THE DEMAGNETIZING FACTOR OF THE RECTANGULAR AND THE ELLIPTIC STRIP

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§ 1. Introduction

In measurement of the permeability and the hysteresis loop of sheets, the ballistic galvanometer is commonly deflected by the impulse which is produced by changing the magnetizing current rapidly. This method is used for the purpose that we can eliminate the effect of the demagnetizing field by means of suitable magnetic yokes or use the specimen in the shape of rings, but it does not give us the reproducible results. Though the reason for the fact has not been made clear in exact figures, many experiments prove that the fact is due to the lagging of the flux inside the specimen and the errors in measured values are presumably unavoidable in this method.

Besides the above one, there is another method which is not so common as the former, but can be used conveniently when the demagnetizing factor of the specimen is previously known. In this method the ballistic galvanometer is deflected by the impulse, which is produced simply by shifting the pick-up coil away from the center of the specimen whithout any change in magnetization of the specimen. As generally recognized this method gives us the reproducible results, but has not been applied when the material is in the form of sheets, simply because the demagnetizing factor is not known except for the case that the specimen is either ellipsoidal[1] or cy-

[1] In this case the demagnetizing factor can be calculated analytically.

lindrical[2] and the sheet can not be put into such a form. To remove the restriction, the following research work has been performed by me.

§ 2. The Demagnetizing Factors of Rectangular Strips in the Region of Discontinuous Magnetization

a) Preliminaries

Let I be the magnetization of the specimen, H the effective field, H_0 the external field and N the demagnetizing factor respectively, there holds the following relation;

$$H = H_0 - NI, \qquad \ldots\ldots\ldots(1)$$

where I is obtained from the deflection of the ballistic galvanometer and H_0 is from the coil constant and the magnitude of the current through the coil. For the case of an ellipsoid N is calculated according to the following formula;

$$N_a = 2\pi abc\int_0^\infty \frac{du}{(a^2+u)\sqrt{(a^2+u)(b^2+u)(c^2+u)}},$$

where a, b and c are the semi-axes of the ellipsoid and the magnetization is along the direction of a-axis. When $a \gg b \gg c$, the above formula can be transformed into the simple one as below;

$$N_a = (4\pi bc/a^2)\ln(4a/b). \qquad \ldots\ldots\ldots(2)$$

Though the ellipsoid is the most convenient form for calculation, it is practically impossible to prepare especially when the material is in the form of sheets. Among many shapes, the rectangular strip is the most convenient to prepare from the material in the form of sheets, which is to be considered at first. Let $2a$, $2b$ and d denote the length, the width and the thickness respectively, and when $a \gg b \gg d$, the demagnetizing factor of the strip seems to approach to that of the ellipsoid with the semi-axes of a, b and $d/2$. Thus it may be convenient to express the demagnetizing factor as follows;

$$N = K(2\pi b/a^2)\ln(4a/b)\cdot d, \qquad \ldots\ldots\ldots(3)$$

where K is a numerical factor to be determined by the following experiments and responsible for the fact that the shape is deviated from the ellipsoid.

In general K would be a function not only of a, b and d but also of I. In the region of discontinuous magnetization, however, where the relation between I and H is almost linear, K would be the function of the dimentions and the slope of the curve. As far as the dimensions are concerned since the principle of similitude must hold, K is dependent only upon the ratios of the dimensions and when d is small compared with a or b, it may be sufficient to take into consideration only the first order term; that is to say, K in (3) is assumed to be dependent only on the ratio of a and b.

Though N depends generally upon the slope of the curve as well as the dimensions

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[2] For this case the demagnetizing factors are known from the experimental data by Shuddenmagen.

except for the case of the ellipsoid, N for the case of the rectangular strip, as far as the first approximation is concerned, may be assumed to be almost independent on the slope of the curve, since the rectangular strip is not so far from the ellipsoid when $a \gg b \gg d$. Accordingly K in (3) would be dependent only on the ratio of a and b, as justified by the following experiments.[3]

b) Independence of K of thickness

If it were possible to prepare several specimens with the same length and the same width but different thickness, made from the *magnetically identical* material, the above assumption would be checked from measurement of I versus H_0 on the specimens. In Schuddenmagen's experiment[4] to determine the demagnetizing factors of cylinders, he prepared a cylinder of pure iron of which the first measurement was carried out and after cutting it carefully on a lathe to reduce the diameter, the second measurement was done of it and so on. From a series of measurements thus carried out, he determined the demagnetizing factors of the cylinders. Though the method similar to his may be adopted with success, it is doubtful that the specimen should not suffer any strain when it is cut and even if it were not the case, it is desirable to ascertain the fact before every measurement. As for the method to ascertain the fact, no description can be found in his report.

Partly on account of the hope to exclude the ambiguity and mainly for the convenience of experimental technique, the following method has been adopted by me. Many specimens of the exactly same length (300 ± 0.5 mm) and width (10 ± 0.1 mm) but of different thickness (from 0.2 mm to 1 mm) are prepared by rolling down a thick sheet of electrolytic iron and annealed in the atmosphere of hydrogen for several hours at the temperature of about 800°C. Then the values of H_0 versus I at several points on the loop which corresponds to the maximum value of $4\pi I$ equal to 10,000 gauss, are precisely measured on each specimen. According to the values of H_0, the coercive forces, the specimens are classified into several groups. The relations between H_0 and I measured on the specimens, different in thickness but belonging to one group, are shown in Fig. 1. From this the relation of H_0 versus d for the same value of I on the ascending branch or on the descending branch of the loop, is obtained as shown in Fig. 2, in which the values of H_0 for I equal to 600 gauss on the ascending branch are plotted against d. The curve shown in Fig. 2 suggests us that H_0 increases linearly with d; that is to say, N is proportional to d or K is independent of d.

Using the relations $H = H_0 - NI$ and $N = K(2\pi b/a^2) \ln (4a/b) \cdot d$, it follows at once that the intercept of the curve with its ordinate gives the effective field and the slope gives the magnitude of NI divided by d, namely the magnitude of $K(2\pi b/a^2) \ln (4a/b) I$, from which K can be calculated by substituting the values of a, b and I. Besides

[3] It would be desirable to justify the above assumption by the analytical treatment as well as by the experimental evidences, but in spite of the author's effort, it is practically impossible to obtain the solution which is accurate enough to be useful in futher discussion.

[4] Shuddenmagen, C. L. B.; Phys. Rev. **31** (1910), 165.

the one shown in Fig. 2 and Fig. 3, the similar curves are obtained for other groups. The value of K calculated for the four groups which have the coercive forces of 0.21 ± 0.01, 0.24 ± 0.01, 0.31 ± 0.01 and 0.33 ± 0.01 oersted respectively, are summerized in the following formula;

$$K = 0.83 \pm 0.01.$$

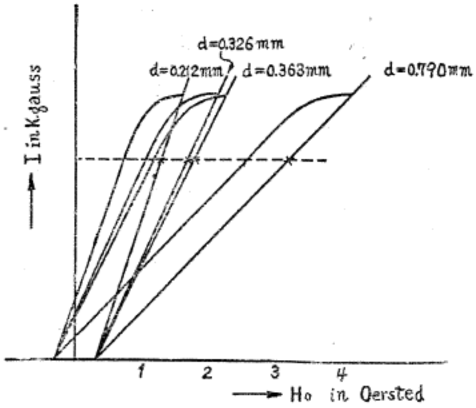


Fig. 1

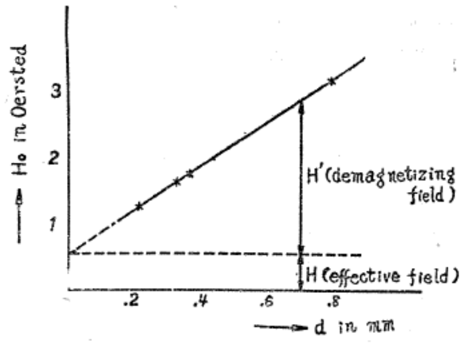


Fig. 2

The method adopted above, is based on the assumption that the loops of any two specimens which have the same magnitude of the coercive force, coincide into a single figure when plotted so as to show the relation between I and H. Though the theory of ferromagnetism does not afford any positive bases supporting the assumtion, it is justified empirically at least for the case of pure iron.[5]

c) *Dependence of K of b/a*

The next step is to determine K as a function of b/a. For this purpose the above method may serve but the following method has been adopted for the sake of convenience. As well known a single crystal of iron exhibits a marked knee on the descending branch of the loop as shown in Fig. 3. The above fact is due to the abrupt change in mechanism of magnetization across the knee, and it is expected that the knee appears where the effective field is almost null, though the remanence, the value of magnetization at the knee varies according to the direction along which it is magnetized; for instance, in the case of iron the values are re-

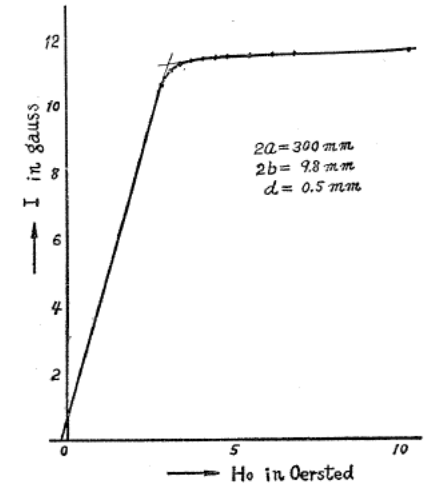


Fig. 3

[5] For instance we can find an evidence supporting the assumption in Yensen's report in which he publishes the values of H_c and hysteresis loss for the maximum value of $4\pi I$ equal to 10,000, measured on many specimens of pure iron. (Yensen, T. D.; Trans. Amer. Inst. Elect. Eng., **43** (1924), 145.)

spectively I_s for the direction of [100], $I_s/\sqrt{2}$ for [110] and $I_s/\sqrt{3}$ for [111], where I_s is the saturation value.

Thus the single crystals of iron which are in the form of long strips, about 10 mm in width and about 0.5 mm in thickness, have been prepared. After the value of H_0 for the point of the knee is precisely measured for a certain value of length, it is diminished in length and annealed, of which the next measurement is made and so on. The relations thus obtained between the length and the external field at the knee are shown in Table 1 for two crystals which have the remanence of 1,160 gauss and 1,420 gauss respectively.

Table 1.

$2a$	30.0 cm	25.0	20.0	15.0	12.1	10.0	7.0	5.0
H_0	3.1 Oe	4.1	5.9	9.7	13.8	18.3	31.9	52.8

$I_r=1.16$ K-gauss, $2b=9.8$ mm, $d=0.5$ mm.

$2a$	30.1 cm	25.0	20.0	15.0	12.0	10.0	7.0	5.0
H_0	3.8 Oe	5.0	7.3	11.8	17.1	23.0	39.8	63.7

$I_r=1.42$ K-gauss, $2b=9.7$ mm, $d=0.5$ mm.

Table 2.

b/a	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
K	0.88	0.84	0.81	0.79	0.77	0.75	0.74	0.73
b/a	0.10	0.11	0.12	0.13	0.14	0.16	0.18	0.20
K	0.71	0.70	0.69	0.68	0.67	0.65	0.63	0.62

From the above data the values of N, accordingly the value of K are obtained against b/a as shown in Table 2. The values of N/d calculated from the values of K are shown in Table 3 for the practical purpose.

Table 3.

$2b\rightarrow$ / $2a\downarrow$	0.5 cm	1.0	1.5	2.0	2.5	3.0	3.5	4.0
5 cm	0.0656	0.0916						
10	0.0215	0.0328	0.0405	0.0460				
15	0.0110	0.0172	0.0218	0.0255	0.0283	0.0307		
20	0.0068	0.0108	0.0139	0.0164	0.0185	0.0203	0.0217	0.0230
25	0.0046	0.0074	0.0097	0.0115	0.0131	0.0145	0.0157	0.0180
30	0.0033	0.0054	0.0071	0.0085	0.0097	0.0108	0.0118	0.0126
35		0.0042	0.0056	0.0067	0.0077	0.0086	0.0093	0.0101
40		0.0034	0.0045	0.0054	0.0062	0.0069	0.0076	0.0082
45		0.0028	0.0037	0.0044	0.0051	0.0057	0.0063	0.0068
50		0.0023	0.0031	0.0037	0.0043	0.0048	0.0053	0.0058
55		0.0020	0.0026	0.0032	0.0037	0.0041	0.0046	0.0049
60		0.0017	0.0023	0.0027	0.0032	0.0037	0.0040	0.0043

N/d (d in mm)

In addition the values of K are checked for some values of length and width. Though trivial disagreement occurs, it is always below the amount of about 2%, so that the values shown in the above tables seem to be fairly satisfactory.

§ 3. The Demagnetizing Factors of Rectangular Strips in the Region of Continuous Magnetization

It is readily seen that the demagnetizing factor of a rectangular strip calculated under the assumption of uniform magnetization becomes $4bd/a^2$ for the case of $a \gg b \gg d$, and thus it is fairly smaller than that for the region of discontinuous magnetization; for instance, substituting the values of $a=150\,\mathrm{mm}$, $b=5\,\mathrm{mm}$ and $d=1\,\mathrm{mm}$, $4bd/a^2$ becomes 0.0008 while Table 3 gives us the value of 0.0054. Thus the demagnetizing factor would decrease in the region of continuous magnetization with the increase of I, since in the state of magnetic saturation the magnetization is uniform.

To get the general idea of the problem, the distribution of the flux along the direction of the length is measured as shown in Fig. 4, where x is the distance from the centre and the ordinate gives the ratio of the flux at x to that at the centre. Since the difference between the magnetic induction and $4\pi I$ is small in the present case, Fig. 4 may be regarded as showing the distribution of the relative values of I; Strictly speaking it is the mean value of I averaged over the cross section at x.

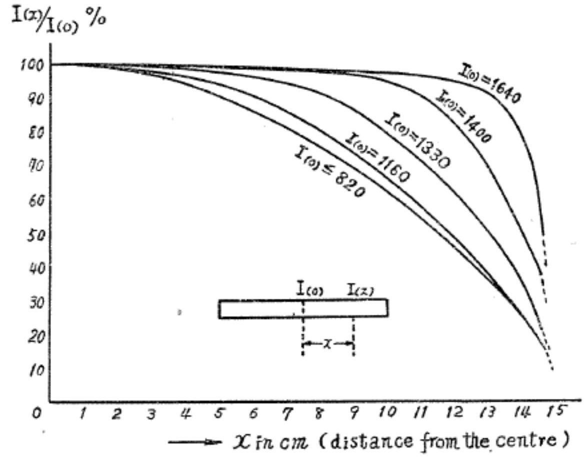


Fig. 4. A rectangular strip of pure iron. $2a=300\,\mathrm{mm}$, $2b=9.8\,\mathrm{mm}$, $d=0.5\,\mathrm{mm}$.

As easily seen, the distribution does not change in the region of discontinuous magnetization, while it varies with increasing I and approaches gradually to the state of uniform magnetization. Similar curves may be obtained on other specimens, but the shapes of the curves as well as their dependence on the magnitude of I at the centre are widely different from specimen to specimen and there is no simple correlation between them; for instance when the specimens are of single crystal, the shapes of the curves for the same magnitude of I at the centre, are quite different according to the orientation of the crystals. Thus as for the polycrystalline specimen the situation depends upon their anisotropy, which makes the problem too complicated to be solved.

The case which can be discussed without any consideration on the above difficulties, is when the strip is so long that K in (3) might vary very slowly with the change in length and accordingly N might vary almost in proportion to $(\ln a)/a^2$. The values of H_0 for the same I, measured gradually diminishing the length of a rectangular strip, are plotted against $(\ln a)/a^2$ as shown in Fig. 5. When extrapolated, the intercept with the ordinate gives the effective field and the difference of H_0 and the effective field gives the demagnetizing field, from which we can obtain K as the function of I as shown in Fig. 6.

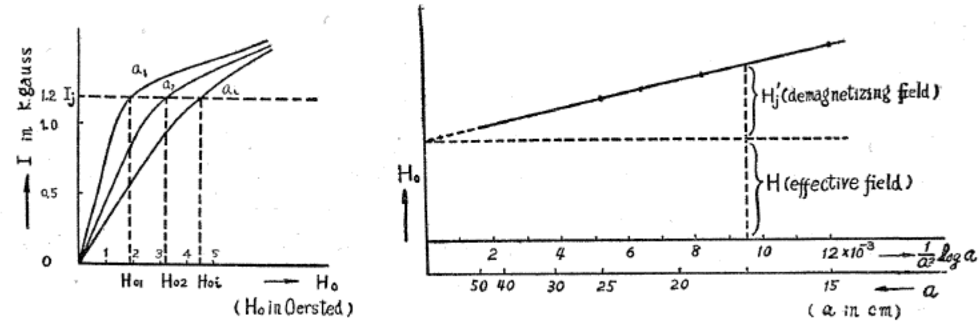


Fig. 5

It must be noticed that the dependence of K on I varies widely from specimen to specimen even if they are equal in dimensions. Thus the results can not be summerized into any compact form as far as the present problem is discussed without any deeper insight into the anisotropic properties of the material.

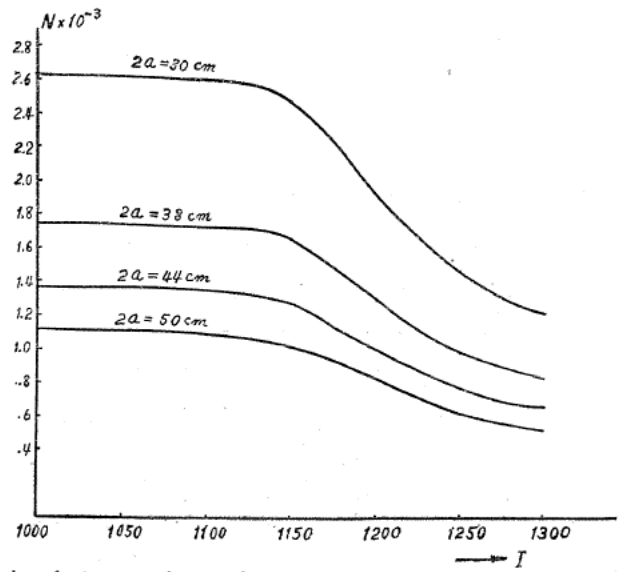


Fig. 6. Demagnetizing factor specimen of pure iron. $d = 0.5$ mm, $2b = 9.74$ mm, $2a = 50 \sim 30$ cm.

§ 4. The Demagnetizing Factors of Elliptic Strips

Though the rectangular strip is the most convenient to prepare from the sheet, its demagnetizing factor varies in the region of continuous magnetization in the complicated way. Thus it is sometimes desirable that the measurement is made on a specimen of such a form that the variation of the demagnetizing factor can be neglected all over the whole range of magnetization. In addition it must be the shape that can be prepared easily from the sheet. Among many shapes that serve for this purpose, the elliptic strip is considered here. The demagnetizing factor of the elliptic strip of 0.5 mm in thickness with the semi-axes of 150 mm and 5 mm, is determined from the measurements of H_0 and I at the knee which appears on the descending branch of the loop of the single crystal. The result is as follows;

$$N = (0.98 \pm 0.01)(2\pi\, b/a^2) \ln (4\, a/b) \cdot d$$

Since the above formula gives the demagnetizing factors for the region of discontinuous magnetization, the next step is to calculate N in the state of saturation. Though the approximate calculation is only possible for this case, it can be shown that N is expressed with the error below the amount of 1% as follows;

$$N = (2\pi\, b/a^2) \ln (4\, a/b)\, d$$

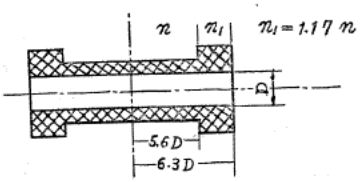
Thus we can use the above formula for the whole range of magnetization if we can disregard the error of about 3%.

It is not always necessary, however, to use the elliptic strip; sometimes the rectangular strip may serve quite satisfactorily. When $a \gg b \gg d$ N is very small and thus the error in N is not serious for the results of the measurements, so that we may neglect entirely its change in the region of continuous magnetization.

§ 5. Some Remarks on the Apparatus

a) *The magnetizing coil and the power supplying device*

Some remarks on the apparatus are to be mentioned here as a supplement to the report of the research work. For the measurement by the method of shifting the pick-up coil, the long magnetizing solenoid is necessary which may produce the



Distance from the centre	3.5 D	4.2D	4.9 D	5.3D	5.6D
Relative intensity of the field ($H(x)/H(o)$)	0.9967	0.9938	0.9886	0.9910	1.0000

Fig. 7

field almost uniform all over the range of about one and half of the length of the specimen. Since some devices such as the movable coil, its guide, the holder of the specimen, etc. should be included inside the solenoid, its diameter becomes fairly large. Thus it is usual in practice that the solenoid is about three times as long as the specimen. It is effective, however, in reducing the length of the solenoid to use the suitable supplementary windings, which not only improve the uniformity of the field but elongate the range of uniform field as shown in Fig. 7.

The magnetizing coil is excited conveniently by the current from the power unit as shown in Fig. 8. The current is varied simply by changing the grid voltage and the effect of the fluctuation in voltage of the a.c. source is eliminated by stabilizing the applied voltage of the screen grid by means of glow tubes.

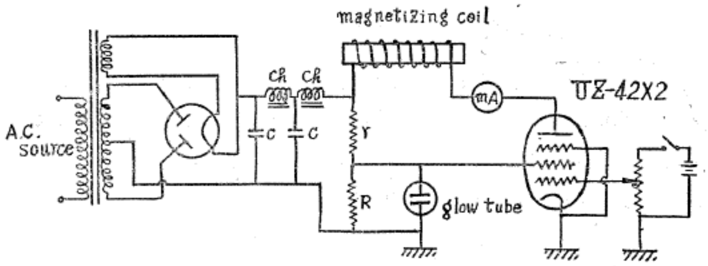


Fig. 8

b) The pick-up coil

Though it is assumed hitherto that I is precisely determined from the deflection of the galvanometer, it is not the case especially when the cross sectional area of the pick-up coil is considerably larger than that of the specimen. As previously mentioned the galvanometer is deflected by the impulse which is produced by the change in flux through the pick-up coil when it is shifted away from the centre of the specimen to an arbitrary position within the range of uniform external field, where the effect of the demagnetizing field is negligibly small. Let Φ_1 denote the flux through the coil before it is shifted, Φ_2 the flux after, A the cross sectional area of the pick-up coil, q that of the specimen, H'_n the normal component of the demagnetizing field measured in opposite direction to I or H_0, and n the number of turns of the coil, and then Φ_1 and Φ_2 are given as follows;

$$\Phi_1 = n(H_0 A + 4\pi I q - \iint_A H'_n df),$$

$$\Phi_2 = n H_0 A,$$

where df is the elementary area of the cross section of the coil. Accordingly the change in flux is as below;

$$\Phi_1 - \Phi_2 = n(4\pi I q - \iint_A H'_n df).$$

If the specimen is uniformly magnetized and A is nearly equal to q, it is assumed H' is also uniform all over the area of A and thus, using the relation $H' = NI$, we can get the following simple relation;

$$\Phi_1 - \Phi_2 = n(4\pi q - NA)I$$

from which I is determined since the deflection of the galvanometer is proportional to $(\Phi_1 - \Phi_2)$.

Since it is not always the case, however, the effect of the term $\iint_A H_n' df$ on the deflection of the galvanometer must be examined. When the deflection which corresponds to the point of the knee on the descending branch of a single crystal is observed, it decreases in general according to the diminution in the length of the crystal. Fig. 9 gives such an extreme example for the above case that we can hardly find it in the practical apparatus. This fact is due to the effect of the term $\iint_A H_n' df$.

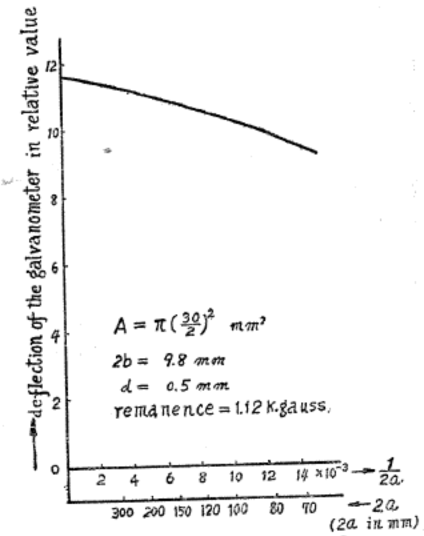


Fig. 9

As for the case of the rectangular strip, since $\iint_A H_n' df$ is always smaller than NIA, thus it can be expressed as follows;

$$\iint_A H_n' df = \alpha NIA,$$

where α is a factor which is unknown but unquestionably smaller than unity. Substituting the relation $N = K(2\pi b/a^2)\ln(4a/b)d$ and $q = 2bd$, we obtain the relation as below;

$$\iint_A H_n' df = \frac{A}{a^2}\left(\alpha K\pi \ln \frac{4a}{b}\right) Iq,$$

hence

$$\Phi_1 - \Phi_2 = 4\pi n I q (1 - \beta),$$

where β is as below;

$$\beta = K\frac{A}{4a^2}\ln\frac{4a}{b} < \frac{A}{4a^2}\ln\frac{4a}{b}.$$

From the above formula we can obtain some knowledge about the problem; for instance when the specimens are long enough the pick-up coil with a large area may be used without any serious error.

c) The connection of the circuit

Finally some remarks on the electric circuit is to be mentioned. The connection diagram is shown in Fig. 10. The mutual inductance and its primary circuit are useful only for the calibration of the sensitivity of the balistic galvanometer.

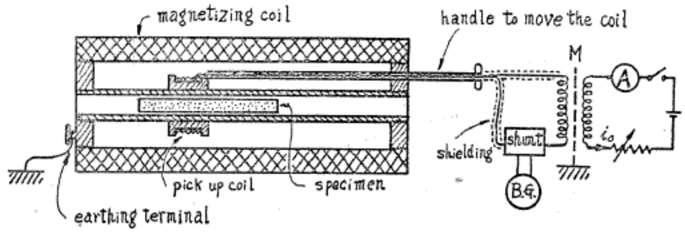


Fig. 10

It seems very important to shield the circuit of the galvanometer entirely from that of the magnetizing coil; owing to the attention paid to the shielding, there occurs no trouble due to the leakage current, even in rainy season.

I wish to express my thanks to Mr. Osamu Yamada, Mr. Susumu Maruse and Mr. Masahiro Kato for their advices and helps which encouraged me throughout the work.