Hidden Local Symmetry and the Loop Effect

(隠れた局所対称性とそのループ効果)

原田正康

Hidden Local Symmetry and the Loop Effect*

Masayasu HARADA

Department of Physics, Nagoya University
Nagoya 464-01, Japan

Abstract

We investigate loop corrections to the successful tree level predictions of the hidden local symmetry in the $SU(2)_L \times SU(2)_R$ nonlinear sigma model: The KSRF I relation as a "low energy theorem" as well as the ρ-coupling universality, the KSRF II relation and the vector meson dominance of the electromagnetic form factor of the pion. We show that these predictions are preserved in the low energy limit, even if we include the loop corrections of the hidden local gauge boson. Most amazingly, the vector meson dominance holds at any momentum, if and only if we take the parameter choice $a = 2$. We further calculate the β functions for the parameters a and the gauge coupling g, and show that the "vector limit" ($a = 1$, $g = 0$) corresponds to an ultraviolet fixed point of the β functions.

*This paper is based on the work done in collaboration with K. Yamawaki[1].

1 Introduction

If we set the masses of u and d quarks equal to zero in QCD, the Lagrangian is invariant under the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ chiral symmetry. The vacuum of the theory spontaneously breaks this symmetry into its subgroup $\mathrm{SU}(2)_\mathrm{V}$ (for a review of the chiral symmetry breaking, see, for example, Ref.[2]). This broken symmetry leads to the three massless Nambu-Goldstone (NG) bosons coupled to the broken axial-vector currents (Nambu-Goldstone theorem), and determines the low energy interactions among such NG bosons (low energy theorems of chiral symmetry) in terms of the decay constant (the strength of the coupling of NG bosons to the broken currents).

In the real world, since the masses of u and d quarks are tiny in comparing with the scale of the theory (for a review of quark masses, see Ref.[3]), the QCD Lagrangian has an approximate $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ chiral symmetry. The pions are identified with the pseudo-NG bosons, and the low energy theorems should be approximately valid. Then, by using the current algebra, the matrix elements for the soft-pion interactions can be determined from the symmetry structure only. We can more easily calculate the same matrix elements from an $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ invariant effective Lagrangian. The non-linear sigma model, which includes the pions only, well describes the low energy pion dynamics.

Recently, the systematic method to consider the pion loop contributions is studied in the non-linear chiral Lagrangian ("Chiral Perturbation Theory"[4, 5, 6] (for a recent review, see Ref.[7])). In the chiral perturbation theory, the pion loop yields predictions beyond that of the low energy theorem, which successfully reproduce the results in the energy region slightly away from the low energy limit. But it has many unknown parameters (finite parts of the counter terms), whose values are not determined from the symmetry structure. Moreover, we cannot use this model in the higher energy region, because of the ρ meson. As pointed out in Refs.[8, 9, 10], the tree-level effects of the ρ meson saturate the above unknown parameters. So we can expect that the effective Lagrangian including the pion and the ρ meson can describe the results in a wider energy region. Actually as shown in Refs.[9, 11, 12], the loop effects of vector mesons are crucial to the $\pi^+ - \pi^0$ mass difference[13].

Now that, the ρ mesons as well as the pions play an important role in the low energy hadron dynamics, so that we introduce the ρ mesons and the pions to an effective theory in a systematic way. Among such approach, a model based on the hidden local symmetry has well succeeded in describing the system which includes the pion and the ρ meson. In this model, the ρ meson is identified with the dynamical gauge boson of the hidden local symmetry, $[\mathrm{SU}(2)_\mathrm{V}]_\mathrm{local}$; (for a review, see Ref.[2]). The Lagrangian is invariant under the $[\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}]_\mathrm{global} \times [\mathrm{SU}(2)_\mathrm{V}]_\mathrm{local}$ symmetry. This symmetry is spontaneously broken into its subgroup $\mathrm{SU}(2)_\mathrm{V}$. Then, the hidden gauge boson (ρ meson) has its own mass through the Higgs mechanism. In the hidden local symmetry, we have only three parameters in the low energy;

1) f_π: the pion decay constant,

2) f_σ: the decay constant of the would-be NG boson absorbed into the hidden gauge boson (ρ meson),

3) g: the hidden gauge coupling constant.

In the low energy experiments for the ρ mesons and the pions, it is important that we investigate the $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ and the ρ-γ transition strength g_ρ as well as the ρ-meson mass m_ρ, which are determined from the above tree parameters. (See sect. 3.) For the parameter choice $a = 2$ (a is defined by $f_\sigma^2 = a f_\pi^2$[14, 15]) in the hidden local symmetry Lagrangian, we can reproduce the following phenomenological facts[16];

1) the ρ-coupling universality, $g_{\rho\pi\pi} = g$[17],

2) the KSRF relation (II), $m_\rho^2 = 2 f_\pi^2 g_{\rho\pi\pi}^2$[18],

3) the ρ meson dominance of the electromagnetic form factor of the pion[17]. Furthermore this model predicts the successful KSRF relation (I)[18],

$$g_\rho = 2g_{\rho\pi\pi}f_\pi^2,$$

as a "low energy theorem" of hidden local symmetry[19].

We can introduce the ρ mesons as antisymmetric tensor fields into the chiral Lagrangian[5, 9]. In such a model, the ρ mesons transforms as the ordinary matter content under the $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ chiral symmetry. Since there exists no more than the chiral symmetry, the parameters g_ρ and $g_{\rho\pi\pi}$ mentioned above (corresponding to F_V and G_V in the notation used in Refs.[5, 9]) are independent. Then the KSRF relation (I), $g_\rho = 2f_\pi^2 g_{\rho\pi\pi}$, is just an input relation from the experiments.

Recently, there are many field theoretical approach to the possible vector resonances in the strongly coupled Higgs sector or the technicolor model. Although it is unlikely[20] that the ρ-like (techni-ρ) resonance exists in the strongly coupled Higgs sector, we do expect such a resonance in the technicolor model. The vector resonances play an essential role to constrain the technicolor model from the experiments[21]. Thus it is important to consider an effective theory including the vector resonances for the technicolor model. Actually, the above hidden local symmetry Lagrangian has been applied to this kind of model ("BESS" model)[22]. For instance, the one doublet technicolor model has the $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$ chiral symmetry with $\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ gauging. The NG bosons, which appear as a consequence of the spontaneous symmetry breaking, are absorbed into W bosons. Then the longitudinal components of W bosons (W_L) correspond to the pions and the vector resonances correspond to the gauge bosons of the $[\mathrm{SU(2)_V}]_{\mathrm{local}}$ hidden local symmetry. Further, the loop contributions of the vector resonances to the vertices among the W boson and fermions have been studied[23].

Now, we think the effective theory as an "effective field theory" and calculate the loop corrections. At that time, we need the many counter terms to renormalize the divergence. Thus we get the renormalization-group equations for the parameters (finite parts of the counter terms), which are fixed by the underlying dynamics. Through these renormalization-group equations, we can get the parameters in the higher energy region, for example, near the cutoff scale, in which the effective theory breaks down. If we determine the parameters from the underlying dynamics in such a high energy region, we can directly relate the low energy parameters to the underlying dynamics. For instance, the dynamics of QCD should have determined the value of the parameter a, which is important to the above successful predictions of hidden local symmetry.

In the QCD case, however, there exist many heavy particles in the higher energy region. So we cannot directly relate the above hidden local symmetry to QCD. In such region, we consider the theory including the heavy particles, for example, the a_1 mesons. (In the generalized hidden local symmetry Lagrangian, we can also include the a_1 mesons as well as ρ mesons[14, 24].) In the low energy effective theory, the contributions from the heavy particles are included in the counter terms. Actually, Gasser and Luetwyler presented[6] that the loop effects of K and η mesons are included in the counter terms in the $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ chiral Lagrangian. (As is mentioned above, the ρ meson contribution almost saturate these counter terms, so that contributions of K and η are negligibly small [9].)

In this paper we take the hidden local symmetry Lagrangian as the effective Lagrangian, and consider the loop effects of its gauge boson (ρ meson). We investigate whether or not the above successful tree predictions of hidden local symmetry survive the loop corrections. In particular, it is important to study the loop effects to the above "low energy theorem", $g_\rho = 2g_{\rho\pi\pi}f_\pi^2$, so that we calculate the loop corrections in the low energy limit as a first step of such an investigation. The "low energy theorem" is a consequence of the hidden local symmetry, hence it is important to use *the full gauge degrees of freedom* (would-be Nambu-Goldstone boson, Faddeev-Popov (FP) ghost as well as ρ meson itself). In the chiral perturbation theory, the pion loop yields no correction in the low energy limit, as proved with the naive power counting in Ref.[4]. In the hidden local symmetry, however,

ρ-meson loops give the corrections even in the low energy limit. Thus, the loop corrections may change the tree level relations in the low energy limit.

We show that such corrections can be absorbed into the suitable counter terms, and hence the tree level relations mentioned above hold at one loop level in the low energy limit. This strongly suggests that the KSRF relation (I), $g_\rho = 2f_\pi^2 g_{\rho\pi\pi}$, may become a true low energy theorem of hidden local symmetry. (Recently, this relation has been proved to all orders based on the BRS symmetry[25].) As far as ρ meson dominance is concerned, we calculate the momentum dependence of the $\gamma\pi\pi$ vertex function, and show that no direct $\gamma\pi\pi$ interaction is induced for the finite photon momentum not restricted to the low energy limit, if and only if we take the parameter choice $a = 2$. This implies that the ρ meson dominance of the electromagnetic form factor of the pion remains valid at one loop order. Further, we get the renormalization group equations for the parameters a and g, and show that in the "idealized" high energy limit this hidden local symmetry model becomes asymptotically the theory which has a higher symmetry ("vector limit"[26]).

In sect. 2, we briefly review the non-linear realization of the chiral symmetry. In sect. 3, we review the hidden local symmetry and summarize its tree-level predictions. The saturation of the low energy parameters are discussed in sect. 4. We study the tree-level ρ-meson contributions to the low energy parameters in the hidden local symmetry. Sect. 5 is a main part of this paper. There, we calculate the one-loop effects of the hidden gauge boson (ρ meson), and investigate corrections to the successful tree-level predictions. Further, we discuss the renormalization-group equations for the parameters a and g, and the relation between the hidden local symmetry and the "vector limit"[26] in sect. 6.

2 Non-Linear Realization of Chiral Symmetry

In this section we discuss the non-linear realization of the chiral symmetry according to Ref.[2].

First we consider the general case where the symmetry group G is spontaneously broken to its subgroup H[24]. In this case we can divide a set of generators $T^A \in \mathcal{G}$ of G into two parts, the unbroken generators $S^\alpha \in \mathcal{H}$ and the broken generators $X^a \in \mathcal{G} - \mathcal{H}$:

$$\{T^A \in \mathcal{G}\} = \{S^\alpha \in \mathcal{H}, X^a \in \mathcal{G} - \mathcal{H}\}. \tag{2.1}$$

We choose these generators to satisfy

$$\langle T^A T^B \rangle = \frac{1}{2}\delta^{AB}, \quad \langle S^\alpha X^a \rangle = 0. \tag{2.2}$$

We note that the symbol $\langle A \rangle$ denotes the trace of a matrix A hereafter. The second equation of Eq.(2.2) implies that the element $[S^\alpha, X^a]$ always lies in $\mathcal{G} - \mathcal{H}$;

$$[\mathcal{H}, \mathcal{G} - \mathcal{H}] \subset \mathcal{G} - \mathcal{H}. \tag{2.3}$$

The coset space G/H is a symmetric space when the condition

$$[\mathcal{G} - \mathcal{H}, \mathcal{G} - \mathcal{H}] \subset \mathcal{H} \tag{2.4}$$

is satisfied. In this case we can define a parity-like transformation τ such that

$$\tau : \mathcal{G} \to \mathcal{G}, \quad \begin{cases} \tau(Y) = +Y, & \text{for } Y \in \mathcal{H}, \\ \tau(Y) = -Y, & \text{for } Y \in \mathcal{G} - \mathcal{H}. \end{cases} \tag{2.5}$$

Now there exist Nambu-Goldstone (NG) bosons, whose number is equal to the dimension of the coset space G/H, $\dim G - \dim H$, and we can identify NG fields as the coordinate of the coset space G/H. Let $\xi(\pi)$ be the "representative" of the coset space G/H, which is parameterized in terms of the NG bosons $\pi(x)$ as

$$\xi(\pi) = e^{i\pi(x)/f_\pi}, \qquad \pi(x) \equiv \sum_{X^a \in \mathcal{G} - \mathcal{H}} \pi^a(x) X^a, \tag{2.6}$$

where f_π denotes a scale parameter (the "decay constant")†.

†We assume that the coset space G/H is irreducible for simplicity, so that we have only one decay constant.

Under the symmetry group G, this transforms as

$$\xi(\pi) \to \xi'(\pi) = h(\pi, g)\xi(\pi)g^\dagger, \quad g \in G, \tag{2.7}$$

where $h(\pi, g) \in H$ is uniquely determined depending on $\pi(x)$ as well as on g.

A fundamental object is the Maurer-Cartan 1-form;

$$\alpha_\mu(\pi) = \frac{1}{i}\partial_\mu \xi(\pi) \cdot \xi^\dagger(\pi). \tag{2.8}$$

This 1-form is a Lie-algebra-valued quantity, so that we can expand this in terms of the generators $\{T^A \in \mathcal{G}\} = \{S^\alpha \in \mathcal{H}, X^a \in \mathcal{G} - \mathcal{H}\}$. We can divide this 1-form into $\alpha_{\mu\|}(\pi) \in \mathcal{H}$ and $\alpha_{\mu\perp}(\pi) \in \mathcal{G} - \mathcal{H}$:

$$\begin{aligned} \alpha_{\mu\|}(\pi) &= 2\left\langle S^\alpha \alpha_\mu(\pi)\right\rangle \cdot S^\alpha, \\ \alpha_{\mu\perp}(\pi) &= 2\left\langle X^a \alpha_\mu(\pi)\right\rangle \cdot X^a. \end{aligned} \tag{2.9}$$

These transform as

$$\begin{aligned} \alpha_{\mu\|}(\pi) &\to \alpha'_{\mu\|}(\pi) = h(\pi, g)\alpha_{\mu\|}(\pi)h^\dagger(\pi, g) + \frac{1}{i}\partial_\mu h(\pi, g) \cdot h^\dagger(\pi, g), \\ \alpha_{\mu\perp}(\pi) &\to \alpha'_{\mu\perp}(\pi) = h(\pi, g)\alpha_{\mu\perp}(\pi)h^\dagger(\pi, g). \end{aligned} \tag{2.10}$$

Only the perpendicular part $\alpha_{\mu\perp}(\pi)$ transforms homogeneously, so that the most general Lagrangian with the lowest derivatives is given by

$$\mathcal{L} = f_\pi^2 \left\langle \left(\alpha_{\mu\perp}(\pi)\right)^2 \right\rangle. \tag{2.11}$$

Now we consider the case where the chiral symmetry $G \equiv \mathrm{SU(N)}_L \times \mathrm{SU(N)}_R$ is spontaneously broken to its subgroup $H \equiv \mathrm{SU(N)}_V$. In this case we can take the quantity $(\xi^\dagger(\pi), \xi(\pi))$ as a "representative" of the coset space G/H, which is parameterized in terms of the NG bosons $\pi(x)$ as

$$\xi(\pi) = e^{i\pi(x)/f_\pi}, \qquad \pi(x) \equiv \sum_a \pi^a(x)T^a, \tag{2.12}$$

where T^a denotes the generator of SU(N) group and f_π denotes the decay constant[‡]. Under the group G, this transforms as

$$\left(\xi^\dagger(\pi), \xi(\pi)\right) \to \left(\xi'^\dagger(\pi), \xi'(\pi)\right)$$
$$= \left(h(\pi, g_L, g_R)\xi^\dagger(\pi)g_L^\dagger, h(\pi, g_L, g_R)\xi(\pi)g_R^\dagger\right) . \quad (2.13)$$

The parallel and perpendicular components of the Maurer-Cartan 1-form are given by

$$\alpha_{\mu\parallel}(\pi) \equiv \frac{\partial_\mu \xi^\dagger(\pi) \cdot \xi(\pi) + \partial_\mu \xi(\pi) \cdot \xi^\dagger(\pi)}{2i},$$
$$\alpha_{\mu\perp}(\pi) \equiv \frac{\partial_\mu \xi^\dagger(\pi) \cdot \xi(\pi) - \partial_\mu \xi(\pi) \cdot \xi^\dagger(\pi)}{2i}. \quad (2.14)$$

These transform as

$$\alpha_{\mu\parallel}(\pi) \to h(\pi, g_L, g_R)\alpha_{\mu\parallel}(\pi)h^\dagger(\pi, g_L, g_R)$$
$$+ i^{-1}h(\pi, g_L, g_R)\partial_\mu h^\dagger(\pi, g_L, g_R) ,$$
$$\alpha_{\mu\perp}(\pi) \to h(\pi, g_L, g_R)\alpha_{\mu\perp}(\pi)h^\dagger(\pi, g_L, g_R) . \quad (2.15)$$

The Lagrangian with the lowest derivatives is given by

$$\mathcal{L} = f_\pi^2 \left\langle \left(\alpha_{\mu\perp}(\pi)\right)^2 \right\rangle . \quad (2.16)$$

Here we define the quantity, $U \equiv (\xi(\pi))^2$, which transforms linearly under the chiral group G;

$$U \to g_L U g_R^\dagger . \quad (2.17)$$

Using this quantity, the Lagrangian now reads

$$\mathcal{L} = \frac{f_\pi^2}{4} \left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle . \quad (2.18)$$

‡In this case, the coset space G/H is irreducible.

3 Hidden Local Symmetry

In this section we briefly review the "hidden local symmetry". Let us start with the $[SU(2)_L \times SU(2)_R]_{global} \times [SU(2)_V]_{local}$ "linear" model[16]. We introduce two SU(2)-matrix valued variables, $\xi_L(x)$ and $\xi_R(x)$, which transform as

$$\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g^\dagger_{L,R}\,, \tag{3.1}$$

where $h(x) \in [SU(2)_V]_{local}$ and $g_{L,R} \in [SU(2)_{L,R}]_{global}$. These variables are parameterized as

$$\begin{aligned}
\xi_L(x) &\equiv \xi(\sigma)\xi^\dagger(\pi),\\
\xi_R(x) &\equiv \xi(\sigma)\xi(\pi),\\
\xi(\sigma) &= e^{i\sigma(x)/f_\sigma}, \qquad [\sigma(x) \equiv \sigma^a(x)\tau^a/2],\\
\xi(\pi) &= e^{i\pi(x)/f_\pi}, \qquad [\pi(x) \equiv \pi^a(x)\tau^a/2],
\end{aligned} \tag{3.2}$$

where π and σ are the pion and the "compensator" (would-be Nambu-Goldstone field) to be "absorbed" into the hidden gauge boson (the ρ meson), respectively, and f_π and f_σ are the corresponding decay constants in the chiral symmetric limit. The covariant derivatives are defined by

$$\begin{aligned}
D_\mu\xi_L(x) &\equiv \partial_\mu\xi_L(x) - igV_\mu(x)\xi_L(x) + i\xi_L(x)\mathcal{L}_\mu(x),\\
D_\mu\xi_R(x) &\equiv \partial_\mu\xi_R(x) - igV_\mu(x)\xi_R(x) + i\xi_R(x)\mathcal{R}_\mu(x),
\end{aligned} \tag{3.3}$$

where g is the gauge coupling constant of the hidden local symmetry, V_μ $(\equiv V^a_\mu \frac{\tau^a}{2})$ the hidden gauge boson field (the ρ meson), and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ denote the gauge fields when we gauge the $[SU(2)_L \times SU(2)_R]_{global}$ symmetry. In this paper we gauge the $U(1)_{em}$ part only, so that we take

$$\mathcal{L}_\mu = \mathcal{R}_\mu = e\mathcal{B}_\mu \frac{\tau^3}{2}, \tag{3.4}$$

where $\mathcal{B}_\mu$ denotes the photon field and e denotes the electromagnetic coupling constant.

The covariantized Maurer-Cartan 1-forms[24] are given by

$$\hat{\alpha}_{\mu\perp}(x) \equiv \frac{D_\mu \xi_L(x) \cdot \xi_L^\dagger(x) - D_\mu \xi_R(x) \cdot \xi_R^\dagger(x)}{2i}\,, \tag{3.5}$$

$$\hat{\alpha}_{\mu\parallel}(x) \equiv \frac{D_\mu \xi_L(x) \cdot \xi_L^\dagger(x) + D_\mu \xi_R(x) \cdot \xi_R^\dagger(x)}{2i}\,, \tag{3.6}$$

which transform as

$$\hat{\alpha}_{\mu\perp,\parallel}(x) \to \hat{\alpha}'_{\mu\perp,\parallel}(x) = h(x)\hat{\alpha}_{\mu\perp,\parallel}(x)h^\dagger(x)\,. \tag{3.7}$$

We can construct the following two invariants with the lowest derivative[§];

$$\begin{aligned} \mathcal{L}_A &= f_\pi^2 \left\langle \left(\hat{\alpha}_{\mu\perp}\right)^2 \right\rangle, \\ \mathcal{L}_V &= f_\pi^2 \left\langle \left(\hat{\alpha}_{\mu\parallel}\right)^2 \right\rangle. \end{aligned} \tag{3.8}$$

Thus we obtain the Lagrangian of the $[SU(2)_L \times SU(2)_R]_{global} \times [SU(2)_V]_{local}$ "linear" model, with the $[SU(2)_L \times SU(2)_R]_{global}$ being partly gauged[16]:

$$\mathcal{L} = \mathcal{L}_A + a\mathcal{L}_V + \mathcal{L}_{kin}(V_\mu), \tag{3.9}$$

where a is a constant, $\mathcal{L}_{kin}(V_\mu)$ denotes the kinetic term of the hidden gauge boson[2]:

$$\begin{aligned} \mathcal{L}_{kin}(V_\mu) &\equiv -\frac{1}{2}\left\langle \left(F_{\mu\nu}^{(V)}\right)^2 \right\rangle, \\ F_{\mu\nu}^{(V)} &= \partial_\mu V_\nu - \partial_\nu V_\mu - ig\left[V_\mu, V_\nu\right]. \end{aligned} \tag{3.10}$$

(The kinetic term of hidden local gauge bosons are possibly induced by the dynamics of QCD. Actually in the extended Nambu-Jona-Lasinio model, which is discussed as an analogue model of QCD, a massive vector bound state appears as a gauge boson of the hidden local symmetry[27].)

Normalizing the kinetic term of σ, we find[14, 15]

$$f_\sigma^2 = a f_\pi^2. \tag{3.11}$$

[§] We impose the parity invariance.

In the "unitary gauge" ($\xi(\sigma) = 1$) this Lagrangian reduces to

$$\mathcal{L} = \langle \partial_\mu \pi \partial^\mu \pi \rangle + \mathcal{L}_{kin}(V_\mu) + m_\rho^2 \langle V_\mu^2 \rangle - 2eg_\rho \langle V_\mu \mathcal{B}^\mu \rangle + 2ig_{\rho\pi\pi} \langle V_\mu [\partial^\mu \pi, \pi] \rangle + 2ig_{\gamma\pi\pi} \langle \mathcal{B}_\mu [\partial^\mu \pi, \pi] \rangle + \cdots, \tag{3.12}$$

where the parameters in Eq.(3.12), the ρ meson mass m_ρ, the ρ-γ transition strength g_ρ, the $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ and the direct $\gamma\pi\pi$ coupling constant $g_{\gamma\pi\pi}$ are given by[16, 19]:

$$m_\rho^2 = ag^2 f_\pi^2, \tag{3.13}$$

$$g_\rho = agf_\pi^2, \tag{3.14}$$

$$g_{\rho\pi\pi} = \frac{1}{2}ag, \tag{3.15}$$

$$g_{\gamma\pi\pi} = \left(1 - \frac{a}{2}\right) e. \tag{3.16}$$

Eqs.(3.14) and (3.15) lead to[19]

$$g_\rho = 2f_\pi^2 g_{\rho\pi\pi}, \tag{3.17}$$

which is nothing but the KSRF relation[18] (version I). Eq.(3.17) is actually independent of the parameter a and hence is the decisive test of the hidden local symmetry[19]. Thus it was conjectured to be a "low energy theorem" of the hidden local symmetry[19] and was then proved at tree level[24]. Moreover, for a parameter choice $a = 2$, the above results reproduce the outstanding phenomenological facts[16]:

(1) $g_{\rho\pi\pi} = g$ (universality of the ρ-couplings)[17]¶,

(2) $m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$ (KSRF II) [18],

(3) $g_{\gamma\pi\pi} = 0$ (ρ meson dominance of the electromagnetic form factor of the pion) [17].

¶ We assume that the ρ-coupling to the nucleon is minimal. (see, for example, Ref.[2].)

4 The ρ-Meson Contribution to the Low Energy Parameters

In this section we discuss the ρ-meson contribution to the low energy effective chiral Lagrangian[8, 9, 10]. Here we think about the tree-level effect of the ρ meson in the hidden local symmetry model. The following analyses are done in the unitary gauge ($\xi(\sigma) = 1$).

From the Lagrangian, Eq.(3.9), the equation of motion of ρ meson is given by

$$V_\mu - \alpha_{\mu\|}(\pi) - \frac{1}{m_\rho^2}\left(\partial^\nu F_{\mu\nu}^{(V)} - ig\left[V^\nu, F_{\mu\nu}^{(V)}\right]\right) = 0, \tag{4.1}$$

where $\alpha_{\mu\|}(\pi)$ is the parallel component of the Maurer-Cartan 1-form given in Eq.(2.14).

	$l_i(m_\rho)$	l_i^V
l_1	-6.0 ± 3.9	-7.3
l_2	5.5 ± 2.8	7.3
l_6	-13.8 ± 1.2	-14.6

Table I: The ρ-meson contribution to the low energy parameters in units of 10^{-3}.

Solving Eq.(4.1) by iteration and substituting the solution into the Lagrangian Eq.(3.9), we get the following $\mathcal{O}(p^4)$ terms;

$$\begin{aligned}
\mathcal{L}_4^V \;=\; & -\frac{1}{16g^2}\left\langle \mathcal{D}_\mu U \mathcal{D}^\mu U^\dagger \right\rangle^2 + \frac{1}{16g^2}\left\langle \mathcal{D}_\mu U \mathcal{D}_\nu U^\dagger \right\rangle \left\langle \mathcal{D}^\mu U \mathcal{D}^\nu U^\dagger \right\rangle \\
& -i\frac{1}{4g^2}\left\langle F_{\mu\nu}^{\mathcal{R}} \mathcal{D}^\mu U^\dagger \mathcal{D}^\nu U + F_{\mu\nu}^{\mathcal{L}} \mathcal{D}^\mu U \mathcal{D}^\nu U^\dagger \right\rangle \\
& -\frac{1}{4g^2}\left\langle F_{\mu\nu}^{\mathcal{R}} U^\dagger F^{\mathcal{L}\mu\nu} U \right\rangle - \frac{1}{8g^2}\left\langle F_{\mu\nu}^{\mathcal{R}} F^{\mathcal{R}\mu\nu} + F_{\mu\nu}^{\mathcal{L}} F^{\mathcal{L}\mu\nu} \right\rangle,
\end{aligned} \tag{4.2}$$

where the covariant derivative $\mathcal{D}_\mu$ is defined by

$$\mathcal{D}_\mu U = \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu, \tag{4.3}$$

and $F^{\mathcal{R}}_{\mu\nu}$ and $F^{\mathcal{L}}_{\mu\nu}$ denote the field strength of the chiral gauge field if we gauge the $[\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R]_{global}$ symmetry. As shown in Refs.[5, 6], the general $\mathcal{O}(p^4)$ Lagrangian, which includes the pion and the external gauge fields, is given by

$$\begin{aligned}\mathcal{L}_4 &= \frac{l_1}{4}\left\langle \mathcal{D}_\mu U \mathcal{D}^\mu U^\dagger \right\rangle^2 + \frac{l_2}{4}\left\langle \mathcal{D}_\mu U \mathcal{D}_\nu U^\dagger \right\rangle \left\langle \mathcal{D}^\mu U \mathcal{D}^\nu U^\dagger \right\rangle \\ &\quad + i\frac{l_6}{2}\left\langle F^{\mathcal{R}}_{\mu\nu}\mathcal{D}^\mu U^\dagger \mathcal{D}^\nu U + F^{\mathcal{L}}_{\mu\nu}\mathcal{D}^\mu U \mathcal{D}^\nu U^\dagger \right\rangle \\ &\quad + l_5\left\langle F^{\mathcal{R}}_{\mu\nu} U^\dagger F^{\mathcal{L}\mu\nu} U \right\rangle + h\left\langle F^{\mathcal{R}}_{\mu\nu}F^{\mathcal{R}\mu\nu} + F^{\mathcal{L}}_{\mu\nu}F^{\mathcal{L}\mu\nu}\right\rangle, \end{aligned} \tag{4.4}$$

where l_1, l_2, l_5 and l_6 are low energy parameters and h is a high energy constant which is irrelevant to the low energy experiments. Comparing Eq.(4.2) with Eq.(4.4), we can easily read off the ρ-meson contributions to the low energy parameters[5];

$$\begin{aligned} l_1^V &= -\frac{af_\pi^2}{4m_\rho^2}, & l_2^V &= \frac{af_\pi^2}{4m_\rho^2}, \\ l_5^V &= -\frac{af_\pi^2}{4m_\rho^2}, & l_6^V &= -\frac{af_\pi^2}{2m_\rho^2}. \end{aligned} \tag{4.5}$$

The a_1-meson contribution to l_5 parameter is important. The ρ-meson contribution is not enough to saturate this parameter.

In Table I, we compare the ρ-meson contributions with the low energy parameters determined by the experiments[5, 9, 10], where we take the parameter choice $a = 2$. These imply that the ρ-meson contribution saturate the low energy parameters, l_1, l_2 and l_6.

5 The One-Loop Corrections

In this section we consider the one-loop effects of the gauge boson of the hidden local symmetry.

First we introduce the R_ξ-gauge-like gauge-fixing terms and FP ghost Lagrangian corresponding to the hidden gauge boson. These are given by (see Appendix A)

$$\mathcal{L}_{GF} = -\frac{1}{\alpha}\left\langle (\partial_\mu V_\mu)^2 \right\rangle + \frac{i}{2}agf_\pi^2\left\langle \partial_\mu V_\mu(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger)\right\rangle$$

$$+\frac{1}{16}\alpha a^2 g^2 f_\pi^4\left[\left\langle(\xi_L-\xi_L^\dagger+\xi_R-\xi_R^\dagger)^2\right\rangle-\frac{1}{2}\left\langle\xi_L-\xi_L^\dagger+\xi_R-\xi_R^\dagger\right\rangle^2\right], \quad (5.1)$$

$$\mathcal{L}_{FP}=i\left\langle\bar{v}\left[2\partial^\mu D_\mu v+\frac{1}{2}\alpha a g^2 f_\pi^2(v\xi_L+\xi_L^\dagger v+v\xi_R+\xi_R^\dagger v)\right]\right\rangle, \quad (5.2)$$

where v denotes the ghost field. In the following calculations we choose the Landau gauge, $\alpha = 0$. In this gauge the would-be NG bosons σ are still massless, no other vector-scalar interactions are created and the ghost field couples only to the gauge fields. Since we are interested in the strong interaction effect, we consider the photon field as the external field and do not consider its loop effect.

For canceling the lowest derivative divergent part we redefine the normalization of the parameters and the fields such that

$$\begin{aligned}
&a = Z_a a_r, \qquad g = Z_g g_r, \qquad e\mathcal{B}_\mu = Z_e e_r \mathcal{B}_{r\mu};\\
&V_\mu = Z_V^{1/2} V_{r\mu}, \qquad \pi = Z_\pi^{1/2}\pi_r, \qquad \sigma = Z_\sigma^{1/2}\sigma_r;\\
&f_\pi = Z_\pi^{1/2} f_{\pi r}, \qquad f_\sigma = Z_\sigma^{1/2} f_{\sigma r}.
\end{aligned} \quad (5.3)$$

We note that from Eq.(3.11), the wave function renormalization constants, Z_π and Z_σ, are related by

$$Z_\sigma = Z_a Z_\pi. \quad (5.4)$$

Further, Z_e corresponds to Z_1/Z_2 in usual QED case, so that from the Ward-Takahashi identity, we can conclude

$$Z_e = 1, \quad (5.5)$$

which implies the electromagnetic charge universality. So we take $Z_e = 1$ hereafter.

We define the ρ meson mass parameter m_ρ by

$$m_\rho^2 \equiv a_r g_r^2 f_{\pi r}^2. \quad (5.6)$$

Hereafter, we denote the pion momentum by k_μ and q_μ, and the ρ meson momentum by p_μ. We also denote the photon momentum by p_μ. In the following calculations we set the pion momentum on the mass-shell, $k^2 = q^2 = 0$.

5.1 The Determination of the Z Factors

There are four independent Z factors in Eq.(5.3), Z_g, Z_V, Z_π and Z_σ. In the low energy calculations, Z_g always appears together with Z_V in the form $Z_V Z_g^2$ and hence we have three independent Z factors in the low energy limit; we use the ρ-propagator, the π-propagator and the σ-propagator to determine these Z factors. Then we determine the counter terms for the $\rho\pi\pi$ vertex, the ρ-γ mixing and the $\gamma\pi\pi$ vertex. Explicit calculations are done in the dimensional regularization scheme. The Feynman rules are shown in Appendix B.

First we calculate the correction to the ρ-meson propagator. The one-loop contributions to the ρ-meson propagator are shown in Fig. 1. These are given by

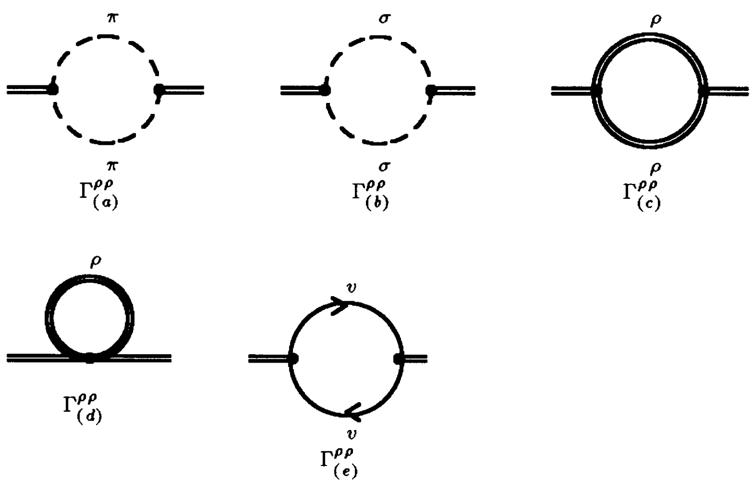


Figure 1: One-particle irreducible graphs contributing to the ρ propagator

$$\Gamma^{\rho}_{(a)} = \left(p_\mu p_\nu - p^2 g_{\mu\nu}\right) \frac{a_r^2}{12} \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{8}{3}\right],$$

$$\Gamma^{\rho}_{(b)} = \left(p_\mu p_\nu - p^2 g_{\mu\nu}\right) \frac{1}{12} \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{8}{3}\right],$$

$$\Gamma^{\rho}_{(c)} = \frac{g_r^2}{(4\pi)^2} \left[g_{\mu\nu} \left\{ 6 m_\rho^2 \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{1}{3} \right) \right.\right.$$

$$
\begin{aligned}
&+ \frac{25}{6} p^2 \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 - \frac{1}{6} \right) + p^2 F_{(c1)}(p^2) \Big\} \\
&- p_\mu p_\nu \left\{ \frac{14}{3} \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 - \frac{11}{84} \right) + F_{(c2)}(p^2) \right\} \Bigg] ,
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^\rho_{(d)} &= -\frac{9}{2} g_{\mu\nu} m_\rho^2 \frac{g_r^2}{(4\pi)^2} \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{1}{6} \right) , \\
\Gamma^\rho_{(e)} &= \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{6} g_{\mu\nu} p^2 \left(\frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{8}{3} \right) \right. \\
&\left. + \frac{1}{3} p_\mu p_\nu \left(\frac{1}{\bar{\epsilon}} - \ln(-p^2) + \frac{1}{2} \right) \right] , \qquad (5.7)
\end{aligned}
$$

where

$$
\frac{1}{\bar{\epsilon}} \equiv \frac{2}{4-n} - \gamma + \ln(4\pi),
$$

$$
[\gamma : \text{Euler constant}, \quad n : \text{the dimension of the integral}] \qquad (5.8)
$$

and $F_{(c1)}(p^2)$ and $F_{(c2)}(p^2)$ denote the complicated functions. In the low energy limit, $p^2 = 0$, these reduce to

$$
\Gamma^\rho_{(a+b+c+d+e)} \underset{p^2 \to 0}{\Rightarrow} \frac{3}{2} \frac{g_r^2}{(4\pi)^2} \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{5}{6} \right) m_\rho^2 g_{\mu\nu} . \qquad (5.9)
$$

From Eq.(5.9) we can determine at $p^2 = 0$ the counter term

$$
(Z_V Z_g^2 Z_a Z_\pi - 1) m_\rho^2 g_{\mu\nu} = -\frac{3}{2} \frac{g_r^2}{(4\pi)^2} \left(\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{5}{6} \right) m_\rho^2 g_{\mu\nu} \qquad (5.10)
$$

in such a way as to obtain the ρ meson mass parameter M_ρ in the low energy limit;

$$
M_\rho^2(p^2 = 0) = m_\rho^2 = a_r g_r^2 f_{\pi r}^2 . \qquad (5.11)
$$

Next, Z_π and Z_σ are determined by renormalizing the wave functions of the π and σ fields at the on-shell point $q^2 = 0$ (remember that σ is massless in the Landau gauge). The one-loop graphs contributing to these propagators are shown in Figs. 2 and 3. The one-loop contributions to the π-propagator are given by

$$
\Gamma^{\pi\pi}_{(a)} \underset{q^2 \to 0}{\Rightarrow} -q^2 \frac{3 a_r^2}{2} \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{5}{6} \right] ,
$$

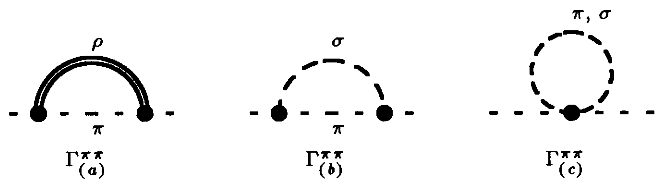


Figure 2: One-loop contributions to the π propagator

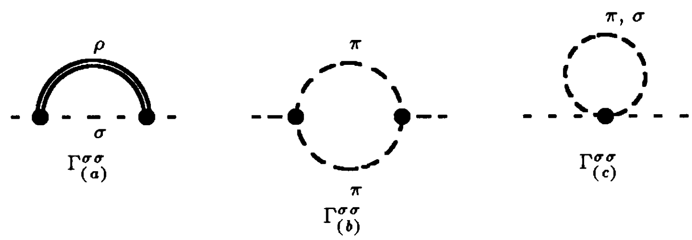


Figure 3: One-loop contributions to the σ propagator

$$
\begin{aligned}
\Gamma^{\pi\pi}_{(b)} &= q^2 \frac{a_r}{2} \frac{q^2}{(4\pi f_{\pi r})^2} \left[\frac{1}{\bar{\epsilon}} - \ln(-q^2) + 2 \right] , \\
\Gamma^{\pi\pi}_{(c)} &= 0.
\end{aligned} \tag{5.12}
$$

Similarly the one-loop contributions to the σ-propagator are given by

$$
\begin{aligned}
\Gamma^{\sigma\sigma}_{(a)} &\underset{q^2\to 0}{\Rightarrow} -q^2 \frac{3}{2} \frac{g^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{5}{6} \right] , \\
\Gamma^{\sigma\sigma}_{(b)} &= q^2 \frac{1}{12} \frac{q^2}{(4\pi f_{\pi r})^2} \left[\frac{1}{\bar{\epsilon}} - \ln(-q^2) + \frac{8}{3} \right] , \\
\Gamma^{\sigma\sigma}_{(c)} &= 0.
\end{aligned} \tag{5.13}
$$

These contributions determine the π and σ wave function renormalization constants:

$$
Z_\pi - 1 \;=\; - \left. \frac{d\Gamma^{\pi\pi}_{(a+b+c)}(q^2)}{dq^2} \right|_{q^2=0} = \frac{3a_r^2}{2} \frac{g_r^2}{(4\pi)^2} \left[\frac{1}{\bar{\epsilon}} - \ln m_\rho^2 + \frac{5}{6} \right] , \tag{5.14}
$$

$$Z_\sigma - 1 \;=\; - \left. \frac{d\Gamma^{\sigma\sigma}_{(a+b+c)}(q^2)}{dq^2} \right|_{q^2=0} = \frac{3}{2}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right]. \tag{5.15}$$

From Eqs.(5.10), (5.14) and (5.15), we can determine the Z factors:

$$Z_a - 1 \;=\; -\frac{3}{2}(a_r^2 - 1)\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right], \tag{5.16}$$

$$Z_\pi - 1 \;=\; \frac{3a_r^2}{2}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right], \tag{5.17}$$

$$Z_V Z_g^2 - 1 \;=\; -3\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right]. \tag{5.18}$$

As is seen from Eq.(5.16), for the parameter choice $a_r = 1$ the parameter a is not renormalized. This implies that in the "vector limit"[26] the loop effect of the ρ meson does not induce the deviation from $a = 1$ in contrast to the expectation of Ref.[26].

These Z factors determine the counter terms for the $\rho\pi\pi$ vertex;

$$i\frac{a_r}{2}g_r(Z_V^{1/2}Z_gZ_aZ_\pi - 1) = 0, \tag{5.19}$$

for the ρ-γ mixing;

$$-\, e_r a_r g_r f_{\pi r}^2 g_{\mu\nu}(Z_e Z_V^{1/2} Z_g Z_a Z_\pi - 1) = 0, \tag{5.20}$$

and for the $\gamma\pi\pi$ vertex;

$$\begin{aligned}
&-ie_r\epsilon_{3bc}(k-q)_\mu\left[Z_eZ_\pi\left(1 - \frac{a_r}{2}Z_a\right) - \left(1 - \frac{a_r}{2}\right)\right] \\
&= -ie_r\epsilon_{3bc}(k-q)_\mu \frac{3a_r(2a_r - 1)}{4}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right].
\end{aligned} \tag{5.21}$$

5.2 The $\rho\pi\pi$ Vertex, the ρ-γ Mixing and the $\gamma\pi\pi$ Vertex

Now that we have determined the counter terms, we can obtain the one-loop corrections by calculating the one-loop graphs contributing to the $\rho\pi\pi$ vertex, the ρ-γ mixing and the $\gamma\pi\pi$ vertex.

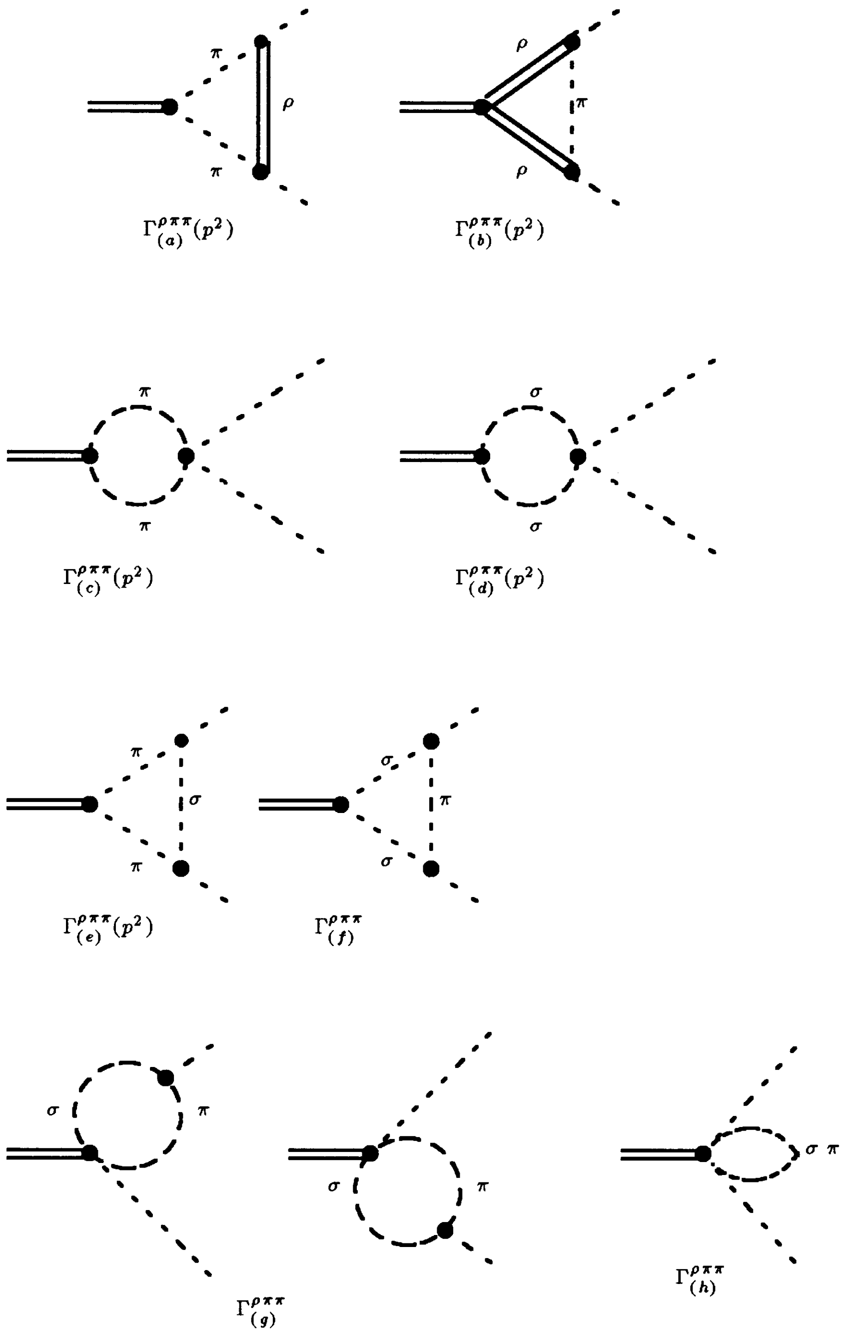


Figure 4: One-particle irreducible graphs contributing to the $\rho\pi\pi$ vertex

First we calculate the correction to the $\rho\pi\pi$ vertex. The one-loop graphs contributing this vertex are shown in Fig. 4. These contributions are given by

$$\begin{aligned}
\Gamma^{\rho\pi\pi}_{(a)} &= ig_r\epsilon_{abc}(k-q)_\mu \frac{a_r^3}{8}\frac{g_r^2}{(4\pi)^2}F_{(a)}(p^2),\\
\Gamma^{\rho\pi\pi}_{(b)} &= ig_r\epsilon_{abc}(k-q)_\mu \frac{a_r^2}{4}\frac{g_r^2}{(4\pi)^2}F_{(b)}(p^2),\\
\Gamma^{\rho\pi\pi}_{(c)} &= ig_r\epsilon_{abc}(k-q)_\mu \frac{a_r(3a_r-4)}{48}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon}-\ln(-p^2)+\frac{8}{3}\right],\\
\Gamma^{\rho\pi\pi}_{(d)} &= -ig_r\epsilon_{abc}(k-q)_\mu \frac{1}{24}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon}-\ln(-p^2)+\frac{8}{3}\right],\\
\Gamma^{\rho\pi\pi}_{(e)} &= 0,\\
\Gamma^{\rho\pi\pi}_{(f)} &= ig_r\epsilon_{abc}(k-q)_\mu \frac{a_r}{48}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon}-\ln(-p^2)+\frac{8}{3}\right],\\
\Gamma^{\rho\pi\pi}_{(g)} &= \Gamma^{\rho\pi\pi}_{(h)} = 0,
\end{aligned} \tag{5.22}$$

where $F_{(a)}(p^2)$ and $F_{(b)}(p^2)$ denote certain complicated functions which have no divergent part and $F_{(a)}(p^2=0)=F_{(b)}(p^2=0)=0$.

From Eq.(5.22) we can easily see that at $p^2=0$ there exist no contributions. From this and Eq.(5.19), we find that the $\rho\pi\pi$ coupling remains the same as the tree level in the low energy limit;

$$g_{\rho\pi\pi}(p^2=0;k^2=0,q^2=0)=\frac{a_r}{2}g_r. \tag{5.23}$$

Eq.(5.23) implies that for $a_r=2$, the universality of the ρ-couplings remains intact in the low energy limit.

Similarly, one-loop graphs contributing to the ρ-γ mixing are shown in Fig. 5. These are given by

$$\begin{aligned}
\Gamma^{\gamma\rho}_{(a)} &= \frac{1}{12}(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{e_r g_r}{(4\pi)^2}\left[\frac{1}{\bar\epsilon}-\ln(-p^2)+\frac{8}{3}\right],\\
\Gamma^{\gamma\rho}_{(b)} &= -\frac{a_r(a_r-2)}{12}(p_\mu p_\nu - p^2 g_{\mu\nu})\frac{e_r g_r}{(4\pi)^2}\left[\frac{1}{\bar\epsilon}-\ln(-p^2)+\frac{8}{3}\right],\\
\Gamma^{\gamma\rho}_{(c)} &= 0,
\end{aligned} \tag{5.24}$$

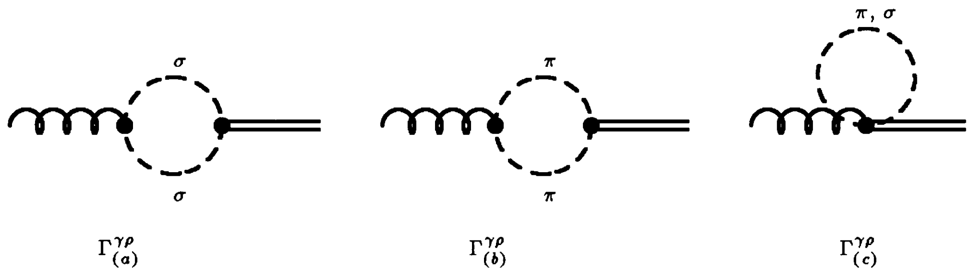


Figure 5: One-particle irreducible graphs contributing to the ρ-γ mixing

which have no contributions at $p^2 = 0$. From this and Eq.(5.20), we find that the ρ-γ mixing also remains the same as its tree level in the low energy limit;

$$g_\rho(p^2 = 0) = a_r g_r f_{\pi r}^2 . \tag{5.25}$$

Next we calculate the one-loop contributions to the $\gamma\pi\pi$ vertex. These graphs are shown in Figs. 6 and 7.

The contributions from the graphs in Fig. 6 are given by

$$\Gamma^{\gamma\pi\pi}_{(a)} = ie_r\epsilon_{3bc}(k-q)_\mu \frac{a_r}{48}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon} - \ln(-p^2) + \frac{8}{3}\right], \tag{5.26.a}$$

$$\Gamma^{\gamma\pi\pi}_{(b)} = -ie_r\epsilon_{3bc}(k-q)_\mu \frac{1}{24}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon} - \ln(-p^2) + \frac{8}{3}\right], \tag{5.26.b}$$

$$\Gamma^{\gamma\pi\pi}_{(c)} = 0, \tag{5.26.c}$$

$$\Gamma^{\gamma\pi\pi}_{(d)} = ie_r\epsilon_{3bc}(k-q)_\mu \frac{9a_r^2}{8}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right], \tag{5.26.d}$$

$$\Gamma^{\gamma\pi\pi}_{(e)} = ie_r\epsilon_{3bc}(k-q)_\mu \frac{3a_r^2}{8}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6} + F_{(e)}(p^2)\right], \tag{5.26.e}$$

$$\Gamma^{\gamma\pi\pi}_{(f)} = -ie_r\epsilon_{3bc}(k-q)_\mu \frac{3a_r}{4}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6} + F_{(e)}(p^2)\right], \tag{5.26.f}$$

$$\Gamma^{\gamma\pi\pi}_{(g)} = 0, \tag{5.26.g}$$

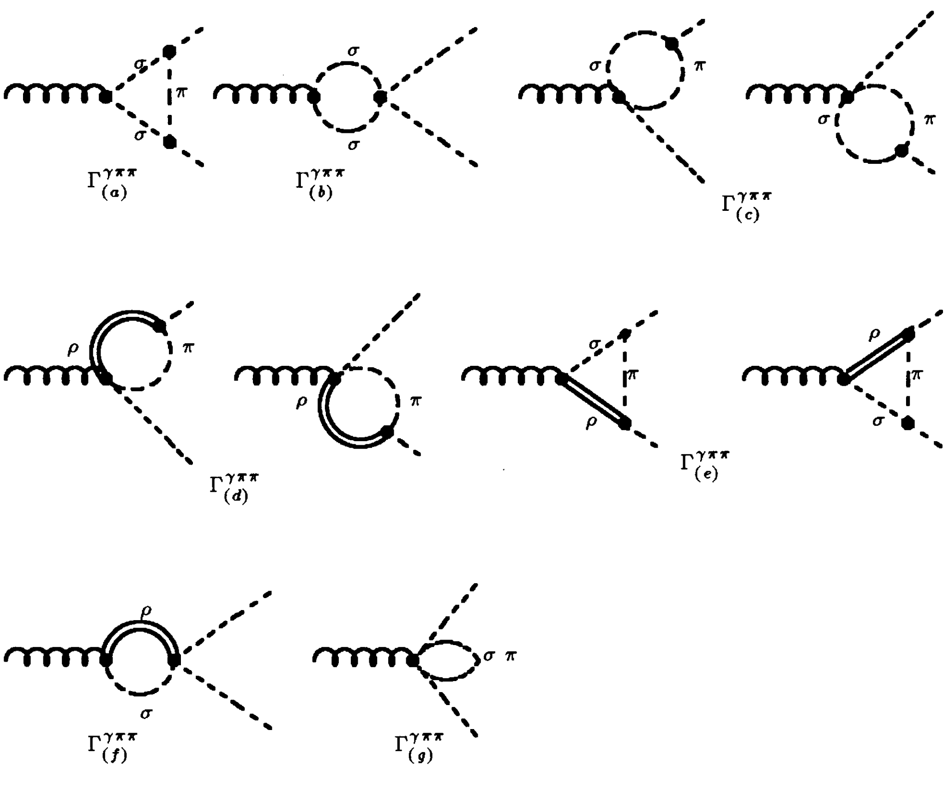


Figure 6: One-particle irreducible graphs contributing to the $\gamma\pi\pi$ vertex which include no tree level direct $\gamma\pi\pi$ vertex

where the function $F_{(e)}(p^2)$ is defined by

$$F_{(e)}(p^2) \equiv -\frac{4}{3}\int_0^1 dx \ln\left(1 - x\frac{p^2}{m_\rho^2}\right) + \frac{2}{3}\int_0^1 y\,dy \int_0^1 dx \ln\left(1 - \frac{xy(1-xy)p^2}{(1-y)m_\rho^2}\right). \tag{5.27}$$

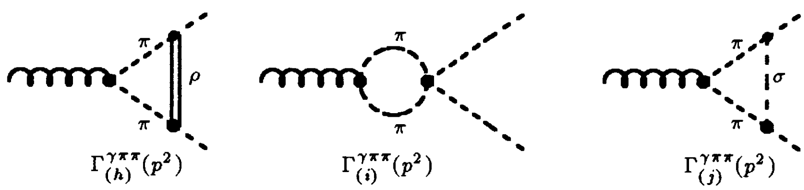


Figure 7: One-particle irreducible graphs contributing to the $\gamma\pi\pi$ vertex which include the tree level direct $\gamma\pi\pi$ vertex

Next we calculate the one-loop graphs through the tree-level direct $\gamma\pi\pi$ vertex (Fig. 7). These contributions are given by

$$\begin{aligned}
\Gamma^{\gamma\pi\pi}_{(h)} &= -ie_r\epsilon_{3bc}(k-q)_\mu \frac{a_r^2(2-a_r)}{8}\frac{g_r^2}{(4\pi)^2}F_{(h)}(p^2),\\
\Gamma^{\gamma\pi\pi}_{(i)} &= ie_r\epsilon_{3bc}(k-q)_\mu \frac{(2-a_r)(3a_r-4)}{48}\frac{p^2}{(4\pi f_{\pi r})^2}\left[\frac{1}{\bar\epsilon} - \ln(-p^2) + \frac{8}{3}\right],\\
\Gamma^{\gamma\pi\pi}_{(j)} &= 0,
\end{aligned} \tag{5.28}$$

where $F_{(h)}(p^2)$ denotes the complicated function defined by

$$\begin{aligned}
F_{(h)}(p^2) = \int_0^1 y\,dy \int_0^1 dx \Bigg[& 2(3xy-2)\ln\frac{(1-y)m_\rho^2 - x(1-x)y^2p^2}{(1-y)m_\rho^2}\\
& + (2xy-1)\frac{(y-2-x(1-x)y^2)p^2}{(1-y)m_\rho^2 - x(1-x)y^2p^2}\Bigg].
\end{aligned} \tag{5.29}$$

In the zero photon momentum limit, $p^2 = 0$, the contribution of these graphs reduce to

$$ie_r\epsilon_{3bc}(k-q)_\mu \frac{3a_r(2a_r-1)}{4}\frac{g_r^2}{(4\pi)^2}\left[\frac{1}{\bar\epsilon} - \ln m_\rho^2 + \frac{5}{6}\right]. \tag{5.30}$$

This is precisely canceled by the counter term, Eq.(5.21). So we get the $\gamma\pi\pi$ vertex function given by

$$
\begin{aligned}
&\Gamma^{\gamma\pi\pi}(p^2; k^2 = q^2 = 0) \\
&= -ie_r\epsilon_{3bc}(k-q)_\mu\left(1-\frac{a_r}{2}\right)\left[\frac{a_r-1}{8}\frac{p^2}{(4\pi f_{\pi r})^2}\left\{\frac{1}{\bar{\epsilon}}-\ln(-p^2)+\frac{8}{3}\right\}\right. \\
&\qquad \left. + \frac{3a_r}{4}\frac{g_r^2}{(4\pi)^2}F_{(e)}(p^2)+\frac{a_r^2}{4}\frac{g_r^2}{(4\pi)^2}F_{(h)}(p^2)\right]. \qquad (5.31)
\end{aligned}
$$

This implies that for $a_r = 2$ there exists no direct $\gamma\pi\pi$ interaction.

5.3 The Results

From Eqs.(5.23) and (5.25), we obtain the desired "low energy theorem" (KSRF I);

$$g_\rho(p_\rho^2 = 0) = 2f_{\pi r}^2 g_{\rho\pi\pi}(p_\rho^2 = 0; p_\pi^2 = 0, p_\pi^2 = 0) \qquad (5.32)$$

at one-loop level.

Eq.(5.23) implies that for $a_r = 2$ the universality of the ρ couplings remains intact in the low energy limit;

$$g_{\rho\pi\pi}(p_\rho^2 = 0; p_\pi^2 = 0, p_\pi^2 = 0) = g_r\,. \qquad (5.33)$$

Combined with Eq.(5.23), Eq.(5.11) yields the KSRF relation (II) for $a_r = 2$ in the low energy limit;

$$M_\rho^2(p_\rho^2 = 0) = 2g_{\rho\pi\pi}^2(p_\rho^2 = 0; p_\pi^2 = 0, p_\pi^2 = 0)f_{\pi r}^2. \qquad (5.34)$$

Moreover, Eq.(5.31) implies that no direct $\gamma\pi\pi$ interaction is induced for the finite photon momentum not restricted to the zero momentum limit, if and only if we take the parameter choice, $a_r = 2$;

$$\Gamma^{\gamma\pi\pi}(p_\gamma^2\,; p_\pi^2 = 0, p_\pi^2 = 0) = 0. \qquad (5.35)$$

6 The Renormalization-Group Equations

In this section we make some comments on the renormalization-group equations for the parameters, a_r and g_r, in the modified minimal subtraction scheme.

From Eq.(5.7), we can get the ρ-meson wave function renormalization constant in this scheme;

$$Z_V - 1 = \frac{51 - a_r^2}{12} \frac{g_r^2}{(4\pi)^2} \frac{1}{\bar{\epsilon}} . \tag{6.1}$$

Further, the Z factors lead to

$$Z_a - 1 = -\frac{3}{2}(a_r^2 - 1)\frac{g_r^2}{(4\pi)^2}\frac{1}{\bar{\epsilon}} , \tag{6.2}$$

$$Z_V Z_g^2 - 1 = -3\frac{g_r^2}{(4\pi)^2}\frac{1}{\bar{\epsilon}} . \tag{6.3}$$

Comparing Eq.(6.1) with Eq.(6.3), the Z factor for the hidden gauge coupling is given by

$$Z_g - 1 = -\frac{87 - a_r^2}{24}\frac{g_r^2}{(4\pi)^2}\frac{1}{\bar{\epsilon}} . \tag{6.4}$$

From Eqs.(6.2) and (6.4), we can get the β functions for a_r and g_r (see Fig. 8);

$$\beta_a(a_r, g_r) \equiv \mu\frac{\partial a_r}{\partial \mu} = -3a_r(a_r^2 - 1)\frac{g_r^2}{(4\pi)^2}, \tag{6.5}$$

$$\beta_g(a_r, g_r) \equiv \mu\frac{\partial g_r}{\partial \mu} = -\frac{87 - a_r^2}{12}\frac{g_r^3}{(4\pi)^2}. \tag{6.6}$$

The β function for a_r, Eq.(6.5), has an ultraviolet fixed point at $a_r = 1$, which corresponds to the fact that the parameter a is not renormalized if we set $a = 1$ from the beginning. Eq.(6.6) implies that the hidden gauge coupling constant g_r is asymptotically free for not so large value of a_r ($a_r < \sqrt{87}$). These imply that for a reasonable value for a_r in the low energy (for example $a_r = 2$), the parameter a_r and the coupling constant g_r go asymptotically to the value of "vector limit"[26] ($a_r = 1$ and $g_r = 0$), i.e., the "vector limit" is realized as the "idealized" high energy limit of the hidden local symmetry.

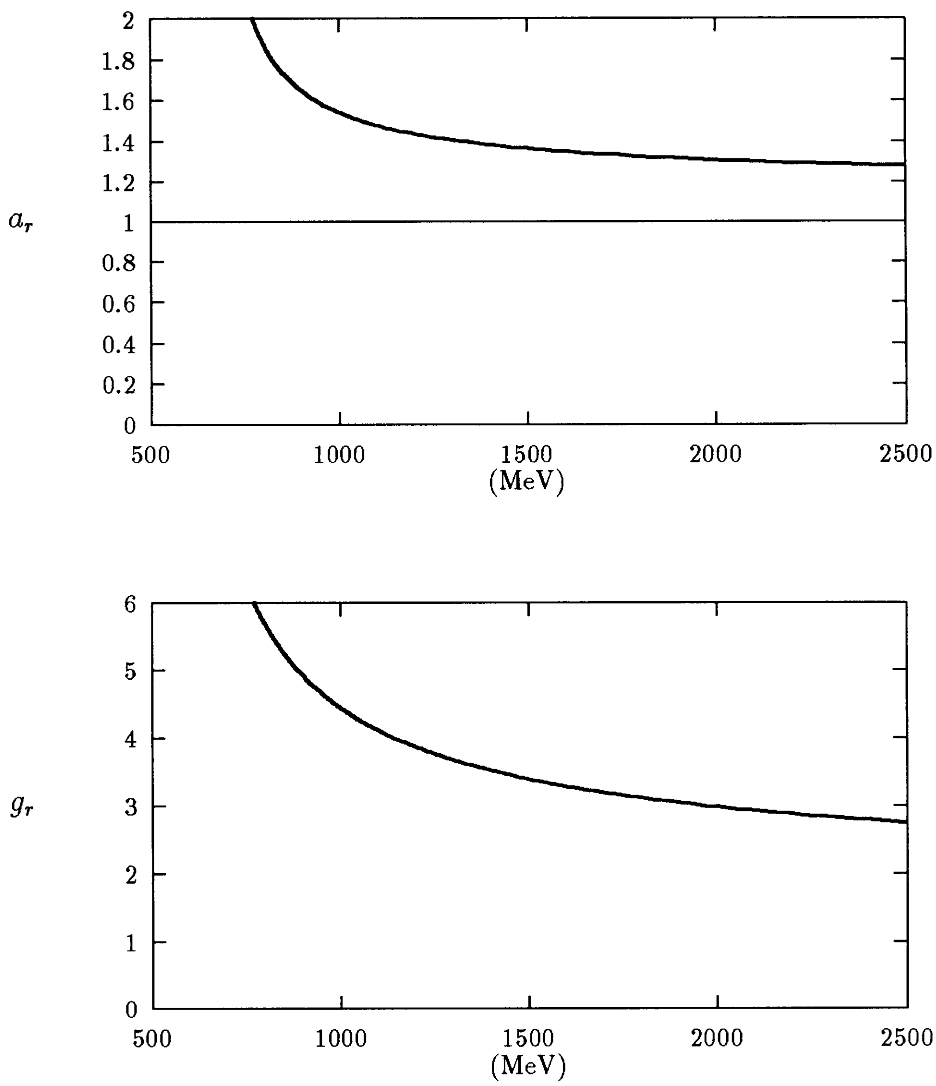


Figure 8: The renormalization-group evolution of the parameters, a_r and g_r. We take $a_r(\mu = m_\rho) = 2.0$ and $g_r(\mu = m_\rho) = 6.0$ as input values.

7 Conclusions and Discussions

We have studied the one-loop corrections of the hidden local gauge boson to the successful tree level predictions. The KSRF relation (I) is satisfied even if we include the one-loop correction of the ρ meson. This suggests that this relation may become a true low energy theorem of the hidden local symmetry[||]. Further, in the parameter choice $a_r = 2$, the KSRF relation (II) and the ρ-coupling universality survive the loop corrections. Most amazingly, there exists no direct $\gamma\pi\pi$ coupling for the finite photon momentum not restricted to the zero momentum limit, if and only if we take the parameter choice $a_r = 2$. This implies that the ρ-meson dominance of the electromagnetic form factor of the pion remains valid, even if we include the loop correction of the ρ meson.

Further, we have studied the renormalization-group equations for a_r and g_r, and shown that the values in the "vector limit" ($a = 1$, $g = 0$) are ultraviolet fixed points of each β function. This implies that the "vector limit" is realized as "idealized" high energy limit of the hidden local symmetry.

As far as tree level is concerned, one might argue[26] that the hidden local symmetry is entirely rotated away by a choice of gauge (unitary gauge). However, this is not true at one-loop level. Actually we have demonstrated that the gauge degrees of freedom (σ, FP ghost as well as ρ meson itself) are essential to the whole successful results mentioned above.

In this paper, we have calculated the loop corrections in the low energy limit except for the $\gamma\pi\pi$ vertex. It is important that we investigate the loop corrections to the above results at higher energy scale, $p^2 \simeq m_\rho^2$, and see whether or not we can use this "effective field theory" up to the a_1-meson mass region. In that scale, however, we should consider the counter terms with higher derivatives ($\mathcal{O}(p^4)$ terms)[**], as considered in the chiral perturbation theory[4, 5, 6].

[||]Actually we have recently proved this relation to all orders based on the BRS symmetry[25].

[**]The structure of these counter terms are investigated by Tanabashi, using a formalism in which the gauge invariance is transparent[28].

After such consideration, we can compare the results with the generalized hidden local Lagrangian which include a_1 meson as well as ρ meson[24, 14]. As such we may relate the unknown parameters to the underlying dynamics, QCD.

It would be very useful that we apply this hidden local symmetry as an "effective field theory" not only to the ordinary technicolor model, but also to the dynamical electroweak symmetry breaking models with large anomalous dimension, such as the walking technicolor[29], the strong ETC technicolor[30] and the top quark condensate models[31], etc., in which the vector resonances may exist.

Acknowledgment

I would like to thank Koichi Yamawaki for collaboration[1] on which this paper is based and for reading this manuscript. Masaharu Tanabashi is also acknowledged for useful discussions. I am also very grateful to A. Ichiro Sanda, Hitoshi Ikemori, Yuki Sugiyama, Noriaki Kitazawa, Masashi Hayakawa, Akihiro Shibata, Daizo Mochinaga, Shogo Tanimura and the other people in the laboratory for encouragement.

Appendices

A The R_ξ Gauge Fixing

In this section we introduce the R_ξ gauge fixing condition which cancels the vector-scalar mixing terms in the hidden local Lagrangian.

First we define the BRS transformations:

$$\begin{aligned}
&\delta_{BRS}\xi_{L,R} = igv\xi_{L,R}, \quad [v : \text{ghost field}],\\
&\delta_{BRS}V_\mu = \partial_\mu v + ig[v, V_\mu] \equiv D_\mu v,\\
&\delta_{BRS}B = 0, \qquad [B : \text{ NL field}],\\
&\delta_{BRS}\bar{v} = iB.
\end{aligned} \tag{A.1}$$

We introduce the gauge fixing plus FP ghost term given by

$$\mathcal{L}_{GF+FP} = -i\delta_{BRS}\left\langle \bar{v}\left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2\left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger\right) + \alpha B\right]\right\rangle, \tag{A.2}$$

where α denotes the gauge parameter.

From the definitions of the BRS transformation, Eq.(A.1), Eq.(A.2) becomes

$$\begin{aligned}
\mathcal{L}_{GF+FP} &= \left\langle B\left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2\left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger\right) + \alpha B\right]\right\rangle\\
&\quad + \left\langle i\bar{v}\left[2\partial^\mu D_\mu v + \frac{i}{2}\alpha a g f_\pi^2\left(v\xi_L + \xi_L^\dagger v + v\xi_R + \xi_R^\dagger v\right)\right]\right\rangle.
\end{aligned} \tag{A.3}$$

The E-L equation for NL field is now given by

$$B^a = \frac{1}{\alpha}\left\langle T^a\left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2\left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger\right)\right]\right\rangle, \tag{A.4}$$

where T^a denotes the generator of the hidden local symmetry. Substituting Eq.(A.4) into Eq.(A.3), we can get the following gauge fixing term and FP ghost term:

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha}\left\langle T^a\left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2\left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger\right)\right]\right\rangle$$

$$\times \left\langle T^a \left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2 \left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger \right) \right] \right\rangle , \qquad (A.5)$$

$$\mathcal{L}_{FP} = i \left\langle \bar{v} \left[2\partial^\mu D_\mu v + \frac{i}{2}\alpha a g f_\pi^2 \left(v\xi_L + \xi_L^\dagger v + v\xi_R + \xi_R^\dagger v \right) \right] \right\rangle . \qquad (A.6)$$

Here using the formula for SU(N) generator T^a;

$$\sum_a \langle T^a A \rangle \langle T^a B \rangle = \frac{1}{2} \langle AB \rangle - \frac{1}{2N} \langle A \rangle \langle B \rangle , \qquad (A.7)$$

Eq.(A.5) reduces to

$$\begin{aligned} \mathcal{L}_{GF} &= -\frac{1}{4\alpha} \left\langle \left[2\partial^\mu V_\mu - \frac{i}{2}\alpha a g f_\pi^2 \left(\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger \right) \right]^2 \right\rangle \\ &\quad + \frac{1}{4N\alpha} \left(\frac{i}{2}\alpha a g f_\pi^2 \right)^2 \left\langle \xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger \right\rangle^2 , \qquad (A.8) \end{aligned}$$

where we use $\langle \partial^\mu V_\mu \rangle = 0$.

Finally we get the following gauge fixing term and FP ghost term in $N = 2$ case:

$$\begin{aligned} \mathcal{L}_{GF} &= -\frac{1}{\alpha} \langle (\partial_\mu V_\mu)^2 \rangle + \frac{i}{2} a g f_\pi^2 \left\langle \partial_\mu V_\mu (\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger) \right\rangle \\ &\quad + \frac{1}{16}\alpha a^2 g^2 f_\pi^4 \left[\left\langle (\xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger)^2 \right\rangle - \frac{1}{2} \left\langle \xi_L - \xi_L^\dagger + \xi_R - \xi_R^\dagger \right\rangle^2 \right] , \qquad (A.9) \\ \mathcal{L}_{FP} &= i \left\langle \bar{v} \left[2\partial^\mu D_\mu v + \frac{1}{2}\alpha a g^2 f_\pi^2 (v\xi_L + \xi_L^\dagger v + v\xi_R + \xi_R^\dagger v) \right] \right\rangle . \qquad (A.10) \end{aligned}$$

B Feynman Rules

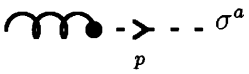
B.1 The Propagators

$$\begin{aligned} D_{\mu\nu}^{ab}(p) &= \delta^{ab} \frac{1}{p^2 - m_\rho^2} \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] \\ D_v^{ab}(p^2) &= \delta^{ab} \frac{i}{-p^2} \\ D_\pi^{ab}(p^2) &= \delta^{ab} \frac{1}{-p^2} \\ D_\sigma^{ab}(p^2) &= \delta^{ab} \frac{1}{-p^2} \end{aligned}$$

B.2 The Vertices

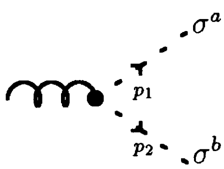
B.2.1 The vertices which include one photon

γ-σ mixing



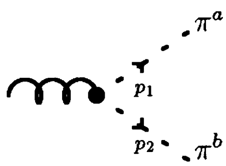
$$ief_\sigma p_\mu \delta^{a3}$$

$\gamma\sigma\sigma$ vertex



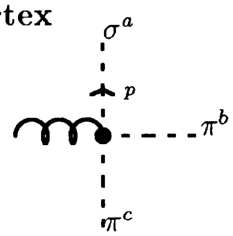
$$-i\frac{1}{2}e\epsilon_{3ab}(p_1 - p_2)_\mu$$

$\gamma\pi\pi$ vertex



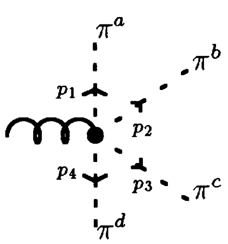
$$-i(1 - \frac{a}{2})e\epsilon_{3ab}(p_1 - p_2)_\mu$$

$\gamma\sigma\pi\pi$ vertex



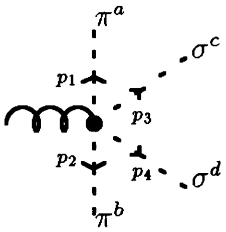
$$ie\frac{a}{2f_\sigma}\left(\delta_{ac}\delta_{b3} + \delta_{ab}\delta_{c3} - 2\delta_{a3}\delta_{bc}\right)p_\mu$$

$\gamma\pi\pi\pi\pi$ vertex



$$-ie\frac{7a-8}{12f_\pi^2}$$
$$\Big[(p_1 - p_2)_\mu\epsilon_{ab3}\delta_{cd} + (p_2 - p_3)_\mu\epsilon_{bc3}\delta_{ad}$$
$$+(p_3 - p_4)_\mu\epsilon_{cd3}\delta_{ab} + (p_4 - p_1)_\mu\epsilon_{da3}\delta_{bc}$$
$$+(p_1 - p_3)_\mu\epsilon_{ac3}\delta_{bd} + (p_2 - p_4)_\mu\epsilon_{bd3}\delta_{ac}\Big]$$

$\gamma\pi\pi\sigma\sigma$ **vertex**



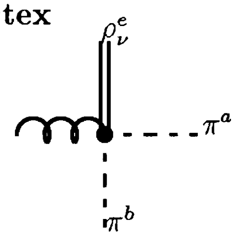
$$-ie\frac{1}{4f_\pi^2}(p_3 - p_4)_\mu$$
$$(\epsilon_{bcd}\delta_{3a} + \epsilon_{acd}\delta_{3a} - 2\epsilon_{3cd}\delta_{ab})$$

B.2.2 The vertices which include one photon and one ρ

ρ-γ **mixing**

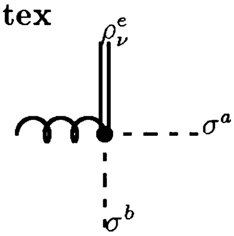
$$-eagf_\pi^2 g_{\mu\nu}\delta^{a3}$$

$\gamma\rho\pi\pi$ **vertex**



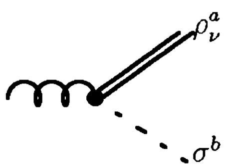
$$e\frac{a}{2}gg_{\mu\nu}$$
$$(2\delta_{e3}\delta_{ab} - \delta_{3a}\delta_{eb} - \delta_{3b}\delta_{ea})$$

$\gamma\rho\sigma\sigma$ **vertex**



$$e\frac{1}{2}gg_{\mu\nu}$$
$$(2\delta_{e3}\delta_{ab} - \delta_{3a}\delta_{eb} - \delta_{3b}\delta_{ea})$$

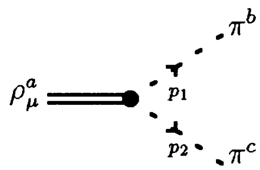
$\gamma\rho\sigma$ **vertex**



$$egf_\sigma g_{\mu\nu}\epsilon_{ab3}$$

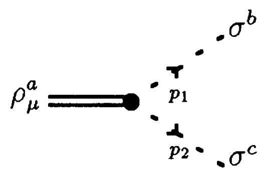
B.2.3 The vertices which include one ρ

$\rho\pi\pi$ vertex



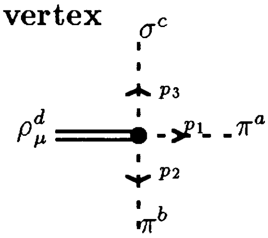
$$-ig\frac{a}{2}\epsilon_{abc}(p_1 - p_2)_\mu$$

$\rho\sigma\sigma$ vertex



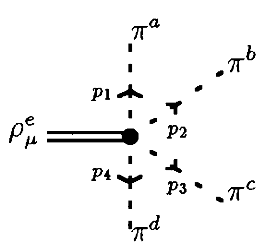
$$-ig\frac{1}{2}\epsilon_{abc}(p_1 - p_2)_\mu$$

$\rho\sigma\pi\pi$ vertex



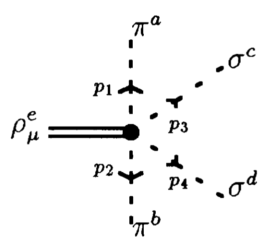
$$-ig\frac{a}{2f_\sigma}\Big[\frac{1}{2}(p_1 + p_2 + p_3)_\mu \delta_{ab}\delta_{cd} + (p_1 - p_2)_\mu (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\Big]$$

$\rho\pi\pi\pi\pi$ vertex



$$ig\frac{a}{12f_\pi^2}\Big[(p_1 - p_2)_\mu \epsilon_{abe}\delta_{cd} + (p_2 - p_3)_\mu \epsilon_{bce}\delta_{ad} + (p_3 - p_4)_\mu \epsilon_{cde}\delta_{ab} + (p_4 - p_1)_\mu \epsilon_{dae}\delta_{bc} + (p_1 - p_3)_\mu \epsilon_{ace}\delta_{bd} + (p_2 - p_4)_\mu \epsilon_{bde}\delta_{ac}\Big]$$

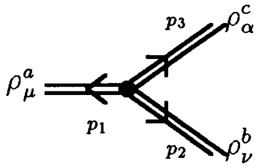
$\rho\pi\pi\sigma\sigma$ vertex



$$-ig\frac{1}{4f_\pi^2}(p_1 - p_2)_\mu (\epsilon_{abd}\delta_{ec} + \epsilon_{abc}\delta_{de} - 2\epsilon_{abe}\delta_{cd})$$

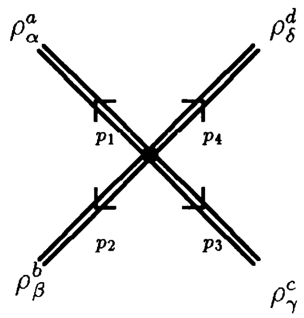
B.2.4 The ρ 3-point and 4-point vertices

ρ 3-point vertex



$$ig\epsilon_{abc}\big[g_{\mu\nu}(p_1 - p_2)_\alpha + g_{\nu\alpha}(p_2 - p_3)_\mu + g_{\alpha\mu}(p_3 - p_1)_\nu\big]$$

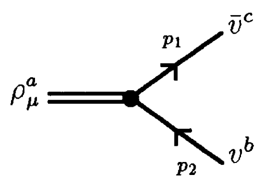
ρ 4-point vertex



$$-g^2\Big[\epsilon_{eab}\epsilon_{ecd}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + \epsilon_{eac}\epsilon_{ebd}(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\gamma\beta}) + \epsilon_{ead}\epsilon_{ebc}(g_{\alpha\beta}g_{\delta\gamma} - g_{\alpha\gamma}g_{\delta\beta})\Big]$$

B.2.5 The vertices which include the ghost field

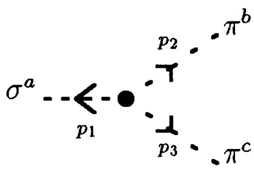
ρ-ghost-ghost vertex



$$g\epsilon_{abc}p_1^\mu$$

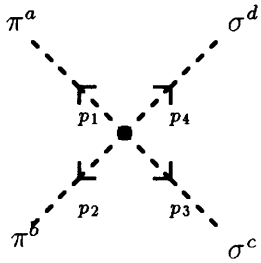
B.2.6 The vertices which include the NG bosons only

$\sigma\pi\pi$ vertex



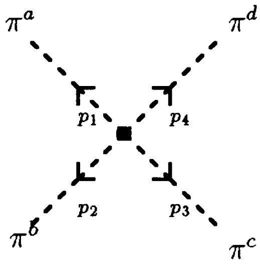
$$-\frac{a}{2f_\sigma}\epsilon_{abc}p_1 \cdot (p_2 - p_3)$$

$\sigma\sigma\pi\pi$ **vertex**



$$\frac{1}{4f_\pi^2}(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})(p_1-p_2)\cdot(p_3-p_4)$$

4π **vertex**



$$-\frac{3a-4}{12f_\pi^2}$$
$$\Big[\delta_{ab}\delta_{cd}\left\{(p_1-p_3)\cdot(p_2-p_4)-(p_1-p_4)\cdot(p_3-p_2)\right\}$$
$$+\delta_{ac}\delta_{bd}\left\{(p_1-p_4)\cdot(p_3-p_2)-(p_1-p_2)\cdot(p_4-p_3)\right\}$$
$$+\delta_{ad}\delta_{bc}\left\{(p_1-p_2)\cdot(p_4-p_3)-(p_1-p_3)\cdot(p_2-p_4)\right\}\Big]$$

References

[1] M. Harada and K. Yamawaki, Phys. Lett. **B297** (1992) 151.

[2] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. **164** (1988) 217.

[3] J. Gasser and H. Leutwyler, Phys. Rep. **C87** (1982) 77.

[4] S. Weinberg, Physica **96A** (1979) 327.

[5] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.)**158** (1984) 142.

[6] J. Gasser and H. Leutwyler, Nucl. Phys. **B250** (1985) 465.

[7] H. Leutwyler, BUTP-91-26, Lectures given at the XXX Internationale Universitätswochen für Kernphysik, Schladming, Austria and at the advanced Theoretical Study Institute in Elementary Particle Physics, Boulder, Colorado (1991).

[8] J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D39** (1989) 1947

[9] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321** (1989) 311.

[10] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. **B223** (1989) 425.

[11] M. Wakamatsu, Ann. Phys. (N.Y.)**193** (1989) 287.

[12] W.A. Bardeen, J. Bijnens and J.-M. Gérard, Phys. Rev. Lett. **62** (1989) 1343.

[13] T. Das, G.S. Guralnik, V.S. Mathur, F.E. Low and J.E. Young, Phys. Rev. Lett. **18** (1967) 759.

[14] M. Bando, T. Fujiwara and K. Yamawaki, Prog. Theor. Phys. **79** (1988) 1140.

[15] K. Yamawaki, in *Proc. 1985 INS Symp. on Composite Models of Quarks and Leptons, Tokyo, Aug. 13 - 15, 1985*, ed. H. Terazawa and M. Yasuè (INS, Univ. of Tokyo, 1985).

[16] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54** (1985) 1215.

[17] J.J. Sakurai, *Currents and Mesons* (Univ. Chicago Press, Chicago, 1969).

[18] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255; Riazuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071.

[19] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. **B259** (1985) 493.

[20] D. Atkinson, M. Harada and A.I. Sanda, Phys. Rev. **D46** (1992) 3884.

[21] See, for example, M.E. Peskin and T. Takeuchi, Phys. Rev. **D46** (1992) 381.

[22] R. Casalbuoni, S. de Curtis, D. Dominici and R. Gatto, Phys. Lett. **B155** (1985) 95; Nucl. Phys. **B282** (1987) 235.

[23] G. Cvetič and R. Kögerler, Phys. Lett. **B217** (1989) 496, Z. Phys. **C48** (1990) 109.

[24] M. Bando, T. Kugo and K. Yamawaki, Prog. Theor. Phys. **73** (1985) 1541.

[25] M. Harada, T. Kugo and K. Yamawaki, in preparation.

[26] H. Georgi, Phys. Rev. Lett. **63** (1989) 1917; Nucl. Phys. **B331** (1990) 311.

[27] See, for example, A. Dhar, R. Shankar and S.R. Wadia, Phys. Rev. **D31** (1985) 3256; D. Ebert and H. Reinhardt, Nucl. Phys. **B271** (1986) 188; J. Bijnens, C. Bruno and E. de Rafael, CERN TH 6521/92, CPT-92/P.2710. See also Refs.[2, 11].

[28] M. Tanabashi, private communication.

[29] B. Holdom, Phys. Lett. **B150** (1985) 301; K. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. **56** (1986) 1335; T. Akiba and T. Yanagida, Phys. Lett. **B169** (1986) 432; T. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. Lett. **57** (1986) 957.

[30] V.A. Miransky and K. Yamawaki, Mod. Phys. Lett. **A4** (1989) 129; K. Matumoto, Prog. Theor. Phys. **81** (1989) 277; T. Appelquist, T. Takeuchi, M. Einhorn and L.C.R. Wijewardhana, Phys. Lett. **B220** (1989) 223.

[31] V.A. Miransky, M. Tanabashi and K. Yamawaki, Phys. Lett. **B221** (1989) 177; Mod. Phys. Lett. **A4** (1989) 1043; Y. Nambu, Chicago preprint EFI 89-08 (1989); W.J. Marciano, Phys. Rev. Lett. **62** (1989) 2793; W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. **D41** (1990) 1647.