ON THE SURGING OF A BLOWER

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1. Introduction

A violent vibration of pressure is apt to occur when a blower or a pump is working at its small discharge or shut-off condition. It is said that the vibration is caused by the praticular part of the characteristic curve — the negative characteristic or the part of increasing head by increasing discharge, such as AB in Fig. 1, — and pressure and discharge vary along the line $ADBEA$ of Fig. 1 during the vibration(1). But in the pipe line if there is no non-return valve, such as check valve or foot valve, the change along EA can not occur and it must be corrected in the way that the cyclic path of a surging is $A'D'BE'A'$, where $E'A'A$ is the characteristic curve for a negative discharge. But in this explanation the inertia force of the fluid column in vibration and the spring force are not taken into account satisfactorily and accordingly the period of vibration is not known.

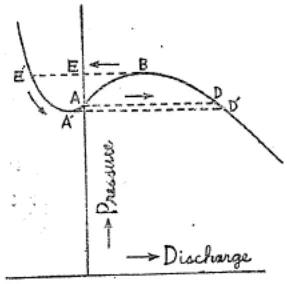

Fig. 1. Cyclic change of pressure and discharge.

The surging of a pump has recently been treated by some authors precisely(2) but compressibility of the fluid in vibration has not been taken into account hitherto. In this paper the sustained vibration of a compessible fluid column was considered, as caused in a pipe line of a blower by the particular part of its characteristic curve.

2. Vibration of a fluid column

Let v be the velocity along the axis of a straight pipe of uniform section (x-direction), ρ the density, p the pressure, and t the time. The equations of motion and of continuity are as follows:-

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad \cdots\cdots\cdots(1)$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x} + \rho\frac{\partial v}{\partial x} = 0. \qquad \cdots\cdots\cdots(2)$$

(1) It may be recognized that the working condition of the pump changes abruptly from B to A while the surging vibration goes along BEA and the difference of head EA is supported by the check valve or foot valve suddenly closed.

(2) R. Dziallas: Untersuchungen an einer Kreiselpumpe mit labiler Kenlinie.
Sumiji Fujii, Trans. of the Japan Society of Mechanical Engineers. Vol. 13, No. 44, 46, and 48.

Assume now v and the variations of p and ρ are small in the case of the small vibration of a fluid column, as usually done, and $v\dfrac{dv}{dx}$ and $v\dfrac{d\rho}{dx}$ may be neglected in comparison with the other terms in the above equations. As the definition of the modulus of elasticity K of a fluid in compression is $\Delta p = -K\Delta V/V$, where V is the volume of a fluid and ΔV the decrease of volume due to increase of pressure Δp, we can write $dp = K d\rho/\rho$ and the above equations are

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad \frac{\partial p}{\partial t} = -K\frac{\partial v}{\partial x}. \qquad \cdots\cdots\cdots(1\,a), (2\,a)$$

Differentiating (1 a) with respect to t and (2 a) with x, we obtain

$$\frac{\partial^2 v}{\partial t^2} = a^2\frac{\partial^2 v}{\partial x^2}, \qquad \cdots\cdots\cdots(3)$$

and also differentiating (1 a) with respect to x and (2 a) with t, we obtain

$$\frac{\partial^2 p}{\partial t^2} = a^2\frac{\partial^2 p}{\partial x^2}, \qquad \cdots\cdots\cdots(4)$$

where $a^2 = K/\rho$ and a is the velocity of propagation of sound in a fluid. Equation (3) or (4) is known as the differential equation of wave motion.

When the surging occurs, the one end of the pipe line, which is either the delivery or the suction side of a blower, is usually closed or almost closed and the other end open. Now we assume the suction side is open and the delivery side closed. Denoting the distance along the pipe from its open end by x, $p = 0$ and $\partial v/\partial x = 0$ at $x = 0$(3). The solutions of (3) and (4) satisfying these boundary conditions can be written by using normal co-ordinates as follows:-

$$p = \sum_s q_s \sin k_s x, \qquad v = u_0 + \sum_s q_s' \cos k_s x,$$

where u_0 is the mean velocity of a fluid in a pipe and q_s and q_s' are the normal co-ordinates, which are the functions of time. And from (2 a) we have $q_s' = \dot{q}_s/(k_s K)$, where $\dot{q}_s = dq_s/dt$. Then the above equations are

$$p = \sum_s q_s \sin k_s x, \qquad v = u_0 + \frac{1}{K}\sum_s \frac{\dot{q}_s}{k_s}\cos k_s x. \qquad \cdots\cdots\cdots(5)$$

Now if the dimensions of the pipe line are given, we can determine the value of k_s. Let l be the length of the pipe of sectional area A. Its one end is open and a chamber of volume V is attached at the other end. The fluid is flowing out from the chamber at a constant rate $u_0 A$. The pressure in the chamber is equal to the pressure at $x = l$ at every moment, provided the wave length of the vibration of the fluid column in the pipe is sufficiently longer than the linear dimensions of the chamber. The excess volume of fluid flowing in over flowing out from the chamber in time Δt is $(Av_{x=l} - Au_0)\Delta t$. Accordingly the rise of pressure Δp in the chamber is $K(Av_{x=l} - Au_0)\Delta t/V$ in time Δt or we can write

$$\left(\frac{\partial p}{\partial t}\right)_{x=l} = K\frac{A(v_{x=l} - u_0)}{V}.$$

Putting equation (5) into this relation we obtain

(3) The origin of the co-ordinate is to be shifted if the mouth correction is necessary.

$$k_s l \tan k_s l = Al/V. \qquad \cdots\cdots\cdots (6)$$

This is the equation for determining k_s and if we put $k_s = \alpha_s s\pi/(2l)$, the value of α_s is shown in Fig. 2 for various values of Al/V, s being odd number only in this case.

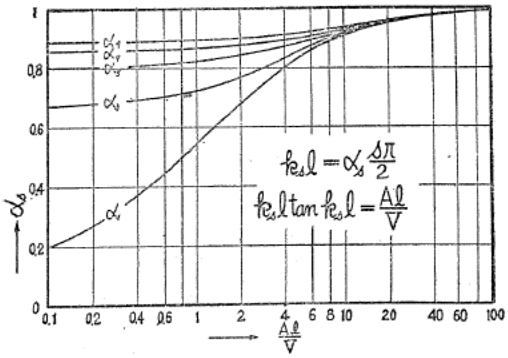

Fig. 2.

We can see from equation (6) that, if Al/V becomes infinitely large, then $\alpha_s = 1$ and the vibration of fluid column in a pipe with one end closed is obtained. In case of $Al/V > 30$ the vibration may be practically regarded as that in a pipe with one end closed(4). When Al/V is small, equation (6) becomes $(k_1 l)^2 \fallingdotseq Al/V$ for $s = 1$ and so α_1 is equal to $(2/\pi)\sqrt{Al/V}$. This shows that when Al/V is less than 0.2, it is practically allowable that the compressibility of the fluid in a pipe may be neglected for the vibration of $s = 1$ and the fluid column in a pipe behaves like a solid piston during the vibration(5). But vibrations other than $s = 1$ are nearly equal to those of fluid column in a pipe with both ends open. The theory of surging of a pump hitherto investigated is applicable to the case of $s = 1$ for $Al/V < 0.2$.

3. The equation of motion for the surging

Let us consider the pipe line as treated in the previous article and shown in Fig. 3, to which a blower is attached at $x = \xi l$. We have known that the vibration of fluid column in the pipe can be expressed in the form of equation (5) using normal co-ordinates q_s. Here we take q_s as generalized co-ordinates and consider that generalized forces act on the vibrating column of fluid at $x = \xi l$ by the blower.

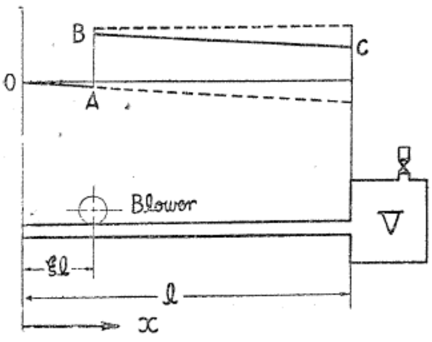

Fig. 3.

Lagrange's equation of motion is given in the form

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} + \frac{\partial F}{\partial \dot{q}_s} + \frac{\partial V_0}{\partial q_s} = Q_s, \qquad \cdots\cdots\cdots (7)$$

where T is the kinetic energy, V_0 the potential energy, F the dissipation function, and Q_s the generalized force having no potential.

As the velocity of fluid in the chamber is very slow as compared with the one in the pipe, the kinetic energy of the fluid in the chamber may be neglected. So the

(4) The difference of periods of vibrations is about 3% when $Al/V = 30$.

(5) The difference of periods of vibrations is about 3% when $Al/V = 0.2$.

kinetic energy of the system is

$$T = \frac{1}{2}\rho A\int_0^l v^2 dx = \frac{1}{2}\rho A\int_0^l \left(u_0 + \frac{1}{K}\sum_s \frac{\dot{q}_s}{k_s}\cos k_s x\right)^2 dx$$
$$= \frac{1}{2}\rho A l u_0^2 + \frac{\rho A u_0}{K}\sum_s \frac{\dot{q}_s}{k_s^2}\sin k_s l + \frac{\rho A l}{4K^2}\sum_s \frac{\dot{q}_s^2}{k_s^2}\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\}. \quad \cdots(8)$$

In this calculation, using equation (6), the following relations may be used.

$$\frac{l}{2} + \frac{\sin 2k_s l}{4k_s} = \frac{l}{2}\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\}, \quad \cdots\cdots\cdots(a)$$

$$\frac{\sin(k_s + k_r)l}{k_s + k_r} + \frac{\sin(k_s - k_r)l}{k_s - k_r} = 0. \quad \cdots\cdots\cdots(b)$$

In these equations both k_s and k_r satisfy equation (6) and $r \neq s$.

When the pressure at x rises by an amount Δp, the fluid of volume Adx is compressed by an amount $A\cdot dx\cdot \Delta p/K$. Accordingly the work of compression is $\int_0^p p(A\,dx/K)dp = (1/2)(A/K)p^2 dx$ and the potential energy of the system is

$$V_0 = \frac{1}{2}\frac{A}{K}\int_0^l p^2 dx + \frac{1}{2}\frac{V}{K}p^2_{x=l}$$
$$= \frac{1}{2}\frac{A}{K}\int_0^l \left(\sum_s q_s \sin k_s x\right)^2 dx + \frac{1}{2}\frac{V}{K}\left(\sum_s q_s \sin k_s l\right)^2$$
$$= \frac{Al}{4K}\sum_s q_s^2\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\}. \quad \cdots\cdots\cdots(9)$$

In this calculation, using equation (6), the following relations may be used.

$$A\left(\frac{l}{2} - \frac{\sin 2k_s l}{4k_s}\right) + V\sin^2 k_s l = \frac{Al}{2}\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\}, \quad \cdots\cdots(c)$$

$$A\,\{k_r \sin k_s l \cos k_r l - (k_r^2/k_s)\cos k_s l \sin k_r l\}\big/(k_s^2 - k_r^2) + V\sin k_s l \sin k_r l = 0.\cdots(d)$$

We assume that the loss of head is proportional to the velocity in the pipe. The pressure drop is $2\varepsilon\rho v dx$ between the distance dx, where 2ε is a constant of proportionality. As the quantity of flow is Av, the loss of energy between dx is $2\varepsilon\rho A v^2 dx$ per unit time. Then the dissipation function is

$$F = \frac{1}{2}\int_0^l 2\varepsilon\rho A v^2 dx = \varepsilon\rho A l u_0^2 + \frac{2\varepsilon\rho A u_0}{K}\sum_s \frac{\dot{q}_s}{k^2}\sin k_s l$$
$$+ \frac{\varepsilon\rho A l}{2K^2}\sum_s \frac{\dot{q}_s^2}{k^2}\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\}. \quad \cdots\cdots\cdots(10)$$

The generalized force from the blower may be determined as the following. When one of the generalized co-ordinates q_s varies by an amount δq_s, the change of pressure is $\delta p = \delta q_s \sin k_s x$ from equation (5). The change of volume between $x = \xi l$ and $x = l$ and at the chamber is given by using (6) as follows:-

$$\delta\mathfrak{V} = \int_{\xi l}^l \frac{\delta p A dx}{K} + \frac{(\delta p)_{x=l}V}{K} = \delta q_s \frac{A\cos k_s \xi l}{k_s K}.$$

As the pressure P acts at $x = \xi l$ by the blower, work done by the pressure P is $P\delta\mathfrak{V}$

while one of the co-ordinates q_s changes by an amount δq_s. Then we can determine generalized force Q_s by putting the work done equal to $Q_s \delta q_s$, thus:-

$$Q_s = \frac{PA\cos k_s \xi l}{k_s K}. \qquad \cdots\cdots\cdots(11)$$

As the characteristic curve of a blower may be represented by a straight line in a small range of vibration of discharge, we can write

$$P = P_0 + \rho m(v_{x=\xi l} - u_0) = P_0 + \frac{\rho m}{K}\sum_s \frac{\dot{q}_s}{k_s}\cos k_s \xi l, \qquad \cdots\cdots\cdots(12)$$

for the small amplitude of vibration in the vicinity of M shown in Fig. 4, where m is constant. But m is no longer constant for the large amplitude of vibration and the discussion will be made later for this case. Inserting equation (12) into (11) we obtain

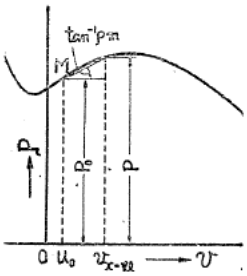

Fig. 4.

$$Q_s = \frac{P_0 A}{k_s K}\cos k_s \xi l + \frac{\rho m A}{k_s K^2}\cos k_s \xi l \sum_s \frac{\dot{q}_r}{k_r}\cos k_r \xi l, \quad \cdots(11\,\mathrm{a})$$

where r includes the case $r = s$.

The equation of motion of the fluid column is obtained by inserting (8), (9), (10), and (11 a) into (7).

$$\ddot{q}_s + 2\varepsilon\dot{q}_s - 2mk_s\sigma_s\cos k_s\xi l\sum_r\frac{\dot{q}_r}{k_r}\cos k_r\xi l + k_s^2 a^2 q_s$$
$$= 2a^2 k_s\sigma_s P_0\cos k_s\xi l - 4\varepsilon u_0 K\sigma_s\sin k_s l. \qquad \cdots\cdots\cdots(13)$$

Here σ_s is given by

$$l\left\{1 + \frac{Al/V}{(Al/V)^2 + k_s^2 l^2}\right\} = \frac{1}{\sigma_s}. \quad \cdots(14)$$

The magnitude of σ_s is shown in Fig. 5 and it is between $1/l$ and $1/(2l)$.

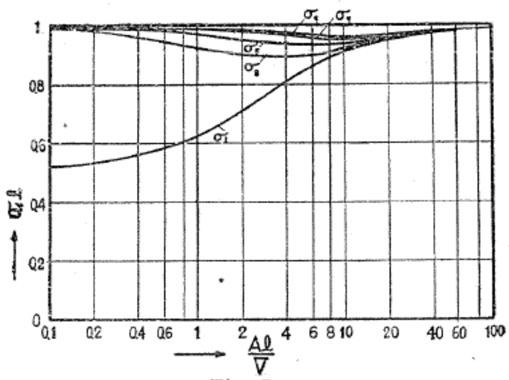

Fig. 5.

4. Its solution

If we assume m is constant, equation (13) is a linear differential equation of second order and its solution is the sum of a particular solution of it and the general solution of the equation, which is the same as (13) except that the right-hand side is zero.

The particular solution of equation (13) is

$$q_s = \frac{2\sigma_s}{k_s}P_0\cos k_s\xi l - \frac{4\varepsilon\rho u_0\sigma_s}{k_s^2}\sin k_s l.$$

The equation which is the same as (13) except that the right-hand side is zero has the solution of the form $q_s = \lambda_s e^{ht}$ and inserting it to this we have

$$\lambda_s h^2 + \lambda_s 2\varepsilon h + 2mk_s\sigma_s h\cos k_s\xi l\sum_r\frac{\lambda_r}{k_r}\cos k_r\xi l + \lambda_s k_s^2 a^2 = 0.$$

Eliminating λ_s we have the so-called "frequency equation" which determines h as follow:-

$$\Delta \equiv \begin{vmatrix} b_{11} & b_{13} & b_{15} \cdots \\ b_{31} & b_{33} & b_{35} \cdots \\ b_{51} & b_{53} & b_{55} \cdots \\ \cdots & \cdots & \cdots \end{vmatrix} = 0$$

where, s and r being odd number,

$$b_{ss} = h^2 + 2(\varepsilon - m\sigma_s \cos^2 k_s \xi l)h + k_s^2 a^2,$$

$$b_{sr} = -2 m\sigma_s \frac{k_s}{k_r} h \cos k_s \xi l \cos k_r \xi l.$$

It is difficult to determine h from the above determinant whose constituents are infinite in number. But if $|\varepsilon - m\sigma_s \cos^2 k_s\xi l|$ and $|m\sigma_s \cos k_s \xi l \cos k_r \xi l|$ are much smaller compared with $k_s a$, we may neglect without serious error the second order and more of smallness in the development of the determinant. It seems that k_s/k_r becomes very large when $s \gg r$. But it may be easily seen that the development contains no more k_s/k_r as the product of this value becomes 1 in $b_{sr}b_{rs}$.

Let $\Delta_1, \Delta_3, \Delta_5, \cdots$ be the first minor of Δ, and

$$\Delta = b_{11}\Delta_1 + b_{13}\Delta_3 + b_{15}\Delta_5 + \cdots\cdots = 0.$$

In this equation all the terms except $b_{11}\Delta_1$ contain the second order or more of smallness and will be neglected. Accordingly the above equation can be written $b_1\Delta_1 = 0$. From this $b_{11} = 0$ or $\Delta_1 = 0$ is obtained. The same process is repeated again to $\Delta_1 = 0$ and so on. Then we obtain finally in a general form as follows:-

$$h^2 - 2(m\sigma_s \cos^2 k_s\xi l - \varepsilon)h + k_s a = 0.$$

This is the equation to determine h and we have

$$h = \nu_s \pm i\omega_s,$$

where

$$\omega_s = k_s^2 a^2 - (m\sigma_s \cos^2 k_s \xi l - \varepsilon)^2, \qquad \cdots\cdots\cdots(15)$$

and

$$\nu_s = m\sigma_s \cos^2 k_s \xi l - \varepsilon. \qquad \cdots\cdots\cdots(16)$$

The general solution of equation (13) is

$$q_s = e^{\nu_s t}(A_s \cos \omega_s t + B_s \sin \omega_s t) + \frac{2\sigma_s}{k_s} P_0 \cos k_s \xi l - \frac{4\varepsilon\rho u_0 \sigma_s}{k_s^2} \sin k_s l.$$

The pressure and the velocity in the pipe are given from equation (5) and the above as follow:-

$$p = \sum_s \left\{ e^{\nu_s t}(A_s \cos \omega_s t + B_s \sin \omega_s t) + \frac{2\sigma_s}{k_s} p_0 \cos k_s \xi l - \frac{4\varepsilon\rho u_0 \sigma_s}{k_s^2} \sin k_s l \right\} \sin k_s x \cdots (17)$$

$$v = u_0 + \frac{1}{K}\sum_s \frac{e^{\nu_s t}}{k_s}\left\{(\nu_s A_s + \omega_s B_s)\cos\omega_s t + (\nu_s B_s - \omega_s A_s)\sin\omega_s t\right\}\cos k_s x. \quad \cdots\cdots (18)$$

Here A_s and B_s are constants to be determined from initial conditions.

It seems from equation (13) that the co-ordinates q_s are not independent to each other but it is known from the above discussions that the degree of interaction of the co-ordinates is so weak that all co-ordinates behave as if they are independent to each others. And it is allowable that all the terms except $s = r$ are neglected in $\sum_s(\dot{q}_r/k_r)\cos k_r\xi l$ of equation (13).

We can see from equations (17) and (18) that the pressure and the velocity in the pipe vibrate with the circular frequency ω_s and the amplitudes of vibrations increase with time when $\nu_s > 0$, that is to say, the vibrations are self-excited when $\nu_s > 0$.

The value of $\sum_s(2\sigma_s/k_s)P_0\cos k_s\xi l\sin k_s x$ in equation (17) represents zero at $0 < x < \xi l$ and P_0 at $\xi l < x < l$, and $\sum_s(4\varepsilon\rho u_0\sigma_s/k_s^2)\sin k_s l\sin k_s x$ represents $2\varepsilon\rho u_0 x$ between $0 < x < l$, which is the pressure drop in the pipe due to loss. These are easily seen from the following. We assume that a function $f(x)$ may be developed in the form $f(x) = \sum_s A_s\sin k_s x$, where k_s satisfies the conditions (6). Multiplying $\sin k_r x$ by both sides and integrating from 0 to l we have

$$\int_0^l f(x)\sin k_r x dx + f(l)\frac{V}{A}\sin k_r l = \sum_s A_s\left\{\int_0^l \sin k_s x\sin k_r x dx + \frac{V}{A}\sin k_s l\sin k_s l\right\}.$$

Using (7), (a) and (d) we obtain that the value of { } in the right-hand side of the above equation is zero at $r \neq s$ and $1/(2\sigma_s)$ at $r = s$. Accordingly we obtain

$$A_s = 2\sigma_s\left\{\int_0^l f(x)\sin k_s x dx + f(l)\frac{V}{A}\sin k_s l\right\}.$$

The afore-mentioned relations are at once obtained by using this equation. And we see from equation (17) that the mean pressure in the pipe is shown by $OABC$ in Fig. 3.

5. The surging

The condition of self-excitation of the fluid column is from the previous article

$$m\sigma_s\cos^2 k_s\xi l > \varepsilon. \quad \cdots\cdots\cdots (19)$$

The value of ε may be estimated by referring Kundt's experiment and the loss of energy at the end of the pipe. But it is recommended that the correct value of ε should be determined by a direct experiment.

We can see from the above equation that if $\cos k_s\xi l$ is zero or very small vibration is not self-excited. This means that when the blower is attached near the node of the vibration of velocity—that is, near the loop of the vibration of pressure,—such a vibration would not be excited. Accordingly when the pipe line is closed at the immediate back of the blower at its discharge side, the surging would not occur.

As the value of σ_s is proportional to $1/l$, the surging would hardly occur in a very long pipe. This means that much energy is necessary for exciting a vibration in a long pipe. The length of the pipe line should be the sum of the actual length of the pipe line and a certain equivalent length corresponding to the motion of fluid in a blower.

The other condition of self-excitation is $m > 0$ and m should be larger than a certain limiting value. We can see from Fig. 4 that m is positive at the part of negative characteristic of a blower. Various methods have been adopted for representing the characteristic curves of blowers. But here we represent them by taking φ and ψ as abscissa and ordinate, which are defined by the following equations.

$$\varphi = \frac{Q}{\pi D b u} = 6.079\,\frac{Q}{n D^3 \Omega}, \qquad \cdots\cdots\cdots(20)$$

$$\psi = \frac{P}{\rho u^2/2} = 729.5\,\frac{P}{\rho n^2 D^2}. \qquad \cdots\cdots\cdots(21)$$

Here Q is the discharge in m^3/s, P the total manometric pressure in kg/m^3, n revolutions per minuite, D, b the outside diameter and breadth of impeller in m respectively, u the peripheral velocity of the impeller in m/s and $\Omega = b/D$. The characteristic curves represented in this way may be drawn approximately in a single curve for the same types of blowers irrespective of its sizes, speed, and kinds of fluids[6].

Let φ_0 and ψ_0 be the values of φ and ψ corresponding to $Q = u_0 A$ and $P = P_0$ respectively, and the characteristic curve of a blower can be represented by the following equation in the neighbourhood of M (see Fig. 6).

$$\psi = \psi_0 + \beta(\varphi - \varphi_0). \qquad \cdots\cdots\cdots(22)$$

Here β is a function of φ_0 and $\varphi - \varphi_0$ for a characteristic curve but it is allowable for small $|\varphi - \varphi_0|$ that β is constant. We have from equations (14), (20), and (21)

$$m = 0.008333\,\frac{An}{\Omega D}\,\beta. \qquad \cdots\cdots\cdots(23)$$

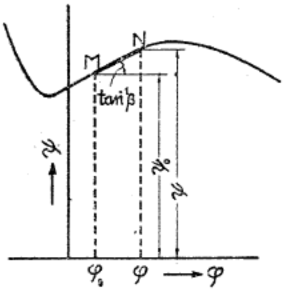

Fig. 6.

And we see that m is proportional to the diameter of the pipe, speed of the blower, and β, and inversly proportional to diameter of the impeller and Ω.

Assiduous efforts have been continued to design for preventing surging a blower which has the characteristic of $\beta < 0$ at every points of discharge. But this kind of blower has very poor efficiency and the characteristics of ordinary blowers have always some range of $\beta > 0$ at the small value of φ. When the working condition of a blower is in this range, vibrations are apt to be excited. And if the diameter of the pipe line is determined by the rule of Q/A = constant, where Q is the quatity of

(6) The effect of compression ratio must be taken into account when it is large.

discharge of a blower at its normal running condition and $Q \propto nD^3\Omega$, we have

$$m\sigma_s \propto n^2 D^2 \beta / l. \qquad \cdots\cdots\cdots(24)$$

Accordingly severe vibrations of surging are liable to occur for high speed blowers or high pressue blowers and it is reasonable that large and high speed blowers have often made severe surgings. If the blower is of multistage, $m \propto zAn\beta/(\Omega D)$, where β is the value per one stage.

If the vibration is produced and its amplitude grows large gradually, m is no more constant but varies with the amplitude. As m becomes zero and negative at large amplitude even if it is positive in the vicinity of its equilibrium position, damping force acts there. So the amplitude of vibration does not exceed a cirtain value.

When the vibration of surging is very large it extends very often to the range of negative discharge and we must investigate the characteristic curve of a blower at negative discharge. We have had not too much of the experimental investigation of the characteristic curves of blowers and pumps at their negative discharges, as far as the writer is aware. Some of them are shown in Fig. 7. The curve A is the characteristic curve per single stage of the small multi-stage turbine pump which is investigated by R. Dziallas[7] and has the impellers of exit angle $\beta_2 = 90°$. The curves B anb C are from Tabusi's experiment[8] which was carried out by using the pump of $D = 300$ mm and $\beta_2 = 30°$. The curve B is the case fitted with guide vanes of a small inlet angle and C the case without guide vane. The curves C lacks the parts at large φ and the curve D is added for the sake of comparison, which is obtained from a nearly similar pump by T. Kasai[9]. It is regrettable not to be able to add in this figure the D. Thoma's experiments[10] for the lack of dimenssions of the impeller. But his experiments covered very wide range of working condition and very useful.

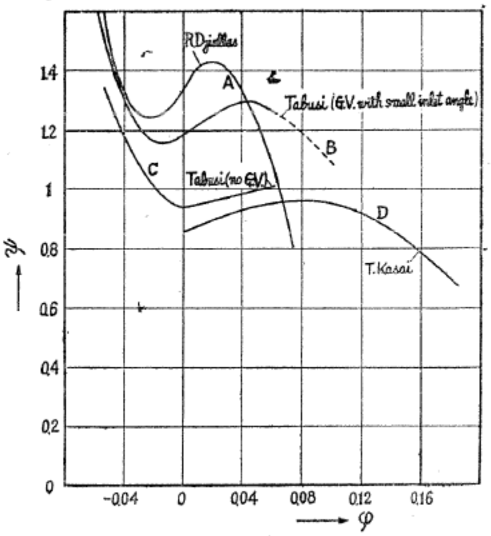

Fig. 7.

6. The amplitude of surging

When we consider the vibration of surging only, there is no objection to put the

(7) See (2).

(8) K. Tabusi, Kikai and Denki, Vol. 5, No. 12.

(9) T. Kasai, The Memoirs of the Faculty of Engineering, Kyushu Imperial University Vol. 8, No. I, 1936.

(10) D. Thoma, Mitt. d. Hyd. Inst. d. T. H. München, H. 4.

right-hand side of equation (13) to zero. And if the small quantities are ignored, all the terms of $\sum_r$ except $r = s$ may be neglected in the eqation, as previously stated. Thus the equations of vibrative motion of fluid column is

$$\ddot{q}_s - 2(m\sigma_s \cos^2 k_s\xi l - \varepsilon)\dot{q}_s + k_s^2 a^2 q_s = 0. \qquad \cdots\cdots\cdots(25)$$

Multiplying $\dot{q}_s$ and integrating we have

$$(1/2)(\dot{q}_s^2 + k_s^2 a^2 q_s^2) = C + 2\int_0^t (m\sigma_s \cos^2 k_s\xi l - \varepsilon)\dot{q}_s^2 dt,$$

where C is a cnstant of integration and is determined from the energy of vibration at $t = 0$. If the second term of the right-hand side in the above equation increases with time, the amplitude grows up, and if it decreases with time, the amplitude fall off gradually and vibratory motions stop finally when the right-hand side of the above equation becomes zero. Accordingly if the last integral of the above equation is zero when integrating over one period, the vibration is sustained with the same amplitude. This is also true for the case of varying m unless the right-hand side of the equation becomes zero in the course of integration. Therefore we can determine the amplitude of surging by the following equation:-

$$\int_0^\tau \left(m\sigma_s \cos^2 k_s\xi l - \varepsilon \right) \dot{q}_s^2 dt = 0. \qquad \cdots\cdots\cdots(26)$$

Here τ is the period of vibration. If m is a function of $\dot{q}_s$, the vibration is non-harmonic but if $m\sigma_s \cos^2 k_s\xi l - \varepsilon$ is small in comparison with $k_s a$, the vibration has very much resemblance to the harmonic, and it is allowable to assume a harmonic vibration in this case.

It is not impossible but very tedious to represent m by a function of $\dot{q}_s$ by means of experimental formula, but it is rather convenient to determine the amplitude by way of graphical integration, using P-Q curve or ψ-φ curve directly.

Now we have from (26)

$$\oint (m\sigma_s \cos^2 k_s\xi l - \varepsilon)\dot{q}_s dq = 0. \qquad \cdots\cdots\cdots(26\,\mathrm{a})$$

Here $\oint$ means integration over one complete oscillation. If the vibration is harmonic and one mode of vibration occurs only(11), we have

$$q_s = Q_s \sin \omega_s t, \qquad \cdots\cdots\cdots(27)$$

where Q_s is the amplitude of vibration which is to be determined.

From equation (27) we obtain

$$\dot{q}_s/\omega_s = Q_s \cos \omega_s t. \qquad \cdots\cdots\cdots(28)$$

Referring to (5), we have

$$v' \equiv v_{x=\xi l} - u_0 = \frac{\dot{q}_s}{k_s K} \cos k_s\xi l. \qquad \cdots\cdots\cdots(29)$$

(11) When the many modes of vibrations start together, the vibrations do not reach the stationary states unless there is the least common multiple in their periods.

Here we put $\omega_s \fallingdotseq k_s a$.

Putting

$$H = \frac{n^2 D^2}{729.5} \frac{K\omega_s}{a} \sigma_s \cos k_s \xi l, \qquad G = \frac{nD^3 \Omega K \omega_s}{6.079\, a \cos k_s \xi l}, \qquad \cdots\cdots\cdots(30)$$

we have

$$\left.\begin{aligned} &\frac{\dot{q}_s}{\omega_s}\Big/G = \varphi - \varphi_0, \qquad \frac{m\sigma_s \cos^2 k_s \xi l \dot{q}_s}{H} = \psi - \psi_0, \\ &\frac{\varepsilon \dot{q}_s}{H} = \varepsilon \frac{\Omega D}{n} \frac{120}{A\sigma_s \cos^2 k_s \xi l}(\varphi - \varphi_0), \end{aligned}\right\} \qquad \cdots\cdots\cdots(31)$$

from (20), (21), (22), (23), and (29). By using these relations we can determine the amplitude of surging from ψ-φ curve of the blower. Now let us draw, in Fig. 8, the horizontal line MA through M, which is on the ψ-φ curve and is the point corresponding to φ_0 and ψ_0. Next draw MB which represents the third of equation (31), that is $\varepsilon\dot{q}_s/H$, and the difference between the ψ-φ curve and MB is $1/H$ of the integrand of equation (26a). But the integration (26a) is with q and so we must transform this curve to the one which is taken q as abscissa. If we read from M the length along MA by using the scale of φ, we see from the first of equation (31) that it represents $(\dot{q}_s/\omega_s)/G$. The relation between $\dot{q}_s/\omega_s$ and $\dot{q}_s$ is shown in equations (28) and (27). Now take $M'A'$ as $\dot{q}_s/\omega_s$-axis and $M'N'$ as q_s-axis perpendicular to $M'A'$. Assuming appropriate value of amplitude Q_s and describing a semi-circle with its centre M' and radius Q_s/G, we can easily transform the point F on the $\dot{q}_s/\omega_s$-axis to the corresponding point E' on the q_s-axis as shown in the figure. The scale of the q_s-axis is $1/G$ as well as the $\dot{q}_s/\omega_s$-axis.

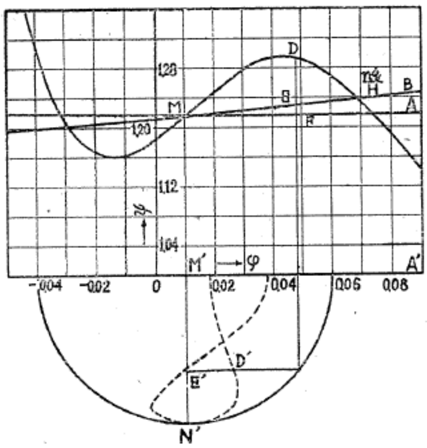

Fig. 8.

Taking $E'D'$ equal to ED, we finally obtain the D'-curve by repeating the same method. The transformation of the curve is enough for the semi-circle as the other is completely symmetrical to the one. The integration of (26a) is performed by measuring the area enclosed with the D' curve. If the area is zero, the assumed Q_s corresponds to the amplitude of sustained vibration. If the area be not zero, the same method will be repeated by assuming $Q_s a$ little large or smaller in accordance with a positive or negative value of the area and finally we can obtain the amplitude of sustained vibration by interpolation.

By using Q_s thus found the amplitudes of vibration of pressure and velocity are

$$Q_s \sin k_s x \quad \text{and} \quad \frac{Q_s \omega_s}{k_s K} \cos k_s x \fallingdotseq \frac{Q_s a}{K} \cos k_s x,$$

at x of the pipe line respectively.

If the values of Q_s are found for the combinations of several points of M and various inclinations of the line MB, we can draw a convenient graph from which the amplitude of surging may immediately be found for the combinations of a similar blower at any working conditions and any pipe lines. Fig. 9 shows an example of this. Here the B-curve of Fig. 7 is used as the characteristic curve of a blower and as the curve lacks some necessary parts the dotted curve is extended appropriately as shown in Fig. 7. We can see from the figure that the amplitude of surging is somewhat larger when the small quantity of discharge is made than when no discharge.

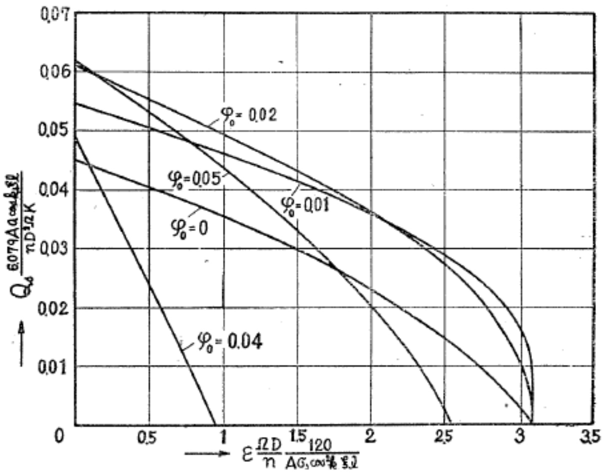

Fig. 9.

Fig. 10 shows the characteristic curve of the blower of $D=300\,\text{mm}$, $\Omega=0.05$ and $n=10000\,\text{r.p.m.}$ for $\rho=0.142\ \text{kgs}^2/\text{m}^4$, whose ψ-φ curve is the same as the B curve of Fig. 7. The pipe of length 5 m and diameter 125 mm is fitted to it. The suction-side of the pipe is open and a chamber of $Al/V=1$ is fitted to the other end of the pipe. The position of the blower is given by $\xi=0.1$. When the blower runs at the condition of no discharge, the surging may occur and its amplitude can be determined from the curve shown by $\varphi_0=0$ in Fig. 9. Variations of pressure and discharge are shown in Fig. 10 at the position of blower (full line) and of air chamber (dotted line) for $s=1, 3, 5,$ and 7. The vibration for $s=9$ may not occur as $\cos k_s\xi l$ is small. The direction of change of pressure and discharge are all in counter.clockwise as well as $s=1$, as shown by arrows, but when $\cos k_s x$ and $\sin k_s x$ are different in sign the direction of change is clock-wise. If the position of blower is $\xi=0.5$ the

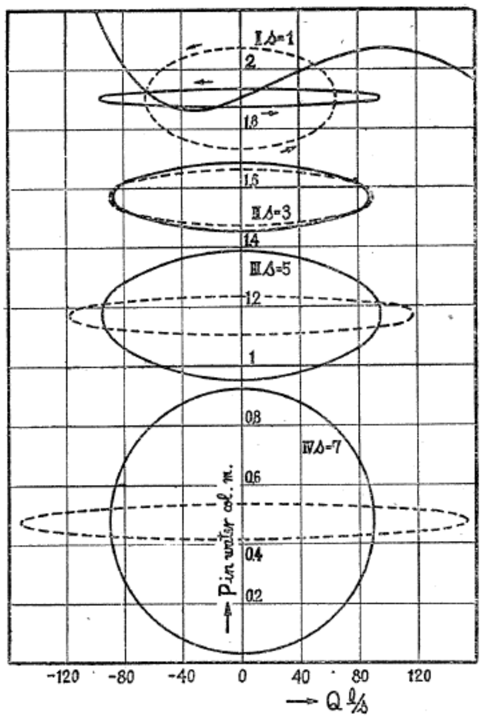

Fig. 10.

vibrations of $s = 3$ and 5 may not occur.

When the P-Q curve of the blower is used, the similar method may be adopted for finding the amplitude of surging. This is clearly understood by comparing the following equations with (31).

$$\left.\begin{aligned} \frac{\dot{q}_s}{\omega_s} &= \frac{K}{aK\cos k_s\xi l}(Q - Q_0), \\ m\sigma_s\cos^2 k_s\xi l\dot{q}_s &= \frac{K\omega_s\sigma_s\cos k_s\xi l}{\rho a}(P - P_0), \\ \varepsilon\dot{q}_s &= \frac{\varepsilon K\omega_s}{aA\cos k_s\xi l}(Q - Q_0). \end{aligned}\right\} \qquad \cdots\cdots\cdots(32)$$

7. Conclusion

When a blower attached to a pipe line works at a point of negative characteristic, the vibrations of each mode will be excited on the fluid column in the pipe. The vibrations is self-excited severly in proportions to the square of the peripheral velocity of the impeller and to the value of β (see equation (22)). But when the blower is situated near the loop of the vibration of the pressure or the node of the vibration of velocity, the vibrations may not be excited. And the vibrations is less excited for a long pipe line and a large coefficient of damping.

When the amplitude of vibration becomes large the self-excitation becomes weak and negative ultimately at the parts of large displacement owing to the characteristic of the blower. So the amplitude does not grow up over a certain value and the vibration is sustained with this amplitude. The amplitude of sustained vibration can be determined graphically from the characteristic curve of the blower including the part of negative discharge. And if the amplitude is represented with no dimension, as shown in Fig. 9, we can find easily from the figure the amlitudes of sustained vibrations of various modes for the combination of blower with various sizes and speed and pipe lines, provided the blowers are similar.