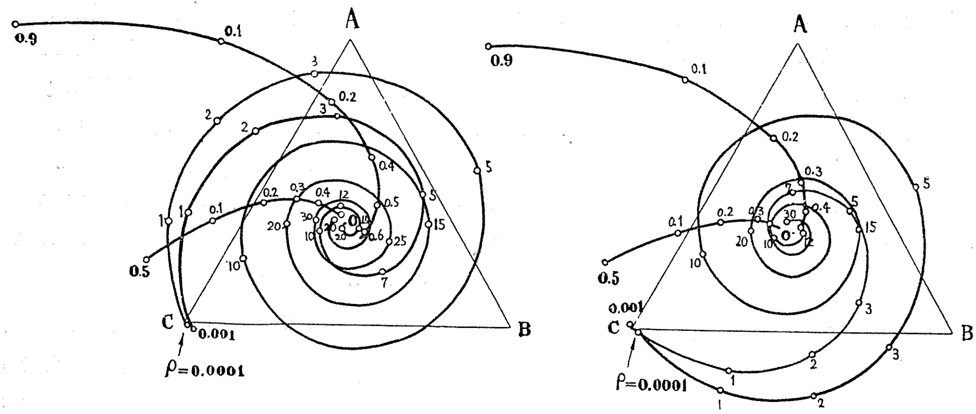


Fig. 4. Calculated vector loci of restoring voltages. The phase C is grounded. Figures shown in the diagrams denote the time from the initial in cycles of source frequency.
a (*left*). Under compensation: $\varepsilon = 0.8$, $\psi = 90°$. *b* (*right*). Over compensation: $\varepsilon = 1.2$, $\psi = 90°$.

and ψ as parameters, and some of them are shown in Fig. 4. If ζ and η are used in place of ε and ρ, the rotating directions and angular velocities are uniquely determined by ζ, but in practice the $\varepsilon - \rho$ method is good for the purpose and easier in numerical calculations. The experimental results obtained by the tracer are compared with those from the electromagnetic oscillograms and theoretical calculations in Fig. 5. Fig. 6 shows the photographs of the vector loci obtained by the tracer.

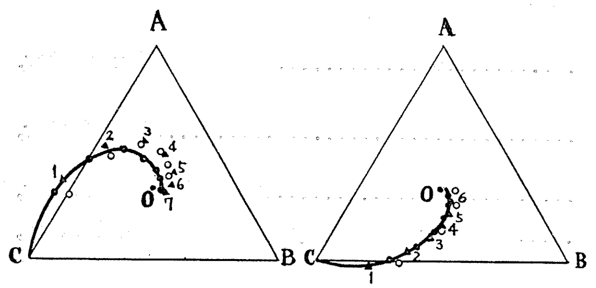


Fig. 5. Comparison of vector loci of restoring voltages measured.
—●— tracer
▲ calculation
○ electromagnetic oscillograms
a (*left*). Under compensation: $\varepsilon = 0.815$, $\rho = 3.78^{-3}$.
b (*right*). Over compensation: $\varepsilon = 1.10$, $\rho = 2.79^{-3}$.

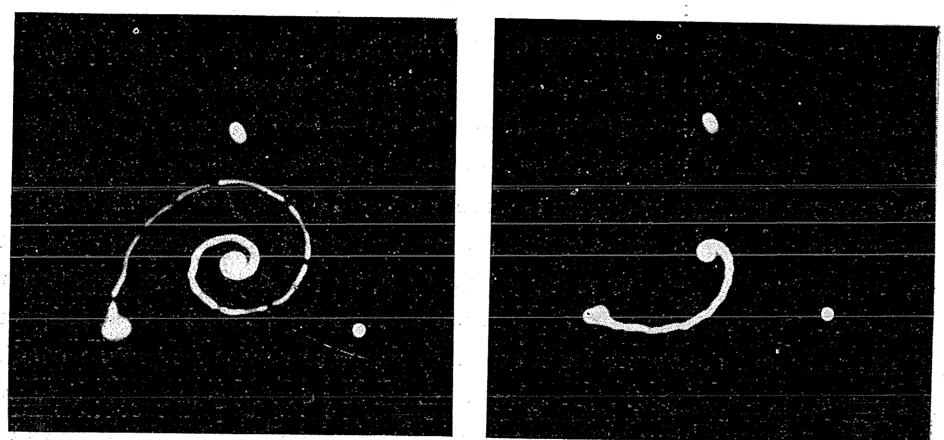


Fig. 6. Photographs of vector loci obtained.
a (*left*). Under compensation: $\varepsilon = 0.740$. *b* (*right*). Over compensation: $\varepsilon = 1.107$.

Extraordinary vector loci are observed in case of one-line-broken faults. The resonance conditions are influenced not only by the zero phase sequence impedances but by positive and negative ones of the whole network observed from the point where the fault takes place. The neutral potential against ground, after the transient oscillations have been ceased to exist, is shown as follows when one of the lines of the no load transmission system is broken at the sending end where the reactor is equipped:

$$\dot{V}_{OA}=\frac{\dot{E}_a}{2}\,\frac{1}{1-(1+j\lambda)\,\varepsilon/\varepsilon_0}\ , \qquad \cdots\cdots(4.2)$$

and the maximum multiplying factor, then, is

$$K_1=\frac{1}{2}\sqrt{1+\frac{1}{\lambda^2}}\ , \qquad \cdots\cdots(4.3)$$

where

$$\lambda=\lambda_r+\lambda_l\ ,$$

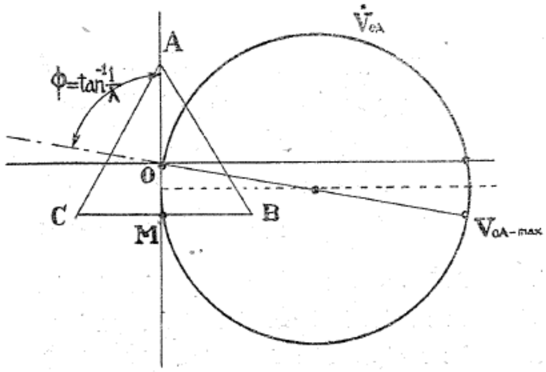
$$\varepsilon_0=\frac{2\,(n+3)}{3\,(n+2)}\ ,$$

$$n=C_0/C\ ,$$

and

E_a: voltage vector of the broken phase,
λ_r : resistance reactance ratio of the reactor (unit),
λ_l : conductance susceptance ratio of the lines (unit),
C_0: line-to-ground capacities per phase,
C : line-to-line capacities.

The value of ε_0 is always less than 1 and is generally 0.8 or so and λ depends upon the construction of the system denoting the practical values of 0.06~0.08. $\dot{V}_{OA}$ of Eq. (4.2) is shown by a circle passes through the points O and M in Fig. 7 in



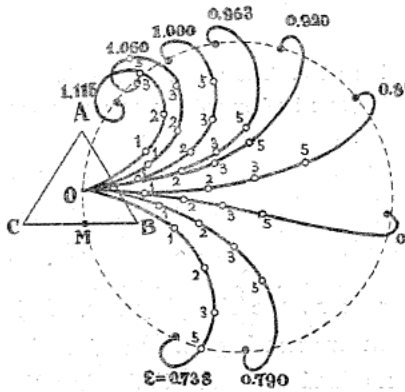


Fig. 7 (*left*). Circle locus of $\dot{V}_{OA}$. The phase A is broken at the sending end of a no load system.

Fig. 8 (*right*). Transient vector loci for one-line-broken faults. These are all observed by the tracer at the different compensation factors on the following conditions: broken phase, A; broken point, sending end; initial phase angles, $\psi=60°$; load, none.

correspondence to the value of ε. The transient vector loci are designated by logarithmic spirals each of which converges to the point on this circle, and thus the examples of the one-line-broken faults at the sending end of the model arrangement of a no load system are analysed into the groups of curves in Fig. 8 by the tracer. The effects of the compensation factors and the point, where the broken fault will occur, are recorded in Fig. 9 for the same model system. Vector loci of resistance loaded system are traced in Fig. 10 where the values of the load current are 11 times

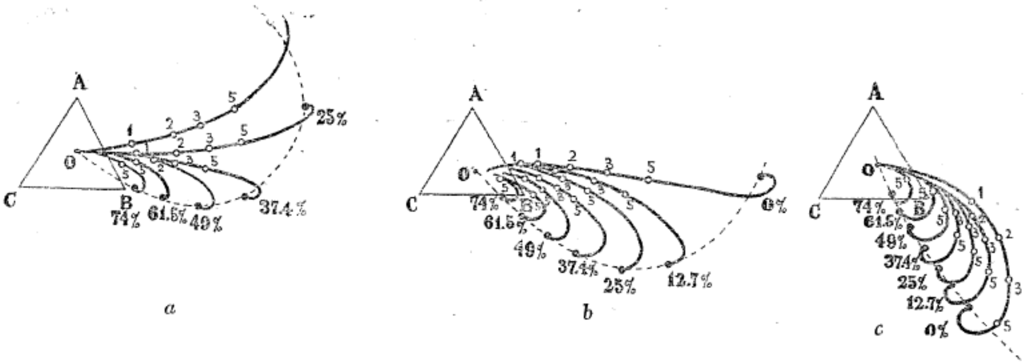
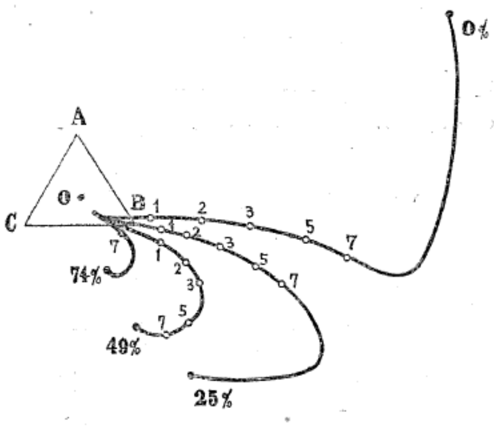


Fig. 9. Transient vector loci for one-line-broken faults, effect of the broken points. The system is the same with that of Fig. 8, the conditions being as follows: broken phase, A; initial phase angles, $\psi=60°$; load, none. The figures shown with % denote the broken points from the sending end for total length.

a. $\varepsilon=0.920$ (over compensation on the broken condition of the sending end),
b. $\varepsilon=0.832$ (resonance on the broken condition of the sending end),
c. $\varepsilon=0.738$ (under compensation).

Fig. 10. Transient vector loci for one-line-broken faults, for loaded system. The resistance load requires 11 times as much current as the charging value. The conditions are: broken phase, A; % figures, broken points; compensation factor, $\varepsilon=0.920$.



as great as the charging one. It is designated that the overvoltages in the loaded system are much greater and various changes of the loci are found comparing with the no load system. Following Eq. (4.3) the reciprocal calculation gives $\lambda=0.0738$ from the practical value of K_1 in a working system of 77 KV.

Insufficient transposition of the lines or residual voltages induced in the zero phase sequence circuit result in the displacement of the neutral potential caused by the series resonance in the ordinary state of the reactor system. The neutral tracer certainly pursues the magnitudes and the phase angles of them. The instantaneous induction by the residual electromotive forces may, of course, be analysed as the transient displacement of the neutral.

About the switching operations of the system including the ground fault neutralizer, it is understood that the instance where the first pole alone is opened is corresponding to the one-line-broken fault and that when two poles are opened, the series resonance takes place in the remaining single phase circuit. These are always accompanied by the neutral displacement corresponding to the compensation factors, line capacities and the load conditions, etc., and hence the abnormal overvoltages which are caused by the switching time differences of the circuit breakers could not be neglected. For the difference of no less than one cycle of the system frequency, the growth of the resonance potential may be evident and some precautions are necessary for the abnormal voltages.

The saturation effect of the reactor is a preferable one and the inductance value is so varied as the terminal voltages do not increase in proportion to the current, but the inversion of the neutral potential raises the terminal voltages to some extent. The ferro-resonance suppresses the overvoltages and the vector loci of the neutral potential are discriminated by singular curves as in Fig. 11. The arc-suppressing

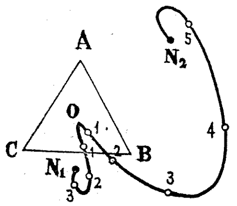


Fig. 11. Transient vector loci for one-line-broken faults, effect of ferro-resonance. The system is broken at the sending end of phase *A*.

N_1: locus without leap,

N_2: locus with leap—the inversion phenomenon.

reactor, in the working system, is more or less saturated at the rating voltage, and if the system is somewhat over compensated for the normal voltage, the locus of the restoring neutral potential from the one-line-grounded fault might rotate counter-clockwise first, but soon turn clockwise at the rate of the decrease of the terminal voltage by the free oscillation. Those facts have been proved to be reliable with field tests.

Such phenomena as the grounded short circuit or the grounded broken-line faults are the multiple accidents analytically consisted of the fundamental phenomena above mentioned, and some of them are to be brought into the quite new situation that might be considered as the result of a second reason caused by the outbreak of the overvoltages. As is always true where there exist two modes of oscillations, one of which is damped free and the other is forced, these take place toward the resonance conditions determined by the newly established system constructions. The transient potentials involving D.C. components, which are found generally at the line side of the broken point, as well as the higher harmonics are originally improper for the tracer to analyse.

5. Experimental Analysis of Induction Motor Characteristics

Conception of Application. The general idea of vectors in electrical engineering is universally utilized for the current circle diagram of an induction motor. It is a well known fact that the line current varies in magnitude as well as in phase angles for a wide range in proportion to the given load, and is shown with the so-called circle diagram. Since the current loci take respective complicated forms as the kinds of rotors, the starting methods and the number of phases, etc. change, the calculation of their characteristics is now one of the most important subjects of discussion in the institutional session. The more complicated the construction is, the more indispensable is to obtain directly the correct vectors, and to the contrary the more effective is the means to investigate the appropriateness of the assumption on the calculation of characteristics. The current voltage transformation is necessary prior to the application of the apparatus to the tracing of the locus. It is convenient for this purpose to take advantage of the voltage drops of the current through a series resistance of extremely minute value, which must be chosen so as not to change the overall characteristics of the motor or not to alter the phase angles between the current and the voltage. The total current locus from the point that corresponds to the locked value to the point that corresponds to the no load running may be recorded by making use of the Prony brake for smaller machines and the suitably prolonged starting period for larger ones. By means of phase-to-phase voltage of two other lines, whose currents are now out of consideration, the imaginary axis of the locus is determined. The above method of direct measurement will promote the efficiency of the individual characteristic test of the mass-produced motors in a short period of hours. The experimental methods of test of motors will be perfect in the case that the tracer is used at the same time with the apparatus by which the torque-versus-speed characteristics are designated.

Examples. The characteristics of an induction motor of the ordinary cage rotor are known as the Heyland's or Hoh's circle diagram. Fig. 12 shows the observed diagram of a 1 HP motor for the 3-phase-200 V-60 cycle source, and it is almost the same with the curve theoretically calculated within the range of the observation. Each example of diagram for deep-slotted and double cage rotor machines of 10 HP or so apparently shows the deviation from the circle in the vicinity of the locked point as the theoretical operation shows. As an example of the single phase driving of the three phase cage-rotor machine, the current characteristics of

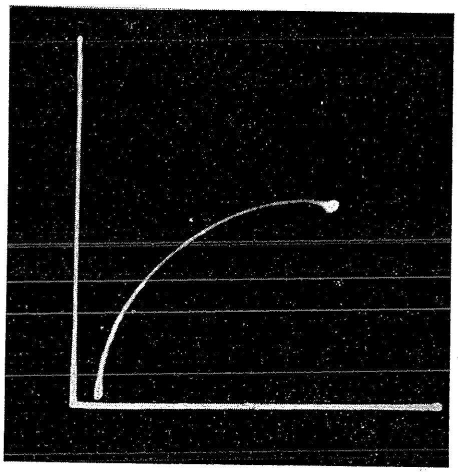


Fig. 12. The photographed circle diagram of an induction motor. The ratings are as follows: Output, 1 HP; source, 3 phase-200 V-60 c/s.

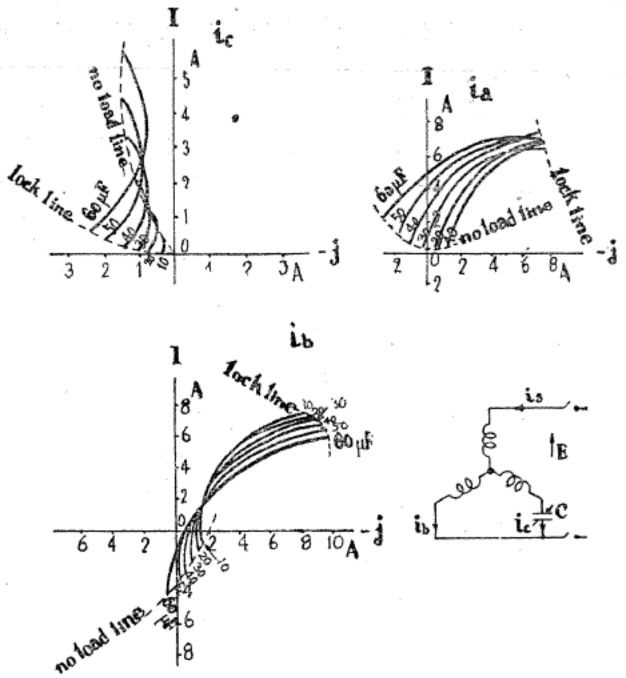


Fig. 13. The circle diagrams of a single phase condenser motor. The motor proper is quite the same with that in Fig. 12.

a condenser-split-phase motor are shown in Fig. 13. The machine is quite the same with that in Fig. 12. Those groups of loci clearly show how the current in each phase varies actually and how relatively the locked and no load current vectors are displaced in correspondence to the condenser values applied. The figures are to give fundamental reference data to the determination of condenser values. The tracer is effectively applied not only to the analysis of induction motor characteristics, but also to that of A.C. commutator machines such as Schrage motors and the like without difficulties.

6. Summary

The neutral potential tracer, as stated above, is a new equipment for the measurement of voltage vectors in the power network, and its main part consists of vacuum and Braun tubes, and it makes possible for us to measure directly the voltages or the currents transformable into the voltages in their vector quantities with low energy input powers. After indispensable adjustments have been made, any supplementary cares are needless except for the phase rotation of the coordinate voltages and the polarity of the phenomenal voltage, and the observation of the vectors is quite easy. The phase voltages are generally taken as the standard vectors. The apparatus is compact and is easy to operate.

The three dimensional observation of the displacement of the neutral potential in an arc-suppressing reactor system and the two dimensional obserbation of the current locus of an induction motor ennable us to determine the fittest way of the application of the tracer to them, as examples show. The abnormal overvoltages caused by the resonance of leakage inductances of generators and transformers with the line capacities of the no load network at the system frequency are enormous both in magnitude and in time duration, and often have been the menaces to the line insulation at the time of system separation following the fault. The relations between the overvoltages and the neutral resisters are understood more exactly by means of the tracer in the resonance tests of the working 154 KV network. Experimental analysis of the fundamental resonance phenomena will be greatly promoted by using the apparatus if it is adopted with the transient analyser by which the free oscillations of the machine windings are observed when the impressed rectangular currents are intercepted. Especially it is effective where the non linear circuit of sine wave voltages are concerned as is the case with the ferro-resonance.

The further extensive applications to the technical development such as those to the A.C. network analyser, to the telemetering devices of phase angles or to the synchronous indicator as a measuring equipment, to the tachometer of the constant speed motor as a watching apparatus, and to the vector analyser in the electrical network as an educational aids are earnestly expected by the author.

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Appendix I. Theory of Vector Composition by Beat Oscillation Method

To each of the two fundamental oscillations

$$A: \ x = C\cos\omega_1 t,$$
$$B: \ y = C\cos\left(\omega_1 t - \frac{\pi}{2}\right)$$

which are perpendicular to each other and of the same amplitude and frequency, the third oscillation of different elements

$$D: \ z = C'\cos(\omega_2 t - \varphi)$$

is composed to obtain beat oscillations. For convenience, we assume $C'/C = \alpha < 1$ and $\alpha^2 \ll 1$ hereafter. The resultant oscillation of A and D is shown as follows,

$$\left.\begin{aligned} R &= \sqrt{C^2 + C'^2 + 2CC'\cos\{(\omega_1 - \omega_2)t + \varphi\}} \cdot \cos\left(\frac{\omega_1 + \omega_2}{2}t - \frac{\varphi}{2} + \theta\right), \\ \theta &= \tan^{-1}\left\{\frac{C - C'}{C + C'}\tan\left(\frac{\omega_1 - \omega_2}{2}t + \frac{\varphi}{2}\right)\right\}. \end{aligned}\right\}$$

The amplitude of the beat oscillation is

$$\overline{R}=\sqrt{C^2+C'^2}\Big[1+\frac{CC'}{C^2+C'^2}\cos\{(\omega_1-\omega_2)t+\varphi\}$$
$$-\frac{1}{8}\Big(\frac{2CC'}{C^2+C'^2}\Big)^2\cos^2\{(\omega_1-\omega_2)t+\varphi\}+\ldots\Big]$$
$$\fallingdotseq C\sqrt{1+\alpha^2}\Big[1+\frac{\alpha}{1+\alpha^2}\cos\{(\omega_1-\omega_2)t+\varphi\}\Big]$$
$$\fallingdotseq C+\alpha C\cos\{(\omega_1-\omega_2)t+\varphi\}. \qquad \ldots\ldots\ldots(I.1)$$

Similary we get as the resultant of B and D

$$\left.\begin{array}{l} S=\sqrt{C^2+C'^2+2CC'\cos\Big\{(\omega_1-\omega_2)t-\Big(\frac{\pi}{2}-\varphi\Big)\Big\}} \\ \qquad\qquad \times\cos\Big(\frac{\omega_1+\omega_2}{2}t-\frac{\pi/2+\varphi}{2}+\gamma\Big), \\ \gamma=\tan^{-1}\Big\{\frac{C-C'}{C+C'}\tan\Big(\frac{\omega_1-\omega_2}{2}t-\frac{\pi/2-\varphi}{2}\Big)\Big\}. \end{array}\right\}$$

The amplitude of the beat oscillation is

$$\overline{S}\fallingdotseq C+\alpha C\sin\{(\omega_1-\omega_2)t+\varphi\}. \qquad \ldots\ldots\ldots(I.2)$$

Now suppose C and ω_1 are constant and C', ω_2 and φ variable.

If we supply $\overline{R}$ and $\overline{S}$ simultaneouly to the ordinate and the abscissa respectively in case of $\omega_1=\omega_2$, as we get

$$\left.\begin{array}{l}\text{variable component of } \overline{R}=\alpha C\cos\varphi \ldots \text{ projection of } C' \text{ upon } A \text{ axis,}\\ \text{variable component of } \overline{S}=\alpha C\sin\varphi \ldots \text{ projection of } C' \text{ upon } B \text{ axis,}\end{array}\right\}$$

the point P of Fig. 101 is to give the vector topography of the oscillation D upon ω_1-plane where the ordinate is the direction of A oscillation and the abscissa is that of B which lags by $\pi/2$.

If $\omega_1-\omega_2=\beta_1$, we get

$$\left.\begin{array}{l}\text{variable component of } \overline{R}=\alpha C\cos(\varphi+\beta_1 t),\\ \text{variable component of } \overline{S}=\alpha C\sin(\varphi+\beta_1 t).\end{array}\right\}$$

For simplicity we assume $C'=\alpha C=\text{const.}$ The case $\beta_1>0$ shows that ω_2 oscillation

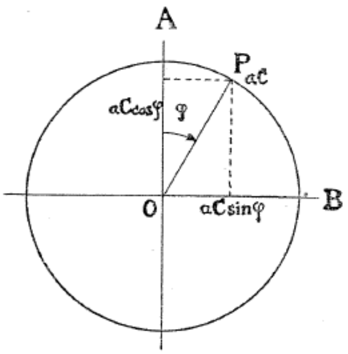

Fig. 101

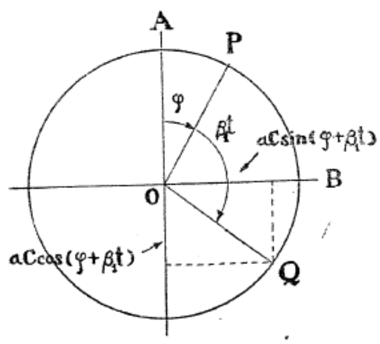

Fig. 102

lags gradually with the lapse of time, and if we suppose the positive direction of $\beta_1 t$ clockwise as in Fig. 102, the rotating direction of vector locus becomes clockwise at $\omega_1 > \omega_2$ and counter-clockwise at $\omega_1 < \omega_2$.

The expansion formula of (I.1) is

$$\overline{R}=C\sqrt{1+\alpha^2}+\frac{C'}{(1+\alpha^2)^{1/2}}\cos\delta-\frac{1}{2}\frac{\alpha C'}{(1+\alpha^2)^{3/2}}\cos^2\delta+\frac{1}{2}\frac{\alpha^2 C'}{(1+\alpha^2)^{5/2}}\cos^3\delta$$
$$-\frac{5}{8}\frac{\alpha^3 C'}{(1+\alpha^2)^{7/2}}\cos^4\delta+\frac{7}{8}\frac{\alpha^4 C'}{(1+\alpha^2)^{9/2}}\cos^5\delta-\frac{21}{16}\frac{\alpha^5 C'}{(1+\alpha^2)^{11/2}}\cos^6\delta$$
$$+\frac{33}{16}\frac{\alpha^6 C'}{(1+\alpha^2)^{13/2}}\cos^7\delta-\frac{429}{64}\frac{\alpha^7 C'}{(1+\alpha^2)^{15/2}}\cos^8\delta+\dots \qquad \text{(I.3)}$$

The value of $\overline{S}$ is obtained from $\overline{R}$ replacing $\cos\delta$ by $\sin\delta$, where $\delta=(\omega_1-\omega_2)t+\varphi$. The terms succeeding to the first two in Eq. (I.3) diminish rapidly with the decrease of α, and also they are alternately positive or negative, so they can be neglected with little errors. The first nine terms of $\overline{R}$ and $\overline{S}$ at the values of $\alpha=0.1\sim1.0$ determine the locus diagrams in correspondence to δ as shown in Fig. 103 assuming

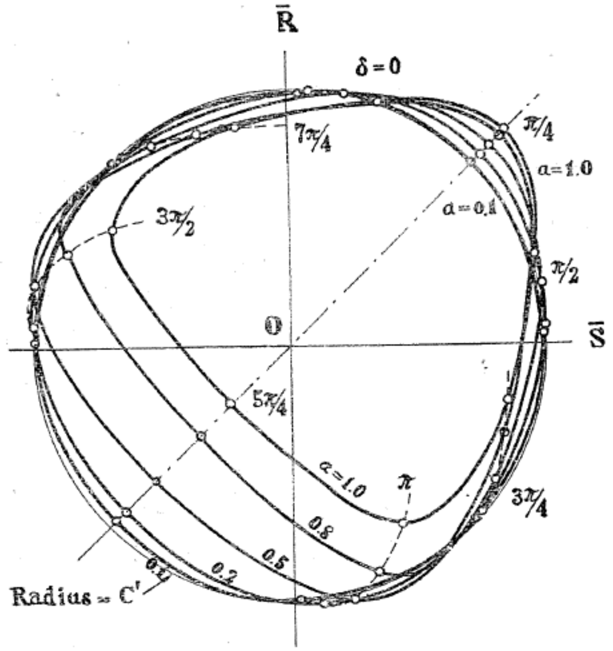

Fig. 103. The composition of vectors for constant C'.
Notations are: $\alpha = C'/C$
$\delta=(\omega_1-\omega_2)t+\varphi$.

C' is costant. From these we know that it is desirable that the beat oscillations should be composed practically on the condition of $\alpha \leqq 0.1$ for obtaining a vector locus. Assuming $\alpha=0.1$, we have the errors of variable components about $0.05\,C'$ and $0.0004\,C'$ for respective coordinate axis.

In an arc-suppressing reactor system, take ω_1 as the angular frequency of the system and ω_2 as that of free oscillation, so the vector loci corresponding to the restoring phenomena of one-line-grounded fault rotate clockwise on the under compensation conditions that $\omega_1 > \omega_2$, and counter-clockwise on the over compensation conditions that $\omega_1 < \omega_2$, these being the same with our common sense.

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