DISTRIBUTION OF TEMPERATURE IN GAS TURBINE ROTOR

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Contents

1. Introduction

A gas turbine rotor may consist of a turbine wheel at one end and a turbo-compressor runner at the other end of the turbine shaft. The distribution of temperature in the surrounding medium varies from place to place. To solve such an actual problem, the wrighter of this papers tried to get the required results, such as, the distribution of temperature, the effect of local cooling, the dissipation of heat and the like, by simplifying the problems to some extent.

2. Given Problems

Fig. 1 (a) shows the construction of the gas turbine rotor under question, A being the turbine disc, B the compressor runner and C the common shaft. In order to get approximate results, the following procedure has been taken:

1) The turbine wheel is assumed, firstly, to be a circular disc of uniform thickness, as shown in Fig. 1 (b).

2) Secondly, the distribution of temperature in the actual turbine wheel of variable thickness, as shown in Fig. 1 (a), is to be determined in a more rigorous way.

3) The heating effect of the moving blades is to be taken into account by determining, theoretically, the equivalent heat transfer coefficient at the peripheral surface of the turbine wheel.

4) The distribution of temperature in the moving blades is to be computed in two ways: a) by assuming the uniform distribution of temperature at any section per-

pendicular to the longitudinal axis of the blades, b) by assuming the blade to be a sectoral column, thus, giving always lower temperature at the inner parts than the temperature at the outer surface at any section.

5) The cooling effect of the turbine shaft as well as the compressor runner is to be taken into account by considering the equivalent heat transfer coefficient at the side surface of the turbine disc to which the shaft is attached.

6) The heat transfer coefficient at the surface of the moving blades is to be estimated by the empirical formula given by Schörner.

7) The heat transfer coefficient at the side surface of the turbine wheel and the compressor runner, as well, is to be estimated by the empirical formula given by Stanton.

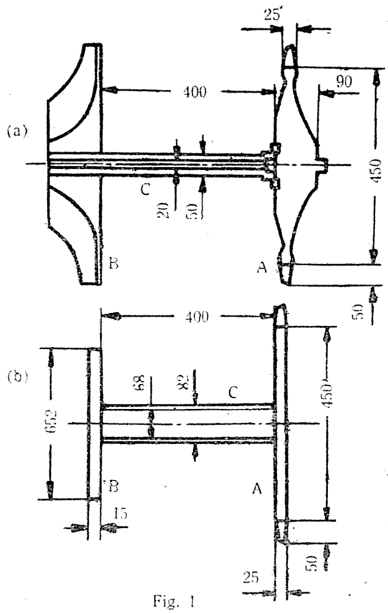


Fig. 1

3. Distribution of Temperature in Moving Blades

In gas turbines, the moving blades taper off gradually from their roots to the tips, but the working edges at the entry and exit of the gas flow are rounded to the great extent, instead of the sharp edges as seen in steam turbine blades. Hence, these may be assumed to be heating fins of uniform section. In such a case, the temperature at the section x is given by the following equation:

$$\theta_x = \vartheta_{gas} - \frac{\vartheta_{gas} - \theta_{x=0}}{2} \cdot \frac{(1-\varepsilon)\cdot e^{-m(x_1-x)} + (1+\varepsilon)\cdot e^{m(x_1-x)}}{\cosh m x_1 + \varepsilon \sinh m x_1}, \quad \cdots\cdots (1)$$

where $m = \sqrt{\dfrac{\alpha}{\lambda_{blade}} \cdot \dfrac{U_2}{F_2}}$;

$\varepsilon = \dfrac{\alpha_{x1}}{m \cdot \lambda_{blade}}$;

λ_{blade} = thermal conductivity for the material of the blade, kcal/m h °C;

α = heat transfer coefficient between the working gas and the side surface of the moving blades, kcal/m^2 h °C;

α_{x1} = Do. between the working gas and the tip surface of the moving blades, kcal/m^2 h °C;

ϑ_{gas} = mean temperature of the working gas;

$\theta_{x=0}$ = temperature of the moving blades at their roots;

x_1 = height of the blades, m;

F_2 = mean area of the blade section, m^2;

U_2 = mean perimeter of the blade section, m.

These are shown in Fig. 2.

When the blade is assumed to be a sectoral column as shown in Fig. 3, the tem-

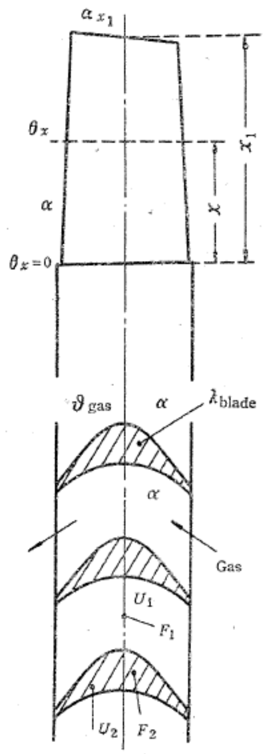


Fig. 2

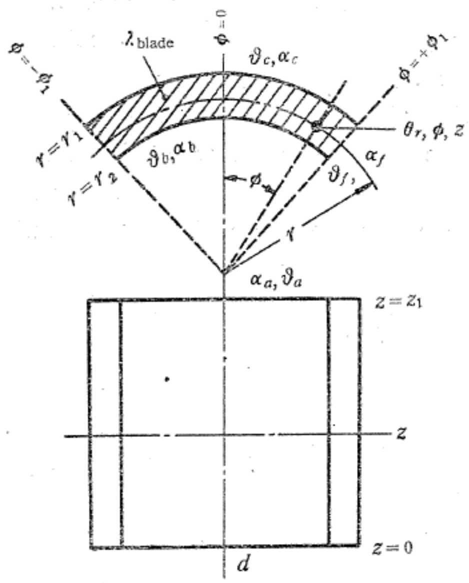


Fig. 3

perature at the distance x from its root, at a radius r and at an angular distance ϕ from its central section, may be expressed by the following equation:

$$\theta_{z,r,\phi} = \vartheta_{gas} + \sum_m \sum_n (A_{mn} e^{-\mu z} + B_{mn} e^{\mu z}) \cos nv \{J_0(mr) - \rho Y_0(mr)\}, \cdots \quad (2)$$

where $v = \frac{r_1 + r_2}{2} \cdot \phi$ and $\mu = \sqrt{m^2 + n^2}$.

In this equation, J_0 and Y_0 denote the ordinary Bessel function of order zero and of the first and the second kind, respectively.

The solution (2) must satisfy the heat transfer conditions at the inner and outer surfaces of the blades. These boundary conditions indicate that m and ρ must be the roots of the following equations:

$$\left. \begin{aligned} m \cdot \frac{J_1(mr_2) - \rho Y_1(mr_2)}{J_0(mr_2) - \rho Y_0(mr_2)} &= -h_b, \\ m \cdot \frac{J_1(mr_1) - \rho Y_1(mr_1)}{J_0(mr_1) - \rho Y_0(mr_1)} &= +h_c, \end{aligned} \right\} \cdots\cdots\cdots (2\,a)$$

where $h_b = \frac{\alpha_b}{\lambda_{blade}}$; $h_c = \frac{\alpha_c}{\lambda_{blade}}$;

α_b = heat transfer coefficient at the working surface of blade;

α_c = Do. at the back surface of the blade.

J_1 and Y_1 represent Bessel functions of order unity.

The solution (2) must also satisfy the heat transfer condition at the edges of the blades, which gives the following condition:

$$n \tan nv_1 = h_f , \qquad \cdots\cdots\cdots (2\,b)$$

where $v_1 = \frac{r_1 + r_2}{2} \cdot \phi_1$, $h_f = \frac{\alpha_f}{\lambda_{blade}}$,

α_f = heat transfer coefficient at the entry and the exit edges of the blade.

Finally, the values of A_{mn} and B_{mn} are to be determined by the boundary conditions at $z = 0$ and $z = z_1$. The surface condition of the heat transfer at the tip of the blade is expressed by

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=z_1} = h_a (\vartheta_a - \theta_{z=z_1}) ,$$

where $h_a = \frac{\alpha_a}{\lambda_{blade}}$.

By substituting the relations given by Eq. (2), we obtain

$$\sum_m \sum_n \mu(-A_{mn}e^{-\mu z_1} + B_{mn}e^{\mu z_1}) \cos nv \cdot \{J_0 - \rho Y_0\}$$
$$+ h_a \sum_m \sum_n (A_{mn}e^{-\mu z_1} + B_{mn}e^{\mu z_1}) \cos nv \cdot \{J_0 - \rho Y_0\} = h_a(\vartheta_a - \vartheta_{gas}) .$$

Multiply the both sides by $\cos nv \cdot dv$ and integrate from $\phi = 0$ to $\phi = \phi_1$, we get

$$\sum_m \left\{ - \left(\frac{\mu}{h_a} - 1\right) A_{mn}e^{-\mu z_1} + \left(\frac{\mu}{h_a} + 1\right) B_{mn}e^{\mu z_1} \right\} = Y_n \cdot (\vartheta_a - \vartheta_{gas}) ,$$

where $$Y_n = \frac{\int_0^{\phi_1} \cos nv \cdot dv}{\int_0^{\phi_1} \cos^2 nv \cdot dv} = \frac{2 \sin nv_1}{nv_1 + \sin nv_1 \cdot \cos nv_1} .$$

Next, multiply the both sides of the last equation by $\{J_0(mr) - \rho Y_0(mr)\} r \cdot dr$ and integrate from $r = r_2$ to $r = r_1$, then we get

$$-\left(\frac{\mu}{h_a} - 1\right) A_{mn}e^{-\mu z_1} + \left(\frac{\mu}{h_a} + 1\right) B_{mn}e^{\mu z_1} = X_{m\rho} \cdot Y_n \cdot (\vartheta_a - \vartheta_{gas}) , \quad \cdots\cdots (2\,c)$$

where $$X_{m\rho} = \frac{\int_{r_2}^{r_1} (J_0 - \rho Y_0) r \cdot dr}{\int_{r_2}^{r_1} (J_0 - \rho Y_0)^2 r \cdot dr} = \frac{\Big[r(J_1 - \rho Y_1) \Big]_{r_2}^{r_1}}{\frac{m}{2} \Big[r^2 \{ (J_0 - \rho Y_0)^2 + (J_1 - \rho Y_1)^2 \} \Big]_{r_2}^{r_1}} .$$

The boundary condition at the root of the blade may be divided into the following three cases:

1) $\theta_{z=0} = \theta_0$ = constant (independent of r and ϕ) ,
2) $\theta_{z=0} = f(r, \phi)$
3) $\left(\frac{\partial\theta}{\partial z}\right)_{z=0}$ = constant for all values of r and ϕ.

When $\theta_{z=0} = \theta_0$ = constant, this must be equal to the right-hand side of Eq. (2) for $z = 0$. Treating the equation, thus obtained, just in the same way as before, we get finally

$$A_{mn} + B_{mn} = - X_{m\rho} \cdot Y_n (\vartheta_a - \theta_0) . \qquad \cdots\cdots\cdots (2\,d)$$

In our case, we may assume $\vartheta_a = \vartheta_{gas}$. Then Eq. (2 c) and (2 d) give the values of A_{mn} and B_{mn} in the following form:

$$\begin{matrix} A_{mn} \\ B_{mn} \end{matrix} = \frac{-\left(\frac{\mu}{h_a} \pm 1\right)e^{\pm\mu z_1}\cdot(\vartheta_{gas} - \theta_0)\cdot X_{m\rho}\cdot Y_n}{\left(\frac{\mu}{h_a}+1\right)e^{\mu z_1} + \left(\frac{\mu}{h_a}-1\right)e^{-\mu z_1}}. \qquad \cdots\cdots\cdots (2\,\mathrm{e})$$

In actual case, $e^{-\mu z_1}$ may be neglected in comparison with $e^{\mu z_1}$. In such a case Eq. (2 e) becomes

$$\left.\begin{aligned} A_{mn} &= -(\vartheta_{gas} - \theta_0)\cdot X_{m\rho}\cdot Y_n\,, \\ B_{mn}\cdot e^{\mu z_1} &= -(\vartheta_{gas} - \theta_0)\cdot X_{m\rho}\cdot Y_n \frac{\frac{\mu}{h_a}-1}{\frac{\mu}{h_a}+1}\, e^{-\mu z_1}\,. \end{aligned}\right\} \qquad \cdots\cdots\cdots (2\,\mathrm{f})$$

In the second case, we get

$$\theta_{z=0} = a_0 + \sum_n a_n \cos nv\,.$$

This must be equal to the right-hand side of Eq. (2) for $z = 0$. Hence we have

$$a_0 + \sum_n a_n \cos nv = \vartheta_{gas} + \sum_m\sum_n (A_{mn} + B_{mn}) \cos nv\cdot\{J_0(mr) - \rho Y_0(mr)\}\,.$$

This relation must hold good for all values of r. Multiply the both sides by $\{J_0(mr) - \rho Y_0(mr)\}r\cdot dr$ and integrate from $r = r_2$ to $r = r_1$, then we get

$$A_{mn} + B_{mn} = -(\vartheta_{gas} - a_0)X_{m\rho}\cdot Y_n + a_n\cdot X_{m\rho}\,. \qquad \cdots\cdots\cdots (2\,\mathrm{g})$$

Combining Eq. (2 c) with (2 g), we obtain

$$\begin{matrix} A_{mn} \\ B_{mn} \end{matrix} = -\frac{\left(\frac{\mu}{h_a} \pm 1\right)e^{\pm\mu z_1}\cdot\{(\vartheta_{gas} - a_0)\cdot Y_n - a_n\}\cdot X_{m\rho}}{\left(\frac{\mu}{h_a}+1\right)e^{\mu z_1} + \left(\frac{\mu}{h_a}-1\right)e^{-\mu z_1}}\,. \qquad \cdots\cdots\cdots (2\,\mathrm{h})$$

In the third case, we may assume that the temperature gradient at the root of the blade is constant for all values of r and ϕ. Then we have

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = \sum_m\sum_n \mu\,(-A_{mn} + B_{mn}) \cos nv\cdot\{J_0(mr) - \rho Y_0(mr)\}.$$

Treat this equation in the same way as before, then we have

$$-A_{mn} + B_{mn} = \left(\frac{\partial\theta}{\partial z}\right)_{z=0}\cdot\frac{X_{m\rho}\cdot Y_n}{\mu}. \qquad \cdots\cdots\cdots (2\,\mathrm{i})$$

Combining Ep. (2 i) with (2 c), we get

$$\begin{matrix} A_{mn} \\ B_{mn} \end{matrix} = \frac{-\left(\frac{\partial\theta}{\partial z}\right)_{z=0}\cdot\left(\frac{\mu}{h_a} \pm 1\right)e^{\pm\mu z_1}}{\left(\frac{\mu}{h_a}+1\right)e^{\mu z_1} + \left(\frac{\mu}{h_a}-1\right)e^{-\mu z_1}}\cdot\frac{X_{m\rho}\cdot Y_n}{\mu}\,. \qquad \cdots\cdots\cdots (2\,\mathrm{j})$$

If we neglect $e^{-\mu z_1}$ as compared with $e^{\mu z_1}$, Eq. (2 j) becoms

$$\left.\begin{aligned} A_{mn} &= -\left(\frac{\partial\theta}{\partial z}\right)_{z=0}\frac{X_{m\rho}\cdot Y_n}{\mu}\,, \\ B_{mn} &= -\left(\frac{\partial\theta}{\partial z}\right)_{z=0}\frac{X_{m\rho}\cdot Y_n}{\mu}\, e^{-2\mu z_1}\,. \end{aligned}\right\} \qquad \cdots\cdots\cdots (2\,\mathrm{k})$$

The value of $\left(\frac{\partial\theta}{\partial z}\right)_{z=0}$ = constant can be determined in the following way. As shown by Eq. (1), the temperature at any section of the blade is given by

$$\theta_z = \vartheta + \frac{\left(\frac{m}{h_a}+1\right)e^{m(z_1-z)} + \left(\frac{m}{h_a}-1\right)e^{-m(z_1-z)}}{\left(\frac{m}{h_a}+1\right)e^{mz_1} + \left(\frac{m}{h_a}-1\right)e^{-mz_1}}(\theta_0 - \vartheta_c),$$

where $m = \sqrt{\frac{\alpha}{\lambda}\cdot\frac{U}{F}}$ which is quite different from the value of m found in Eq. (2). Derive the expression for θ_z of the equation for $\frac{\partial\theta}{\partial z}$ and put $z = 0$, then we get

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = -\frac{\left(\frac{m}{h_a}+1\right)e^{mz_1} - \left(\frac{m}{h_a}-1\right)e^{-mz_1}}{\left(\frac{m}{h_a}+1\right)e^{mz_1} + \left(\frac{m}{h_a}-1\right)e^{-mz_1}}\cdot m(\theta_0 - \vartheta_c). \quad \cdots\cdots\ (2\,1)$$

Or, neglecting $e^{-\mu z}$, this expression becomes

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = m\cdot(\vartheta_c - \theta_0). \qquad \cdots\cdots\cdots\ (2\,\mathrm{m})$$

4. Equivalent Heat Transfer Coefficient at the Periphery of Turbine Disc

As shown in Fig. 2, the sectional area of the blade is F_2 m², the perimeter of the same section U_2 m, the area of the clear surface at the periphery of the turbine disc F_1 m². Simplifying Eq. (1), we get the expression for the temperature difference between the working gas and the blade surface

$$\vartheta_{gas} - \theta_x = (\vartheta_{gas} - \theta_0)\frac{\cosh m(x_1 - x)}{\cosh mx_1}.$$

Hence the heat transferred to the moving blade from the working gas is given by

$$Q_2 = \alpha\int_0^{x_1} U_2(\vartheta_{gas} - \theta_0)\frac{\cosh m(x_1 - x)}{\cosh mx_1}\,dx = \alpha\cdot U_2\frac{\vartheta_{gas} - \theta_0}{m}\tanh mx_1.$$

On the other hand, the heat transferred to the turbine disc through the surface F_1 is given by

$$Q_1 = \alpha\cdot F_1(\vartheta_{gas} - \theta_0).$$

Let α' be the required equivalent heat transfer coefficient at the peripheral surface of the turbine disc, then it may be given by

$$Q = \alpha'(F_1 + F_2)\cdot(\vartheta_{gas} - \theta_0).$$

Putting $Q = Q_1 + Q_2$, we obtain

$$\alpha' = \alpha\left\{\frac{F_1}{F_1 + F_2} + \frac{U_2}{m(F_1 + F_2)}\tanh mx_1\right\} \quad \mathrm{kcal/m^2\,h^\circ C}, \quad \cdots\cdots\cdots\ (3)$$

where α stands for the heat transfer coefficient between the blade surface and the working gas.

5. Distribution of Temperature in Turbine Disc

Assuming the turbine disc to be a circular disc of uniform thickness as shown in Fig. 4, we get the expression for the temperature at a point (r, z) as follows:

$$\theta_{r,z} = \vartheta_c + \sum_p (A_p e^{-pz} + B_p e^{pz}) \cdot J_0(pr) . \quad \cdots (4)$$

The surface condition at the periphery $r = r_1$ gives

$$p \frac{J_1(pr_1)}{J_0(pr_1)} = h_c = \frac{\alpha_c}{\lambda_{disc}}, \quad \cdots\cdots\cdots (4\,a)$$

which enables us to determine the every value of p.

The solution (4) must satisfy the surface condition at $z = 0$. The temperature of the gas at the left-hand side of the disc can be generally expressed by

$$\vartheta_a = f_a(r) .$$

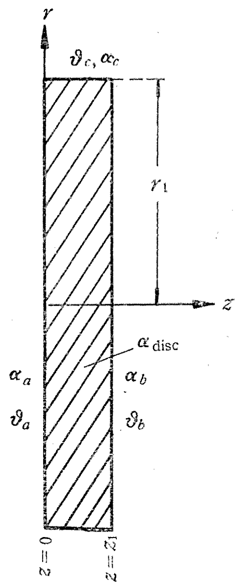


Fig. 4

The surface condition gives

$$\sum_p p(-A_p + B_p) \cdot J_0(pr) + h_a\{f_a(r) - \vartheta_c - \sum_p (A_p + B_p) \cdot J_0(pr)\} = 0 ,$$

where $h_a = \alpha_a / \lambda_{disc}$.

Multiply the both sides of this equation by $J_0(pr) r \cdot dr$ and integrate from $r = 0$ to $r = r_1$, then we get

$$\left(\frac{p}{h_a} + 1\right) \cdot A_p - \left(\frac{p}{h_a} - 1\right) \cdot B_p = \varphi_{ap} - \vartheta_c \cdot X_p , \quad \cdots\cdots\cdots (4\,b)$$

where $X_p = \dfrac{\int_0^{r_1} J_0(pr) r \cdot dr}{\int_0^{r_1} [J_0(pr)]^2 r \cdot dr} = \dfrac{2}{pr_1} \dfrac{J_1(pr_1)}{J_0^2(pr_1) + J_1^2(pr_1)}$,

$$\varphi_{ap} = \frac{\int_0^{r_1} f_a(r) \cdot J_0(pr) r \cdot dr}{\text{〃}} .$$

Similarly, the surface condition at $z = z_1$ gives

$$\left(\frac{p}{h_b} - 1\right) \cdot A_p \cdot e^{-pz_1} - \left(\frac{p}{h_b} + 1\right) \cdot B_p \cdot e^{pz_1} = \vartheta_c \cdot X_p - \varphi_{bp} , \quad \cdots\cdots\cdots (4\,c)$$

where $\vartheta_b = f_b(r)$, $h_b = \alpha_b / \lambda_{disc}$,

$$\varphi_{bp} = \frac{\int_0^{r_1} f_b(r) \cdot J_0(pr) r \cdot dr}{\int_0^{r_1} [J_0(pr)]^2 r \cdot dr} .$$

Combine Eq. (4 c) with (4 b), then we obtain

$$\begin{matrix} A_p \\ B_p \end{matrix} = \frac{(\varphi_{ap} - \vartheta_c \cdot X_p) \cdot \left(\frac{p}{h_b} \pm 1\right) e^{\pm pz_1} - (\vartheta_c \cdot X_p - \varphi_{bp}) \cdot \left(\frac{p}{h_a} \mp 1\right)}{\left(\frac{p}{h_a} + 1\right)\left(\frac{p}{h_b} + 1\right) e^{pz_1} - \left(\frac{p}{h_a} - 1\right)\left(\frac{p}{h_b} - 1\right) e^{-pz_1}} . \quad \cdots\cdots (4\,b)$$

The integration for the value of φ_{ap} or φ_{bp} may be possible if the temperature $\vartheta_a = f_a(r)$ or $\vartheta_b = f_b(r)$ can be expressed in the form of the so-called Bessel-Fourier series which has been specially developed by the writer of this papers.

Apart from such analytical solutions, we proceed to compute the actual case of local cooling by blowing cold air jets at the temperature ϑ_{a_2} or ϑ_{b_2}, whereas the temperature of the gas in contact with the other parts of the disc surface remains constant at ϑ_{a_1} or ϑ_{b_1} as shown in Fig. 5. In such a case, the integration is quite easily done as follows:

$$\begin{aligned}\varphi_{\substack{ap\\bp}} &= \int_0^{r_1} f_{\substack{a\\b}}(r)\cdot J_0(pr)\,r\cdot dr \Big/ \int_0^{r_1} [J_0(pr)]^2 r\cdot dr \\ &= \Big[\vartheta_{\substack{a_1\\b_1}}\Big\{\int_0^{r_2} J_0(pr)\,r\cdot dr + \int_{r_3}^{r_1} J_0(pr)\,r\cdot dr\Big\} + \vartheta_{\substack{a_2\\b_2}}\int_{r_2}^{r_3} J_0(pr)\,r\cdot dr\Big] \div \int_0^{r_1}[J_0(pr)]^2 r\cdot dr \\ &= \Big[\vartheta_{\substack{a_1\\b_1}}\frac{r_2}{p}J_1(pr_2) + \vartheta_{\substack{a_1\\b_1}}\Big\{\frac{r_1}{p}J_1(pr_1) - \frac{r_3}{p}J_1(pr_3)\Big\} \\ &\quad + \vartheta_{\substack{a_2\\b_2}}\Big\{\frac{r_3}{p}J_1(pr_3) - \frac{r_2}{p}J_1(pr_2)\Big\}\Big] \div \Big[\frac{r_1^2}{2}\Big\{J_0^2(pr_1) + J_1^2(pr_1)\Big\}\Big]. \qquad \cdots\cdots\cdots (4\,\mathrm{e})\end{aligned}$$

The solution for the case of a turbine disc of variable thickness, variable gas temperature and variable heat transfer coefficient is somewhat toilsome. At any rate, the temperature of the disc is expressed evidently by Eq. (4). The surface condition of heat transfer can be generally given by

$$\left(\frac{\partial\theta}{\partial n}\right)_0 = h(\vartheta - \theta_0)\,, \qquad \cdots\cdots\cdots (5)$$

where $h = \alpha/\lambda_{disc}$; θ_0 = surface temperature of the disc at the point under question; $\left(\frac{\partial\theta}{\partial n}\right)_0$ = normal temperature gradient at the same point.

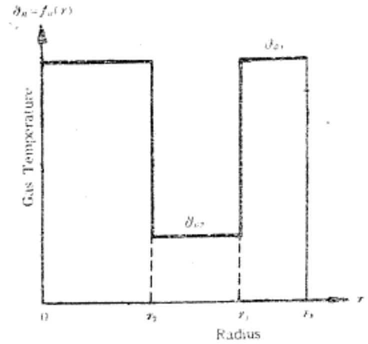


Fig. 5

The normal temperature gradient at the disc surface is expressed by

$$\frac{\partial\theta}{\partial n} = \frac{\partial\theta}{\partial z}\cos\delta + \frac{\partial\theta}{\partial r}\sin\delta\,, \qquad \cdots\cdots\cdots (5\,\mathrm{a})$$

where δ denotes the angular inclination of the normal at this point to z-axis, as shown in Fig. 6.

Let Grad. θ denote the resultant of the two principal components of the temperature gradient $\frac{\partial\theta}{\partial r}$ and $\frac{\partial\theta}{\partial z}$, then we have

$$\frac{\partial\theta}{\partial r} = \mathrm{Grad.}\,\theta\cdot\sin\beta$$

and $$\frac{\partial\theta}{\partial z} = \mathrm{Grad.}\,\theta\cdot\cos\beta\,.$$

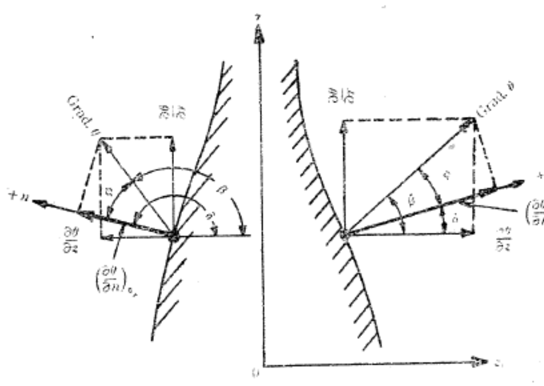


Fig. 6

Hence we get

$$\frac{\partial\theta}{\partial n} = \text{Grad.}\,\theta\,(\cos\beta\cos\delta + \sin\beta\sin\delta) = \text{Grad.}\,\theta\cdot\cos(\beta-\delta) = \text{Grad.}\,\theta\cdot\cos\phi\,, \quad (5\,\text{b})$$

where ϕ = angle between Grad. θ and $\frac{\partial\theta}{\partial n}$,

β = angle between Grad. θ and z-axis.

Applying Eq. (5) to the various points, $2n$ points in all, at the surface of the turbine disc on both sides, we have $2n$ equations involving n pairs of A_p and B_p. Solve these simultaneous equations for A_p's and B_p's.

6. Distribution of Temperature in Turbine Shaft

The surface condition of the turbine shaft is shown in Fig. 7. The heat transfer coefficient α_e at the inner surface of the hollow shaft may be assumed to be infinitely small, since the hollow shaft is made air-tight at both ends. In such a case, the temperature of the shaft may be expressed by

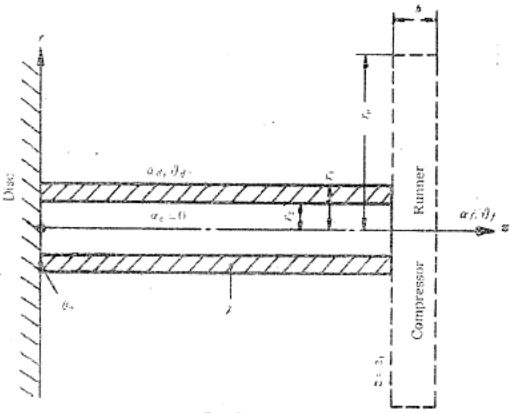


Fig. 7

$$\theta = \vartheta_d + (A\cdot e^{-mz} + B\cdot e^{mz})\,, \quad (6)$$

where $m = \sqrt{\frac{\alpha_d}{\lambda}\frac{U}{F}}$; $U = 2\pi r_1$,

$F = \pi(r_1^2 - r_2^2)$;

r_1 = outer radius of the hollow shaft;

r_2 = inner radius of the hollow shaft;

α_d = heat transfer coefficient at the outer surface of the shaft;

λ = thermal conductivity of the shaft material;

ϑ_d = temperature of the medium in contact with the outer surface of the shaft.

The solution (6) must satisfy the boundary conditions at $z = 0$ and $z = z_1$. These conditions will give two simultaneous equations involving A and B, which give

$$\begin{matrix} A \\ B \end{matrix} = \frac{(\theta_0 - \vartheta_d)\left(\frac{m}{h_f} \pm 1\right)e^{\pm mz_1} \pm (\vartheta_d - \vartheta_f)}{\left(\frac{m}{h_f} + 1\right)e^{mz_1} + \left(\frac{m}{h_f} - 1\right)e^{-mz_1}}, \qquad \cdots\cdots\cdots (6\,\text{a})$$

where $h_f = \alpha_f/\lambda$; ϑ_f = temperature of the surrounding medium in contact with the end surface of the shaft at $z = z_1$; θ_0 = temperature of the shaft at $z = 0$.

The temperature gradient at the section $z = 0$ is given by

$$\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = -m(A - B)\,.$$

And the heat quantity flowing through this section is expressed by

$$Q_{z=0} = \lambda m(A - B)F\,.$$

On the other hand, the heat transferred at this section may be expressed in the form of Newton's cooling equation as follows:

$$Q_{z=0} = \alpha'(\theta_0 - \vartheta_c)F\,,$$

where α' stands for the equivalent heat transfer coefficient.

Equate the right-hand sides of these two equations, then we get the expression for the equivalent heat transfer coefficient reduced to at the outer surface of the turbine disc.

$$\alpha' = \frac{\lambda m(A-B)}{\theta_0 - \vartheta_c} = \lambda m \frac{\left\{\left(\frac{m}{h_f}+1\right)e^{mz_1} + \frac{\vartheta_d - \vartheta_f}{\theta_0 - \vartheta_d}\right\} - \left\{\left(\frac{m}{h_f}-1\right)e^{-mz_1} - \frac{\vartheta_d - \vartheta_f}{\theta_0 - \vartheta_d}\right\}}{\left(\frac{m}{h_f}+1\right)e^{mz_1} + \left(\frac{m}{h_f}-1\right)e^{-mz_1}}\,. \qquad \cdots\cdots\cdots (6\,\text{b})$$

7. Cooling Effect of Compressor Runner

On the other end of the hollow shaft, a compressor runner is fixed. The runner has the impeller vanes which add the cooling surface so much to the base area of the disc itself. Standing on this point of view, we may reduce the runner to a simple circular disc of uniform thickness, as shown in Fig. 7.

The temperature of the disc at radius r is given by

$$\theta_r = \vartheta_g + AI_0(\mu r) + BK_0(\mu r)\,, \qquad \cdots\cdots\cdots (7)$$

where $\mu = \sqrt{\frac{2\alpha}{\lambda b}}$; λ = thermal conductivity of the runner disc, kcal/m h °C;

α = heat transfer coefficient between the impeller vane surface and the flowing air; b = thickness of the reduced disc.

The values of A and B are to be determined by the boundary conditions at the inner surface $r = r_1$ and the outer surface $r = r_0$ of the disc. The results of the computations are as follows:

$$\begin{matrix} A \\ B \end{matrix} = \frac{\frac{\varphi_k}{\varphi_i}\cdot(\theta_0 - \vartheta_g)}{\varphi_k I_0(\mu r_1) + \varphi_i K_0(\mu r_1)}\,, \qquad \cdots\cdots\cdots (7\,\text{a})$$

where $\varphi_i = \frac{\mu}{h_0} I_1(\mu r_0) + I_0(\mu r_0)$;

$\varphi_k = \frac{\mu}{h_0} K_1(\mu r_0) - K_0(\mu r_0)$;

I_0, K_0, I_1, K_1 = Bessel functions of imaginary argument, the first and second kind, order zero and unity, respectively; $h_0 = \alpha_0/\lambda$; ϑ_g = temperature of the cooling air; α_0 = heat transfer coefficient at the periphery of the disc; θ_0 = temperature of the disc at $r = r_1$.

The dissipation of heat from the side surface of the disc is given by

$$Q_1 = \frac{4\pi\alpha}{\mu} \frac{\varphi_k\{r_0 I_1(\mu r_0) - r_1 I_1(\mu r_1)\} - \varphi_i\{r_0 K_1(\mu r_0) - r_1 K_1(\mu r_1)\}}{\varphi_k I_0(\mu r_1) + \varphi_i K_0(\mu r_1)} (\theta_0 - \vartheta_g)\,. \qquad \cdots\cdots\cdots (7\,\text{b})$$

The dissipation of heat from the peripheral surface $r = r_0$ of the disc is given by

$$Q_2 = \alpha_0 \cdot 2\pi r_0 b\{AI_0(\mu r_0) + BK_0(\mu r_0)\}\,. \qquad \cdots\cdots\cdots (7\,c)$$

Next, the dissipation of heat from the end surface of the shaft which is to be considered solid in this case is expressed by

$$Q_3 = \alpha_f \pi r_1^2(\theta_{z=z1} - \vartheta_f)\,. \qquad \cdots\cdots\cdots (7\,d)$$

If the total heat $(Q_1 + Q_2 + Q_3)$ is to be dissipated from the end surface $\pi(r_1^2 - r_2^2)$ of the hollow shaft, we have the following relation which enables us to determine the value of the equivalent heat transfer coefficient α_f'.

$$Q_1 + Q_2 + Q_3 = \alpha_f' \pi(r_1^2 - r_2^2)\cdot(\theta_{z=z1} - \vartheta_f)\,. \qquad \cdots\cdots\cdots (7\,e)$$

Since $\theta_{z=z1} = \theta_0$, Eq. (7 e) gives

$$\alpha_f' = \alpha_f \frac{r_1^2}{r_1^2 - r_2^2} + \frac{4\alpha}{\mu(r_1^2 - r_2^2)}\cdot\frac{\varphi_k\{r_0 I_1(\mu r_0) - r_1 I_1(\mu r_1)\} - \varphi_i\{r_0 K_1(\mu r_0) - r_1 K_1(\mu r_1)\}}{\varphi_k I_0(\mu r_1) + \varphi_i K_0(\mu r_1)}$$
$$+ \frac{2\alpha_0 b r_0}{r_1^2 - r_2^2}\cdot\frac{\varphi_k I_0(\mu r_0) + \varphi_i K_0(\mu r_0)}{\varphi_k I_0(\mu r_1) + \varphi_i K_0(\mu r_1)}\,. \qquad \cdots\cdots\cdots (7\,f)$$

Thus, we see that the value $h_f' = \alpha_f'/\lambda$ must be used instead of the actual value $h_f = \alpha_f/\lambda$ in order to take account of the cooling effect of the compressor runner.

8. Heat Transfer Coefficient between Moving Blades and Working Gas

Schörner (Luftf.-Forschung, Bd. 15, 1938) studied on the transfer of heat between the working gas and the moving blade surface and came to the conclusion that the Nusselt number $Nu = \frac{\alpha\cdot d}{\lambda}$ is proportional to 0.86 power of Péclet number $Pé = \frac{w\cdot d}{a}$. According to his experimental researches, the proportionality constant varies from 0.0235 to 0.0265. Taking the mean value 0.0250, we may put

$$\alpha_{gas} = 0.0250\,\frac{\lambda_{gas}}{d}\left(\frac{w\cdot d}{a}\right)^{0.86}, \qquad \cdots\cdots\cdots (8)$$

where $d = \frac{4f}{u}$ = hydraulic mean diameter, m; f = sectional area for the gas flow through the blabe passage, m^2; u = perimeter of the section for the gas flow, m; w = mean gas velocity relative to the working surface of the moving blades, m/h; $a = \frac{\lambda_{gas}}{C_p\cdot\gamma}$ = thermometric conductivity, m^2/h; λ_{gas} = thermal conductivity of the working gas, kcal/m h°C; C_p = specific heat of the working gas at constant pressure, kcal/kg °C; γ = specific weight of the working gas, kg/m^3.

The specific weight of the working gas at temperature T°K and pressure P kg/m^2 is given by

$$\gamma = \frac{M}{22.41}\cdot\frac{273.2}{T}\cdot\frac{P}{P_0},$$

where P_0 = 10,332 kg/m^2; M = molecular weight of the working gas. Thus, we see that the denominator of the expression of a is given by

$$C_p\cdot\gamma = \frac{\mathfrak{C}_p}{22.41\cdot\frac{T}{273.2}\cdot\frac{P_0}{P}}, \qquad \cdots\cdots\cdots (8\,a)$$

where $\mathfrak{C}_p$ = molecular specific heat of the working gas at constant pressure, kcal/mol °C.

The molecular specific heat of the working gas may be given by

$$\mathfrak{C}_p = \sum_i \frac{n_i}{N} \mathfrak{C}_{pi}, \qquad \cdots\cdots\cdots (8\,b)$$

where $\mathfrak{C}_{pi}$ = molecular specific heat at constant pressure of any constituent gas; n_i = number of mols of the same gas; N = total number of mols of the working gas as a whole mixture.

The thermal conductivity of gases can be expressed in the following form:

$$\lambda = \lambda_{273} \frac{273 + C}{T + C} \left(\frac{T}{273}\right)^{1.5} \quad \text{kcal/m h °C}, \qquad \cdots\cdots\cdots (9)$$

where λ_{273} and C are the characteristic constants for individual gas. For the constituent gases of the working gas under question, we have the following data:

Gas	λ_{273}	C
O_2 and N_2	0.0203	144
CO_2	0.01212	519
H_2O	0.0140	581

For the gaseous mixture, the thermal conductivity is given by

$$\lambda_{gas} = \sum_i \frac{n_i}{N} \lambda_i . \qquad \cdots\cdots\cdots (9\,a)$$

In these equations, the temperature T°K must be the mean temperature ranging from the gas temperature ϑ_{gas}°K to the surface temperature θ_w°K. Standing on the experimental point of view, we may assume the temperature distribution as follows:

$$T \cdot y = \text{constant},$$

where y denotes the normal distance from the solid surface. Then we have

$$T_m = \frac{\log_e \vartheta_{gas}/\theta_w}{1/\theta_w - 1/\vartheta_{gas}}, \qquad \cdots\cdots\cdots (10)$$

which is the mean temperature under question.

9. Heat Transfer Coefficient at the Surface of Turbine Disc

Stanton gives the following empirical formula for the heat transfer coefficient between air stream and fixed plates:

$$\alpha_{air} = 1.18(1 + 0.00075\, T_m) w^{0.73} \quad \text{kcal/m}^2\text{ h °C}, \qquad \cdots\cdots\cdots (11)$$

where T_m = mean temperature, °K and w = air velocity, km/h.

In the case of a rotating disc, the air velocity may be assumed to be equal to the tangential velocity of the disc surface, which is evidently proportional to the radius r. Thus, we have

$$\alpha = f(r),$$

for the variation of the heat transfer coefficient along the disc surface. The mean value of α is given by

$$\alpha_{mean} = \frac{1}{\pi r_1^2} \int_0^{r_1} f(r) \cdot 2\pi r \cdot dr .$$

10. Calculated Results

The foregoing theories have been applied to the case of a gas turbine used in turbo-jet propulsion.

1) Characteristics of the working gas

The ratio by weight of gasoline to combustion air is 1 : 85. Assuming the chemical formula for the gasoline used for C_7H_{16}, the burnt products have been found to consist of the following constituents:

Component	CO_2	H_2O	O_2	N_2	Mixture
Mols	7	8	50.5	232.2	297.7
Mol ratio $\frac{n_i}{N}$	0.0235	0.0269	0.1697	0.7799	1.0000
$\mathfrak{C}_p$ kcal/mol °C	9.37	7.28	5.22	5.22	5.37
λ kcal/m h °C	0.0419	0.0495	0.0565	0.0565	0.0559

The surface temperature of the moving blades $\theta_w = 758°K$. The gas temperature varies from 660°C at the entry to 570°C at the exit of the blade passage, giving the maen value $\vartheta_{gas} = 615°C = 888°K$. Substituting these values in Eq. (10), we have $T_m = 825°K$.

The values of $\mathfrak{C}_p$ for the constituent gases at this mean temperature are given in the table above. The molecular specific heat $\mathfrak{C}_{p\,gas}$ for the working gas can be estimated by Eq. (8 b) as given in the last column. The thermal conductivity of the individual gas can be estimated by Eq. (9) and that of the working gas by Eq. (9 a). The figures thus obtained are also tabulated above.

From Eq. (8 a), we have $C_p \cdot \gamma = 0.0800$, hence the thermometric conductivity of the working gas $a = 0.699$. The relative velocity of the flowing gas being 445 m/s at the entry and 230 m/s at the exit, the mean velocity will be $w = 338$ m/s = 1215 km/h.

The sectional area for the gas flow through the blade passage is 649 mm² at the inlet, 345 mm² at the outlet and 426 mm² at the halfway section. Taking the mean out of these three, we obtain $f = 473$ mm². In similar way, the perimeter of the section for the gas flow varies from 112 to 109 mm, giving the mean perimeter $u = 110$ mm. Thus, we can estimate the value of the hydraulic mean diameter $d = 0.0172$ m. The Péclet number becomes $Pé = 29{,}900$ and $\lambda/d = 3.746$.

The heat transfer coefficient between the working gas and the blade surface is now to be estimated by Eq. (8) by making use of these values, namely, $\alpha_{gas} = 574$ kcal/m²h °C.

2) Equivalent heat transfer coefficients

The thermal conductivity of the moving blade is $\lambda_{blade} = 31.0$ kcal/mh °C at 550°C. In Fig. 2, $F_2 = 70$ mm² and $U_2 = 38.6$ mm, whence we have $m = \sqrt{\frac{\alpha_{gas}}{\lambda_{blade}} \frac{U_2}{F_2}} = 101\ \mathrm{m}^{-1}$. The height of the moving blades being $x_1 = 0.050$ m, we have $mx_1 = 5.056$, $\varepsilon = 0.183$ for Eq. (1).

Next, the area of the clear surface at the periphery of the turbine disc $F_1 = 310$ mm², hence we have $\frac{F_1}{F_1 + F_2} = 0.816$ and $\frac{U_2}{F_1 + F_2} = 101.6\ \mathrm{m}^{-1}$. Substituting these numerical values, thus obtained, in Eq. (3), we can evaluate the equivalent heat transfer coefficient at the periphery of the turbine disc, namely, $\alpha' = 1045\,\mathrm{kcal/m^2h^\circ C}$, which shows an increase by 82 % due to the heating effect of the turbine blades as heating fins.

In Fig. 7, $r_0 = 0.326\,\mathrm{m}$, $r_1 = 0.041\,\mathrm{m}$, $r_2 = 0.034\,\mathrm{m}$, $b = 0.015\,\mathrm{m}$. The peripheral speed being 130 m/s at $r = r_0$ and 63 m/s at $r = r_1$, the mean speed may take $w = 96.5\,\mathrm{m/s} = 347\,\mathrm{km/h}$. The mean temperature of the air through the compressor runner $T = 328^\circ\mathrm{K}$. By making use of these data, Stanton's formula gives $\alpha = 292\,\mathrm{kcal/m^2h^\circ C}$.

The thermal conductivity of the material for the compressor runner may take $\lambda = 30$. Then we have $h = h_0 = 9.73\,\mathrm{m}$; $\mu = \sqrt{\dfrac{2\alpha}{\lambda b}} = 36.0\,\mathrm{m^{-1}}$. The values of Bessel functions involved in Eq. (7 a) are as follows:

Argument	I_0	I_1	K_0	K_1
$\mu r_1 = 1.477$	1.624,4	0.959,1	0.223,2	0.290,9
$\mu r_0 = 11.73$	14,635	13,997	$2.915,2 \times 10^{-6}$	$3.037,0 \times 10^{-6}$

By making use of these numerical values, we have

$$\varphi_i = 533{,}100 \quad \text{and} \quad \varphi_k = 0.1154 \times 10^{-3}.$$

Substituting these values in Eq. (7 e), we have $\alpha' = 3330\,\mathrm{kcal/m^2h^\circ C}$. The cooling effect of the compressor runner has been thus estimated as an increased heat transfer coefficient α_f' instead of actaul value $\alpha_f = 30$ at the end surface of the shaft itself, their ratio being 111.

Thirdly, we proceed to compute the cooling effect of the turbine shaft on the temperature distribution of the turbine disc proper. In Fig. (7), $\theta_0 = 227^\circ\mathrm{C}$, $\vartheta_f = 25^\circ\mathrm{C}$, $\vartheta_d = 100^\circ\mathrm{C}$. The heat transfer coefficient at the outer surface of the shaft is given by Stanton's formula as $\alpha_d = 59.4$, corresponding to the mean temperature $T_m = 403^\circ\mathrm{K}$ and the air velocity $w = 151\,\mathrm{km/h}$. The thermal conductivity of the material for the shaft being $\lambda = 31.4$, we have $h_d = 1.89$, $h_f = 111$, and the latter corresponds to $\alpha' = 3330$ as explained above. The perimeter of the outer surface of the hollow shaft $U = 0.157\,\mathrm{m}$, the sectional area $F = 0.00165\,\mathrm{m^2}$, hence $m = \sqrt{\dfrac{\alpha_d}{\lambda}\dfrac{U}{F}} = 13.4\,\mathrm{m^{-1}}$. The length of the turbine shaft $z_1 = 0.400\,\mathrm{m}$, and then $mz_1 = 5.37$. By putting these values in Eq. (6 e), we get the value of the equivalent heat transfer coefficient $\alpha' = 421\,\mathrm{kcal/m^2h^\circ C}$. On the other hand, Stanton's formula gives the actual heat transfer coefficient at this part of the disc surface as $\alpha = 96$. Thus, we see that the cooling effect of the turbine shaft with the compressor runner at its other end increases the heat transfer coefficient 4.4 times as much.

In the fourth place, we have to consider the cooling effect of the projection at the center of the turbine disc as shown in Fig. 1 (a). The dimensions and the boundary conditions are given in Fig. 8. The temperature distribution of this projection is given by Eq. (6), and the equivalent heat transfer coefficient by Eq. (6 e). Thus, by taking $\vartheta_c = \vartheta_b$, we obtain

$$\alpha' = \lambda m \frac{\left(\frac{m}{h_b}+1\right)e^{mz_1} - \left(\frac{m}{h_b}-1\right)e^{-mz_1}}{\left(\frac{m}{h_b}+1\right)e^{mz_1} + \left(\frac{m}{h_b}-1\right)e^{-mz_1}}.$$

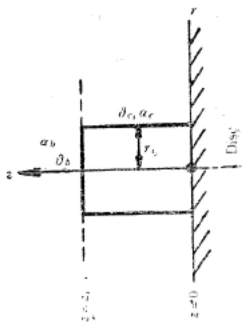


Fig. 8

Stanton's formula gives $\alpha_c = 41$ and $\alpha_b = 30$ in our case. The thermal conductivity $\lambda = 31.7$, hence $h_b = 0.945$, $mz_1 = 0.308$ for $z_1 = 0.021$ m. The equivalent heat transfer coefficient $\alpha' = 166$ kcal/m²h°C, showing that this projection increases the heat transfer coefficient from 30 to 166, i.e., 5.5 times as much.

3) Distribution of temperature

The following eight cases have been worked out.

Case I and II: The temperature ϑ_c of the working gas was first found to be 615°C when the edges of the moving blades were partly damaged. Improving the compression efficiency by replacing the original centrifugal compressor with a propeller type, the temperature of the working gas was reduced to 515°C, and the fuel-air ratio remains the same.

Case III and IV: The temperature ϑ_a of the fluid in contact with the downstream-side of the turbine disc was at first found to be 400°C, which was lowered to 250°C by supplying some quantity of cooling air.

Case V and VI: The temperature ϑ_b of the fluid in contact with the upstream-side of the turbine disc remained unaltered at 300°C. In order to cool the turbine disc positively, cooling air at 100°C or 200°C was tried to blow against the surface of the disc on both sides. This local cooling was carried out at the rim of the turbine disc ranging from $r = 0.20$ to $r = 0.22$ m, whereas the outside radius of the disc $r = 0.225$ m.

Case VII and VIII: In order to get better lubricating condition at the turbine shaft bearings, the temperature of the shaft was tried to be lowered by blowing some quantity of cooling air at 100°C or 200°C against the both sides of the disc. This was done over the central part of the disc ranging from the center to $r = 0.04$ m. The temperature of the fluid in contact with the outer surface of the turbine shaft remained constant at 100°C and at 25°C at the end surface.

Boundary conditions

Case	Working gas ϑ_c°C	Down-stream-side ϑ_a°C	Upstream-side ϑ_b°C
I	615	400	300
II	515	〃	〃
III	615	250	〃
IV	515	〃	〃

The calculated results are shown in diagrams in four plates.

a) Plate 1 and 2 corresponding to Case I, II, III and IV.

Plate 1 shows the surface temperature of the turbine disc of uniform thickness of 25 mm. Lowering the gas temperature affects distinctly the temperature distribution near the wheel rim, but little effect can be seen near the disc center. The temperature of the moving blades is affected most distinctly, differing by about 1.5°C at their tip surface from the temperature of the working gas.

In Plate 2, the distribution of temperature in the disc are shown with groups of isothermals or isothermal surfaces. Heat flows into the disc through the peripheral surface of the disc and flows out of the disc into the surrounding fluid on both sides of the disc. But this tendency will subside immediately. Toward the center of the disc, heat flows from one side to the other due to the temperature difference between the fluids on both sides of the disc.

Plate 1 also shows the distribution of temperature in the shaft. The temperature drops rapidly at first, but the dropping rate of temperature will soon become very gradual. Toward the other end of the shaft, the temperature drops very rapidly once more. This is due to the cooling effect of the cmpressor runner.

The distribution of temperature in the compressor runner can be determined by Eq. (7). The followings are some of the results of calculations:

Radius, mm	41	60	80	100	120	140	180	326
Case I	51.0	31.7	27.9	26.3	25.6	25.3	25.1	25.0
Case III	49.3	31.0	27.6	26.1	25.5	25.2	25.1	25.0

b) Plate 3 and 4 corresponding to Case V, VI, VII, and VIII.

As stated above, the distribution of temperature in the disc will become more uniform when the disc is locally cooled by blowing cold air against the side surface near the wheel rim (Case V and VI).

The temperature of the shaft at the left-hand bearing has been lowered by blowing cold air against the central part of the turbine disc (Case VII and VIII). The boundary conditions are shown in the following table:

Case	Gas ϑ_c°C	Downstream-side ϑ_a°C		Upstream-side ϑ_b°C
V	615	For $r = 200\sim220$ mm	200°	200°
		Other parts	400°	300°
VI	〃	For $r = 200\sim220$ mm	100°	100°
		Other parts	400°	300°
VII	〃	For $r = 0\sim40$ mm	200°	200°
		Other parts	400°	300°
VIII	〃	For $r = 0\sim40$ mm	100°	100°
		Other parts	400°	300°

4) Three-dimensional analysis of the distribution of temperature in moving blades.

In Fig. 3, we are given $r_1 = 0.0236$ m, $r_2 = 0.0190$ m, $\phi_1 = 48°24' = 0.845$ rad., $\vartheta_a = \vartheta_b = \vartheta_c = \vartheta_f = 615$°C, $\alpha_{gas} = 574$ kcal/m²h°C, $\lambda = 21.0$ kcal/m h °C, $h_a = h_b = h_c = h_f = 27.332\ \mathrm{m}^{-1}$.

The caluculated values of $\left(\frac{\partial\theta}{\partial z}\right)_{z=0} = 13310$°C/m by Eq. (2 m). Starting from this datum, the values of the coefficients A_{mn} and B_{mn} by Eq. (2 k) have been determined and the temperature distribution in the blades by Eq. (2). Some details of the calculated results are given in the following table.

These results show that the temperature of the blade is lowest at the middle of its section. Approaching to the edges of the blade ($\phi_1 = 48°24'$), and also to the outer surface ($r = 0.0236$ m) or the inner surface ($r = 0.019$ m), the temperature increases gradually. This variation in temperature distribution is seen most distinctly at the

Temperature distribution in turbine blades

Rad. mm	z mm / ϕ	0	10	20	30
19	0°	499.6	576.2	602.4	611.0
	20°	498.2	577.1	602.6	〃
	48°24′	510.8	583.0	604.9	611.8
21	0°	493.3	575.1	602.1	610.8
	20°	495.0	576.1	602.5	611.0
	48°24′	508.0	582.0	604.6	611.7
23.6	0°	497.1	576.4	602.5	611.0
	20°	498.8	577.3	602.9	611.2
	48°24′	511.3	583.3	604.9	611.8

z, mm	θ°C
0	500.0
10	578.9
20	603.6
30	611.4
40	613.9
50	614.6
88	615.0

root of the blade ($z = 0$), subsiding rapidly as z increases.

In conclusion, we may use Eq. (1) for the sake of simple calculations without much error, except for special purposes. The figures in the other table obtained from Eq. (1) will show you a good comparison.

11. Heating and Cooling Effect on Moving Blades

Suppose the case that an aircraft begins hell-diving, the gas turbine, having run with full power till now, will cease the internal combustion and only the cold air at higher altitudes will pass through the turbine blading. Such a sudden cooling of the moving blades will cause local shrinkage at the periphery of the turbine disc, resulting in radial cracks in the disc at its rim. Here arises another problem to be solved.

The temperature of the blade at any instant will be given by

$$\theta = \vartheta_0 + \sum_m \sum_n \sum_\mu C_\mu \{J_0(mr) - \rho Y_0(mr)\} \cos nv \cdot \cos \nu z \cdot e^{-\mu^2 at}, \quad \cdots\cdots (12)$$

where $\mu = \sqrt{m^2 + n^2 + \nu^2}$ and ϑ_0 = temperature of cold air through the blade passage.

The values of m and ρ in pairs are given by Eq. (2a) and that of n by Eq. (2b), as explained before. The boundary condition at the tip of the blade ($z = z_1$) gives the following relation:

$$\nu \tan \nu z_1 = h_a, \quad \cdots\cdots\cdots (12\,\mathrm{a})$$

which determines the every value of ν.

At the beginning of the cooling, the temperature distribution in the moving blades is the same as given by Eq. (2), which means the stationary temperature distribution after a long-time running. Thus, we have

$$\vartheta + \sum_m \sum_n (A_{mn} \cdot e^{-\sqrt{m^2+n^2}\,z} + B_{mn} \cdot e^{\sqrt{m^2+n^2}\,z}) \{J_0(mr) - \rho Y_0(mr)\} \cos nv$$
$$= \vartheta + \sum_m \sum_n \sum_\nu C_\mu \{J_0(mr) - \rho Y_0(mr)\} \cos nv \cdot \cos \nu z .$$

This equation must hold good for all values of r, z and v. Treating this equation just in the same way as before, we get

$$C_\mu = (\vartheta_a - \vartheta_0) \cdot X_{m\rho} \cdot Y_n \cdot Y_\nu + A_{mn} W_{-\nu} + B_{mn} W_{+\nu}, \quad \cdots\cdots\cdots (12\,\mathrm{b})$$

where $X_{m\rho}$ and Y_n are the same as before and Y_ν may be obtained by putting νz_1

for nv_1 in the expression of Y_n. Furthermore

$$W_{\pm\nu} = \frac{\int_0^{z_1} e^{\pm\sqrt{m^2+n^2}\,z} \cdot \cos \nu z \cdot dz}{\int_0^{z_1} \cos^2 \nu z \cdot dz},$$

where the denominater is equal to $\frac{\nu z_1 + \sin \nu z_1 \cos \nu z_1}{2\nu}$ and the numerater

$$\frac{1}{\mu^2}\{(\pm\sqrt{m^2+n^2} \cos \nu z_1 + \nu \sin \nu z_1) \cdot e^{\pm\sqrt{m^2+n^2}\,z} \mp \sqrt{m^2+n^2}\}.$$

12. Résumé

The foregoing theories and calculations lead us to the following conclusions:

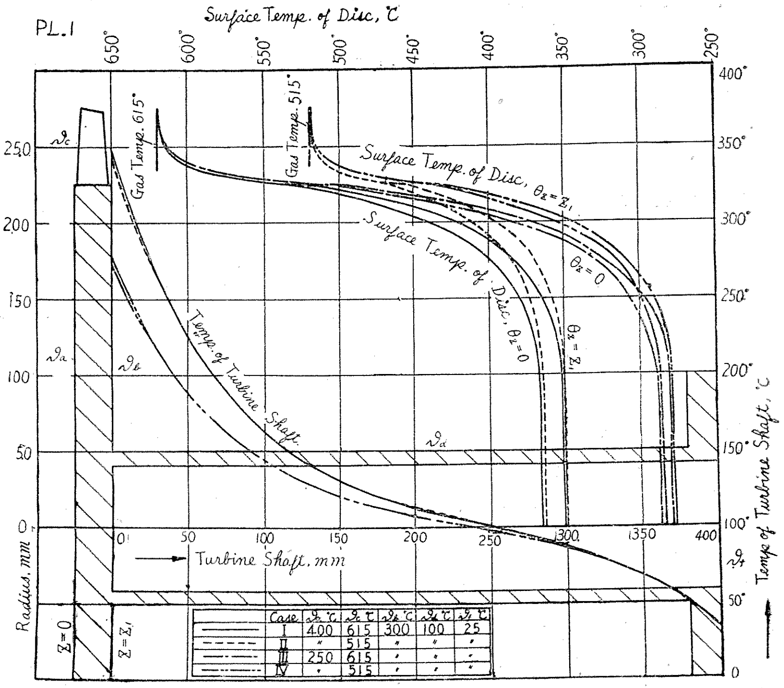
1) The distributions of temperature, thermal deformations and also thermal stresses can be estimated accurately to some extent which affords more reliable data to the practical designers.

2) The quantity of heat to be carried away for cooling can be worked out. This will furnish to the designers a certain starting point in designing the cooling system.

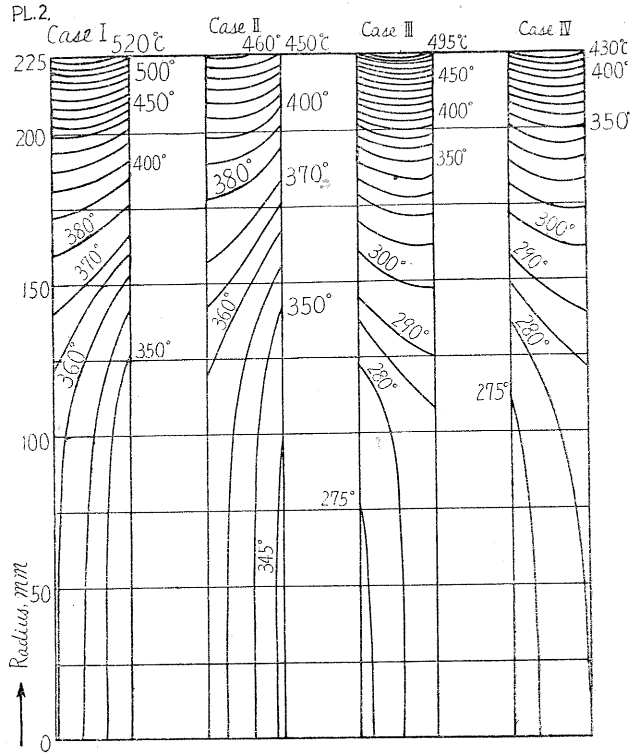
3) Experimental researches can be replaced by the foregoing computations of this kind, resulting in great saving of time and labour.

The writer of this papers looks forward to more detailed and reliable data about the heat transfer coefficient for the gas stream at high speed along solid surfaces, especially at higher temperatures.

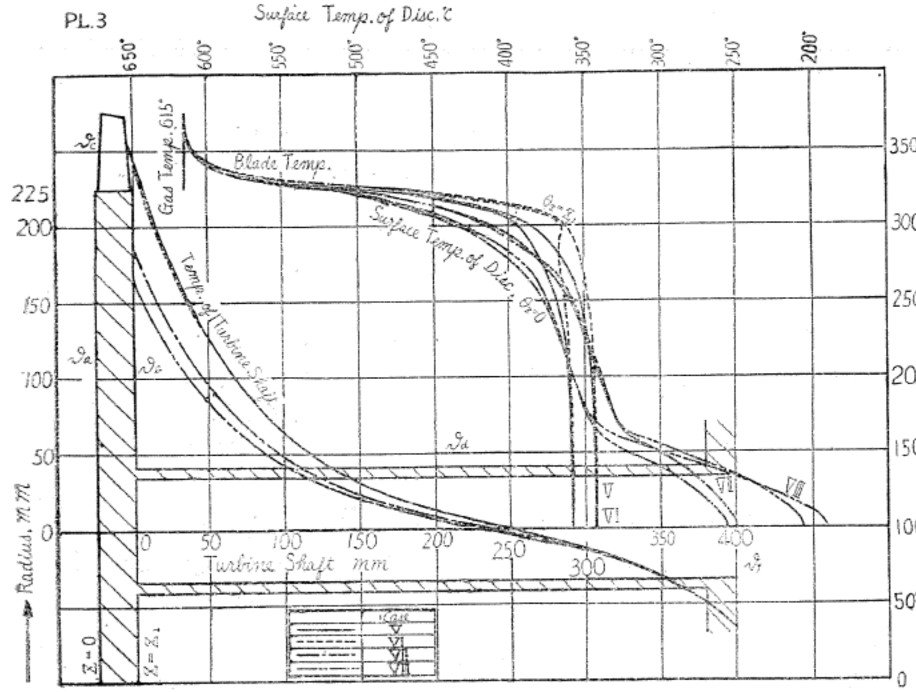
PL. 1


Surface Temp. of Disc, °C
Gas Temp. 615°
Gas Temp. 515°
Surface Temp. of Disc, $\theta_z = z_1$
Surface Temp. of Disc, $\theta_z = 0$
$\theta_z = 0$
$\theta_z = z_1$
Temp. of Turbine Shaft.
Turbine Shaft, mm
Radius, mm
Temp. of Turbine Shaft, °C
z = 0
z = z1
Case I 400 615 300 100 25
II 515
III 250 615
IV 515

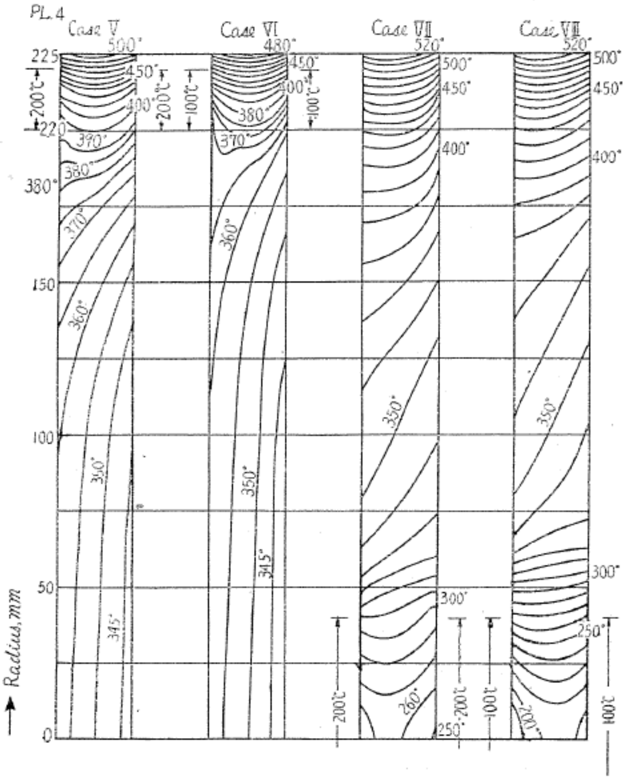
PL. 2.


Case I 520°C
Case II 460° 450°C
Case III 495°C
Case IV 430°C
Radius, mm

PL. 3


Surface Temp. of Disc. ℃
Gas Temp. 615°
Blade Temp.
Surface Temp. of Disc.
Temp. of Turbine Shaft
Turbine Shaft mm
Radius, mm
Case
V
VI
VII
VIII

PL. 4


Case V
Case VI
Case VII
Case VIII
Radius, mm