A SIMPLIFIED TECHNIQUE OF ESTIMATING STEADY STATE PERFORMANCE OF ON-OFF CONTROL SYSTEMS

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Introduction

Since about 1950, it has been very interesting and important ploblems to design optimaly an on-off controller for a given controlled ellement so as to make the transient response to step input faster and less oscillatory by the deliberate introduction of the velocity feed back circuit, as stated in the historical resumé of T. J. Higgins.[1]

In this paper, there is a new technique of estimating the steady state performance of the second order on-off control system having velocity feed back circuit. The idea playing an important part in this technique is *the virtual switching line.* The virtual switching line is so established that, if it be used as a practical switching line, the trajectories indicating the system performance are all closed curves, namely, the system adoptingit as a real switching line becomes a conservative one. Taking the intersecting points of the real (designed) switching line with the virtual one decided by the given controlled ellement, we can easily get the points where the manipulated variable is altered its value in the steady state oscillation.

In further reports, it will be indicated that the virtual switching line is a very usefull tool to attack the steady state response of on-off control system to sinusoidal input.

On-off Control Systems

The block diagram of on-off control system to be treated here is indicated in Fig. 1. The transfer function $G(s)$ is of the form $N(s)/M(s)$, where $M(s)$ is the second order integral function of s, and $N(s)$ first or zeroth order. Computing device is adopted to take the value of the function $F(e, \dot{e})$ from the error signal $e(t)$, where the function $F(e, \dot{e})$ is decided by such design criteria as introduced by I. Bogner and F. Kazda,[2] R. Oldenberger[3] and so on.[4] Corresponding to the

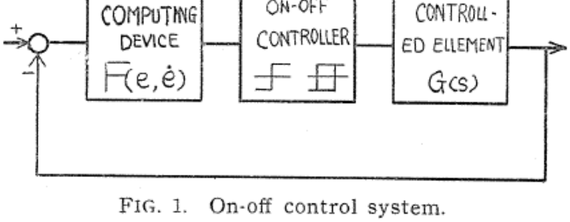

FIG. 1. On-off control system.

sign of the function $F(e, \dot{e})$, the manipulated variable (output of on-off controller) takes one of two constants already designed.

For the sake of simpicity, we assume that these two constants are of the same dimension and the opposite sign, and without loss of generality, we are able to make these two constants be $+1$ and -1, thus the manipulated variable is

$$F(e, \dot{e})/|F(e, \dot{e})|.$$

Adopting the time constant (or one of the time constants) of controlled ellement for the unit of time, and selecting appropriately the unit of the controlled variable, the transfer functions to be treated here can be simplified as follows;

$$\frac{\beta s+1}{s^2}, \quad \frac{1}{(s+1)\ (\alpha s+1)}, \quad \frac{\beta s+1}{s(s+1)} \text{ etc.}$$

where α can be settled equal to or smaller than 1 and β may be neglected.

Phase Plane and Virtual Switching Line

Phase plane is devided into two parts by a line decided by the function $F(e, \dot{e})$, and this line is called here the real switching line. On each side of this line, the shape of the trajectories is decided by the transfer function of controlled element. By the assumption introduced for the manipulated variable in last section, these two groups of trajectories are symmetrical with each other with respect to the origin of phase plane.

Let the trajectory started from a point $(e_0, \dot{e}_0)$ on the real switching line and passing through on one side of it be indicated by

$$f(e, e|e_0, \dot{e}_0)=0. \tag{1}$$

In the case of the generation of self-sustained oscillation, this trajectory intersects with the real switching line at a point $(-e_0, -\dot{e}_0)$, and then a trajectory started from $(-e_0, -\dot{e}_0)$ passing through on the other side of the real switching line arrives at $(e_0, \dot{e}_0)$.

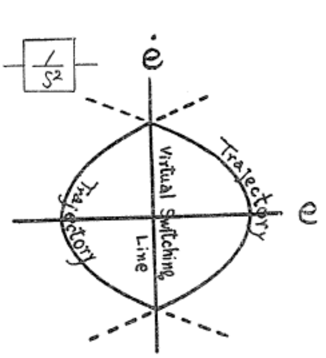

FIG. 2. Trajectories and virtual switching line (1).
Eq. of trajectory:
$e \pm \frac{1}{2}(\dot{e})^2 = \text{const.}$

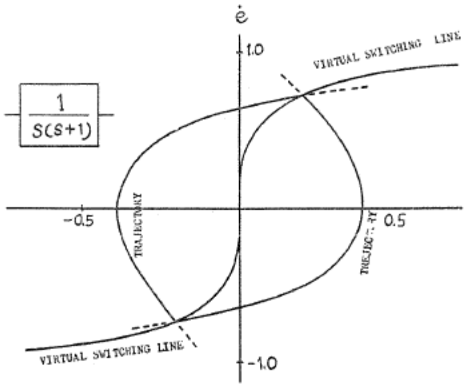

FIG. 3. Trajectories and virtual switching line (2).
Eq. of trajectory:
$\log(1+\dot{e}) - (e+\dot{e}) = \text{const.}$

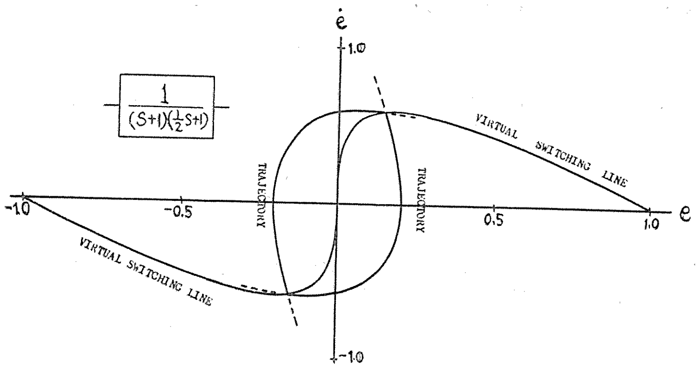

FIG. 4. Trajectories and virtual switching line (3)
Eq. of trajectory:

$$(e+\dot{e}) = \text{const.} \left(\frac{e}{\alpha}+e\right)^{1/\alpha} \qquad \alpha<1$$

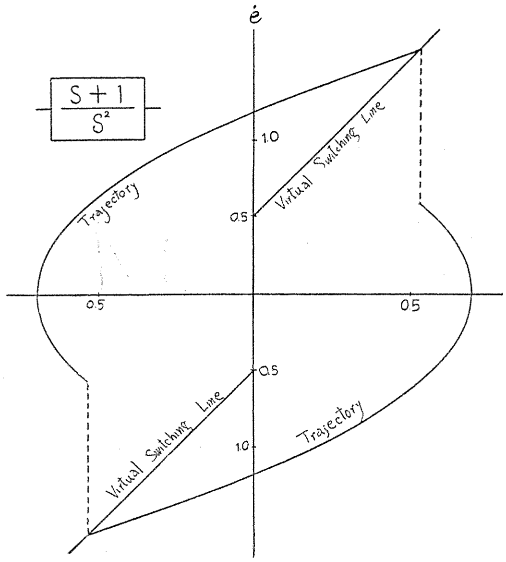

FIG. 5. Trajectories and virtual switching line (4).

Thus, the equation of the virtual switching line is

$$f(-e,\ -\dot{e}\,|\,e,\ \dot{e}) = 0 \tag{2}$$

from Eq. (1), by the definition given in the introduction of this report. It is noted that the virtual switching line is established by the transfer function of the given controlled element, for instance, the transfer function $\frac{1}{S^2}$ requires the $\dot{e}$-axis as the virtual awitching line, as stated in Fig. 2.

Examples of virtual switching line are indicted in Figs. 3, 4, 5, where the transfer functions of controlled element and equations of trajectories are stated together.

The time for travelling a representative point along a trajectory (1) from $(e_0, \dot{e}_0)$ to $(-e_0, -\dot{e}_0)$ (*i.e.* a half period of self-sustained oscillation) is indicated by

$$\tau = \int_{e_0}^{-e_0} \frac{de}{\dot{e}}. \tag{3}$$

These values, estimating from (1) and (3), are written on the virtual switching lines in Fig. 8, 10, 12 in the next section.

Steady State Performance

Let us consider the case where the real switching line is symmetrical with respect to the origin of phase plane.

A closed trajectory representing a self-sustained oscillation, goes through the intersecting points of the real switching line with the virtual switching line. At the intersecting points, the manipulated variable is removed from one constant to the other. We can easily determine the period of self sustained oscillation generating at this system from the value τ previously estimated.

Intersecting the real switching line with the virtual one as stated in Fig. 6 *a*), the self-sustained oscillation generated at this system is stable. On the other hand, a self-sustained oscillation corresponding to an intersecting point of Fig. 6 *b*) is unstable.

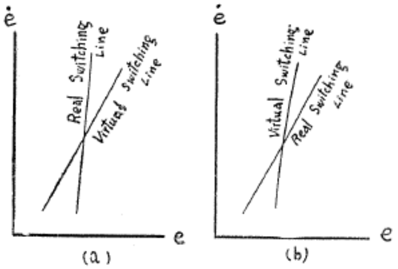

FIG. 6. Stability

Example 1. System, whose controlled ellement is $\frac{1}{s^2}$, without Velocity Feedback Circuit, with Hysteresis.

The virtual switching line of the system does not intersect with the real switching lines which are straight lines parallel to $\dot{e}$-axis. Therefore, any steady state oscillation does not exist.

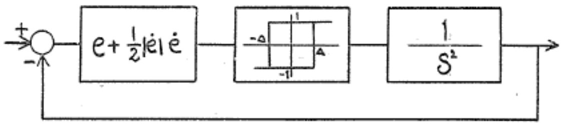

FIG. 7. On-off control system with hysteresis (Ex. 2).

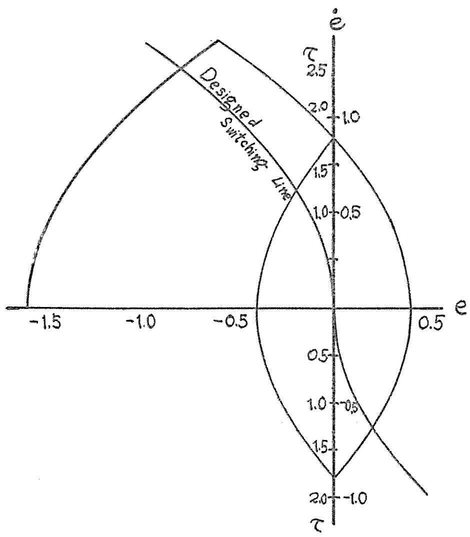

FIG. 8. Performance of the system of Fig. 7.

Example 2. Self-Sustained Oscillation Caused by Hysterisis of the System Having Optimal Designed Velocity Feedback Circuit.

We treat here an on-off control system indicated in Fig. 7. The virtual switching line of this system is $\dot{e}$-axis, and the real one

$$e+\frac{1}{2}(\dot{e})^2=\Delta \text{ or } e-\frac{1}{2}(\dot{e})^2=-\Delta$$

for the upper or lower half of the phase plane.

The manipulated variable is removed at a point $(0, -\sqrt{2\Delta})$ or $(0, \sqrt{2\Delta})$ as stated in Fig. 8, while the period of the self-sustained oscillation is 2.5.

After one switching action is performed, a representative point draws a closed curve, no matter where it may be started.

Example 3. Self-Sustained Oscillation due to Dead Time of the System without Velocity Feedback Circuit.

We consider the system stated in Fig. 9. The switching is performed at dead time after a representative point passed across the designed switching line, *i.e.* $\dot{e}$-axis. The real switching line is, therefore, the straight lines

$$e\cdot\exp(-L)-\dot{e}(1-\exp(-L))=1-\exp(-L)-L\cdot\exp(-L)$$

as stated in Fig. 10, where L's are dead time.

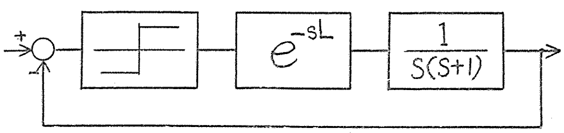

FIG. 9. On-off control system with dead time (Ex. 3).

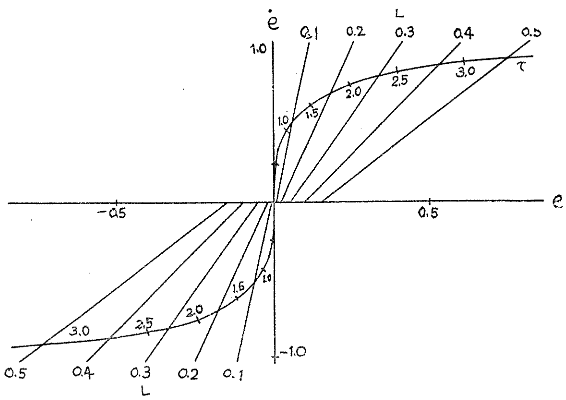

FIG. 10. Real and virtual switching lines of Ex. 3.

From the intersecting points, we can get the self-sustained oscillations.

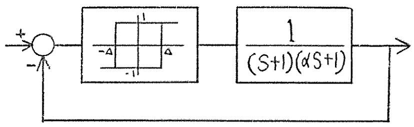

FIG. 11. On-off control system with hysteresis.

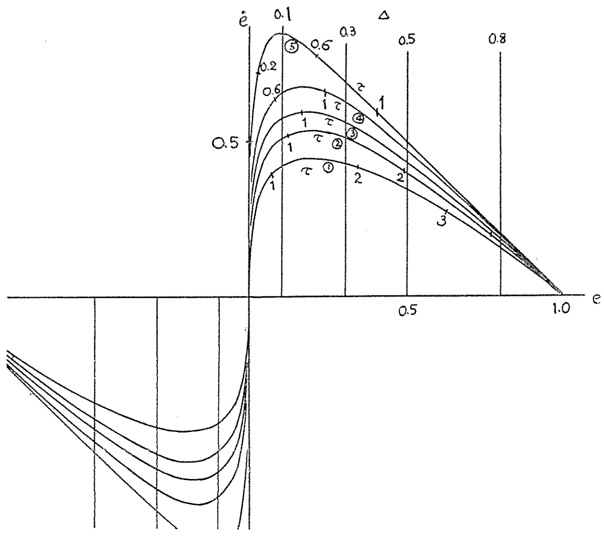

FIG. 12. Performance of the system of Fig. 9.
① $\alpha=1.0$, ② $\alpha=2/3$, ③ $\alpha=1/2$, ④ $\alpha=1/3$, ⑤ $\alpha=1/10$

Example 4. Amplitude of Self-Sustained Oscillation due to Dead Time of the System Having Velocity Feedback Circuit.

The amplitude of the self-sustained oscillation generating at the system whose controlled element is $\frac{e^{-sL}}{s^2}$ is easily estimated for the case where the real switching line is

$$e+\frac{1}{2}|\dot{e}|\dot{e}=0 \text{ or } e+\nu\dot{e}=0,$$

where ν is aconstant.

The amplitude is

$$0.34\ L^2 \text{ or } 0.25\ L^2\left(\frac{L-2\nu}{L-\nu}\right)^2 \quad (\text{for } L\leq\nu)$$

respectively, as stated in Fig. 13.

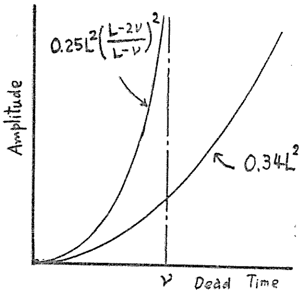

Fig. 13. Relations between amplitude and dead time.

Concluding Remarks

The author introduced a technique for rapid estimation of steady state performance of the second order on-off control system having symmetrical on-off controller without dead zone and the switching line symmetrical with respect to the origin of phase plane. In designing an on-off control system, drawing the virtual switching line on a transparent paper, putting it on a real awitching line, he got the properties of self-sustained oscillation generated in this system.

References

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