ON THE POSITIVE COLUMN OF ELECTRICAL DISCHARGE WITHOUT WALLS

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Contents

I. Introduction

Until recent years, the theoretical works on positive column of electrical discharge were presented only for the glow in low pressure tube[1] and for the thermal ionization arc at high pressures without walls,[2)3)4] for example in the atmosphere. The other discharge column except the above has not yet been employed, one could not have computation for it, whether ionization would be caused thermally or electrically and also electron loss be by diffusion or recombination in every case.[5] These problems are involved in the various types of discharges appeared in the region through the whole range of the self-sustained discharge in the atmosphere from the smallest current glow discharge to the large current pure thermal arc discharge, and also in the discharges at moderate pressure.

In this paper, the author will present a theoretical treatise on general discharge column in free space and its application to the evaluation of discharge characteristics in the air, comparing with the experimental data.

II. General View

As for the column above described, however, there are some difficulties as below mentioned, owing to which we have had a defect in the general view of discharge column. Now our problem may be summarized as the relations between following quantities.

(i) Balance of the charged particles: the positive column may be regarded as a neutral plasma, so that the electron density equals to that of the positive ion, denoted n/cm^3, n can be given by a function of electron temperatre T_e and gas temperature T_g, i.e.,

$$n=f_1(T_e,\ T_g).$$

(ii) Potential gradient E: By the gas kinetics, E can be expressed by T_e and T_g, as

$$E=f_2(T_e,\ T_g)$$

if the energy loss fraction κ of the electron by collision with molecule is known.

(iii) Current density j:

$$j=nek_eE.$$

The electron mobility k_e in this is a function of T_e and T_g in the given gas and pressure, so that

$$j=f_3(n,\ T_e,\ T_g).$$

(iv) Discharging current I: We have at once,

$$I=\pi R^2 j=\int_0^R 2\pi j_r r dr,$$

where R is the radius of the column.

(v) Energy Balance: Electrical input per unit length EI equals to the loss per unit length.

From these five relations, giving one of the seven unknown E, j, I, R, T_e, T_g and n, usually E or I, being easily measurable, all the others should be solved. At low pressures, the ionization is caused by the electron impact accelerated in electric field (called field ionization, which depends only on the electron temperature) and the impact caused mostly inelastically, so that no thermal loss appears. Then, the gas temperature will be kept essentially at a known constant value. Also, the radius of column is given by the tube size. Hence, even if we except the relation (v) because of the difficulty of computation for the energy loss of luminous radiation, we can solve the other five unknown from the four equations by giving one of them quantity. This is the theory of the glow discharge column at low pressure.[1)]

While in the thermal column, usually sustained in free space, R remains unknown, one may approximate it as isothermal plasma, i.e. $T_e \cong T_g$, resulted from the sufficiently good transference of electron energy to gas molecule. And in addition to this favarable condition, the ionization depends only on T_g (called thermal ionization). At high pressures, owing to the low value of electron energy obtained from the field in his one mean free path, an electron loses his energy chiefly by elastic collision, in other words, electrical input is transformed into heat, the thermal motion of the gas molecules, which is in balance with cooling rate givien by a function of T_g and R. Hence we have for the relation (v) $EI = f_4(T_g, R)$. Thus it seems to be soluble, on account of six unknown. But it does not to be so, because the assumption $T_e = T_g$ means an ideal case of no electric field, $E=0$, then $j=0$ which should not be the case. Therefore we must set the relations (iii): $j = nek_eE$ and $T_e \cong T_g$, $E \neq 0$ in the place of (ii) and (iii), giving up the relation (ii). Consequently, if $T_e = T_g$, the one unknown being eliminated, it would diminish one relation, one of (ii) or (iii), and if $T_e \neq T_g$, it would add one unknown. In each case one can not solve this.

For this trouble, several theories have been proposed under an assumption. Stone, Lamar and Compton[4)] had carried out the calculation of the temperature distribution etc, without assumption, but this is somewhat inadequate to be adopted to general theory.

The glow discharge in the atmosphere has the mediate natures of the above two typical forms of discharge, containing the mechanism of either thermal- or field-ionization, diffusion- or recombination-loss, and elastic- or inelastic-collision as the case may be. Also, being sustained in free space, R becomes unknown. Under these difficult and confused conditions we should treat the general theory of positive column.

In this paper, we may confine the problem to the thermal loss column, because of the lack of materials for the energy relation in the luminous radiation column. Of course it might be able to apply this to those other columns, if whose energy relation could be obtained.

We will conclude present section on the measurable quantities of the column to be compared with the theoretical calculation. The measurable quantities in free space of high pressures are E, I, R, j, and T_g, and by the aid of gas kinetics we can know the electron mobility, so that one may decide the remaining two quantities, T_e and n, if the gaseous constant (mean free path of electron and κ) is known.

The direct measurement of n and T_e would not be attained unless a new method like of a micro wave technique is found in place of Langmuir's probe method, which is unsuitable for high pressure and high temperature. In this paper later, the validity of formulae will be ascertained as to E, I, and R or j, since those can be measured easily and reliably.

III. General Relaions

1. Potential gradient and current density

We have now to obtain the relation between electron temperature T_e, gas temperature T_g and field strength E. From the motion of the electron in gas, we have the following relation [E. u. S. I. 185] [L. 183.]*

$$T_e=\frac{T_g}{2}+\sqrt{\left(\frac{T_g}{2}\right)^2+\left(\frac{\sqrt[4]{\pi}\, eE\lambda_e}{2\sqrt{2}\, k\sqrt{\kappa}}\right)^2}, \qquad \cdots\cdots\cdots(1)$$

where λ_e is the mean free path of electron and k, Boltzmann's constant. From this, expressing E in V/cm

$$E=1.83\times10^{-4}\frac{\sqrt{\kappa}}{\lambda_e}\cdot T_e\sqrt{1-\frac{T_g}{T_e}}. \qquad \cdots\cdots\cdots(1a)$$

Denoting the electron mean free path of 0°C, 760 mm Hg, by λ_0, that of T_g°K, p mm Hg is

$$\lambda_e=\lambda_0(760/p)(T_g/273), \qquad \cdots\cdots\cdots(2)$$

with this, we have

$$E=0.05\left(\frac{p}{760}\right)\left(\frac{\sqrt{\kappa}}{\lambda_0}\right)\left(\frac{T_e}{T_g}\right)\sqrt{1-\frac{T_g}{T_e}}. \qquad \cdots\cdots\cdots(1b)$$

The energy loss fraction κ is in general of a complex property, especially at an inelastic collision with polyatomic gas, but in this case of the air it may be considered to be a constant 2×10^{-3} in the range of electron temperature 5,000$\sim$10,000°K as shown in Fig. 1. And also λ_0 may be a constant 2.92×10^{-5} cm for the air at about $T_e=10^4$°K. [L. 181]. Substituting these values in (1b), we have for the air

$$E=0.1\, p\left(\frac{T_e}{T_g}\right)\sqrt{1-\frac{T_g}{T_e}} \quad \text{V/cm} \quad (p \text{ in mm Hg}). \qquad \cdots\cdots\cdots(1c)$$

Our next step is to derive the current density

$$j=nek_eE, \qquad \cdots\cdots\cdots(3)$$

in which n is mean electron density and k_e is electron mobility given by [L. 186].

$$k_e=\frac{0.815\, e}{\sqrt{3m_ek}}\cdot\frac{\lambda_e}{\sqrt{T_e}}, \qquad \cdots\cdots\cdots(4)$$

where m_e is electron mass. From these we have

* As it is often quoted, the books below are denoted in this paper as follows:

[E. u. S.]: Engel u. Steenbeck: Physik und Technik der Elektrische Gasentladungen. Bd I. (1933), Bd II. (1934).

[L.] : Loeb: Fundamental Processes of Electrical Discharge in Gases (1939).

$$j=0.64\times10^{-13}\sqrt{\kappa}\,n\sqrt{T_e}\sqrt{1-\frac{T_g}{T_e}}\ \ \mathrm{A/cm^2}, \qquad \cdots\cdots\cdots(3a)$$

which is independent of λ_e, and also for the air

$$j=2.9\times10^{-15}n\ \sqrt{T_e}\sqrt{1-\frac{T_g}{T_e}}\ \ \mathrm{A/cm^2}. \qquad \cdots\cdots\cdots(3b)$$

2. Energy balance relation

In high pressures, the electrical input EI to the column of unit length is transfered to thermal motion of molecules by an elastic impact of electron. While, the thermal loss per unit length is assumed to be caused by gaseous thermal conduction, the equilibrium may be written as

$$EI=-2\pi\int_0^R\frac{d}{dr}\left(r\Theta\frac{dT_g}{dr}\right)dr=-2\pi R\left[\Theta\frac{dT_g}{dr}\right]_{r=R}, \qquad \cdots\cdots\cdots(5)$$

where Θ is the thermal conductivity of the gas, a function of T_g. Commonly T_g means average gas temperature, but here means temperature distribution. The energy balance condition may consist of three factors, radius R, thermal conductivity or temperature and temperature gradient at the circumference. The relations between them are very complex and was caluculated for N_2 gas,[4] but not resulted to a general theory.

Now we shall apply to this column what is ordinarily called minimun principle,[2] which states that the column may select his radius and temperature distribution so as to make his loss or input minimum for the given circumferential temperature. So that

$$\frac{\partial(EI)}{\partial R}+\frac{\partial(EI)}{\partial(dT_g/dr)_{r=R}}\cdot\frac{d(dT_g/dr)_{r=R}}{dr}=0, \qquad \cdots\cdots\cdots(6)$$

or

$$R\left(\frac{dT_g}{dr}\right)_{r=R}=F,\ \ \text{constant}, \qquad \cdots\cdots\cdots(6a)$$

where F is a constant, independent of R and temperature gradient, but a function of circumferential temperature. Here we assume F to be also independent of T_g, then F is a constant associated with gas, and may be experimentally determined as described later (Eq. 7a). Thus we can see that when the radial temperature gradient or the cooling rate becomes greater, the thermal column contracts his radius of himself in order to prevent the thermal loss, that agrees with the facts in arc phenomena.

We have at once, combining (5) with (6a),

$$EI=-2\pi F\Theta_{r=R}. \qquad \cdots\cdots\cdots(5a)$$

The thermal loss shall be determined only by the thermal conduction at the circumference.

Ordinarily Θ is proportional to the thermal velocity of the molecule or the square root of the gas temperature within the temperature at which the molecules cause remarkable thermal dissociation. Hence one can put the following,

$$\Theta_{r=R} \propto \sqrt{T_{g_{r=R}}} \propto \sqrt{T_{g_{r=0}}} \propto \sqrt{T_{g_{\mathrm{mean}}}}\,. \qquad \cdots\cdots\cdots(5b)$$

approximately when the temperature difference between center and circumference is not large. (The calculated temperature distribution in reference (4) shows nearly this relation.)

Then we have

$$EI = C\sqrt{T_g}\,. \qquad \cdots\cdots\cdots(5c)$$

This shall be held approximately for the general thermal conduction column,* when the temperature does not cause considerable dissociation of gas molecule (for N_2 up to 3500°K),[4] even if the central temperature or mean temperature is remarkably high.

From the formulae (1a), (3a) and (5c), the constant C is given by,

$$C = 4.2\times 10^{-18}\frac{\pi}{\lambda_0}pR^2 n\kappa\left(\frac{T_e}{T_g}\right)^{3/2}\left(1-\frac{T_g}{T_e}\right) \quad \mathrm{W/cm/degree^{1/2}}. \qquad \cdots\cdots(7)$$

Here the constant C shall be originally determined by the nature of gas (thermal conductivity), howewer in the present stage, for the lack of the knowlege about temperature distribution, we can not settle the value. Therefore the value of C must be decided from the experimental data which gives

$$C = 0.68 \qquad \cdots\cdots\cdots(7a)$$

for the air, 760 mm Hg, and, then

$$EI = 0.68\sqrt{T_g}\,. \qquad \cdots\cdots\cdots(5d)$$

3. *Electron temperature*

We are now interested in finding how much the electron temperature shall be in the column under various conditions. To do this we may deal with the loss fraction κ and the constant C of last section.

Remembering the fact that the derivation above should have been obtained simply by the thermal equilibrium of thermal component of input with cooling loss, we see that this relation is quitely indifferent to the discharge quantities E, I, T_e, T_g, n, and R. In so far as κ is the loss fraction of electron at elastic collision,** C in eqs. (5d) and (7) should be kept at a constant value, peculiar to the gas, for any value of κ.

* According to reference (4) convection can be neglected within the current of several hundred mA in the air. Also luminous radiation may be negligible small. [E. u. S. II. 88 (164)]

** In the present case, inasmuch as the electrical input are all changed into heat, κ should be of pure elastic collision. However, we may extend it to the following. In the high pressure discharge, the loss fraction κ involves not only that of elastic collision, but also that of inelastic collision to some extent as a mean. What is called here elastic collision loss means the energy loss of the electron transformed to the thermal motion of molecules by a collision, being caused of course by elastic collision as well as by inelastic collision in such a manner that the activated molecule may give his internal energy (of rotation, vibration, dissociation and ionization) to molecule, transfering to translational energy at a collision of each other. Thus the actual loss fraction will ordinarily far exceed that of pure elastic collision, and we may regard it as a sort of loss fraction of elastic collision.

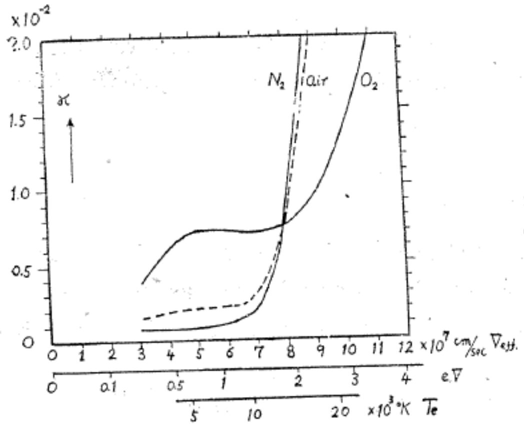

Fig. 1. Loss fraction κ, for N_2 and O_2 from [E. u. S. I. 187 abb. 101] and for the air calculated by their weighted mean.

As the dependence of κ on T_e is shown in Fig. 1, κ will increase steeply when T_e becomes over about 10,000°K in the air. This increase may be resulted by inelastic collision. The produced activated molecules, much more likely dissociated atoms may generate and transport the heat by means of their reduction to normal state of molecule in the way of their radial diffusion. This means the apparent increase of loss fraction at elastic collision as well as the increase of thermal conduction. The steep increase of κ corresponds to the increase of thermal conductivity of gas.

In the preceeding section, such increase of thermal conduction has not been considered, but now for the state in which the electron temperature is over 10,000°K for the air, C becomes a function of electron temperature. Hence we must take account of the electron temperature as well as gas temperature for the thermal balance.

Since the thermal column have the nature, minimizing the thermal loss, the electron temperature must be considered in first rank if this becomes the predominant factor for the thermal loss. In this case it is somewhat difficult to show quantitatively how the electron temperature will concern with it.

However, seeing that the gas temperature have only such a slight effect on the loss like $\sqrt{T_g}$, the electron temperature may be a predominant factor in the thermal balance at the point of steep increase of κ or thermal conductivity. With respect to the electron temperature, the discharge column will not be sustained at such a state resulting higher thermal loss.

Even if T_e exceeds 10,000°K, it must be brought back again to this value. In the columns just after the breakdown, their electron temperatures are expected to be several ten thousands degree owing to the high electric field, while they are non-thermal columns; however from the reason above, T_e shall be suppressed to about 10,000°K when they become the steady thermal state. By experimental data in later section, this value may be precisely determined as

$$T_e = 9800°\text{K}. \qquad \cdots\cdots\cdots (8)$$

4. Balance of charged particles

In our problem are involved the mechanisms of the electron generation by thermal and field ionization and that of the electron loss by diffusion and recombination. These combinations make four sorts of columns, of which diffusion-thermal ionization column may be excluded in this paper according to the unimportance to the actual discharge here.

(a) Field ionization-diffusion column

In the case of low gas temperature and low electron density, thermal ionization and electron recombination have not a essential role for the discharge, therefore this item can be applied for the small current discharge.

We may take ambipolar diffusion owing to the maintainance of neutral plasma unless it consists of extremely low electron density. Now making balance of the rate of loss of electrons (ions) due to diffusion with the generation of electrons (ions), we have well known relation for the radial distribution of electron density n. [E. u. S. II. 84], [L. 588]

$$\frac{d^2n}{dr^2}+\frac{1}{r}\frac{dn}{dr}+\frac{Z_e}{D_a}\cdot n=0, \qquad \cdots\cdots\cdots(9)$$

where D_a expresses an ambipolar diffusion coefficient, and Z_e, a number of ionization per electron per second. Here, Z_e, D_a are assumed to be uniform in radial direction at first approximation. The solution of equation (9) is known as

$$n=n_0J_0\left[\sqrt{\frac{Z_e}{D_a}}\,r\right], \qquad \cdots\cdots\cdots(10)$$

where J_0 is a Bessel function of zero order and n_0 is the electron density at the centre $r=0$, which remains undetermined. In equation (10) $n=0$ gives the radius R of the column as

$$R=2.405\sqrt{\frac{D_a}{Z_e}}. \qquad \cdots\cdots\cdots(11)$$

It is at first sight somewhat wonderful that the radius has been determined by the particle condition, while the radius in free space should be in principle by the thermal relation. This is due to the favourable coincidence of thermal condition with particle condition for the radius. That is, at the outside of the column, T_g may be lower than that in the column, and T_e according to the relation (1a) which holds in the outside as well. As the lowering of T_e decreases the ionization exponentially, n in the outside of the column becomes very small. Both the rapidly fallings of T_g and n give the same boundary of the column.

The total number of electron n_t contained in 1 cm length of the column is

$$n_t=\int_0^R 2\pi rndr=0.43\ \pi R^2n_0. \qquad \cdots\cdots\cdots(12)$$

The mean electron density* $\bar{n}$ is at once given by

* We may denote usually in this paper n for the mean density of electron.

$$\bar{n}=n_t/\pi R^2=0.43\ n_0. \qquad \cdots\cdots\cdots(12a)$$

Now let us compute the radius. Denoting k_{+0} cm/sec/V/cm for the mobility of positive ion at 0°C, 760 mm Hg, and assuming $k_+ \ll k_e$, and positive ion temperature equals to gas temperature we have for D_a

$$D_a=\frac{D_+k_e+D_ek_+}{k_++k_e}=\frac{300kT_e}{e}k_{+0}\frac{T_e+T_g}{T_e}\sqrt{\frac{T_g}{273}}\cdot\frac{760}{p}. \qquad \cdots\cdots\cdots(13)$$

For nitrogen molecule, putting $k_{+0}=1.28$ [E. u. S. I. 182]

$$D_a=8.5\times10^{-2}\frac{1}{p}(T_e+T_g)\sqrt{\frac{T_g}{273}}. \qquad \cdots\cdots\cdots(13a)$$

Next, Z_e, is given by Steenbeck [E. u. S. I. 89] as follows,

$$Z_e=\frac{600apv_j}{\sqrt{\pi}}\sqrt{\frac{2kT_e}{m_e}}\left[1+\frac{2kT_e}{ev_j}\right]e^{-ev_j/kT_e}, \qquad \cdots\cdots\cdots(14)^*$$

where a is a coefficient of ionization probability and v_j, the ionization potential of gas molecule. Putting

$$x=\frac{ev_j}{kTe} \quad (\text{usually } x\gg1), \qquad \cdots\cdots\cdots(15)$$

and substituting (13a) and (14) into (11), we have the radius for N_2

$$\left.\begin{aligned} R&=R_0\left(\frac{T_e+T_g}{T_e}\right)^{1/2}\left(\frac{T_g}{273}\right)^{1/4} \quad \text{cm},\\ R_0&=\frac{2.405}{2.86\times10^2}\cdot\frac{1}{p}\cdot\frac{e^{x/2}}{x^{1/4}\{1+(2/x)\}^{1/2}} \quad (p \text{ in mm Hg}), \end{aligned}\right\} \qquad \cdots\cdots\cdots(11a)$$

assuming $a=0.3$ $v_j=15.8$ V.

Thus, from the particle condition, we have not obtained the relation of electron density unexpectedly, but the radius relation.

(b) Field ionization-recombination column.

As it may be naturally well considered that no negative ions exist in the column, we may take the electron recombination here. The electron recombination coefficient ρ is supposed to be a function of T_e and T_g for the given gas and pressure, which will be derived in the next chapter. It will be seen at once that the equilibrium of the electron density may be given by

$$\frac{dn}{dt}=nZ_e-n^2\rho=0 \quad \text{or} \quad n=\frac{Z_e}{\rho}. \qquad \cdots\cdots\cdots(16)$$

This shall be adopted for high density discharge plasma.

* For the sake of later reference, it may be written also, taking $ev_j \gg 2kT_e$,

$$Z_e=6.17\times10^5\cdot av_jT_e^{1/2}e^{-ev_j/kT_e} \quad \begin{pmatrix} v_j & \text{V} \\ p & \text{mm Hg} \end{pmatrix}. \qquad \cdots\cdots\cdots(14a)$$

For N_2 giving $a=0.3$, $v_j=15.8$, then

$$Z_e=2.9\times10^6\cdot pT_e^{1/2}e^{-ev_j/kT_e}. \qquad \cdots\cdots\cdots(14b)$$

(c) Thermal ionization-recombination column

At the high temperature, mutual collision of gas molecules by their thermal motion, causes ionization. Let Z_T be the total number of ionization per second per cubic cm, we may write the equilibrium of the particle as

$$\frac{dn}{dt}=Z_T-n^2\rho=0 \quad \text{or} \quad n=\sqrt{\frac{Z_T}{\rho}}. \qquad \cdots\cdots\cdots(17)$$

The evalution for Z_T shall be carried out in the succeeding section.

Since, in this column the field intensity becomes lower, the field ionization term of electron, nZ_e, can be neglected, much less the ionization by thermal motion of electron, so that, when $T_e=T_g$ we have $nZ_e \ll Z_T$ as in the later described.

IV. Electron Recombination Coefficient

We have so far considered the role of the electron recombination coefficient for the maintainance of the positive column. The recombination coefficient for each molecule and molecular states can be computed by the quantum mechanical calculation, but the resulting formula does not present the simple form of the dependency on the molecular constants and macroscopic states as we desire here for our problems. Hence it is intended at present from the classical model to obtain a formula, especially adaptable to the state of high pressure and high temperature, which has not been yet treated and to apply it for the calculation of the discharge column.

1. Ionization by thermal motion of molecules

At high temperature, the ionization by thermal motion of molecules must be taken into consideration. When the relative velocity v (or relative kinetic energy $\frac{1}{2}\mu v^2$) exceeds a specified value at the collision between two molecules of mass, m_1 and m_2 respectively (μ=reduced mass$=m_1m_2/(m_1+m_2)$), the ionization may be assumed to take place. For simplicity, taking $m_1=m_2=m_g$, and let denote molecular radius σ cm, and gas density N/cm^3, we obtain the number of collision dZ_T per cubic cm and per second, in which the magnitude of the relative velocity lies in the range v, $v+dv$,

$$dZ_{T-1}=8\pi^2\sigma^2N^2\left(\frac{\mu}{2\pi kT_g}\right)^{3/2}v^3e^{-(1/2)\mu v^2/kT_g}dv. \qquad \cdots\cdots\cdots(18)^{6)}$$

For the total number Z_T of collision per cubic cm and per second in which the energy of relative motion exceeds the value,

$$\frac{1}{2}\mu v^2=\frac{1}{4}m_gv^2=ev_j \qquad \cdots\cdots\cdots(19)$$

are obtained by integration of (18) from v of (19) to ∞

$$Z_{T-1}=4N^2\sigma^2\left(\frac{4\pi kT_g}{m_g}\right)^{1/2}e^{-ev_j/kT_g}\left(1+\frac{ev_j}{kT_g}\right), \qquad \cdots\cdots\cdots(20)$$

which gives the number of ionization.*

* In this case we may consider ionization probability f_i, but as shown in the next section, one can see that f_i should be about unity from the experimental data of the electron recombination coefficient.

We shall also obtain the number of collision in which the translational energy in the direction of the line of centres at the moment of impact exceeds the above specified value, as following;

$$Z_{T-2}=4N^2\sigma^2\left(\frac{4\pi kT_g}{m_g}\right)^{1/2}e^{-ev_j/kT_g}. \qquad \cdots\cdots\cdots(21)$$

We must select either (20) or (21) for the ionization comparing with experiment. As later discussed in the derivation of recombination coefficient, the former will be suitable for the mechanism. Usually $ev_j \gg kT_g$, this may also be written by

$$Z_{T-1}=4N^2\sigma^2\left(\frac{4\pi kT_g}{m_g}\right)^{1/2}\frac{ev_j}{kT_g}e^{-ev_j/kT_g} \qquad \cdots\cdots\cdots(20a)^*$$

$$=1.75\times10^{35}\frac{\sigma^2 v_j}{\sqrt{m_g}}p^2 T_g^{-5/2}e^{-ev_j/300kT_g}. \qquad \cdots\cdots\cdots(20b)$$

(p in mm Hg, v_j in V.)

For N_2, putting $\sigma^2=3.5\times10^{-16}$ cm^2, $v_j=15{,}8$ V, $m_g=46.6\times10^{-24}$ g, we have

$$Z_{T-1}=1.42\times10^{32}p^2 T_g^{-5/2}e^{-ev_j/300kT_g}. \qquad \cdots\cdots\cdots(20c)$$

2. *Saha's equation of thermal ionization*

According to the thermodynamical equilibrium applied to thermally ionized state of gas, Saha has treated it as a chemical reaction of

$$\begin{pmatrix}\text{1 mol}\\ \text{gas}\end{pmatrix}+\begin{pmatrix}\text{ionization}\\ \text{energy for}\\ \text{1 mol}\end{pmatrix}\rightleftarrows\begin{pmatrix}\text{1 mol}\\ \text{electron}\end{pmatrix}+\begin{pmatrix}\text{1 mol}\\ \text{positive ion}\end{pmatrix}, \qquad \cdots\cdots\cdots(23)$$

and has derived the ionization degree $c=n/N$,

$$\frac{c^2}{1-c^2}=\frac{(2\pi m_e)^{3/2}}{h^3 p}(kT)^{5/2}e^{-ev_j/kT}, \qquad \cdots\cdots\cdots(24)$$

where h is Planck's constant and p in dyne/cm^2.

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* For the number of thermal ionization, similar formulae were proposed by analogous calculation in [E. u. S. I. 85] as below,

$$dZ=\frac{N}{\sqrt{2}\lambda_g}\left(\frac{v}{w}\right)^6 e^{-(v/w)^2}dv, \qquad \cdots\cdots\cdots(104)(364)$$

$$Z=\frac{4f_i}{\sqrt{2mg}}\frac{N}{\lambda_g}\frac{(ev_j)^{5/2}}{(kT_g)^2}e^{-2ev_j/kT_g}, \qquad \cdots\cdots\cdots(106)$$

where w is the most probable velocity, λ_g, mean free path and f_i the probability of ionization. Modifying our formulae by substitution of the relations

$$kT_g=\frac{1}{2}m_g w^2, \qquad \sigma^2=\frac{1}{4\sqrt{2}\,\pi N\lambda_g}, \qquad \cdots\cdots\cdots(22)$$

we have

$$dZ_{T-1}=\frac{N}{2\sqrt{\pi}\,\lambda_g}\left(\frac{v}{w}\right)^3 e^{-v^2/2w^2}dv, \qquad \cdots\cdots\cdots(18a)$$

etc.

Comparing dZ_{T-1}, dZ_{T-2}, consequently Z_{T-1}, Z_{T-2} with (104) and (106) respectively, it will see that the both do not coincide with each other and the derivations from these in several pages in that book are all essentially different from our results.

Uaually being $c \ll 1$, the equation (24) simply may be written

$$n^2 = 2.4\times10^{-4}\frac{N^2}{p}T^{5/2}e^{-ev_j/kT_g} \quad \text{and} \quad p = NkT. \qquad \cdots\cdots\cdots (25)$$

While, as the reaction (23) may be regarded as the equilibrium between ionization and recombination process in an isothermal plasma, one can write another form for it.

$$\frac{dn}{dt} = Z_T + nZ_e - n^2\rho = 0. \qquad \cdots\cdots\cdots (26)$$

The produced electron in this case should not concern with ionization as well as in the chemical reaction of (23). Then we may take $Z_e=0$, and have

$$n^2\rho = Z_T, \qquad \cdots\cdots\cdots (27)$$

which corresponds to Saha's equation (24) or (25).

3. Electron recombination coefficient

Now our step is to derive the electron recombination coefficient. Putting n^2 of (24) or (25) and Z_T of (20) or (21) into (27), we heve at once the electron recombination coefficient

$$\rho = \rho_1 = \frac{Z_{T-1}}{n^2} = \frac{4\sigma^2 h^3 p}{\pi\sqrt{2m_e^3 m_g}\,k^2T^2}\left(\frac{ev_j}{kT}+1\right). \qquad \cdots\cdots\cdots (28)$$

For $ev_j \gg kT_g$ (p in mm Hg, v_j in V),

$$\rho_1 = 7.85\frac{\sigma^2 v_j}{\sqrt{m_g}}\cdot\frac{p}{T^3}, \qquad \cdots\cdots\cdots (28a)$$

and

$$\rho = \rho_2 = \frac{Z_{T-2}}{n^2} = \frac{4\sigma^2 h^3 p}{\pi\sqrt{2m_e^3 m_g}\,k^2T^2}. \qquad \cdots\cdots\cdots (29)$$

For N_2 gas

$$\rho_1 = \frac{4.85\,p_{\text{atm}}}{T^3} = \frac{6.4\times10^{-3}p_{\text{mm Hg}}}{T^3}, \qquad \cdots\cdots\cdots (28b)$$

$$\rho_2 = \frac{2.55\times10^{-5}p_{\text{atm}}}{T^2}. \qquad \cdots\cdots\cdots (29a)$$

Table 1. Recombination coefficient
p=760 mm Hg, N_2

T°K	3,000	4,000	6,000
ρ_1	1.8×10^{-10}	0.8×10^{-10}	0.23×10^{-10}
ρ_2	2.8×10^{-12}	—	0.7×10^{-12}

In high temperature arc, comparing the calculated values of ρ by the above formulae, as shown in Table 1, with the known value $(1\sim2)\times10^{-10}$ [7] in the air, ρ_1 gives a sufficiently good agreement with this value. Hence we may adopt rather Z_{T-1} mechanism than Z_{T-2}.

However, in the expressions (28) etc., is contained only the temperature T, whereas the plasma in the actual discharge is not isothermal, as the electric field exsists and obviously will depend on T_e and T_g. In the above derivation for the isothermal plasma, T represents T_e, T_g and p/Nk. These are not separable in the thermodynamical treatment. We must therefore decide this separation of T into T_e and T_g, based on the experimental results and quantum mechanical calculation. In the measurements of recombination at low pressure and low temperature of Cs and Hg vapor the value of coefficient ρ is kept to be of the order of magnitude of 10^{-10}. And in general ρ is proportional to T_e^{-n} in which n are proposed to be the value from $\frac{1}{2}$ to 2 according to various investigations;[8] and ρ increases slightly with pressure. Referring to these results, we may take $n=\frac{1}{2}$ rather than others, as shown in Table 2.

Table 2. Recombination coefficient calculated for N_2

T_e T_g p	3,000°K 300°K 0.76 mm Hg	1,000°K 300°K 76 μ
$\rho' = 6.4\times10^{-3} p T_e^{-2} T_g^{-1}$	1.7×10^{-12}	1.5×10^{-12}
$\rho'' = 6.4\times10^{-3} p T_e^{-3/2} T_g^{-3/2}$	5.5×10^{-12}	3.2×10^{-12}
$\rho''' = 6.4\times10^{-3} p T_e^{-1} T_g^{-2}$	1.7×10^{-11}	5.3×10^{-12}
$\rho'''' = 6.4\times10^{-3} p T_e^{-1/2} T_g^{-5/2}$	0.6×10^{-10}	1.1×10^{-11}

That is, the recombination coefficient may be expressed by

$$\rho = 7.85 \frac{\sigma^2 v_j}{\sqrt{m_g}} \frac{p}{T_e^{1/2} T_g^{5/2}} \qquad (v_j \text{ in volt, } p \text{ in mm Hg}), \qquad \cdots\cdots(30)$$

and for N_2

$$\rho = \frac{6.4\times10^{-3} p_{\mathrm{mm\,Hg}}}{T_e^{1/2} T_g^{5/2}}. \qquad \cdots\cdots\cdots(30a)$$

V. Calculation of Discharge Characteristics in the Atmosphere

From the several relations between the quantities of positive columns and gaseous constants obtained in the preceeding sections, one can calculate the discharge characteristics for the given gas and pressure in free space. We will treat here the discharge in the air for which it is favourable because of the presence of experimental data. It may be no objection that we will take the constants for either N_2 or air for convenience, since they have analogous natures. However we must make distinction between them in the case as of ionization, which is affected exponentially by the ionization potential v_j. In such a case one should take strictly the value for the air. Now we may start with the smallest current glow discharge which are stably sustained in the atmosphere after the breakdown, and then, the calculations will be carried out successively to the larger current up to the arc discharge of several amperes.

1. *Field ionization-diffusion column*

At first we may consider the field ionization-diffusion column for the small current discharge, for which the electron density and the gas temperature shall be comparatively low. Then the problem is reduced to solve simultaneously the following relations for the given gas and pressure. In the air of $p=760$ mm Hg,

$$E(T_e,\ T_g)=0.1p\frac{T_e}{T_g}\sqrt{1-\frac{T_g}{T_e}}\quad \text{V/cm}, \qquad \cdots\cdots\cdots(1c)$$

$$j(n,\ T_e,\ T_g)=2.9\times10^{-15}n\sqrt{T_e}\sqrt{1-\frac{T_g}{T_e}}\quad \text{A/cm}^2, \qquad \cdots\cdots\cdots(3b)$$

$$I=\pi R^2 j, \qquad \cdots\cdots\cdots(31)$$

$$EI=0.68\sqrt{T_g}, \qquad \cdots\cdots\cdots(5d)$$

$$T_e=9{,}800^\circ\text{K}, \qquad \cdots\cdots\cdots(8)$$

$$R(T_e,\ T_g). \qquad \cdots\cdots\cdots(11a)$$

From (1c), (5d) and (8), one can obtain E-I characteristic taking T_g as parameter, as well T_g-R from (8) and (11a), and n, j from (3b) and (31) respectively. These results are shown in Table 3, comparing with experimental data. A very good agreement has been obtained except for the radius of the current of more than about 100 mA.

Table 3. Calculated values for the air 1 atm.
$T_e=9{,}800^\circ$K $v_j=15.0$ V

T_g°K	E V/cm	I mA	observed	R mm	observed	n/cm³
480	1,500	9.0	10	0.52	0.55	3.3×10^{12}
800	900	21.4	20	0.58	0.60	7.3×10^{12}
1,000	705	31.3	30	0.63	0.65	1.1×10^{13}
1,180	590	38.5	40	0.68	0.69	1.35×10^{13}
1,380	500	50.0	50	0.72	0.72	1.5×10^{13}
1,570	440	59.5	60	0.76	0.75	1.4×10^{13}
1,900	360	82.5	80	0.80	0.80	1.32×10^{13}
2,200	300	106	100	0.85	0.81	2.0×10^{13}
3,700	150	260	270	(1.02)	0.70	6.0×10^{13}

Considering of raduius, we will evaluate exactly the electron temparature. As described in the preceeding section, the electron temperature should be about 10,000°K. This value can be determined by means of the sensibly dependent nature of R upon T_e as in (11a). When T_e is nearly equal to 10,000°K, we can know the rough value of T_g from the experimental data with aid of the formula, (1c) or (5d.). With this value of T_g and the relation (11a), it will be seen that $x=ev_j/kT_e=18$ shall be fit for the measured value of R. That gives $T_e=9{,}800^\circ$K and $R_0=4.5\times10^{-2}$ cm for the air of $v_j=15.0$ V.

When the current becomes larger, the radius varies but with slight increase. Consequently the electron density becomes such rapidly larger that the electron recombination mechanism should be taken into account. Supposing that $I=100$ mA, $j=5$ A/cm², $E/p=0.5$, electron velocity $v=10^6$ cm/sec and $n=j/ev=3\times10^{13}$/cm³, the number of electron loss per unit time and unit volume by recombination $n^2\rho$ equals to about 10^{17}, taking $\rho=10^{10}$. While, from $T_e=10^4$, $k_{+0}=1$, $Z_e=\frac{2.4^2 D_a}{R^2}=\frac{2.4^2\times300\times kT_e\times k_{+0}}{R^2}$ $\cong2\times10^3$, we know also the number of electron generation per unit time and per unit volume nZ_e is nearly equal to 6×10^{16}. As the supposed condition is just the point of transition from diffusion to recombination column, the column of the current beyond 100 mA must be treated as that of recombination.

2. Field ionization-recombination column

For this column, the relations shall be settled as below.

$$E(T_e,\ T_g), \qquad \cdots\cdots(1c)$$

$$j(n,\ T_e,\ T_g), \qquad \cdots\cdots(3b)$$

$$I=\pi R^2 j, \qquad \cdots\cdots(31)$$

$$EI=0.68\sqrt{T_g}, \qquad \cdots\cdots(5d)$$

$$T_e=9{,}800°\mathrm{K}, \qquad \cdots\cdots(8)$$

$$n=Z_e/\rho, \qquad \cdots\cdots(16)$$

$$\rho(T_e\ T_g)=6.4\times10^{-3} p T_e^{-1/2} T_g^{-5/2}, \qquad \cdots\cdots(30a)$$

$$Z_e(T_e). \qquad \cdots\cdots(14b)$$

E-I characteristic may be got in quite the same way as the previous column. But the radius R or the current density j is different from that. Let n be expressed as a function of T_e and T_g from (16) (30a) (14b), $n=F_1\ (T_e,\ T_g)$, and substitute this in (3b), then we have $j=F_2(T_e,\ T_g)$ or $I=\pi R^2 F_2(T_e,\ T_g)$, while combining (1c) with (5d) we have also $I\fallingdotseq F_3(T_e,\ T_g)$. Eliminating I from both the current relations, we have the radius for N_2

$$R^2=\frac{1.65\times10^6}{pT_e^{3/2}T_g(T_e-T_g)}\cdot e^{ev_j/kT_e}. \qquad \cdots\cdots(32)$$

For the air of 760 mm Hg, $v_j=15.0$ V, we have also

$$R^2=\frac{1.44\times10^5}{T_g(9{,}800-T_g)}. \qquad \cdots\cdots(32a)$$

The numerical results calculated by these relations are shown in Table 4 comparing with experimental data. It indicates fairly good agreement with theory.

Teble 4. Calculated values, air 1 atm. $T_e=9{,}800°$K, $v_j=15.0$ V

T_g°K	E V/cm calculated	observed	I mA	R mm calculated	observed	n/cm^3
2,000	330	330	92	0.96	0.85	1.7×10^{13}
2,500	255	250	133	0.88	0.80	2.9×10^{13}
3,000	208	205	178	0.84	0.70	4.4×10^{13}
3,500	170	170	237	0.81	0.70	5.8×10^{13}
4,000	143	146	300	0.785	0.75	6.8×10^{13}
4,500	120	117	380	0.775	0.85	7.8×10^{13}
5,000	105	105	460	0.77	0.95	8.5×10^{13}
5,500	89	85	565	0.772	1.10	8.9×10^{13}
6,000	78	75	670	0.79	1.26	9.0×10^{13}

Thus, in this region moreover, the greater the current is, the higher the gas temperature becomes, T_e being at 9,800°K constantly kept. When the current becomes about 670 mA, the gas temperature reaches to the value of 6,000°K, which are ordinarily that of the thermal ionization arc discharge. We may in the next stage concern with the thermal ionization column.

3. Thermal ionization-recombination column

In the region of an arc discharge the column is ordinarily considered to be isothermal plasma, i.e., $T_e=T_g$, and is used to solve by means of Saha's equation. However, for the region of the small current arc discharge, T_e becomes not yet equal to T_g, therefore the applicableness of Saha's equation to it is somewhat doubtful, at least is not completely correct. We now deal with this last column by the method as used before.

For the balance of particles, we have

$$Z_T=n^2\rho, \qquad \cdots\cdots\cdots(17)$$

$$Z_T(T_g), \qquad \cdots\cdots\cdots(20c)$$

$$\rho(T_e,\ T_g). \qquad \cdots\cdots\cdots(30a)$$

And for N_2, from these,

$$n=1.49\times10^{17}p^{1/2}T_e^{1/4}e^{-ev_j/2kT_g}. \qquad \cdots\cdots\cdots(33)$$

The energy balance condition (5b) shall be held in this column, unless the circumferential temperature of column does not exceed that of dissociation. In larger current, the convection plays a role for the loss of column, in which we do not concern ourselves here. We may suppose an arc without convection such as "Wälzbogen" and a small current arc for the object of the problems.

Then the relations on E, I and j are the same as before. In order to compute the radius, substituting (33) into (3b) and by (31), we have $I=R^2\phi_1(T_e,\ T_g)$, while from (1c) and (5d), having also $I=\phi_2(T_e,\ T_g)$, R can be expressed in terms of T_e and T_g as follows

$$R^2=\frac{5\times10^{-3}}{p^{3/2}}\cdot\frac{T_g^{3/2}e^{ev_j/2kT_g}}{T_e^{7/4}(1-T_g/T_e)} \qquad \cdots\cdots\cdots(34)$$

for the air.

If in this case the gas temperature were increasing with increasing current as before, the radius should so rapidly (exponentially) decrease, that that should not be inhibitorily realized according to the studies on the thermal balance of an arc column in various papers published. Whereas the decrease of gas temperature with increasing current should not be permitted owing to the balance of particles. Therefore the gas temperature results in being kept at a constant value. As seen later, the gas temperature at the transition point from the field ionization to the thermal ionization column is about 5,700°K, we may take the arc temperature about

$$T_g=6{,}000^\circ\mathrm{K} \qquad \cdots\cdots\cdots(35)$$

constant, independent of the current.

This value is supported by the various measurements[9] in which the fact may be such well summarized that the gas temperature is kept about 6,000°K constantly over the wide range of the current. Then it will be seen at once that the falling of the field strength with increasing current will be caused by the falling of the

Table 5. Calculated values, air 760 mm Hg, T_g=6,000°K

T_e°K	E V/cm		I	R mm							n/cm³
	calculated	observed	Amp	calculated v_j= 15.8	15	14	13.6	13	12	observed	
9,800	78	75	0.67	3.75	2.38	1.47	1.18	0.89	0.52	1.24	9.0×10^{13}
9,000	65	65	0.80	4.5	2.85	1.75	1.42	1.05	0.63	1.48	8.9×10^{13}
8,000	50.5	56	1.04	5.7	3.6	2.22	1.80	1.35	0.80	1.80	8.4×10^{13}
7,000	34.3	—	1.52	8.4	5.3	3.28	2.65	2.00	1.18	—	8.1×10^{13}
6,500	23.0	—	2.28	11.8	7.4	4.6	3.70	2.80	1.64	—	7.6×10^{13}
6,200	13.7	—	3.85	20.6			6.50			—	7.4×10^{13}

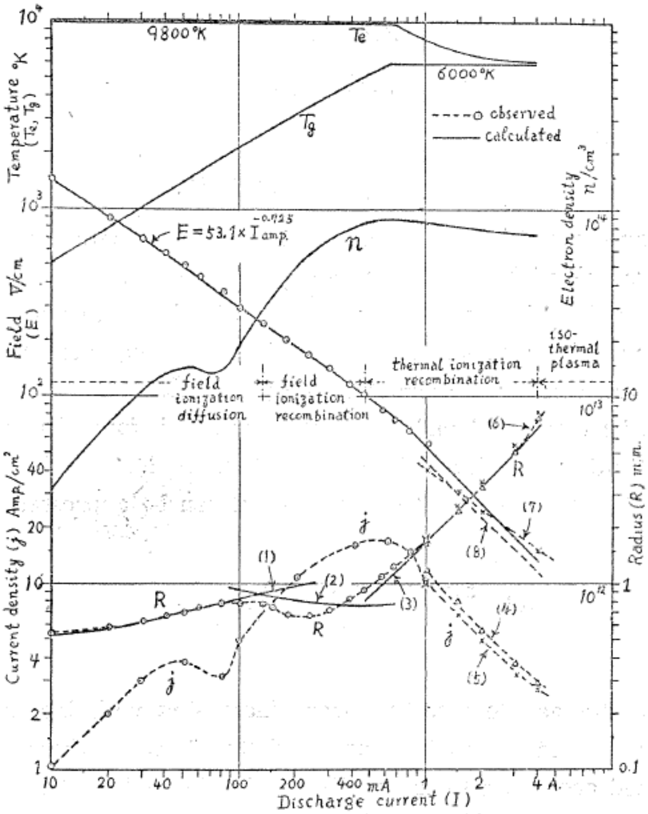

Fig. 2. Discharge characteristics, calculated and observed, for the air of 1 atmosphere.

electron temperature. Taking T_e as parameter, we have E-I characteristic and others as shown in Table 5. The data above 1 Amp. are those to which as the author has not observed, we may cite the following investigations inserted in Fig. 2. In Fig. 2, the curves (4), (6), (7) are for "Wälzbogen" in the tube of 6 cm diameter [E. u. S. II. 148, 149], the curve (5) is the calculated value for N_2 by K. T. Compton and his co-investigators[4] and the curve (8) is the observed value with copper oxide cathode by Suits.[10]

Table 5 has been calculated, assuming the ionzation potential v_j is 13.6 V, according to the dissociation and the mixture by the metal vapor.

The greater the current is, the less the electorn temperature becomes. For the current of 4 A, the difference between T_e and T_g becomes about 150°C, that is just supported by the investigations of White and Mannkopf.[11] The column of the current of more than about 4 A may be regarded practically as an isothermal plasma.

In conclusion, on account of the constant gas temperature in the region here treated, the ionization rate does not vary with the current, hence, the increase of current may be provided with the increase of radius.

VI. Classification of the Positive Columns
(Boundaries between columns)

In the course of the calculation for the characteristics of the various columns in the preceeding sections, we have pointed out the exsistence of each boundary between them. It may be desirable and important to classify those various sorts of columns lied between two typical forms of discharges, the pure thermal arc and the low pressure glow. The author have here to deliver the method to give the boundaries for the example of the case of the air. The thermal ionization-diffusion column is here outside the problem. Now we may have beforehand the expectation that by the characteristic natures of the radius for each column rough boundaries may be given as those which are the cross points of the curve (1) and (2), (2) and (3) in Fig. 2 respectively, that are $I=130$ mA and 430 mA.

1. Boundary between diffusion and recombination columns of field ionization

When both the diffusion and the recombination mechanisms coexist in a column, the distribution of electron density must satisfy the equation.

$$\frac{d^2n}{dr^2}+\frac{1}{r}\frac{dn}{dr}+n\left(\frac{Z_e}{D_a}-\frac{n\rho}{D_a}\right)=0. \qquad \cdots\cdots\cdots(36)$$

Since this equation can not be solved in general, we may approximate it such that at the boundary a half of the generated electron by field ionization corresponds to the diffusion loss of electron and the other half to the recombination loss. Then

$$\frac{Z_e}{2}n-n^2\rho=0 \qquad \text{or} \qquad n=Z_e/2\rho \qquad \cdots\cdots\cdots(37)$$

and

$$\frac{d^2n}{dr^2}+\frac{1}{r}\frac{dn}{dr}+n\frac{Z_e}{2D_a}=0 \qquad \text{or} \qquad R=2.405\sqrt{\frac{2D_a}{Z_e}} \qquad \cdots\cdots\cdots(38)$$

are the conditions for the boundary. Combining (38) with $I=\pi R^2 j$, and substituting (3b): $j(n, T_e, T_g)$, and (37): $n(T_e, T_g)$ to it, we have at once the relation $I=F_1(T_e, T_g)$. On the other hand, eliminating E from (1c): $E(T_e, T_g)$ and (5d): $EI(T_g)$ we have also another relation $I=F_2(T_e, T_g)$. Eliminating I from the above two current formulae we can obtain the following algebraic equation of third order for (T_e/T_g) which is the relation to be satisfied at the boundary. That is, for the air

$$\left(\frac{T_e}{T_g}\right)^3-\left(\frac{T_e}{T_g}\right)-1.58\times10^{14}pT_g^{-9/2}=0. \qquad \cdots\cdots\cdots(39)$$

This equation has at least within $T_g=6{,}000°\mathrm{K}$ one real root with respect to T_e/T_g which leads a value of T_e, giving T_g. Since the field strength E is a function of only T_e/T_g, presented in (1c), the relation of E to T_g which must be satisfied at the boundary can be found. While, the discharge characteristic of $T_g : E$ have been already known, the cross point of the both curves of $T_g : E$ give the boundary which is as presented in Fig. 3, $E=255$ V/cm, $T_g=2{,}500°\mathrm{K}$, $I=134$ mA, predicted above. In this case, since we have known $T_e=9{,}800°\mathrm{K}$, from the relation between T_g and T_e/T_g we have $T_e/T_g=3.95$ which leads simply $T_g=2{,}500°\mathrm{K}$.

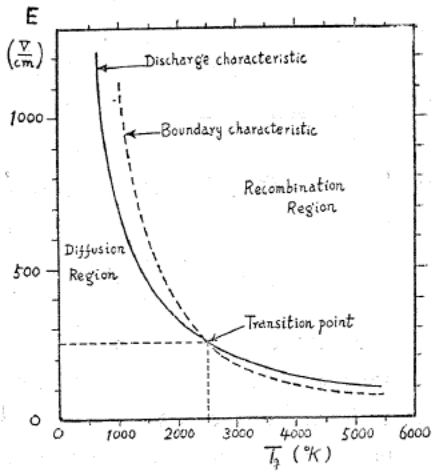

Fig. 3. Transition point from diffusion- to recombination-column of field ionization, calculated.

2. *Boundary between field- and thermal-ionization columns of recombination*

For the steady state of the recombination column, coexisted with both field and thermal ionization, we have

$$\frac{dn}{dt}=Z_e n+Z_T-n^2\rho=0. \qquad \cdots\cdots\cdots(40)$$

At the boundary, equating the both ionization rate to each other,

$$Z_e n=Z_T. \qquad \cdots\cdots\cdots(41)$$

Eliminating n from these and the relations $Z_e(T_e)$, $Z_T(T_g)$ and $\rho(T_e,\ T_g)$, we have $F(T_e,\ T_g)=0$ or

$$1.57\times10^{23}p^2\,T_g^{-5}e^{-ev_j/kT_g}=T_e e^{-ev_j/kT_e}, \qquad \cdots\cdots\cdots(42)$$

for the air which expresses the relation of T_e to T_g, satisfied at the boundary, as indicated numerically in Table 6. In order to transform the temperature relation to

Table 6. Relation between T_g and T_e as well as between E and j which must be satisfied at the boundary.
(calculated for various values of ionization potential)

T_g°K	T_e°K	E V/cm	j A/cm² v_j 15.8	14.8	13.8	10.8	7.8 V
2,000	2,620	48.5	2.8×10^{-14}	—	—	—	4.5×10^{-4}
2,500	3,400	53.0	3.8×10^{-10}	—	—	—	4.9×10^{-2}
3,000	4,300	58.5	2.2×10^{-8}	—	—	3.7×10^{-3}	1.3
3,500	5,300	67	2.4×10^{-6}	—	—	9.5×10^{-2}	13.7
4,000	6,300	71	6.9×10^{-4}	2.9×10^{-3}	1.5×10^{-2}	9.6×10^{-1}	82
4,500	7,500	—	—	—	—	—	—
5,000	8,500	82	9.3×10^{-2}	3.4×10^{-1}	1.1	32	1100
5,500	9,800	—	—	—	—	—	—
6,000	11,000	96	2.56	6.7	19	340	6400
6,500	12,500	—	—	—	—	—	—
7,000	14,000	107	29	52	160	1900	23000

that of the easily measurable quantities, substiuting $n(T_e,\ T_g)$ of (40) or (41) into $j(n,\ T_e,\ T_g)$ we have

$$j=6.07\times10^{2}p^{1/2}T_g^{3/4}\left(\frac{T_e}{T_g}\right)^{1/4}\sqrt{\frac{T_e}{T_g}-1}\ e^{-ev_j/2kT_g}, \qquad \cdots\cdots\cdots(43)$$

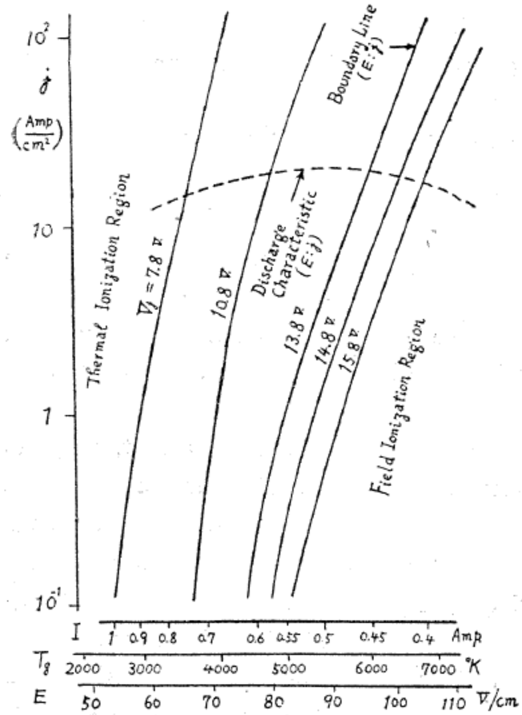

Fig. 4. Transition point from field- to thermal-ionization column, calculated.

while, with (43) and $E(T_e,\ T_g)$, eliminating T_e, we can obtain the characteristic $E:j$ with a parameter of T_g, which must be satisfied for the boundary. With this, we can determine the boundary point with aid of the discharge characteristic $E:j$, as indicated in Fig. 4 and Table 6. Owing to the dissociation, chemical reaction and mixing by vapor of metal electrode we may evaluate approximately the ionization potential $v_j=13.8$ V lowered down from the normal state. Then, we have for the boundary

$$T_g=5{,}700^\circ\mathrm{K},\quad I=0.47\ \mathrm{A},\quad E=92\ \mathrm{V/cm}$$

that have been already expected.

It will be seen from the results that in the region of thermal ionization, being $Z_e n \ll Z_T$, the ionization by electron collision due to electric field as well as thermal motion can be neglected, much more for the condition $T_e \cong T_g$, and it may be correct that the Saha's equation in which the ionization by electron is not taken into account, was applied hitherto to the isothermal plasma such as an arc column.

VII. Considerations

1. Comparison with Experiments

As we have made in each column, the comparison between calculated values and experimental data is summarized in Fig. 2. As hardly any data is presented for the current less than one ampere, the author's obseved data is indicated for small current in the figure. The experimental method is as follows: in order to prevent the instability of burning due to the steep negative characteristic for the small current, the high vacuum diode tube (KN-156) was inserted in series in the discharging circuit as the ballast resistance with its high value in the temperature staturation state, therefore consisting of constant current circuit. The copper electrodes of several millimeter diameter were arranged virtically on behalf of the stable realization. (Horizontal arrangement does not require the amendment of the data. That means no effect of the convection.) The potential gradient was measured by the change of burning voltage with changing of the gap from 1 cm to 6 cm at the given constant current condition. The radius was determined photographically and by the telescopic observation. In the case of small gap length, the gap becomes the smaller, the smaller the radius, becomes. When the gap length is made longer gradually, at first the radius grows, but soon saturated, keeping a constant value, which we will adopt for the radius. For much longer gap the radius of the upper part of the column expands and then goes in confusion.

Therefore, the radius here, will be taken at the middle point of the gap of 1 cm length, which also equals to that at the point apart about 1 cm from the under electrode (cathode) of the longer gap.

With increasing current, the potential gradient falls monotonically obeying to the experimental formula $E=53.1\times I_{\mathrm{amp.}}^{-0.723}$ V/cm, and the radius at first increases slowly and then contracts, and again increases, as we have seen just at the calculation. The later increase corresponds to the usual arc and the contraction was observed by Thoma and Heer,[12] but with no experimental data.

Such fact that the radius or the current density undulates with current, suggests to us the complex mechanism of the column as mentioned. Further more, that fact is also supported by the spectroscopical observation, which shows that at the ordinary gap length, of the current of less than about 100 mA the spectrum is N_2 first positive, second positive, N_2^+ and $NO(\gamma)$ bands, while of the current of more than that, O_2-Runge, N_2^+ and $NO(\gamma)$ diverge. One can see that the initiation of the contraction of the column coincides to the characteristical divergence of the spectrum O_2-Runge band as well as the initiation of the recombination mechanism. When the gap length becomes smaller than nearly 0.5 cm, the column appears somewhat different from the ordinary one, i.e., the spectrum corresponds to that for smaller current, accompanied with the contraction of radius and increase of potential gradient, as if the current became smaller. At the small gap length, because of the addition of the cooling effect by the electrodes, the thermal balance of the column does not be expressed only by the gaseous nature. The inclination of $\log E$ to $\log I$ differs from that at the ordinary gap. The relation may be considered as the form of $EI=\alpha T_g^{1/2}+\beta T_g$ with which we have not concerned.

At the calculation of the radius, we have taken $v_j=15$ volt as the ionization potential of the air in the recombination-field ionization column, and $v_j=13.6$ volt for the recombination- thermal ionization, taking account of the lowering of the ionization potential with increasing current as well as rising temperature. If we may take here the continuous drooping characteristic of the ionization potential with increasing current, the calculated curves (2) and (3) in Fig. 2 shall be such improved that the curve (2) may be somewhat lowered for the larger current side and the curve (3) be risen for the smaller current side. Consequently the agreement with the observation should be attained more sufficiently by these precise calculations.

2. Temperature etc.

We now concern with the results of current characteristics of gas and electron temperature. Remembering that the field which is a function of T_e/T_g, presents a falling nature with increasing current, we would consider essentially for the falling of the field the falling of T_e, but T_g being kept constant refered to the low pressure discharge. However in this case, as described in the preceeding section, T_e have already fallen down to the constant value of 9,800°K, independent of current, owing to the necessity of thermal balance, then now the lowering of the field should be secondarily caused by the rise of T_g. If submitting to the nature of itself, T_e falls with increasing current, the radius should contract in the diffusion column by the equation (11a), and expand in the field ionization recombination column by the equation (32), conflicting with the observation. While the assumption that T_e rises with increasing current, does not be admitted, we must conclude the constancy of electron temperature, which has been yielded from the consideration of the nature of gas molecule in the preceeding section.

In consequence of the independency of T_e with current, the lowering of E will be caused by the gas temperature rise, accompanying with the slow increase of the radius in the diffusion column. In the recombination column of field ionization, the contraction of the radius with increasing current is resulted also by the temperature rise such that the current increase may be yielded by the much more increase of current density j or the product of electron density n and electron mobility k_e, against the decrease of the field strength. The temperature rise of course provides the increase of electron mobility and also electron density in accordance with the behavior of the recombination coefficient expressed by the forumla (30a).

Thus the gas temperature of the positive column rises with increasing current, as if the cathode fall rises in abnormal glow, in order te prepare himself for the innings to the thermal arc discharge. And finally it reaches to the constant temperature of 6.000°K which is here settled only by a rough estimation. The constancy of the arc temperature will not be kept strictly even in the formula (34) because of the variations of ionization potential of the gas in the various discharge conditions.

Now in the thermal ionization column, T_g being constant, EI must be constant, independent of T_g and R. At first sight this appears to oppose to the premise and the fact that EI is a function of T_g as well as R, as considered for the application of minimum principle. However, in this case EI becomes only a function of R and application of minimum principle to that i.e., $d(EI)/dR=0$ leads EI constant, independent of R and of course, of T_g. There is no confliction.

For the large current arc the convection, the dissociation and others which are not taken into account in this derivation, are affecting our evalution. In spite of this, our equation can be held roughly up to several amperes and especially the constancy of gas temperature consists fairly well with the observation.

The value of C in the equation (5d) or (7a) can be checked by substitution of the values of R, n, T_e and T_g into the equation (7). The calculation presents a agreement with the value 0.68 over the current range from 10 mA to 3 A, as shown in Fig. 5. That verifies the minimum principle.

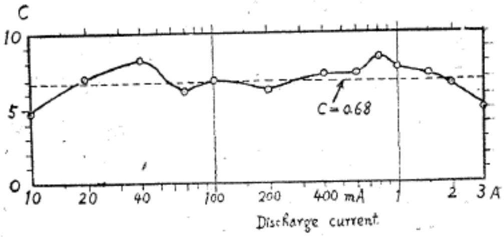

Fig. 5. Calculated *C*.

The proposed formula for the electron recombination coefficcient fits for the practical use, notwithstanding the derivation from classical model and consequently theoretically of not exact representation. The appropriateness of the separation of T_e and T_g in the derivation may be also checked by the evaluation of the column radius. Table 7 indicates the comparison between the calculated values in various separations and the observed values. One can see that the proposed formula may be suitable as well.

Table 7. Calculated values of R for the various recombination formulae, 1 atm, air, T_e=9,800°K

T_g°K	R in mm calculated			R in mm observed
	$\rho=k\cdot T_e^{-1/2}\,T_g^{-5/2}$	$\rho=k\cdot T_e^{-1}\,T_g^{-2}$	$\rho=k\cdot T_e^{-2}\,T_g^{-1}$	
3,000 4,000 6,000	0.84 0.785 1.18 (thermal ioniz.) (0.79) (field ioniz.)	0.63 0.61 1.10 (therm. ioniz.)	0.35 0.39 0.98 (therm. ioniz.)	0.70 (178 mA) 0.75 (300 mA) 1.26 (670 mA)

Conclusion

The theoretical treatment of the positive column and its application to the evaluation of discharge characteristics in the atmosphere through the whole current range of the selfsustanined discharge from the minimum glow discharge to the large current arc have been carried out. The calculation has presented a good agreement with the experimental data. As a summary, it makes clear that, starting from the steady glow of minimum current just after the breakdown, the columns transfer gradationally from the field ionization-diffusion-, mediately field ionization-recombination-, to thermal ionization-recombination one and ultimately isothermal plasma, with increasing current to an arc discharge. In computation of the recombination column, the formula of recombination coefficient has been proposed and, provided the fitness for practical calculation.

As one of the remarkable results yielded from the theoretical consideration, it may be pointed out that the electron temperature is kept at a constant value, about 10,000°K in the atmosphere, in the region of field ionization, while the gas temperature is kept at a constant value, about 6,000°K in the thermal ionization column. That seems to be of a inmature model, but the constancy must be held pretty well owing to the conclusion of the theoretical consideration except at the neighbourhood of their boundary.

In this papar we have not concerned ourselves in the pressure effect. The experiment over the pressure range of several atmospheres showed similar characteristics of field to current and radius to current at one atmosphere. Therefore we may treat that similarly.

For the high frequency discharge, inasmuch as the author has already obtained the relations* between the field E and the electron temperature T_e, and the electron temperature and the ionization rate Z_e per electron per second, one can treat similarly to the D.C. column. The calculation as well as the observation has shown that the column is not much different from that of D.C.

We will conclude this paper on the region in which the discharge of the current between about several micro amperes just after the breakdown and several milliamperes of steady state discharge is involved. The discharges in this region are somewhat difficult to realize because of their instability. From the preceeding calculations, it may be predicted, that the gas temperature may decrease with decreasing current, at last to the room temperature when the current becomes about 1 mA, if the exterpolation may be allowed, and does not go down below it. Since the field strength must rise with decreasing current, the electron temperature can not be held at the constant value of 10,000°K. Then at present step, the gas temperature being kept to the room temperature, the electron temperature should increase with decreasing current, so that the inelastic collision will predominate just like in the column of low pressure glow discharge but in free space. Thus the column will become essentially different from the thermal column which is maintained by the thermal energy balance. Further, the lowering of the electron density might cause a transition of diffusion mechanism from ambipolar to plain one. In the extreme case this region may be connected with a streamer of plasma state which is very important problem for the streamer breakbown mechanism left to hereafter.

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* "The Electron Temperature in High Frequency Field,"
(K. YAMAMOTO and K. NOBATA),
Published in Japanese May 1949.

We have derived by the similar method of K. T. Compton's electron mobility calculation,

$$u(u-\Omega)=\frac{E^2}{2b/\lambda^2}\left[1-\cos\varphi\cos(2\omega t-\varphi)-e^{-At}\sin^2\varphi\right],$$

where u, Ω = energy in eV of electron and gas molecule respectively in high frequency field $E\sin\omega t$,

$$A=3abg/2\lambda_e,\quad \tan\varphi=2\omega/A,\quad a=0.75\sqrt{\frac{3\pi}{16}}e/m_e,$$

$$b=\frac{8}{3}\frac{m_e}{m_g}\cdot\frac{16}{3\pi\times0.75},\quad g=\text{const.}=1\sim4,$$

and also taking account of Ramsauer effect, $\lambda_e=\beta\sqrt{u}$, we have also

$$u-\Omega=\frac{E^2}{2b/\beta^2}\left[1-\cos\theta\ \cos(2\omega t-\theta)-e^{-abt/\beta}\ \sin^2\theta\right]$$

Z_e has been calculated numerically with the aid of the electron energy above indicated.

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