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PHOTOELASTIC EXAMINATION OF NEUBER'S SOLUTION FOR NOTCHED-BAR UNDER TENSION OR BENDING

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Abstract

Neuber's formula of the stress consentration factor for a notched-bar under tension or bending is examined by means of the photoelastic method. Experimental results indicate that his formula can be used for an accurate evaluation of the stress consentration factor only with caution since it gives a small value for notches of medium depth and a large value for shallow notches.

Introduction

It is well known that Neuber's formula for hyperbolic notches is based on an interpolation between limiting theoretical values. According to his formula, the stress concentration factor always increases with increase of the depth of notch, the ratio between the base radius of the notch and the minimum section of the bar being kept constant. His results concerning the stress concentration factor have long been maintained, in general, without any explicitly defined basis and his solution is widely used in practice, since more exact information is not yet available.

However, recent mathematical investigations for parallel-sided *U*-notches indicate that the stress concentration factor of a tension bar is larger, in some instance, for semi-circular notches[1] than for infinitely deep notches[2].

Also in bending, the theoretical results[3][4] for semi-circular notches show higher values, in some instances, than those for infinitely deep hyperbolic notches[5].

Prof. Ōkubo pointed this out by comparing the stress concentration factors for a semi-circular notch and an infinitely deep hyperbolic notch; but he ascribed this to the difference in shape between *U*- and hyperbolic notches[6].

Kikukawa made further investigation of this stress concentration factor on a bar having *U*-notches under tension or bending by means of electric resistance strain-gauges[7][8]. He also observed that the stress concentration factor for notches of medium depth is larger than for deep notches.

In view of the higher accuracy of the recent mathematical results, the present author considers that Neuber's prosedure of interpolation is not always valid and that further investigation is expected. Photoelastic experiments are presented here in order to examine this consideration.

Experiment

Dimension of test pieces used for tension and bending tests are given in Figs. 1 and 2, and Tables 1 and 2, respectively. They were very accurately cut by fine endmill out of carefully annealed plate of epoxy-resin, 6 mm thick. The irregularity of plate thickness was within 0.02 mm. The main dimensions, namely depth and base radius of notches, minimum section of test-pieces, were measured precisely by comperator. In general, the cutting errors for the nominal value of the ratio a/ρ were within 0.5 per cent. Consequently, errors in cutting did not affect the experimental results.

The residual stress by machining can hardly be found at the base of notch. The load is so adjusted as to act accurately on the middle plane of the plate and most uniformly at terminal sections. By observing the fringe pattern, it was ascertained that the terminal effect was entirely eliminated.

In tension test, the bending moment acting on the plane of the plate is eliminated by a small travelling weight moving perpendicularly to the direction of load. In bending, by using the pre-adjusted equipment, possible errors occuring from any loading different from pure bending are eliminated. To reduce both the errors arising from the creep of the material and the change of base radius during stressing, the maximum stress in a specimen was limited to within 0.8 kg/mm^2, which is far below the proportional limit for epoxy-resin. The loading apparatus used in the experiments are shown in Figs. 3 and 4.

In tension test, the fringe order n_1 at the base of notches was measured by Sénarmont's method, increasing and decreasing loads stepwise, and in bending test by Tardy's method, increasing load stepwise. To eliminate the effect of initial stress, the ratio of n_1 to σ_1 is measured, where σ_1 is the nominal stress based on

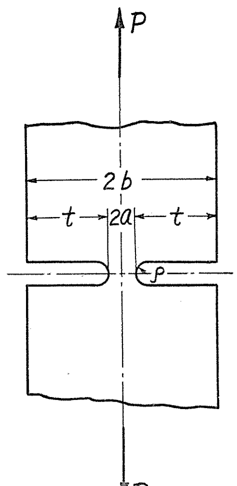

FIG. 1. Test piece (Tension test).

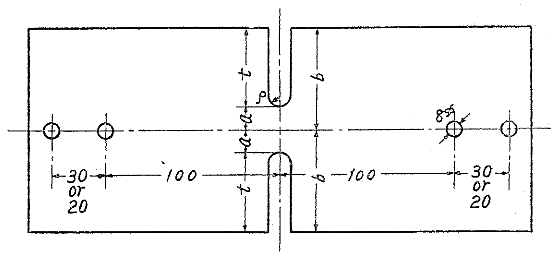

FIG. 2. Test piece (Bending test).

TABLE 1. Dimension of Test Pieces (Tension Test)

($a/\rho=1.48$)

No.	1	2	3	4	5	6	7	8	9	10
b mm	8.39	9.19	9.89	12.48	15.00	16.91	20.92	27.02	37.42	69.96
a mm	7.50	7.52	7.50	7.56	7.50	7.41	7.50	7.51	7.46	7.47
t/b	0.106	0.184	0.250	0.394	0.500	0.562	0.642	0.722	0.801	0.893

($a/\rho=2.00$)

No.	1	2	3	4	5	6	7	8	9	10
b mm	11.01	11.80	13.34	14.92	16.51	19.91	24.83	33.61	49.84	80.15
a mm	10.20	10.16	10.16	10.20	10.12	9.79	9.79	10.01	10.01	10.12
t/b	0.073	0.139	0.239	0.316	0.387	0.508	0.606	0.702	0.799	0.874

($a/\rho=4.02$)

No.	1	2	3	4	5	6	7	8	9	10
b mm	17.00	17.40	18.45	19.99	22.99	26.33	31.91	40.02	53.27	73.60
a mm	16.42	16.42	16.42	16.38	16.38	16.38	16.31	16.31	16.48	16.48
t/b	0.035	0.057	0.108	0.181	0.288	0.378	0.489	0.593	0.691	0.776

TABLE 2. Dimension of Test Pieces (Bending Test)

($a/\rho=1.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
b mm	5.24	5.52	5.76	5.96	6.27	6.68	7.03	7.20	7.72	7.75	8.26	9.09	10.05	12.37	16.62	30.05	49.93
a mm	5.07	5.03	5.01	5.01	4.98	5.09	4.98	4.96	5.02	4.98	5.05	5.00	5.04	5.04	5.01	5.05	5.07
t/b	0.032	0.089	0.130	0.159	0.206	0.238	0.292	0.311	0.349	0.357	0.388	0.449	0.499	0.593	0.698	0.831	0.898

($a/\rho=1.50$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
b mm	7.97	8.44	9.14	9.33	10.64	12.44	14.04	14.74	16.17	16.71	18.57	21.52	25.06	30.05	49.93	74.66
a mm	7.42	7.64	7.57	7.48	7.46	7.57	7.56	7.64	7.66	7.59	7.64	7.60	7.63	7.55	7.58	7.58
t/b	0.068	0.095	0.171	0.198	0.298	0.391	0.461	0.482	0.526	0.545	0.588	0.646	0.696	0.748	0.848	0.899

($a/\rho=3.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13
b mm	16.37	17.04	18.94	21.01	23.11	23.56	24.95	26.85	28.46	37.75	49.86	60.09	74.66
a mm	15.12	15.09	15.11	15.00	15.33	15.17	15.20	15.17	15.14	15.10	15.19	15.12	14.92
t/b	0.076	0.114	0.202	0.258	0.337	0.356	0.391	0.435	0.468	0.600	0.695	0.748	0.800

($a/\rho=4.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
b mm	16.87	17.72	19.01	20.21	22.95	23.67	26.65	26.99	32.52	35.64	40.34	53.50	60.09	80.12
a mm	16.13	16.23	15.95	16.19	16.28	16.09	16.26	16.01	16.19	16.13	16.17	16.21	16.31	16.20
t/b	0.044	0.084	0.161	0.198	0.291	0.320	0.389	0.406	0.502	0.548	0.599	0.697	0.728	0.798

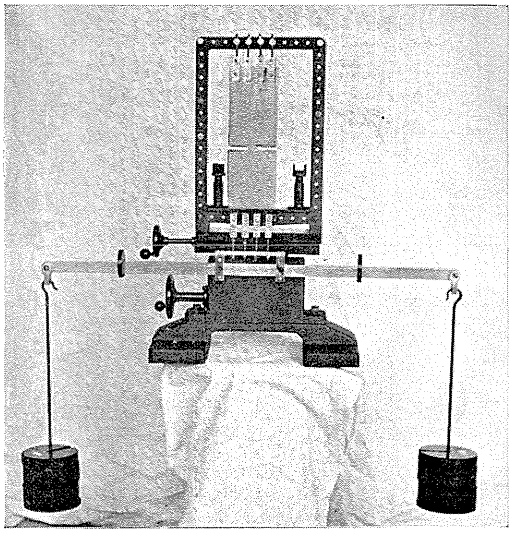

FIG. 3. Loading apparatus (Tension test).

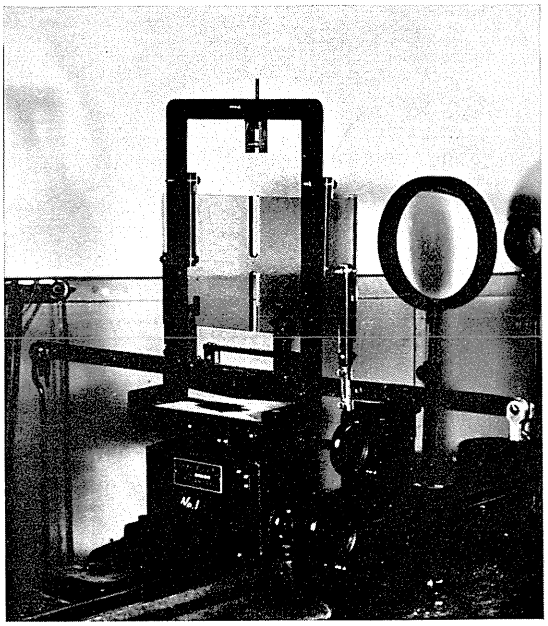

FIG. 4. Loading apparatus (Bending test).

the minimum section of the plate. To obtain the most reliable value for n_1/σ_1, a number of loading steps are employed and measurements are repeated two or three times for every test piece. The more the number of loading steps, the more reliable the value n/σ is, because the errors arising from erroneous loading can be eliminated by plotting a curve based on a large number of measuring points. While, in the photographic method usually employed, these errors can hardly be detected. Typical curves showing the relation between n_1 and σ_1 thus obtained are shown in Figs. 5 and 6.*

For the determination of n_1 corresponding to any assigned value of σ_1, the fringe orders are equated for the notches of both sides within the difference of 0.02 by adjusting the loading and the mean value is taken. In every test, measurement is finished before the edge effect comes out.

In the same manner, the corresponding ratio n_2/σ_2 was obtained for a celibration bar cut from the same plate as the test piece.

The stress concentration factor α for a notch is then found from the relation

$$\alpha = (n_1/\sigma_1)/(n_2/\sigma_2) \tag{1}$$

Fringe order at the base of notches is usually determined by extrapolation from a fringe photograph. Since the extrapolation method is not always reliable, the following method has been adopted in the present experiment.

In the measurement of the fractional order of the fringe, a monochromatic light is now usually used. In the present experiment, however, a mercury lamp without filter was used to get a brighter field. Then the fringe is divided into

* σ_1 corresponds to the difference of the applied load and a pre-load.

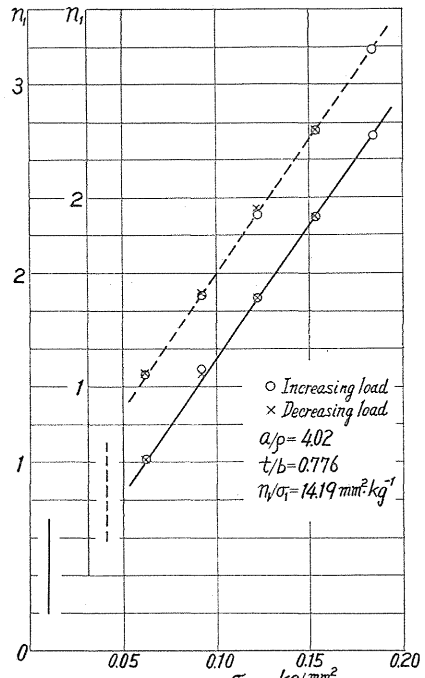

FIG. 5. n_1–σ_1 relation (Tension test).

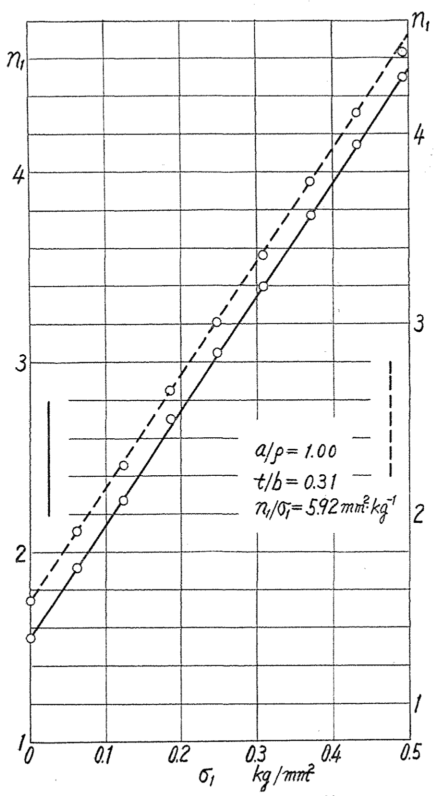

FIG. 6. n_1–σ_1 relation (Bending test).

TABLE 3. Values of Stress Concentration Factors (Tension Test)

(a/ρ=1.48)

No.	1	2	3	4	5	6	7	8	9	10
n_1/σ_1(mm²·kg⁻¹)	7.72	8.18	8.76	9.98	9.89	9.84	9.69	9.71	9.51	9.51
n_2/σ_2(mm²·kg⁻¹)	5.38	5.39	5.43	5.42	5.42	5.49	5.45	5.49	5.42	5.42
α	1.44	1.52	1.61	1.84	1.82	1.79	1.78	1.77	1.76	1.75

(a/ρ=2.00)

No.	1	2	3	4	5	6	7	8	9	10
n_1/σ_1(mm²·kg⁻¹)	7.81	8.78	10.00	10.75	11.12	10.91	10.61	10.55	10.49	10.59
n_2/σ_2(mm²·kg⁻¹)	5.36	5.49	5.49	5.36	5.35	5.32	5.32	5.34	5.34	5.35
α	1.46	1.60	1.82	2.00	2.08	2.05	1.99	1.98	1.97	1.98

(a/ρ=4.02)

No.	1	2	3	4	5	6	7	8	9	10
n_1/σ_1(mm²·kg⁻¹)	8.07	8.62	10.25	12.26	14.27	14.88	14.35	14.20	14.31	14.19
n_2/σ_2(mm²·kg⁻¹)	5.42	5.42	5.42	5.32	5.32	5.32	5.31	5.31	5.35	5.35
α	1.49	1.59	1.89	2.31	2.69	2.80	2.70	2.67	2.67	2.65

TABLE 4. Values of Stress Concentration Factors (Bending Test)

($a/\rho=1.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	1 3	14	15	16	17
$n_1/\sigma_1(\mathrm{mm^2 \cdot kg^{-1}})$	5.03	5.32	5.43	5.57	5.88	5.70	5.95	5.92	6.01	6.15	6.29	6.38	6.46	6.15	6.46	6.42	6.41
$n_2/\sigma_2(\mathrm{mm^2 \cdot kg^{-1}})$	4.75	4.75	4.85	4.75	4.99	4.75	4.80	4.75	4.75	4.80	4.82	4.79	4.80	4.55	4.78	4.86	4.77
α	1.06	1.12	1.12	1.17	1.18	1.20	1.24	1.25	1.27	1.28	1.30	1.33	1.34	1.35	1.35	1.32	1.34

($a/\rho=1.50$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$n_1/\sigma_1(\mathrm{mm^2 \cdot kg^{-1}})$	5.73	6.15	6.66	6.70	7.06	7.26	7.23	6.82	7.20	7.16	7.23	7.16	7.18	7.14	7.11	7.09
$n_2/\sigma_2(\mathrm{mm^2 \cdot kg^{-1}})$	4.73	4.79	4.85	4.80	4.85	4.80	4.80	4.51	4.79	4.78	4.77	4.80	4.81	4.86	4.77	4.80
α	1.21	1.28	1.37	1.40	1.46	1.51	1.50	1.51	1.50	1.50	1.52	1.49	1.49	1.47	1.49	1.48

($a/\rho=3.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13
$n_1/\sigma_1(\mathrm{mm^2 \cdot kg^{-1}})$	7.23	7.95	8.52	8.81	9.06	9.31	8.77	9.22	9.12	8.97	8.90	8.99	9.00
$n_2/\sigma_2(\mathrm{mm^2 \cdot kg^{-1}})$	4.85	4.99	4.85	4.80	4.79	4.79	4.51	4.80	4.78	4.80	4.75	4.77	4.80
α	1.49	1.59	1.76	1.84	1.89	1.94	1.95	1.92	1.91	1.87	1.88	1.88	1.87

($a/\rho=4.00$)

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$n_1/\sigma_1(\mathrm{mm^2 \cdot kg^{-1}})$	7.59	8.51	9.06	9.61	9.66	10.30	9.74	10.36	9.98	10.24	10.05	9.84	9.79	9.81
$n_2/\sigma_2(\mathrm{mm^2 \cdot kg^{-1}})$	4.79	4.99	4.85	4.80	4.55	4.88	4.51	4.80	4.78	4.81	4.80	4.75	4.77	4.80
α	1.58	1.70	1.87	2.00	2.12	2.15	2.16	2.16	2.09	2.13	2.10	2.07	2.05	2.04

some colored bands. The width of the colored band from bluish- to reddish-violet (tint of passage) is very narrow and corresponds to only one degree of the rotation of the analyser. Consequently, an accurate value for the fringe order is obtained by rotating the analyser to have the colored band met the base of notch.

The ratio of n_1/σ_1 to n_2/σ_2 is more important for the evaluation of α than their absolute values. Accordingly, observed errors arising from a mixing of light having different wave lengths are canselled out of the final results.

The experimental results thus obtained are given in Tables 3 and 4, and they are plotted in Figs. 7 and 8.

Discussion and Conclusion

Several authors have determined the stress concentration factors for notched bars photoelastically[9] and Heywood introduced an empirical formula for α by analysing those results[10]. In Figs. 7 and 8 are given the value of α calculated from his empirical formula, the corresponding values based on Neuber[5], and the experimental values by Kikukawa[7)8)].

In the figures, we observe that the difference between the maximum value of α and the value of α for an infinitely deep notch is up to 5 per cent, which corresponds only to 0.2 fringe order even for maximum stressing. Such a small difference can hardly be distinguished by mere visual comparison of fringe patterns,

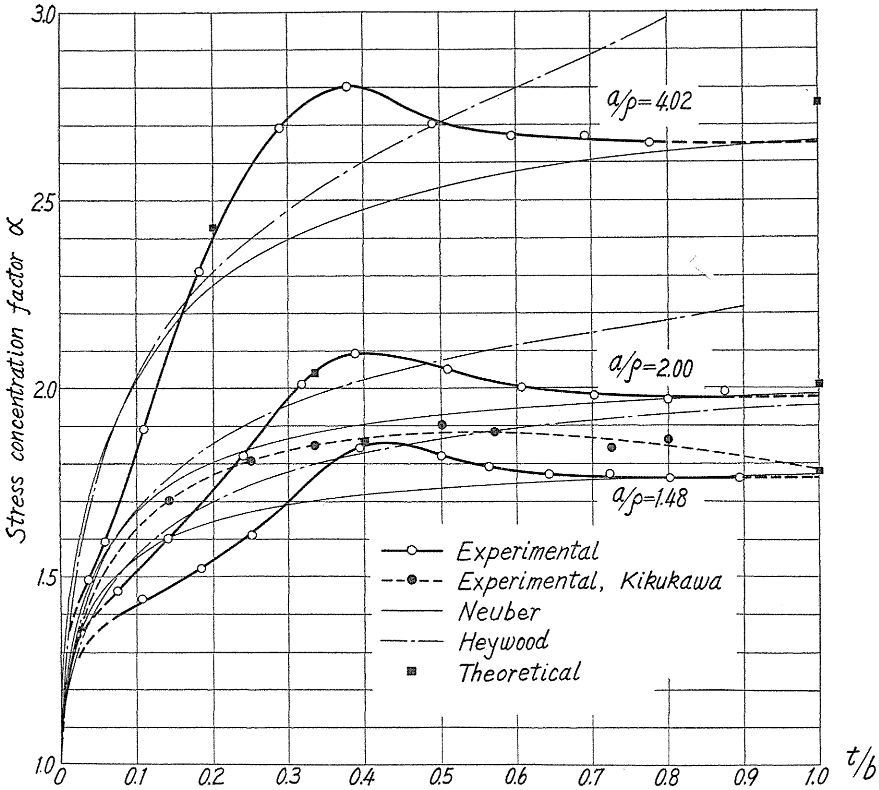

FIG. 7. α–t/b curves (Tension test).

since even with the same test piece, every measured value of fringe order for any assigned value of load is not always constant; a difference up to 0.2 fringe order is found in some instances, although the ratio n/σ derived from the curve of n, σ remains almost constant.

There is definite agreement between the experimental results herein obtained and theoretical results for semi-circular notches. This shows the high accuracy of the experimental results.

Neuber's solution, however, agrees with the experimental results only in the limiting case of $t=\infty$ in tension. In bending, while, the experimental results for large value of t are slightly larger than Neuber's values. In comparison with the results herein obtained, his solution gives much lower values for notches of medium depth and higher values for shallow notches. Accordingly, his curves actually cross the experimental curves at some points of t/b, for instance, at $t/b=0.16$ for $a/\rho=4.02$ in tension, and at $t/b=0.137$ for $a/\rho=4.0$ in bending.

Recently, by means of the electroplating method, Ōkubo and Kikuchi obtained the stress concentration factors for shafts having circumferential U-grooves which had been submitted to bending[11]. It is interesting to note that Neuber's curve also crosses their experimental curve at $t/b=0.135$ for $a/\rho=4.1$, where, a, b denote the minimum and gross radii of the shaft, respectively.

From the experimental results herein obtained, we can conclude that the stress

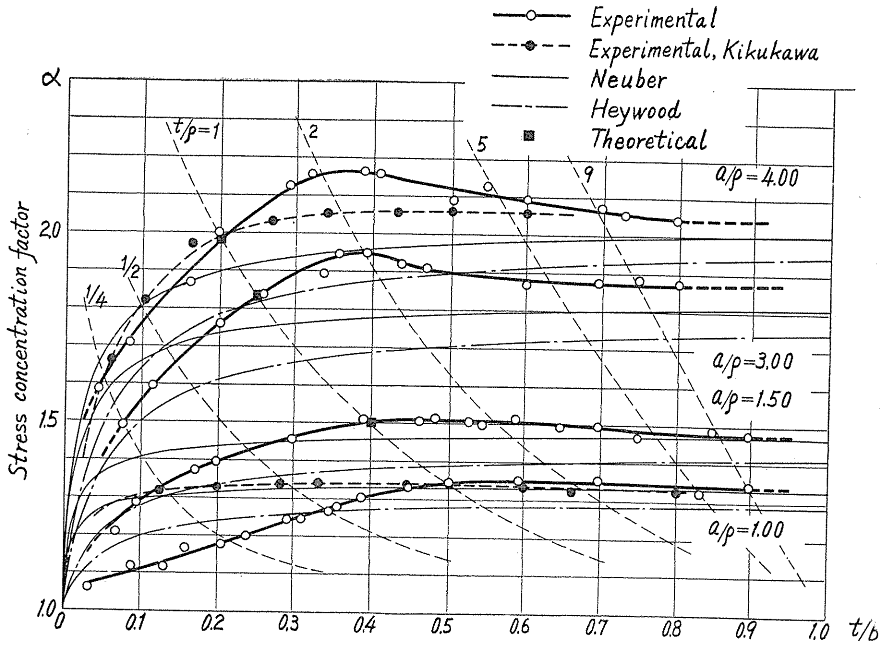

FIG. 8. α–t/b curves (Bending test).

concentration factor does not always increase with increase of the depth of notch and that it is larger for notches of medium depth than for infinitely deep notches, provided that the ratio between the base radius of the notch and the minimum section of the bar is constant. We can also conclude that, in some instances, Neuber's solution cannot be used without perceptible error.

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