EFFECTS OF ALTITUDES ON THE COOLING CHARACTERISTICS OF AIR-COOLED AERO-ENGINES

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Contents

1. Introduction

The cooling characteristics, such as the temperature of finned cylinders, the power required for air-cooling etc., can be determined by the wind-tunnel experiments at ground level. Starting from the data thus obtained, we may deduce the variations of these items when an aircraft fitted with the engines under question flies at higher altitudes. The object of this analytical study is to derive some general formulae relating to these factors. The diagrams given at the end of this paper will furnish to the practical designers some facilities in computing such problems here treated. The numerical calculations have been based upon the "Tokyo Standard Atmosphere" as well as the so-called "International Standard Atmosphere." The former is the yearly mean conditions of the atmosphere observed at Tateno near Tokyo. These diagrams can be applied to cylinder heads with ordinary cast fins and also with sheet metal fins cast in the cylinder heads.

2. Heat Transfer from Finned Cylinder

Let T_m°K denote the mean absolute temperature of the outside surface of the cylinder to which the fins are attached and ϑ_m°K the mean absolute temperature of the cooling air passing around the cylinder. Then the heat dissipation is given by

$$Q = U \cdot F\,(T_m - \vartheta_m)\ \text{kcal/h}$$

where $F=$ the outside area of the cylinder, m^2,

$U=$ the surface coefficient of heat transfer based upon the outside area F of the cylinder to which the fins are attached, kcal/m^{2}h°C.

Then the relation between the cooling heats Q_0 near the ground surface and Q_z

at a height of z meters is given by

$$\frac{Q_z}{Q_0} = \frac{U_z}{U_0} \cdot \frac{T_{mz} - \vartheta_{mz}}{T_{m0} - \vartheta_{m0}}. \qquad \cdots\cdots\cdots (a)$$

Next, the temperature efficiency of the cooling surface can be defined by

$$\eta = \frac{\vartheta_2 - \vartheta_1}{T_m - \vartheta_1}, \qquad \cdots\cdots\cdots (b)$$

where ϑ_1 and ϑ_2 are the absolute temperature of the cooling air at the entry and the exit of the passage between the cylinder fins.

Then the temperature rise $\Delta\vartheta$ of the cooling air can be expressed by the following relation:

$$\Delta\vartheta = \vartheta_2 - \vartheta_1 = \eta\,(T_m - \vartheta_1) = \eta\,(r - 1)\,\vartheta_1, \qquad \cdots\cdots\cdots (c)$$

where r may be called factor of cylinder temperature, as defined by

$$r = T_m/\vartheta_1. \qquad \cdots\cdots\cdots (d)$$

Assuming a linear law of temperature rise of the cooling air stream, the mean temperature of the cooling air may be expressed by

$$\vartheta_m = \tfrac{1}{2}(\vartheta_1 + \vartheta_2) = (1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta r)\,\vartheta_1. \qquad \cdots\cdots\cdots (e)$$

Combining Eq. (d) with (e), we get the mean temperature difference between the hot cylinder surface and the cooling air as follows:

$$T_m - \vartheta_m = (1 - \tfrac{1}{2}\eta)(r - 1)\,\vartheta_1.$$

Thus, the height factor of the mean temperature difference is expressed by

$$\frac{T_{mz} - \vartheta_{mz}}{T_{m0} - \vartheta_{m0}} = \frac{1 - \frac{1}{2}\eta_z}{1 - \frac{1}{2}\eta_0} \cdot \frac{r_z - 1}{r_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}}. \qquad \cdots\cdots\cdots (f)$$

The relation between the local heat transfer coefficient α and the equivalent heat transfer coefficient U for the finned surface with rectangular section, as shown Fig.1, may be expressed by

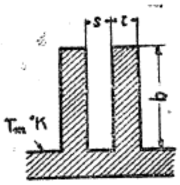


Fig. 1.

$$U = \frac{\alpha}{s+t}\left\{\frac{2}{\sqrt{\frac{2\alpha}{kt}}} \cdot \tanh\sqrt{\frac{2\alpha}{kt}}\left(b + \frac{t}{2}\right) + s\right\}$$

$$\fallingdotseq \frac{\sqrt{2kt\alpha}}{s+t} \equiv C \cdot \alpha^{0.5}, \qquad \cdots\cdots\cdots (g)$$

where s = free space between the fins, m,
t = thickness of fins, m,
b = height of fins, m,
k = thermal conductivity of the finned metal cylinder, kcal/m h°C.

Then the height factor of the equivalent heat transfer coefficient is given by

$$\frac{U_z}{U_0} = \left(\frac{\alpha_z}{\alpha_0}\right)^{0.5}. \qquad \cdots\cdots\cdots (h)$$

By the law of similarity in the heat transfer phenomena, we can express the Nusselt numder $Nu = \frac{\alpha \cdot d}{\lambda}$ as a function of the Pécelet number $Pé = \frac{u \cdot d}{a}$ in the following form:

$$\frac{\alpha \cdot d}{\lambda} = C_1 \left(\frac{u \cdot d}{a} \right)^m , \qquad \cdots\cdots\cdots (i)$$

where d = hydraulic diameter, m,

u = air velocity, m/s,

$a = \frac{\lambda}{C_p \cdot \gamma}$ = thermometric conductivity of the cooling air, m/h,

γ = specific weight of air, kg/m³,

C_p = specific heat at constant pressure of air, kcal/kg°C.

On the other hand, the time-rate of flow of the cooling air is given by

$$G = \gamma \cdot u \cdot S \text{ kg/s},$$

where S = sectional area for the air stream, m².

Thus, we see that

$$\frac{u}{a} = \frac{C_p \cdot \gamma \cdot u}{\lambda} = \frac{C_p}{\lambda} \cdot \frac{G}{S} .$$

Since the specific heat of the atmospheric air is given by

$$\frac{C_{pz}}{C_{p0}} = 1 - 0.000,000,11\, z.$$

We may assume $C_{pz} = C_{p0}$, practically constant at all heights.

Then the height factor for the local heat transfer coefficient can be expressed by the following relation:

$$\frac{\alpha_z}{\alpha_0} = \left(\frac{\lambda_z}{\lambda_0} \right)^{1-m} \cdot \left(\frac{G_z}{G_0} \right)^m . \qquad \cdots\cdots\cdots (j)$$

The thermal conductivity of air at an absolute temperature T°K is given by

$$\lambda = 0.001,67 \frac{1 + 0.000,194\, T}{1 + \frac{117}{T}} \sqrt{T} \doteqdot C_2 T^{0.8}. \qquad \cdots\cdots\cdots (k)$$

In our case, the temperature T may be the mean temperature between the cooling air and the hot surface. Thus, combining Eq. (d) with (e), we get

$$T = \tfrac{1}{2} (T_m + \vartheta_m) = \tfrac{1}{2} \{ (1 - \tfrac{1}{2} \eta) + (1 + \tfrac{1}{2} \eta) r \} \vartheta_1 .$$

Put this relation into Eq. (k), then we get the height factor for the thermal conductivity of the cooling air as follows:

$$\frac{\lambda_z}{\lambda_0} = \left(\frac{T_z}{T_0} \right)^{0.8} = \left\{ \frac{(1 - \frac{1}{2} \eta_z) + (1 + \frac{1}{2} \eta_z) r_z}{(1 - \frac{1}{2} \eta_0) + (1 + \frac{1}{2} \eta_0) r_0} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} \right\}^{0.8} . \qquad \cdots\cdots\cdots (l)$$

Substituting Eq. (l), (j), (k) and (f) into (a), we obtain the expression for the height factor of the heat dissipation

$$\frac{Q_z}{Q_0} = \left\{ \frac{(1 - \frac{1}{2} \eta_z) + (1 + \frac{1}{2} \eta_z) r_z}{(1 - \frac{1}{2} \eta_0) + (1 + \frac{1}{2} \eta_0) r_0} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} \right\}^{0.4(1-m)} \cdot \left(\frac{G_z}{G_0} \right)^{0.5\, m} \cdot \frac{1 - \frac{1}{2} \eta_z}{1 - \frac{1}{2} \eta_0} \cdot \frac{r_z - 1}{r_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} . \qquad \cdots\cdots\cdots (1\,a)$$

The heat dissipation Q from the cylinder surface must be equal to the increase of heat content of the cooling air. This will be proportional to the mass flow G

kg/s multiplied by the temperature rise $\Delta\vartheta$ of the cooling air. Thus, we have

$$Q = 3{,}600\, C_p \cdot G \cdot \Delta\vartheta \text{ kcal/h.}$$

Then the height factor for the cooling heat becomes

$$\frac{Q_z}{Q_0} = \frac{G_z}{G_0} \cdot \frac{\Delta\vartheta_z}{\Delta\vartheta_0} = \frac{G_z}{G_0} \cdot \frac{\eta_z}{\eta_0} \cdot \frac{r_z - 1}{r_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}}. \qquad \cdots\cdots\cdots(1\,\mathrm{b})$$

The values of the height factors for the heat dissipation given by Ep. (1 a) and (1 b) must be equal to each other at any instant. Equating the right-hand sides of these equations, we get the following relation for the quantity of air flow G, in terms of the temperature efficiency η, the cylinder temperature factor r and the inlet temperature ϑ_1 of the cooling air.

$$\frac{G_z}{G_0} = \left\{ \frac{(1 - \frac{1}{2}\eta_z) + (1 + \frac{1}{2}\eta_z) r_z}{(1 - \frac{1}{2}\eta_0) + (1 + \frac{1}{2}\eta_0) r_0} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} \right\}^{\frac{0.4(1-m)}{1-0.5m}} \cdot \left\{ \frac{1/\eta_z - \frac{1}{2}}{1/\eta_0 - \frac{1}{2}} \right\}^{\frac{1}{1-0.5m}}. \qquad \cdots\cdots\cdots(2)$$

The equation, thus obtained, shows the direct relation of the temperature ϑ_1°K and quantity G kg/s of the cooling air to its temperature rise and the cylinder temperature, these being expressed by Eq. (c) and (d), respectively.

Next, we proceed to derive a similar expression as Eq. (2) in terms of the temperature rise factor of the cooling air. This factor may be defined by

$$\xi = \vartheta_2 / \vartheta_1. \qquad \cdots\cdots\cdots(\mathrm{m})$$

Then the temperature rise of the cooling air is

$$\Delta\vartheta = \vartheta_2 - \vartheta_1 = (\xi - 1)\vartheta_1. \qquad \cdots\cdots\cdots(\mathrm{n})$$

Equating the right-hand side of Eq. (n) to that of (c), we get the following relation:

$$\eta(r - 1) = \xi - 1 \qquad \text{or} \qquad r = 1 + \frac{\xi - 1}{\eta}. \qquad \cdots\cdots\cdots(3)$$

Substitute this relation in Eq. (1 b) and (2), and we get the following two equations:

$$\frac{Q_z}{Q_0} = \frac{G_z}{G_0} \cdot \frac{\xi_z - 1}{\xi_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}}, \qquad \cdots\cdots\cdots(4)$$

$$\frac{G_z}{G_0} = \left\{ \frac{(1 - \frac{1}{2}\eta_z) + (1/\eta_z - \frac{1}{2})(\eta_z + \xi_z - 1)}{(1 - \frac{1}{2}\eta_0) + (1/\eta_0 - \frac{1}{2})(\eta_0 + \xi_0 - 1)} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} \right\}^{\frac{0.4(1-m)}{1-0.5m}} \cdot \left\{ \frac{1/\eta_z - \frac{1}{2}}{1/\eta_0 - \frac{1}{2}} \right\}^{\frac{1}{1-0.5m}}. \qquad \cdots\cdots\cdots(5)$$

According to the results of experimental researches on the aero-engines by several authorities, we may take $m = 0.75$. Then the values of the exponents become

$$\frac{0.4(1-m)}{1 - 0.5\,m} = 0.16 \qquad \text{and} \qquad \frac{1}{1 - 0.5\,m} = 1.6\,.$$

If $Q_z/Q_0 = 1$, then the mean temperature of the finned cylinder $t_m = T_m - 273$°C is given by

$$t_m = \theta_{1z} + (t_{m0} - \theta_{10}) \cdot \frac{\eta_z}{\eta_0} \cdot \frac{G_0}{G_z}, \qquad \cdots\cdots\cdots(6)$$

where $\theta_1 = \vartheta_1 - 273$ = initial temperature of cooling air at the entry.

3. Power Required for Air Cooling

The principle of conservation of energy applied to the flowing air between section 1 and 2 shown in Fig. 2 gives

$$I_1 + A\frac{u_1^2}{2g} + q = I_2 + A\frac{u_2^2}{2g} \text{ kcal/kg,} \qquad \cdots\cdots\cdots(a)$$

where I_1 and I_2 stand for the enthalpy and u_1 and u_2 for the velocity of cooling air just before and just after the engine cylinder, respectively. The heat $q = Q/G$ kcal/kg denotes the heat given to the unit mass of cooling air from the hot cylinder surface.

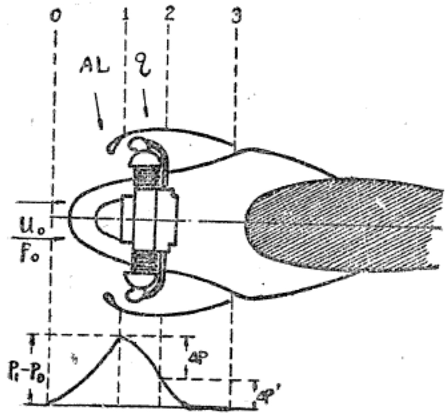


Fig. 2.

On the other hand, the general relation for the heat supplied to the air is

$$d(q + q_r) = dI - AvdP.$$

By integration, we get

$$I_2 - I_1 = (q + q_r) + A\int_{P_1}^{P_2} vdP, \quad \cdots(b)$$

where q_r denotes the friction heat corresponding to the work done W_r against the frictional resistance. Thus

$$q_r = AW_r \text{ kcal/kg,} \qquad \cdots\cdots\cdots(c)$$

where A stands for heat equivalent of mechanical work. Combining Eq. (a), (b) and (c), we obtain

$$\int_{P_2}^{P_1} vdP = W_r + \frac{u_2^2 - u_1^2}{2g} \text{ mkg/kg.} \qquad \cdots\cdots\cdots(d)$$

For the sake of simplicity, we may put practically as

$$\int_{P_2}^{P_1} vdP = \tfrac{1}{2}(v_1 + v_2)\Delta P, \qquad \cdots\cdots\cdots(e)$$

where $\Delta P = P_1 - P_2$ kg/m^2 = pressure drop between the sections 1 and 2, as shown in Fig. 2.

The characteristic equation for the cooling air as an ideal gas will give the following relations:

$$P_1v_1 = R\vartheta_1 \text{ at section 1,}$$
$$P_2v_2 = R\vartheta_2 \;\;〃 \quad 〃 \quad 2.$$

Hence, we have

$$\frac{v_2}{v_1} \cdot \frac{P_1 - \Delta P}{P_1} = \frac{\vartheta_2}{\vartheta_1}.$$

In the case of aero-engines, the value of the ratio $\Delta P/P_1$ hardly exceed over 2%, which may be neglected as compared with unity. Thus, we get

$$\frac{v_2}{v_1} \doteqdot \frac{\vartheta_2}{\vartheta_1} = \xi = 1 - \eta + \eta r. \qquad \cdots\cdots\cdots(f)$$

The mean specific volume of the cooling air is, therefore,

$$\tfrac{1}{2}(v_1 + v_2) = v_1(1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta r)\,, \qquad \cdots\cdots\cdots(g)$$

which enables us to evaluate the value of the left-hand side of Eq. (d) as follows:

$$\int_{P_2}^{P_1} vdP = (1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta r)v_1 \cdot \Delta P \text{ mkg/kg.}$$

Next, the equation of continuity for the air stream between the sections 1 and 2 gives

$$\frac{u_1 S_1}{v_1} = \frac{u_2 S_2}{v_2} = G \text{ kg/s,}$$

which gives the equation for the velocity ratio of the air stream at the inlet and exit

$$\frac{u_2}{u_1} = \frac{S_1}{S_2} \cdot \frac{v_1}{v_2}\,,$$

where the sectional areas S_1 and S_2 for the air stream at the sections 1 and 2 may be assumed to be equal to each other. In such a case, we have

$$\frac{u_2}{u_1} = \frac{v_2}{v_1} = 1 - \eta + \eta r\,. \qquad \cdots\cdots\cdots(h)$$

Thus, the second term in the right-hand side of Eq. (d) becomes

$$\frac{u_2^2 - u_1^2}{2g} = \left\{(1 - \eta + \eta r)^2 - 1\right\} \cdot \frac{u_1^2}{2g}$$

$$= 2\eta(r - 1)(1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta r) \cdot \frac{u_1^2}{2g} \qquad \cdots\cdots\cdots(i)$$

which indicates the increase of kinetic energy of the cooling air.

Lastly, the work done W_r against the frictional resistance is to be considered. The analogy between heat transfer and skin friction, originally given by Reynolds and afterwards developed by Prandtl gives the following relation:

$$\frac{\text{Heat dissipation by convection}}{\text{Resisting force by friction}} = \frac{g \cdot C_p(T_m - \vartheta_m)}{u_m} \cdot \frac{1}{1 + \varphi(Pr - 1)}\,.$$

In our case, the heat transfer is

$$\text{Heat dissipated} = \frac{\alpha F(T_m - \vartheta_m)}{3{,}600} \text{ kcal/s.}$$

On the other hand, the resisting force is expressed by the product of work done W_r mkg/kg and the mass flow G kg/s divided by the mean velocity of air stream u_m m/s. Thus

$$\text{Resisting force} = \frac{G \cdot W_r}{u_m} \text{ kg.}$$

Substituting these relations into the analogy epuation above mentioned, we obtain

$$\frac{\alpha F}{3{,}600\, GW_r} = \frac{g \cdot C_p}{u_m^2} \cdot \frac{1}{1 + \varphi(Pr - 1)}\,. \qquad \cdots\cdots\cdots(j)$$

Next, consider the heat exchange between the hot gas at rest and the cold air stream, as shown in Fig. 3. The temperature of the hot gas remains constant at T_m, while the temperature of the cold air varies from ϑ_1 at the entry and to ϑ_2 at the exit. In such a case, the ratio of the temperature dffer-ence between the two fluids at the inlet and the outlet is expressed by following relation:

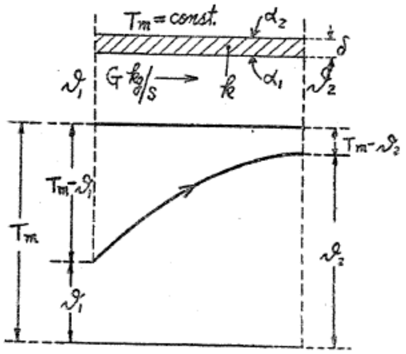


Fig. 3.

$$\frac{T_m - \vartheta_2}{T_m - \vartheta_1} = e^{-\frac{\kappa \cdot F}{3,600\, G \cdot C_p}},$$

in which C_p stands for the specific heat at constant pressure of the cooling air. The heat transmission coefficient κ is given by the relation as

$$\frac{1}{\kappa} = \frac{1}{\alpha_1} + \frac{\delta}{k} + \frac{1}{\alpha_2}$$

in ordinary conception. If the heat transfer coefficient at the gas side α_2 increases infinitely large, we have $1/\alpha_2 = 0$ and the temperation of the partition wall will become equal to the gas temperature T_m. Furthermore, if the thickness of the partition wall δ tends to zero, the value of δ/k will become zero and the surface temperature of the partition wall at the air side will be equal to T_m. This is the case of air-cooled cylinder under question. Thus we have

$$\frac{T_m - \vartheta_2}{T_m - \vartheta_1} = e^{-\frac{\alpha F}{3,600\, G \cdot C_p}}.$$

The temperature efficiency is, therefore,

$$\eta = \frac{\vartheta_2 - \vartheta_1}{T_m - \vartheta_1} = 1 - e^{-\frac{\alpha F}{3,600\, G \cdot C_p}}$$

or

$$\log_e \frac{1}{1-\eta} = \frac{\alpha F}{3,600\, G \cdot C_p}.$$

By substitution of Eq. (k) in (j), we get the required expression of the work done W_r mkg/kg against the frictional resistance in terms of the temperature efficiency η as follows:

$$W_r = \frac{u_m^2}{2g}\left\{ 1 + \varphi(Pr - 1) \right\}\log_e \frac{1}{1-\eta}$$

$$= \frac{u_1^2}{2g}\left(1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta r\right)^2 K \log_{10} \frac{1}{1-\eta},$$

where $K = 2 \times 2.3026\, \{1 + \varphi(Pr - 1)\}$

The Prandtl number Pr for air is equal to 0.725, which is practically constant. The factor φ is given by

$$\varphi = 1.4\, Pr^{-0.185} \cdot Re^{-0.1} \qquad \cdots\cdots\cdots (m)$$

The calculated values of φ and K are as follows:

Re	10^3	10^4	10^5
φ	0.7447	0.5915	0.4699
K	3.662	3.856	4.010

For the flow of cooling air through the space between the cylinder fins, the Reynolds Number Re may vary from 10^3 to 10^4. Thus, we may take the mean value $K = 3.76$

Before proceeding further, we may check the results thus obtained.

Take the temperature efficiency $\eta = 0.25$ which is the case of ordinary cast-finned cylinder head of light alloy, the mean absolute temperature of cylinder head $T_m = 273 + 190°\mathrm{K}$ and the absolute temperature of the cooling air at the entry $\vartheta_1 = 273 + 15°\mathrm{K}$, then we have

$$\gamma = \frac{273 + 190}{273 + 15} = 1.608.$$

Substituting these values in Eq. (l) and (i), we get

$$W_r = 0.579 \frac{u_1^2}{2g} \qquad \text{and} \qquad \frac{u_2^2 - u_1^2}{2g} = 0.327 \frac{u_1^2}{2g},$$

and

$$\frac{\text{Increase in kinetic energy}}{\text{Work done against resistance}} = \frac{0.327}{0.579} = \frac{1}{1.768}.$$

The result of this calculation shows that, in computing the work done required for air cooling, both the loss of head due to the increase of kinetic energy and the loss of head due to the skin friction are to be taken into account, and neither of which is to be neglected.

Substitute the relations given by Eq. (e), (g), (i) and (l) in Eq. (d), and we get the expression for the pressure drop as follows:

$$\Delta P = \gamma_1 \frac{u_1^2}{2g}\left\{(1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta\gamma)K \log_{10}\frac{1}{1-\eta} + 2\eta(\gamma - 1)\right\}$$

$$= \gamma_1 \frac{u_1^2}{2g}\left\{\tfrac{1}{2}(\xi - 1)K \log_{10}\frac{1}{1-\eta} + 2(\xi - 1)\right\} \text{ kg/m}^2 \text{ or mm Aq.}$$

And, the height factor for the pressure drop becomes

$$\frac{\Delta P_z}{\Delta P_0} = \frac{\gamma_{1z}}{\gamma_{10}} \cdot \left(\frac{u_{1z}}{u_{10}}\right)^2 \frac{(1 - \frac{1}{2}\eta_z + \frac{1}{2}\eta_z\gamma_z)K \log_{10}\dfrac{1}{1-\eta_z} + 2\eta_z(\gamma_z - 1)}{(1 - \frac{1}{2}\eta_0 + \frac{1}{2}\eta_0\gamma_0)K \log_{10}\dfrac{1}{1-\eta_0} + 2\eta_0(\gamma_0 - 1)}$$

$$= \frac{\gamma_{1z}}{\gamma_{10}} \cdot \left(\frac{u_{1z}}{u_{10}}\right)^2 \frac{\frac{1}{2}(\xi_z - 1)K \log_{10}\dfrac{1}{1-\eta_z} + 2(\xi_z - 1)}{\frac{1}{2}(\xi_0 - 1)K \log_{10}\dfrac{1}{1-\eta_0} + 2(\xi_0 - 1)}.$$

The horse power HP_c theoretically required to force the cooling air through the finned space around the cylinder is next to be computed. Since the work done by the expansion of 1 kg of flowing air is $\int vdP$ mkg/kg, the corresponding horse power for the mass flow G kg/s will be

$$HP_c = \frac{1}{75} G \int vdP = \frac{1}{75} Gv_m \cdot \Delta P = \frac{1}{75} V_m \cdot \Delta P, \qquad \cdots\cdots\cdots(9)$$

where V_m = mean volume of air flow in m³/s. Hence, the height factor for the horse power is

$$\frac{HP_{cz}}{HP_{c0}} = \frac{V_{mz}}{V_{m0}} \cdot \frac{\Delta P_z}{\Delta P_0} . \qquad \cdots\cdots\cdots(\text{n})$$

The mean volume V_m is given by the product of the mean specific volume $v_m \doteqdot \frac{1}{2}(v_1 + v_2)$ m³/kg and the mass flow G kg/s. Thus

$$V_m = \tfrac{1}{2}(v_1 + v_2)G = (1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta\gamma)v_1 G = \frac{1 - \frac{1}{2}\eta + \frac{1}{2}\eta\gamma}{\gamma_1} G = (1 - \tfrac{1}{2}\eta + \tfrac{1}{2}\eta\gamma) V_1 \text{ m}^3/\text{s}, \qquad \cdots\cdots\cdots(\text{o})$$

where V_1 m³/s stands for the volume of air passing through the section 1 (Fig. 2) per second. Hence, the height factor for the volume of cooling air becomes

$$\frac{V_{1z}}{V_{10}} = \frac{u_{1z}}{u_{10}} = \frac{G_z}{G_0} \cdot \left(\frac{\gamma_{1z}}{\gamma_{10}}\right)^{-1}, \qquad \cdots\cdots\cdots(10)$$

and

$$\frac{V_{mz}}{V_{m0}} = \frac{V_{1z}}{V_{10}} \cdot \frac{1 - \frac{1}{2}\eta_z + \frac{1}{2}\eta_z\gamma_z}{1 - \frac{1}{2}\eta_0 + \frac{1}{2}\eta_0\gamma_0} = \frac{V_{1z}}{V_{10}} \cdot \frac{\xi_z + 1}{\xi_0 + 1} . \qquad \cdots\cdots\cdots(\text{p})$$

By putting these relations in Eq. (n), we get

$$\frac{HP_{cz}}{HP_{c0}} = \frac{u_{1z}}{u_{10}} \cdot \frac{1 - \frac{1}{2}\eta_z + \frac{1}{2}\eta_z\gamma_z}{1 - \frac{1}{2}\eta_0 + \frac{1}{2}\eta_0\gamma_0} \cdot \frac{\Delta P_z}{\Delta P_0}$$

$$= \frac{u_{1z}}{u_{10}} \cdot \frac{\xi_z + 1}{\xi_0 + 1} \cdot \frac{\Delta P_z}{\Delta P_0} . \qquad \cdots\cdots\cdots(11)$$

4. Tokyo Standard Atmosphere

According to the meteorological observations at Tateno near Tokyo, the mean atmospheric temperature in November, as shown in Diagram F, coincides with that of the International Standard Atmosphere up to the height of 11 km. From the height of 11 km to 17 km, the atmospheric temperature decreases at the rate of 2.25°C per km, whereas, in the case of the International Standard Atmosphere, it is considered to remain constant at -56.5°C. The same inclination of the temperature drop has been observed through the whole year. Thus, we propose the Tokyo Standard Atmosphere, in which the variation of the atmospheric temperature is given by the following two linear laws:

$$\vartheta_z = \vartheta_0 - 0.006{,}5\, z \text{ °K} \qquad \text{for} \qquad 0 \leqq z \leqq 11{,}000 \text{ m},$$

$$\vartheta_z = \vartheta_0 - 56.50 - 0.002{,}25\,(z - 11{,}000) \text{ °K} \qquad \text{for} \qquad 11{,}000 \leqq z \leqq 17{,}000 \text{ m},$$

where ϑ_0 = atmospheric temperature at the ground level, °K = 288°K,
z = height, m.

Consider a vertical column of air at an altitude of z m. The pressure at any level is due to the weight of the air column above, then we have

$$dP = -\frac{dz}{v}, \qquad \cdots\cdots\cdots(\text{a})$$

where P = pressure in kg/m² or mm Aq,
v = specific volume, m³/kg.

The polytropic law $Pv^n = \text{constant}$ holds good in the atmospheric condition. By differentiating this equation, we get

$$vdP + nPdv = 0. \qquad \cdots\cdots\cdots(b)$$

On the other hand we have the characteristic equation for the atmospheric air as a perfect gas

$$Pv = R\vartheta.$$

By differentiation, we get

$$vdP + Pdv = Rd\vartheta. \qquad \cdots\cdots\cdots(c)$$

Elliminating dv between Eq. (b) and (c), we obtain

$$nRd\vartheta = (n-1)vdP,$$

which gives the value of the specific volume v.

Substitute the relation thus obtained in Eq. (a), then we get

$$d\vartheta = -\frac{n-1}{n} \cdot \frac{1}{R}\,dz,$$

which gives dy integration

$$\vartheta_z = \vartheta_0 - \frac{n-1}{n} \cdot \frac{1}{R}\,z. \qquad \cdots\cdots\cdots(d)$$

Eq. (d) being identical with the linear laws given above, we have the following relations:

$$\frac{n-1}{n} \cdot \frac{1}{R} = 0.006{,}5 \quad \text{for} \quad 0 \leqq z \leqq 11{,}000\ \text{m},$$

$$\frac{n'-1}{n'} \cdot \frac{1}{R} = 0.002{,}25 \quad \text{for} \quad 11{,}000 \leqq z \leqq 17{,}000\ \text{m}.$$

The value of the gas constant for air is $R = 29.269$, and we get

$$n = 1.235 \quad \text{and} \quad n' = 1.070.$$

The value of n' thus obtained being nearly equal to unity, we may assume an isothermal change instead of a polytropic change. But, as far as the cooling characteristics are concerned, the temperature drop of atmospheric air with altitudes plays a remarkable part.

Be that as it may, we proceed to compute the relative density of atmospheric air as follows: Let γ_z and γ_{11} be the specific density at the altitudes z m and 11,000 m, respectively. Then we have

$$\frac{\gamma_z}{\gamma_{11}} = \left(\frac{\vartheta_z}{\vartheta_{11}}\right)^{\frac{1}{n'-1}}.$$

The relative density becomes

$$\frac{\gamma_z}{\gamma_0} = \frac{\gamma_{11}}{\gamma_0}\,\frac{\gamma_z}{\gamma_{11}} = \frac{\gamma_{11}}{\gamma_0}\left(\frac{\vartheta_z}{\vartheta_0}\right)^{\frac{1}{n'-1}}.$$

The International Standard Atmosphere gives

$$\frac{\gamma_z}{\gamma_0} = \frac{\gamma_{11}}{\gamma_0} \cdot e^{-\frac{z-11{,}000}{216.5\times 29.269}}.$$

The following table shows the results of calculations.

Heihgt z m	International Std. Atmos.		Tokyo Std. Atmos.	
	Temp. °C	Relative density γ_z/γ_0	Temp. °C	Relative density γ_z/γ_0
11,000	−56.50	0.2971	−56.50	0.2971
12,000	〃	0.2538	−58.75	0.2562
13,000	〃	0.2167	−61.00	0.2206
14,000	〃	0.1851	−63.25	0.1896
15,000	〃	0.1581	−65.50	0.1626
16,000	〃	0.1351	−67.75	0.1394
17,000	〃	0.1154	−70.00	0.1192

Thus, we find little difference in the density of the atmospheric air, but some difference in the atmespheric temperature effects so much on the cooling characteristics, e.g., the mass flow of the air, the horse power required for cooling, heat dissipation etc.

5. Reduction to the Running Conditions in the Standard Atmosphere

The foregoing theories can be applied with facilities to the practical problems by graphical solutions. For example, consider Q_z/Q_0-z diagrams in curves corresponding to constant values of η, r, ξ or ϑ. It is repuired to find the value $(Q_z/Q_0)'$ corresponding to the actual running conditions, $\eta+d\eta$, $r+dr$ and $\vartheta+d\vartheta$ or $\eta+d\eta$, $\xi+d\xi$ and $\vartheta+d\vartheta$. In such a case, take, by interpolations, the values $(Q_z/Q_0)_{\eta+d\eta}$, $(Q_z/Q_0)_{r+dr}$, $(Q_z/Q_0)_{\xi+d\xi}$ and $(Q_z/Q_0)_{\vartheta+d\vartheta}$ and substitute them in the following relations. Then we will have the required value of $(Q_z/Q_0)'$.

$$\left(\frac{Q_z}{Q_0}\right)'=\left(\frac{Q_z}{Q_0}\right)_{\eta+d\eta}+\left(\frac{Q_z}{Q_0}\right)_{r+dr}+\left(\frac{Q_z}{Q_0}\right)_{\vartheta+d\vartheta}-2$$

or

$$\left(\frac{Q_z}{Q_0}\right)'=\left(\frac{Q_z}{Q_0}\right)_{\eta+d\eta}+\left(\frac{Q_z}{Q_0}\right)_{\xi+d\xi}+\left(\frac{Q_z}{Q_0}\right)_{\vartheta+d\vartheta}-2.$$

The diagrams given at the end of this paper, have been prepared for the case $Q_z/Q_0=1$, i.e., the heat dissipation remains constant at all heights. If we confine ourselves to such cases, the reduction procedure will become much simpler.

First of all, the observed values from the actual engine test at the ground level must be reduced to the standard atmospheric conditions. The following notations are used to distinguish the reduced values from the observed ones.

Running condition	Observed values	Reduced values
Atmos. press. mm Hg	h	760
Atmos. temp. °K	ϑ_{10}'	$\vartheta_{10}=288$
Specific weight, kg/m³	γ_{10}'	$\gamma_{10}=1.2249$
Heat dissipation, kcal/h	Q_0'	Q_0
Temp. efficiency	η_0'	η_0
Temp. rise factor	ξ_0'	ξ_0
Cylinder temp. factor	r_0'	r_0
Cooling air inlet vel., m/s	u_{10}'	u_{10}

In many cases, it will be required to reduce the observed data obtained from the actual engine test at any given atmospheric conditions to the corresponding values at the standard atmospheric conditions. In such a case, it will be quite natural to assume $Q_0'/Q_0 = 1$ and $u_{10}'/u_{10} = 1$. With these assumptions, we have from Eq. (4).

$$\frac{\gamma_{10}'}{\gamma_{10}} \cdot \frac{\xi_0' - 1}{\xi_0 - 1} \cdot \frac{\vartheta_{10}'}{\vartheta_{10}} = 1. \qquad \cdots\cdots\cdots(a)$$

On the other hand, the characteristic equation for the perfect gas, will give the following relation:

$$\frac{\gamma_{10}'}{\gamma_{10}} \cdot \frac{\vartheta_{10}'}{\vartheta_{10}} = \frac{h}{760}.$$

Put this relation into Eq. (a), then we obtain

$$\frac{\xi_0' - 1}{\xi_0 - 1} = \frac{760}{h}, \qquad \cdots\cdots\cdots(b)$$

which gives the value of ξ_0 at the standard atmospheric condition.

Combining Eq. (4) with (5), we get

$$\frac{h}{760} \cdot \frac{288}{\vartheta_{10}'} = \left\{\frac{(1 - \frac{1}{2}\eta_0') + (1/\eta_0' + \frac{1}{2})(\eta_0' + \xi_0' - 1)}{(1 - \frac{1}{2}\eta_0) + (1/\eta_0 + \frac{1}{2})(\eta_0 + \xi_0 - 1)} \cdot \frac{\vartheta_{10}'}{\vartheta_{10}}\right\}^{0.16} \cdot \left\{\frac{1/\eta_0' - \frac{1}{2}}{1/\eta_0 - \frac{1}{2}}\right\}^{1.6}, \qquad \cdots\cdots\cdots(12)$$

which gives the unknown quantity of η_0.

Diagram E enables us to determine the value of η_0 by graphical solution. Take, for example, the case of an engine tested under the atmospheric conditions of 790 mm Hg and 35°C. As the results of the test, we have, say, $\xi_0' = 1.120$ and $\eta_0' = 0.25$. What would be the corresponding values ξ_0 and η_0 under the conditions 760 mm Hg and 15°C?

To solve this problem, we proceed as follows: Find the points A and B on the horizontal axis corresponding to 760 mm and 790 mm Hg, respectively. On the vertical axis lies a fixed point P. The length $\overline{OP}$ is equal to $\overline{OA}$. Connect the points P and A and also P and B.

Next, take the length $\overline{OP}$ to be equal to $(\xi_0' - 1)$. Draw two straight lines passing through p: one parallel to PA and the other parallel to PB. The two points of intersection of these two straight lines with the horizontal axis being a and b, respectively, we see that the length $\overline{oa}$ is equal to $(\xi_0' - 1)$ and $\overline{ob}$ to $(\xi_0 - 1)$. This is no more than the graphical solution of Eq. (b). The result of the solution is $\xi_0 = 1.125$, in our case.

Next, take the logarithms of the both sides of Eq. (12), then we get

$$\log f(\eta_0, \xi_0) = \log f(\eta_0', \xi_0') - \log(h/760) - (1 + 0.16)\log(288/\vartheta_{10}'), \qquad \cdots\cdots(12')$$

where $f(\eta_0,\xi_0) = \{(1 - \frac{1}{2}\eta_0) + (\frac{1}{2}\eta_0 + \frac{1}{2})(\eta_0 + \xi_0 - 1)\}^{0.16} \cdot (1/\eta_0 - \frac{1}{2})^{1.6}$,

and $f(\eta_0', \xi_0') = \{(1 - \frac{1}{2}\eta_0') + (1/\eta_0' + \frac{1}{2})(\eta_0' + \xi_0' - 1)\}^{0.16} \cdot (1/\eta_0' - \frac{1}{2})^{1.6}$.

The curve corresponding to a certain constant value of η_0 indicates the value of $f(\eta_0, \xi_0)$ which varies with η_0 and ξ_0. Thus, the point c indicates $\eta_0' = 0.25$ and $\xi_0' = 1.120$, which is the intersection of the curve $\eta_0' = 0.25 =$ constant and the vertical line $\xi_0' = 1.120 =$ constant.

The two curves on the upper part of Diagram E represent the terms log $(h/760)$ and $(1+0.16)\log(288/\vartheta_{10}')$, respectively. The lengths $\overline{cd}$ and $\overline{de}$ correspond to the amounts of correction for the atmospheric pressure of 790 mm Hg and temperature 39°C.

Starting from the point c above plotted, take the length $+\overline{cd}$ on the elongation of the vertical $\overline{bc}$. Next, take the length $\overline{de}$ downwards. Passing through the point e, thus plotted, draw a straight line $\overline{ef}$ parallel to the horizontal axis, intersecting at f with the vertical passing through the point a, which represents $\xi_0 = 1.125$.

From the position of the point f on the diagram, we can find the value $\eta_0 = 0.244$ corresponding to the point f, which may be determined by interpolation.

Finally, the value of r_0 is determined by the relation:

$$\eta_0(r_0 - 1) = \xi_0 - 1,$$

which gives $r_0 = 1.512$, whereas $r_0' = 1.480$. The mean cylinder temperature is given by

$$t_{m0} = 288\, r_0 - 273 = 162.5°\text{C} \quad \text{at 760 mm Hg and 15°C,}$$

and $$t_{m0}' = (273+39)\, r_0' - 273 = 188.8°\text{C} \quad \text{at 790 mm Hg and 39°C.}$$

6. Construction of Diagrams and their Applications

In order to avoid complicated calculations, some diagrams have been prepared for graphical solutions. Practical designers will find much facilities in computing the problems concerning to the air-cooling of the aero-engines at higher altitudes. All curves in full lines correspond to Tokyo Standard Atmosphere and those in dotted lines correspond to the stratosphere, the temperature of which being constant at − 56.5°C.

The procedure of the construction of these diagrams is to be here explained.

1) Engine test at ground level

In order to find the cooling characteristics of an engine under question at higher altitudes, it must be, first of all, tested in a wind-tunnel under the atmospheric conditions at the ground level. The atmospheric conditions need not be the same as those of the Standard Atmosphere. In such cases, the observed values can be reduced to those under the Standard Atmosphere.

At any rate, the items to be observed are as follows:

a) Inlet temperature of the cooling air $= \vartheta_{10}'$ °K,

b) Outlet 〃 〃 〃 〃 〃 $= \vartheta_{20}'$ °K,

Temperature rise of the cooling air $= \vartheta_{20}' - \vartheta_{10}' = \Delta\vartheta_0'$,

Temperature rise factor $= \xi_0' = \vartheta_{20}'/\vartheta_{10}'$,

c) Mean surface temperature of the cylinder $= T_{m0}'$ °K,

Cylinder temperature factor $= r_0' = T_{m0}'/\vartheta_{10}'$,

Temperature efficiency $= \eta_0' = (\vartheta_{20}' - \vartheta_{10}')/(T_{m0}' - \vartheta_{10}')$,

d) Mass flow of the cooling air $= G_0'$ kg/s,

Volume of the cooling air at the inlet $= V_{10}' = v_{10}' \cdot G_0'$ m^3/s, where v_{01}' = specific volume of the cooling air at the entry.

Heat dissipation $= Q_0' = 3{,}600 G_0' \cdot C_p \cdot \Delta\vartheta_0'$ kcal/h,

where $C_p = 0.240$ kcal/kg °C.

e) Pressure drop at the baffle plates $= \Delta P_0'$ mm Aq or kg/m².

The inlet velocity of the cooling air u_{10}' can be determined by substituting the values of η_0', r_0' and $\Delta P_0'$ or η_0', ξ_0' and $\Delta P_0'$ above obtained in Eq. (7).

The cooling resistance horse power $= HP_{c0}'$

$$= \frac{1}{75}\left(1 - \tfrac{1}{2}\eta_0' + \tfrac{1}{2}\eta_0' r_0'\right) v_{10}' \cdot G_0'.$$

2) Cooling characteristics reduced to the Standard Atmosphere

It is most probable in actual case to assume $Q_0 = Q_0'$ and $u_{10} = u_{10}'$. Starting with these assumptions, we can determine the valves of η_0, r_0 and ξ_0 under the Standard Atmosphere by the graphical methods explained before. Then the cooling characteristics reduced to the Standard Atmosphere can be obtained as follows:

a) Mass flow of the cooling air $= G_0$ kg/s

We have

$$\vartheta_{10} = 288°\text{K}, \qquad \vartheta_{20} = 288\,\xi_0, \qquad \Delta\vartheta_0 = \vartheta_{20} - \vartheta_{10} = 288\,(\xi_0 - 1),$$

$$Q_0 = Q_0' = 3{,}600\, G_0 \cdot C_p \cdot \Delta\vartheta_0 \text{ kcal/h},$$

which gives the value of G_0.

b) Pressure drop $= \Delta P_0$ mm Aq or kg/m²

Eq. (8) will give the following relation which enables us to determine the value of ΔP_0.

$$\frac{\Delta P_0}{\Delta P_0'} = \frac{\gamma_{10}}{\gamma_{10}'} \cdot \frac{\frac{1}{2}(\xi_0 + 1) K \log_{10}(1/(1-\eta_0)) + 2(\xi_0 - 1)}{\frac{1}{2}(\xi_0' + 1) K \log_{10}(1/(1-\eta_0')) + 2(\xi_0' - 1)},$$

where γ_{10}' and γ_{10} are the specific weights of the atmospheric air corresponding to the specifiic volume v_{10}' and v_{10}, respectively.

c) Power required for air-cooling $= HP_{c0}$

The relation given by Eq. (11) enables us to determine the value of HP_{c0} as follows:

$$\frac{HP_{c0}}{HP_{c0}'} = \frac{\xi_0 + 1}{\xi_0' + 1} \cdot \frac{\Delta P_0}{\Delta P_0'}.$$

3) Cooling characteristics at higher levels

The cooling characteristics, viz., η_z and ξ_z at higher altitudes can be determined graphically by the principle of interpolation as shown in Fig. 4. The outlet temperature of the cooling air, the mean cylinder temperature etc. will be then obtained by the following relations:

$$\vartheta_{2z} = \xi_z \cdot \vartheta_{1z},$$

$$T_{mz} = r_z \cdot \vartheta_{1z} \quad \text{or} \quad = \vartheta_{1z} + \frac{\Delta\vartheta_z}{\eta_z}.$$

Other items will be found directly from the diagrams, the construction of which is explained in the following article.

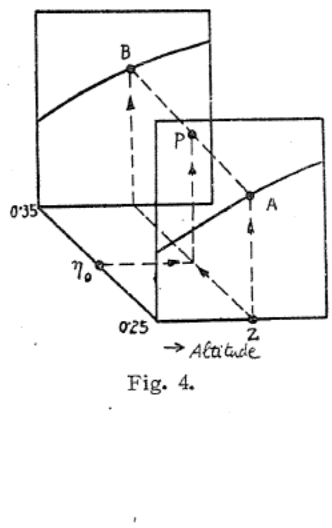


Fig. 4.

4) Construction of the diagrams

All diagrams given at the end of this paper have been constructed on the condition $Q_z/Q_0 = 1$.

a) When the outlet temperaure of the cooling air remains constant at all heights, i.e., $\vartheta_{2z} = \vartheta_{20}$.

In this case, the value of $\xi_z = \vartheta_{2z}/\vartheta_{1z} = \vartheta_{20}/\vartheta_{1z}$ can be determined, first of all. Substitute the relation $Q_z/Q_0 = 1$ in Eq. (4), and we obtain

$$\frac{G_z}{G_0} = \left(\frac{\xi_z - 1}{\xi_0 - 1}\right)^{-1} \cdot \left(\frac{\vartheta_{1z}}{\vartheta_{10}}\right)^{-1}. \qquad \cdots\cdots\cdots (a)$$

The combination of Eq. (a) and Eq. (5) gives

$$\left\{\frac{(1-\frac{1}{2}\eta_z) + (1/\eta_z + \frac{1}{2})(\eta_z + \xi_z - 1)}{(1-\frac{1}{2}\eta_0) + (1/\eta_0 + \frac{1}{2})(\eta_0 + \xi_0 - 1)} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}}\right\}^{0.16} \cdot \left\{\frac{1/\eta_z - \frac{1}{2}}{1/\eta_0 - \frac{1}{2}}\right\}^{1.6} \cdot \frac{\xi_z - 1}{\xi_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} = 1, \qquad \cdots\cdots (b)$$

which enable us to find the value of η_z at any given heihgt for the condition ϑ_{2z} = constant and Q_z = constant at all heights. The results of calculatoins are shown in Diagram A 1.

The cylinder temperature is given by

$$t_{mz} = T_{mz} - 273 = \left(\vartheta_{1z} - 273 + \frac{\vartheta_{2z} - \vartheta_{1z}}{\eta_z}\right),$$

which is shown in Diagram A 2.

Diagram A 3 shows the variation of the mass flow of the cooling air G_z/G_0 which can be worked out by Eq. (a). Eq. (10) gives the value $V_{1z}/V_{10} = u_{1z}/u_{10}$ as shown in Diagram A 4.

Further, Eq. (8) and (11) will give the values $\Delta P_z/\Delta P_0$ and HP_{cz}/HP_{c0} which are shown in Diagram A 5 and A 6, respectively.

b) When the mean cylinder temperature is to be kept constant at all heights, i.e., $T_{mz} = T_{m0}$. In this case, the value of cylinder temperature factor r_z at a given height of z m can be calculated by the following relation:

$$r_z = \frac{T_{mz}}{\vartheta_{1z}} = \frac{T_{m0}}{\vartheta_{10}}.$$

Next, Eq. (1 b) becomes

$$\frac{G_z}{G_0} = \frac{\eta_0}{\eta_z} \cdot \frac{r_0 - 1}{r_z - 1} \cdot \frac{\vartheta_{10}}{\vartheta_{1z}}, \qquad \cdots\cdots\cdots (c)$$

if $Q_z/Q_0 = 1$. Substituting this relation in Eq. (2), we get

$$\left\{\frac{(1-\frac{1}{2}\eta_z) + (1+\frac{1}{2}\eta_z)r_z}{(1-\frac{1}{2}\eta_0) + (1+\frac{1}{2}\eta_0)r_0} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}}\right\}^{0.6} \cdot \left\{\frac{1/\eta_z - \frac{1}{2}}{1/\eta_0 - \frac{1}{2}}\right\}^{1.6} \cdot \frac{\eta_z}{\eta_0} \cdot \frac{r_z - 1}{r_0 - 1} \cdot \frac{\vartheta_{1z}}{\vartheta_{10}} = 1. \cdots (d)$$

The last equation (d) gives the value of η_z as shown in Diagram B 1.

The value of G_z/G_0 can be determined by Eq. (c). The results of calculations are shown in Diagram B 2.

The values of $\frac{V_{1z}}{V_{10}} = \frac{u_{1z}}{u_{10}}$, $\frac{\Delta P_z}{\Delta P_0}$ and $\frac{HP_{cz}}{HP_{c0}}$ have been worked out just in the same way as cxplained above. They are shown in Diagram B 3, B 4 and B 5, respectively.

Lastly, the value of $\xi_z = 1 + \eta_z(r_z - 1)$ can be determined, which gives the outlet temperature of the cooling air by the following relation:

$$\vartheta_{2z} = \xi_z \cdot \vartheta_{1z}.$$

The variations of ϑ_{2z} are shwon in Diagram B 6.

c) When the pressure drop at the baffle plates remains constant at all heights, i.e., $\Delta P_z = \Delta P_0$. This condition seems most likely to take place in actual cases. To solve this problem, combine, firstly, Eq. (8) and (10). Then we get the following general relation:

$$\frac{\Delta P_z}{\Delta P_0} = \left(\frac{\gamma_{1z}}{\gamma_{10}}\right)^{-1} \cdot \left(\frac{G_z}{G_0}\right)^2 \cdot \frac{\frac{1}{2}(\xi_z+1)K\log_{10}(1/(1-\eta_z))+2(\xi_z-1)}{\frac{1}{2}(\xi_0+1)K\log_{10}(1/(1-\eta_0))+2(\xi_0-1)}. \quad \cdots\cdots(13)$$

Conbine Eq. (13) and (4) on the condition $\frac{\Delta P_z}{\Delta P_0} = 1$ and $\frac{Q_z}{Q_0} = 1$, then the following relation will be derived.

$$\frac{\gamma_{1z}}{\gamma_{10}} \cdot \left(\frac{\xi_z-1}{\xi_0-1}\right)^2 \cdot \left(\frac{\vartheta_{1z}}{\vartheta_{10}}\right)^2 = \frac{\frac{1}{2}(\xi_z+1)K\log_{10}(1/(1-\eta_z))+2(\xi_z-1)}{\frac{1}{2}(\xi_0+1)K\log_{10}(1/(1-\eta_0))+2(\xi_0-1)}. \quad \cdots(e)$$

The two simultaneous equations (e) and (b) involving two unknowns η_z and ξ_z will give these values. The procedure of the computation is as follows:

Put

$$A_z = \frac{\gamma_{1z}}{\gamma_{10}} \cdot \left(\frac{\vartheta_{1z}}{\vartheta_{10}}\right)^2 \frac{\frac{1}{2}(\xi_0+1)K\log_{10}(1/(1-\eta_0))+2(\xi_0-1)}{(\xi_0-1)^2}.$$

A_z is the function of the height z only. Then Eq. (e) can be written in the following form of quadratic equation in ξ_z.

$$\xi_z^2 - 2b_z \cdot \xi_z + c_z = 0, \quad \cdots\cdots\cdots(f)$$

where

$$b_z = 1 + \frac{K}{4A_z}\log_{10}\frac{1}{1-\eta_z} + \frac{1}{A_z},$$

$$c_z = 1 - \frac{K}{2A_z}\log_{10}\frac{1}{1-\eta_z} + \frac{2}{A_z}.$$

The solution of Eq. (f) is

$$\xi_z = b_z + \sqrt{b_z^2 - c_z} > 1. \quad \cdots\cdots\cdots(g)$$

Starting the values η_0 and r_0 obtained from the ground level tests, we proceed as follows:

i) Determine the value $\xi_0 = 1 + \eta_0(r_0 - 1)$

ii) By making use of the values η_0 and ξ_0 thus obtained, calculate the values A_z for all heights.

iii) Work out several groups of η_z and ξ_z satisfying Eq. (g). This will be done by trials for appropriate values of η_z.

iv) Substitute the values of η_z and ξ_z in every group successively, then you will find the final combination of η_z and ξ_z which satisfy Eq. (e) and (b), simultaneously.

Once the values of η_z and ξ_z are determined, all the cooling characteristics can be estimated in the same way as before. Thus we get

Diagram C 1 for η_z, Diagram C 2 for t_{mz}°C,
Diagram C 3 for G_z/G_0, Diagram C 4 for $V_{1z}/V_{10} = u_{1z}/u_{10}$,
Diagram C 5 for HP_{cz}/HP_{c0}, Diagram C 6 for ϑ_{2z}.

7. Comparison of Cooling Characteristics between the Three Cases. viz., ϑ_{2z} = constant, T_{mz} = constant and ΔP_z = constant

In order to compare the results thus obtained, another set of diagrams has been prepared. They are applicable to the case that the mean cylinder temperature at the ground level is taken as $t_{m0} = 190°C$ and the temperature efficiency at the ground level as $\eta_0 = 0.25$, 0.35 and 0.45.

Diagram D 1 for η_z, Diagram D 2 for $T_{mz} = 273 + t_{mz}$,
Diagram D 3 for G_z/G_0, Diagram D 4 for $V_{1z}/V_{10} = u_{1z}/u_{10}$,
Diagram D 5 for $\Delta P_z/\Delta P_0$, Diagram D 6 for HP_{cz}/HP_{c0},
Diagram D 7 for ϑ_{2z}.

These diagrams give a good comparison of the general tendency of the cooling characteristics at higher altitudes. Let us enumerate the results of computations:

a) The temperature efficiency η_z increases with altitudes. In the case ΔP_z = constant, the tendency of increase of η_z is most remarkable.

b) The mean cylinder temperature t_{mz} rises most rapidly in the case ΔP_z = constant. It is advisable to increase ΔP_z by opening the flaps at the rear end of the engine cowling, so as to keep the cylinder temperature in desirable limits at higher altitudes.

c) The smaller the value of η_0 at the ground level, the better cooling characteristics at higher altitudes.

d) Better cooling characteristics can be expected at higher altitudes, if we keep the cylinder temperature t_{m0} at the ground level as low as possible.

e) There are intimate relations between the mass flow G_z and other cooling characteristics. For example, the pressure drop ΔP_z at higher altitudes must be increased definitely, if we expect to keep the cylinder temperature not to rise over a certain reasonable limit.

f) The horse power required for air cooling HP_{cz} increases with altitudes. In the case ΔP_z = constant, however, this horse power remains practically constant as high as 7,000 m. This result coincides with our experience in the case of aero-engines with $\eta_0 = 0.25$ and lower value of t_{m0}.

g) The temperature rise $\Delta\vartheta$ of the cooling air is very close to the cooling characteristics, as seen clearly in Diagram D 7. In the case ΔP = constant, the mass flow of cooling air decreases so much at higher altitudes, resulting in rapid rise of the cylinder temperature. On the contrary, if the cylinder temperature is to be kept constant, so much quantity of cooling air must be passed at higher altitudes, resulting in little change in the outlet temperature of the cooling air.

8. Comparison between the Calculated Results and the Experimental Data

There are two procedures of experimental researches to ascertain the validity of the theories here treated. The one is to undertake wind-tunnel tests with the same atmospheric conditions as those at higher altitudes. Some fragmental rerults, though valuable, indicates the adoptability of my theories above given.

The other is to observe the cooling characteristics on the flying aircraft at higher altitudes. One of such experiments was carefully and deliberately conducted by Mitsubishi Aero-Engine Research Institute at Nagoya. The followings are the results reported by Mr. Aso, one of the research member of that institute. Fig. 5 shows the temperature of the rear plug seat at the heights 1, 2, · · ·, 10, 11 km as ordinate and the pressure drop ΔP_z as abscissa. At all heights, the decrease in ΔP_z results in the diminution of the mass flow of the cooling air, and, consequently, the temperature rise of the cylinder. On the basis of these results obtained from this experiment, we get a good comparison between the theory and practice. In Fig. 6, the curve in full line is the cylinder temperature curve for $\eta_0 = 0.25$ and $t_{m0} = 170°C$ selected from Diagram C 2. The plotted dots correspond to the values read from Fig. 5, as the intersection of the curves with the vertical showing $\Delta P = 186$ mm Aq = constant at all heights. As far as this experiment is concerned, we find a good accord of the theory with the facts.

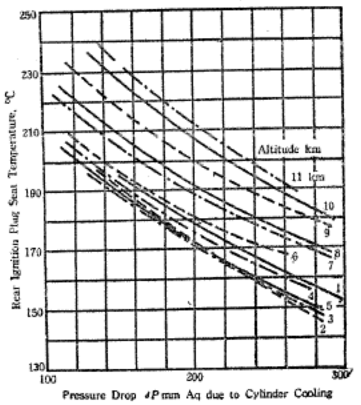


Fig. 5. Observed cyl. temp. at various altitudes

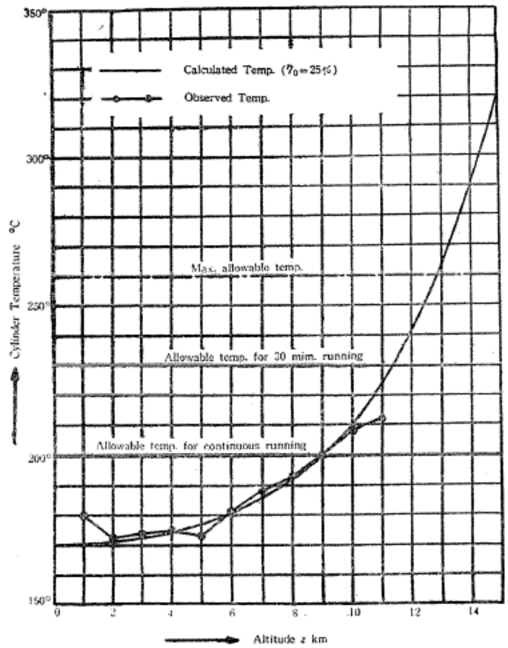


Fig. 6. Variations of cylinder temperature with altitudes when the pressure drop ΔP is kept constant

9. Résumé

The cooling characteristics may be the function of the following factors:

Q = heat dissipation, T_m = mean cylinder temperature, ϑ_1 = inlet temperature of cooling air, ϑ_2 = outlet temperature of cooling air, G = mass flow of cooling air, u_1 = inlet velocity of cooling air, ΔP = pressure drop, HP_c = power required for air cooling, etc.

The foregoing theories will make clear the relations between them. Any two of them can be the independent variables, e.g., Q and T, Q and ΔP, and the like.

The followings are the general conclusions of this study:

1) The temperatures are expressed as functions of the atmospheric air, namely:

η = temperature efficiency = $(\vartheta_2 - \vartheta_1)/(T_m - \vartheta_1) = (\xi - 1)/(\gamma - 1)$

ξ = temperature rise factor = ϑ_2/ϑ_1

γ = cylinder temperature factor = T_m/ϑ_1

Out of these three, we can take any two of them as the independent variables. In this way, we can avoid the confusion of the temperature scale, viz., °C or °F.

All the variables above mentioned are expressed in the form of ratio, such as, Q_z/Q_0, $\Delta P_z/\Delta P_0$, $\vartheta_{1z}/\vartheta_{10}$ etc. Thus, we need not be worried about the units and dimensions, as experienced in the emperical formulae.

2) Eq. (2) or (5) gives the direct relation between the mass flow and the cooling characteristics.

3) The complicated relations between the heat dissipation and the pressure loss can be expressed in such a simple formula as Eq. (7).

4) The Tokyo Standard Atmosphere has been newly introduced.

5) Some diagrams have been prepared in order to give facilities to the practical designers in computing the problems of this kind.

Acknowledgements

The author of this paper wishes to express his hearty gratitudes to Dr. Kōtaro Uhira in working together all the time, especially, in deriving the relation between the equivalent heat transfer coefficient for finned surface and the local heat transfer coefficient. He owes a debt of heartfelt thanks to Mr. Aso who made a generous offer of the experimental data to ascertain the validity of his theories.

Notes to the Annexed Diagrams

PL. A 1, PL. A 2, PL. A 3, PL. A 4, PL. A 5 and PL. A 6:
Heat Dissipation = Q = Constant, Exit Temperature of Cooling Air = ϑ_2 = Constant.

PL. B 1, PL. B 2, PL. B 3, PL. B 4 and PL. B 5:
Heat Dissipation = Q = Constant, Mean Outside Temperature of Cylinder Wall = t_m = Constant.

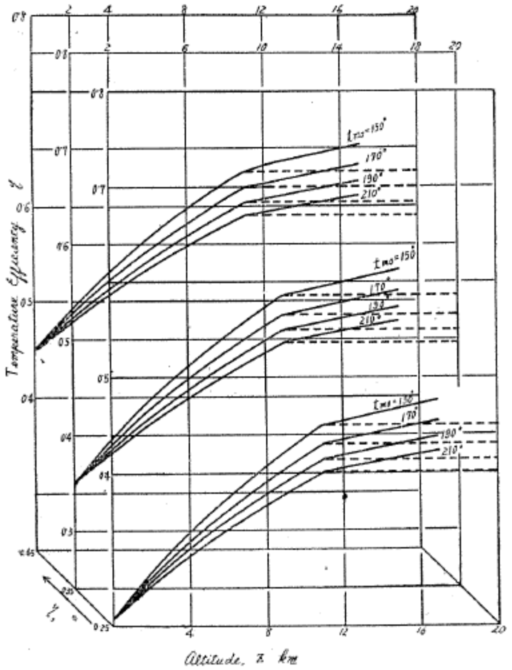
PL. C 1, PL. C 2, PL. C 3, PL. C 4, PL. C 5 and PL. C 6:
Heat Dissipation = Q = Constant, Pressure Drop = ΔP = Constant.

PL. D 1, PL D 2, PL. D 3, PL. D 4, PL. D 5, PL. D 6 and PL. D 7:
Heat Dissipation = Q = Constant, Mean Outside Temperature of Cylinder Wall at Ground Level = t_{m0} = 190°C.

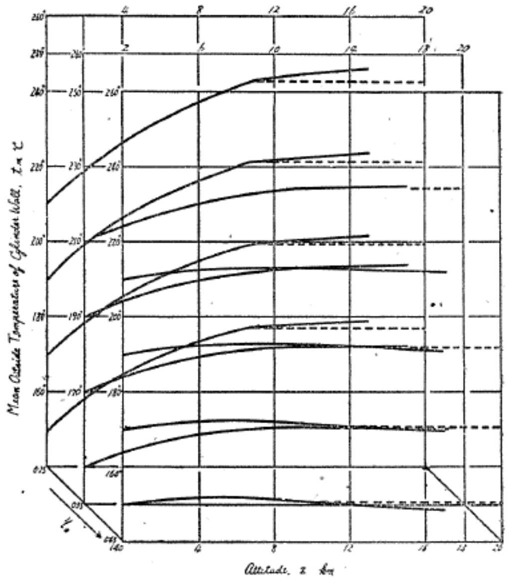
PL. E. Correction Chart for Ground Level Test:
Heat Dissipation = Q = Constant, Inlet Velocity of Cooling Air = u_1 = Constant.

PL. F. Tokyo Standard Atmosphere.

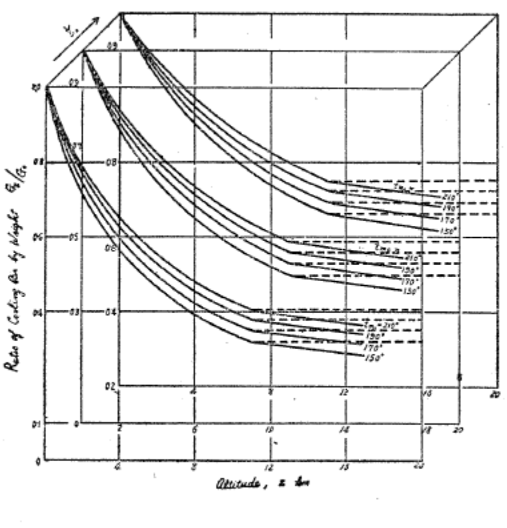
A 1.


Temperature Efficiency
Altitude, z km

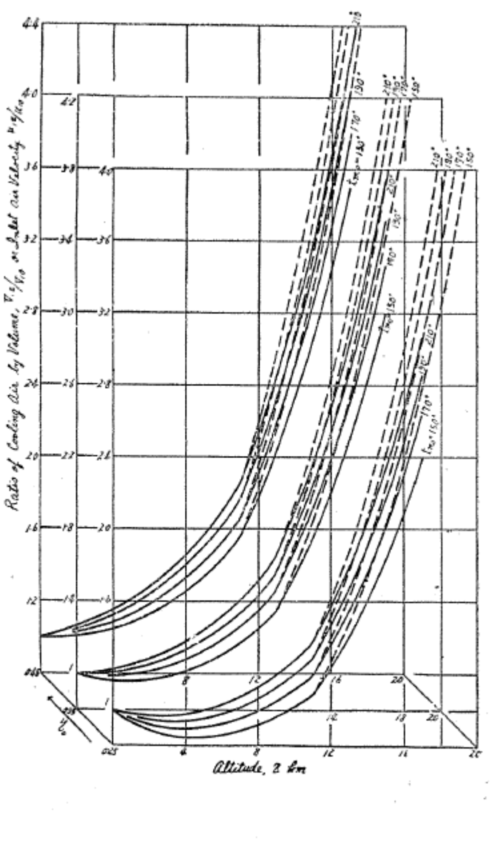
A 2.


Mean Outside Temperature of Cylinder Wall
Altitude, z km

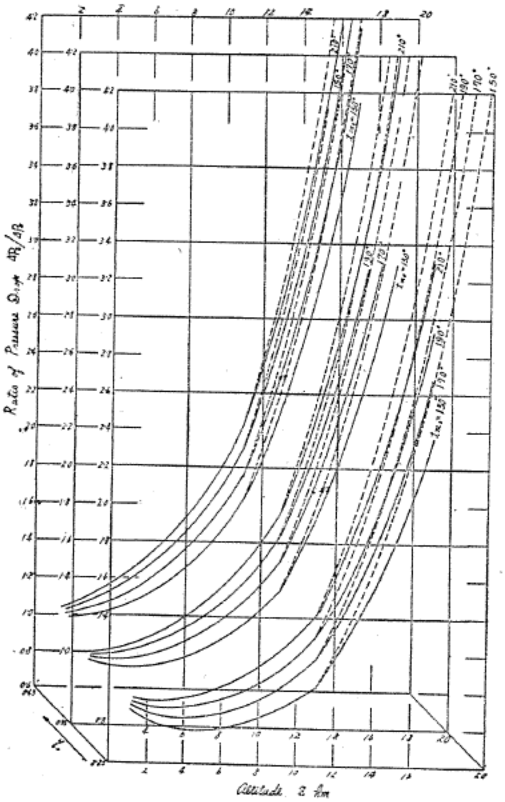
A 3.


Ratio of Cooling Air by Weight
Altitude, z km

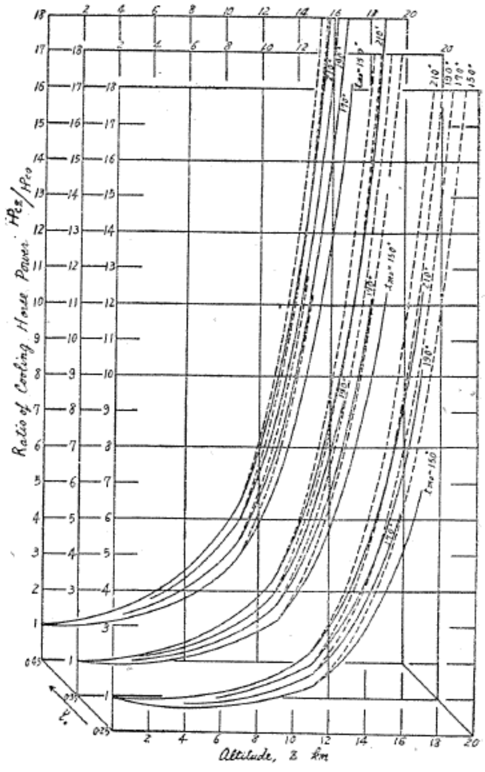
A 4.


Ratio of Cooling Air by Volume
Altitude, z km

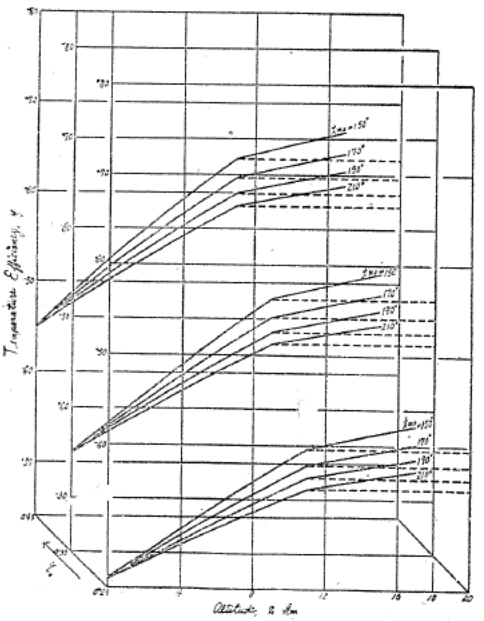
A 5.



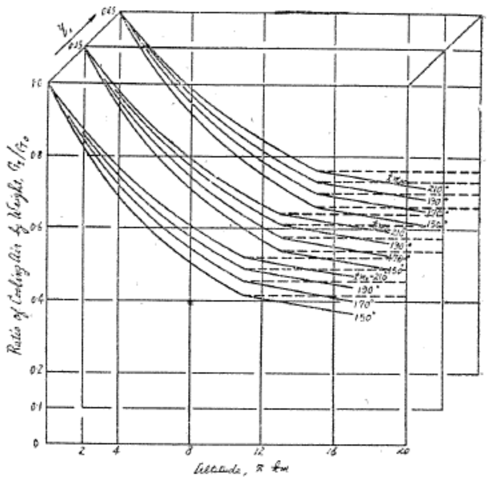
A 6.



B 1.



B 2.



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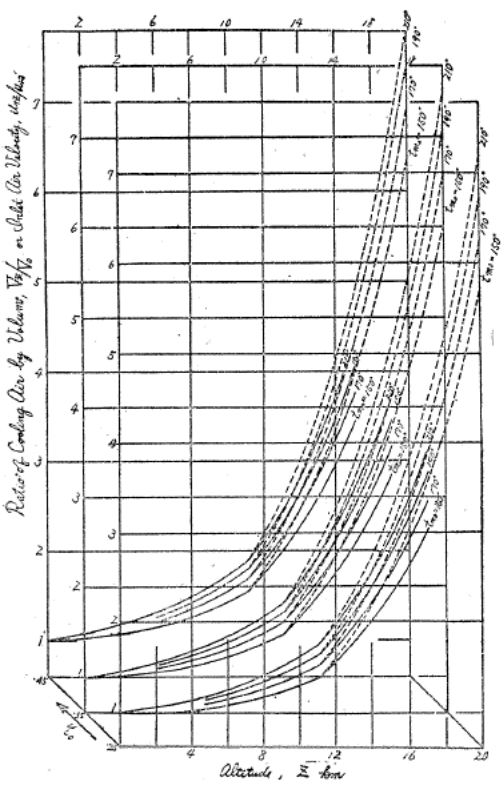
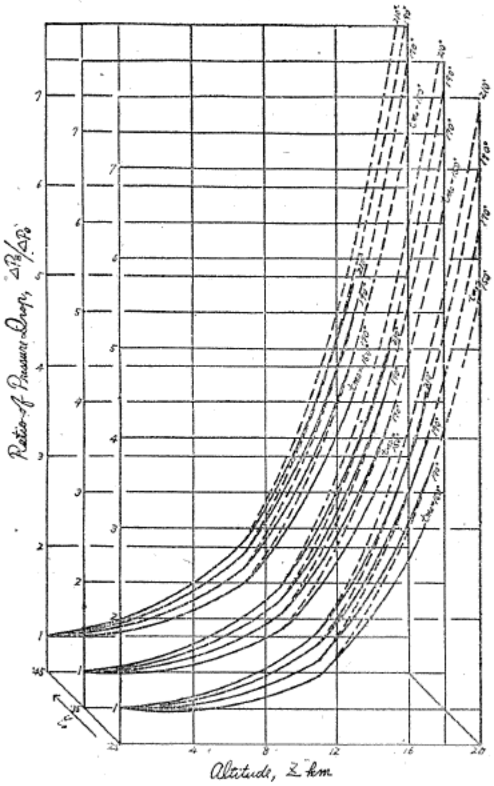
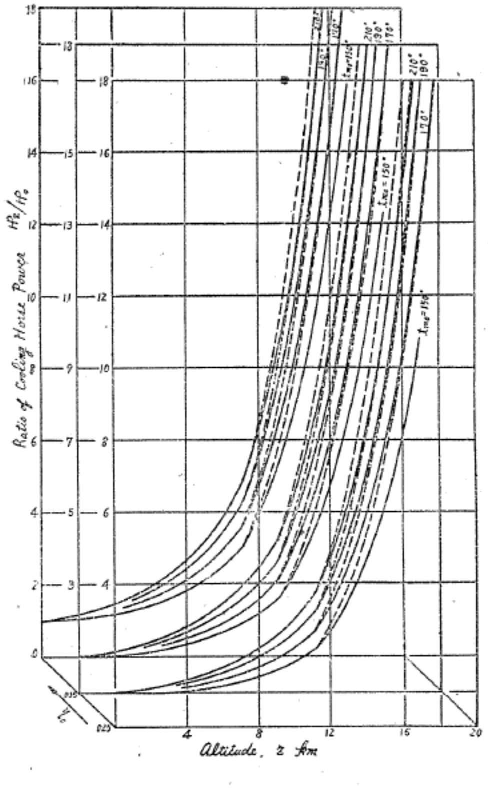

Altitude, Z km

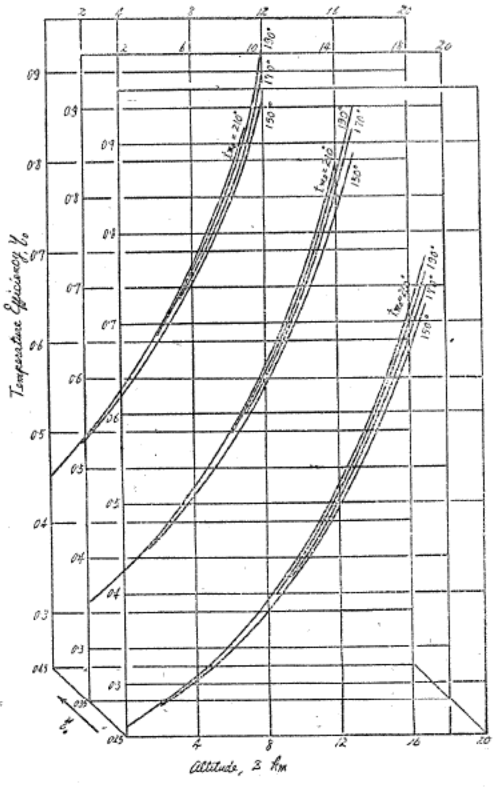
B 4.


Ratio of Pressure Drop
Altitude, Z km

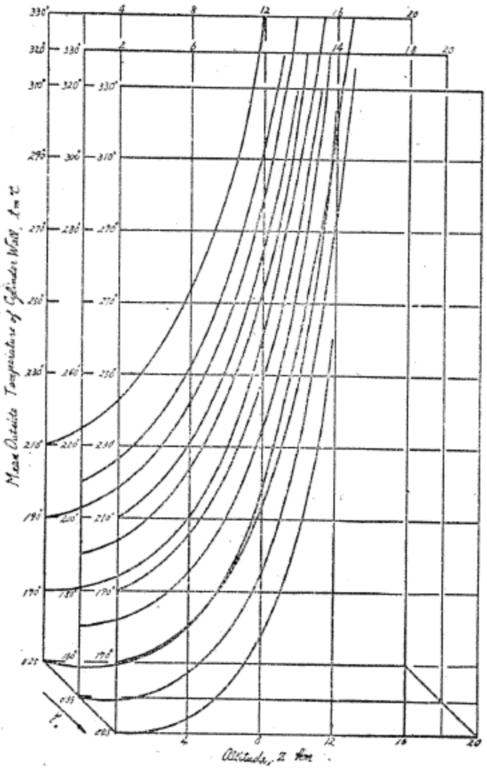
B 5.


Ratio of Cooling Horse Power
Altitude, z km

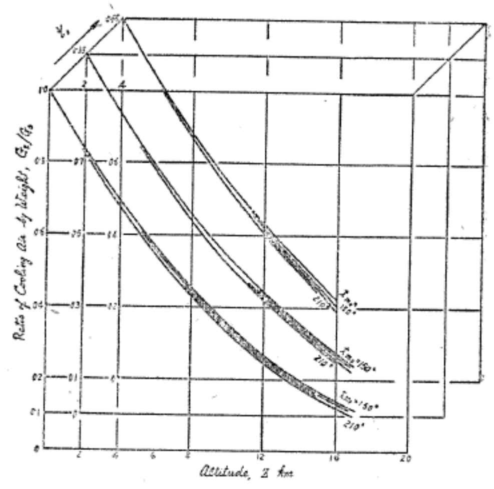
C 1.


Temperature Efficiency
Altitude, z km

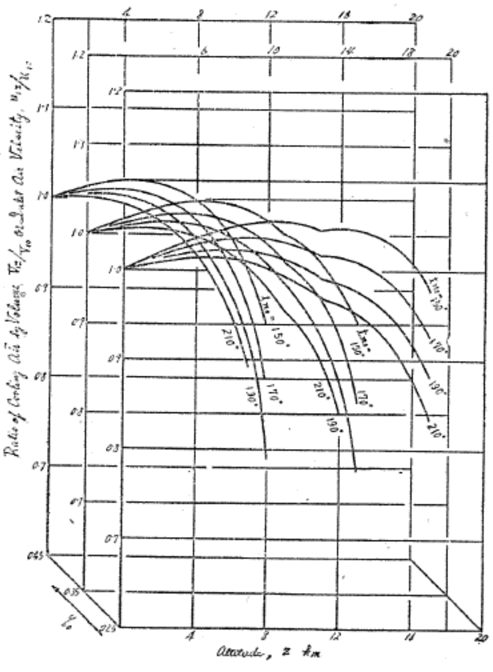
C 2.


Altitude, z km
4
8
12
16
20

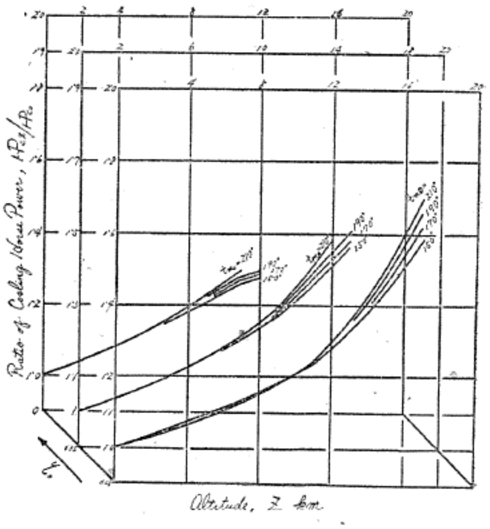
C 3.


Ratio of Cooling Air by Weight, G_z/G_o
Altitude, z km

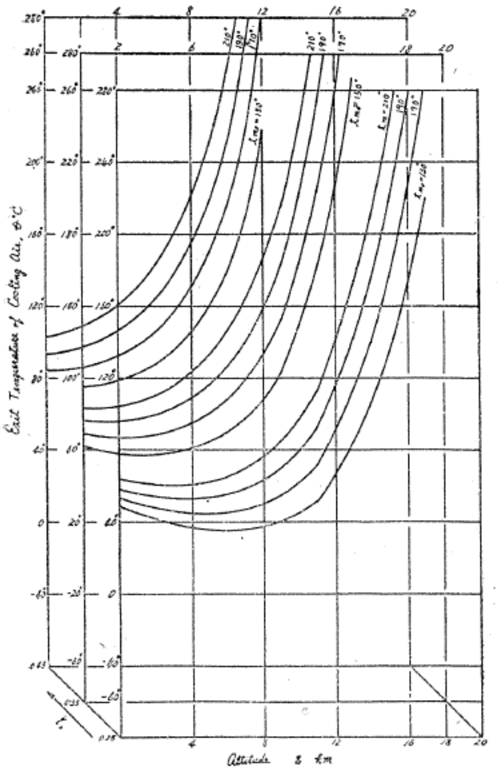
C 4.


Altitude, z km
4
8
12
16
20

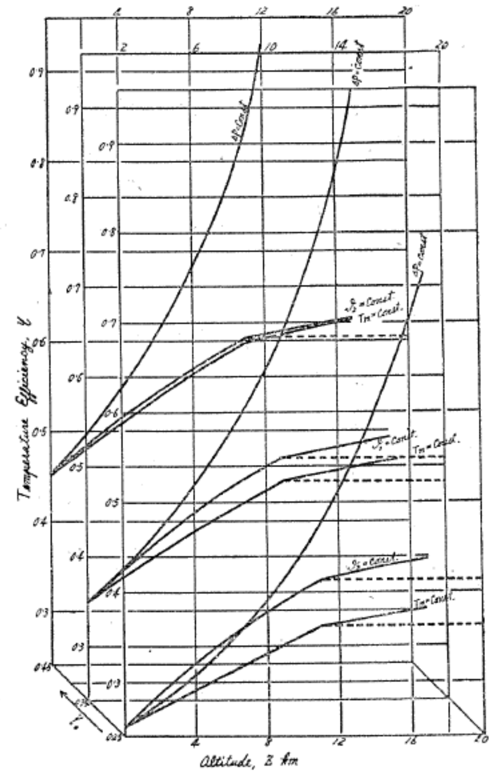
C 5.


Ratio of Cooling Horse Power, HP_{cz}/HP_{co}
Altitude, Z km

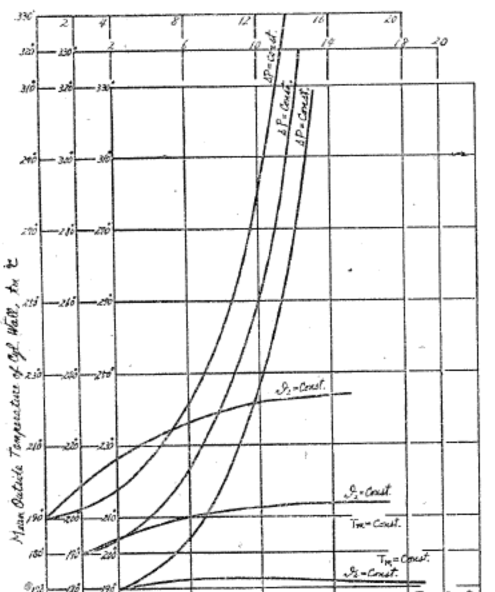
C 6.


Exit Temperature of Cooling Air, θ°C
Altitude z km

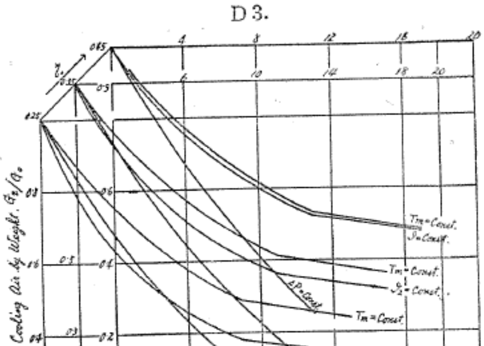
D 1.


Temperature Efficiency, ε
ΔP=Const.
Tm=Const.
Altitude, z km

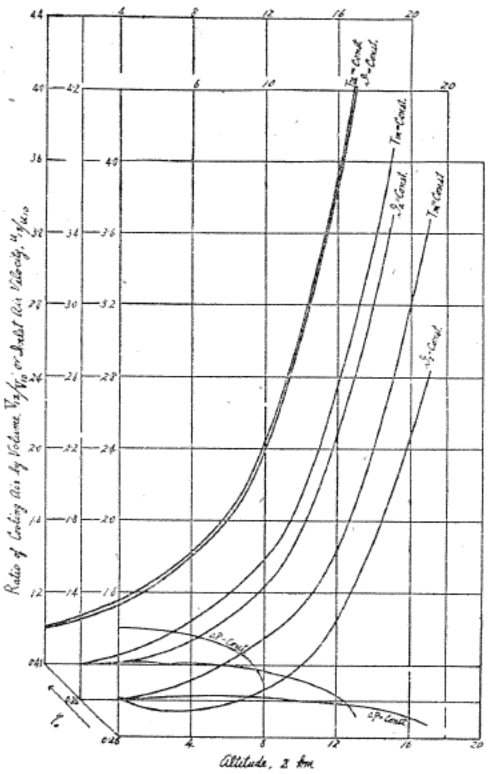
D 2.


Mean Outside Temperature of Cyl. Wall, tm °C
ΔP=Const.
Tm=Const.
Altitude, z km

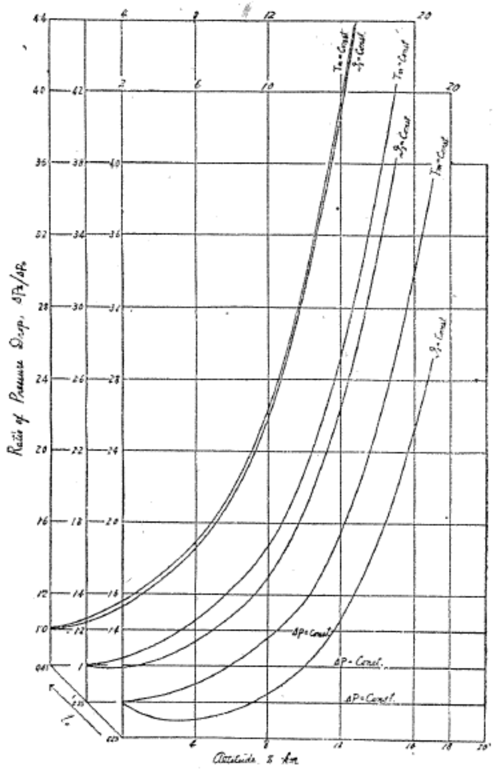
D 3.


Ratio of Cooling Air by Weight, G2/G0
Tm=Const.
ΔP=Const.
Altitude, z km

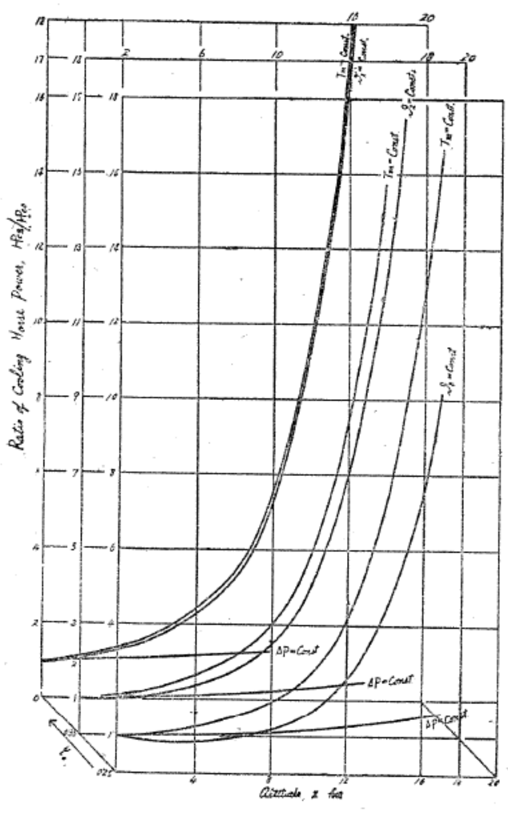
D 4.


Altitude, z km
ΔP=Const.
Tm=Const.

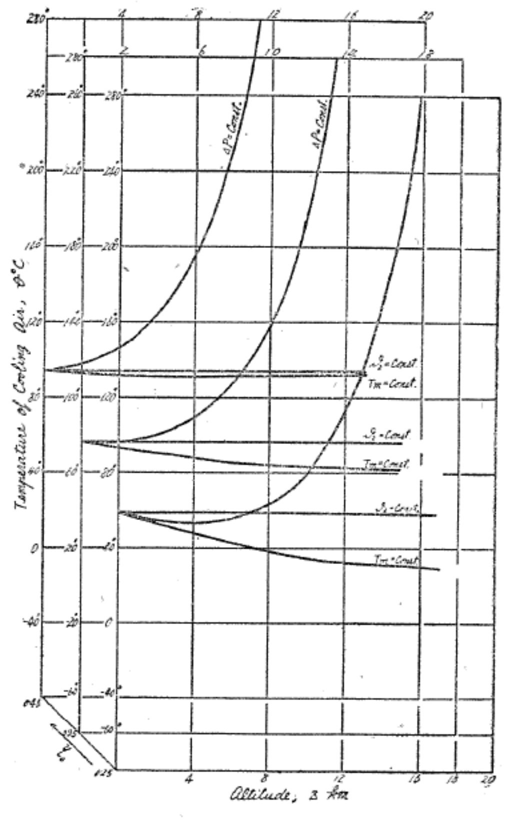
D 5.


Ratio of Pressure Drop
Altitude z km
ΔP=Const.
Tm=Const.

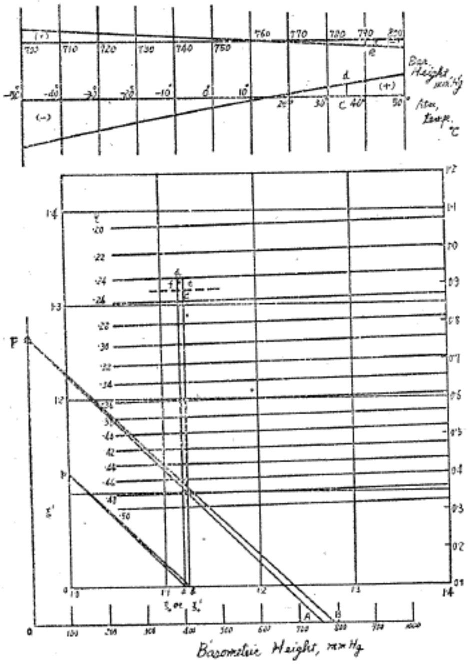
D 6.


Ratio of Cooling Horse Power
Altitude, z km
ΔP=Const.
Tm=Const.

D 7.


Temperature of Cooling Air, θ°C
Altitude, z km
ΔP=Const.
Tm=Const.

E.



1) $\begin{cases} OA = 760 \text{ mm Hg} \\ OB = h = 790 \text{ mm Hg} \end{cases}$

2) $OP = \xi_0' - 1$

3) $pa \parallel PA$, $pb \parallel PB$

4) $\begin{cases} oa = \xi_0' - 1°, \ \xi_0' = 1.120 \\ ob = \xi_0 - 1°, \ \xi_0 = 1.125 \end{cases}$

5) c corresponds for $\eta_0' = 0.25$

6) cd = Correction for atm. temp. 35°C

7) de = Correction for bar. h't 790 mm Hg

8) $ef \parallel ab$

9) f corresponds for the temp. eff. $\eta_0(= 0.244)$ at the ground level for Standard Atmosphere

F.