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Collective behavior of ion Bernstein waves in a multi-ion-species plasma

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Collective behavior of ion Bernstein waves propagating perpendicular to an external magnetic field is studied with attention to the effect of multiple-ion species. In a thermal-equilibrium, multi-ion-species plasma, a great number of Bernstein waves are excited near the harmonics of many different ion cyclotron frequencies. The autocorrelation function of the quasimode consisting of these waves is initially damped and is not recovered to its initial value. This is predicted by the theory and is confirmed by numerical calculations and by particle simulations. It is also demonstrated by particle simulations that a perpendicular macroscopic disturbance is damped in a multi-ion-species plasma. The electric-field energy associated with this disturbance is significantly reduced and is transferred to the ions, indicating that the presence of multiple-ion species affects the energy transport. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1712977]

I. INTRODUCTION

It was generally thought that waves propagating perpendicular to a magnetic field in a collisionless plasma are not damped.[1] It has been shown, however, that a perpendicular magnetosonic pulse is damped in a multi-ion-species plasma.[2–5] (Periodic waves are not damped even in this case.) The damping is due to heavy-ion acceleration caused by the transverse electric field in the wave.[6,7] It was also shown that perpendicular magnetosonic pulses in an electron-positron-ion plasma are damped.[8] The presence of multiple particle species thus significantly affects the wave propagation[9–11] and damping.

Perpendicular electrostatic waves were first discussed with kinetic theory by Bernstein,[1] and it was proved there that each Bernstein mode with frequency $\omega \simeq n|\Omega_j|$ is undamped in a collisionless plasma, where Ω_j is the cyclotron frequency of ions or electrons and n is an integer. It was then shown with theory and simulations[12,13] that in the limit as the magnetic field approaches zero, the quasimode consisting of the electron Bernstein waves becomes a single damped mode; its damping rate is identical to the Landau damping rate of an electron plasma wave in an unmagnetized plasma. This damping is caused by the phase mixing of the electron Bernstein waves with an infinite set of closely spaced real frequencies. When the magnetic field is finite, the quasimode shows periodic behavior with the electron cyclotron period $2\pi/|\Omega_e|$. The recurrence is due to the phase coherence established with this time period.

For ion Bernstein waves with $\omega \simeq n\Omega_i$, the recurrence of the quasimode is also expected to occur in a single-ion-species plasma. The recurrence time will be the ion cyclotron period $2\pi/\Omega_i$. However, space plasmas usually have many ion species. Moreover, each ion species has many different ionic charge states. Numerous different ion cyclotron frequencies and their harmonics will thus exist in such plasmas. The collective behavior of the ion Bernstein waves could then be quite different from that in a single-ion-species plasma with one ionic charge state.

In this paper, we study the collective behavior of ion Bernstein waves with particular attention to the effect of multiple-ion species. Preliminary results were reported in Ref. 14.

In Sec. II, we describe a linear theory of ion Bernstein waves in a multi-ion-species plasma. In Sec. III, we numerically obtain power spectra of the ion Bernstein waves in a thermal-equilibrium plasma. We then calculate autocorrelation functions of quasimodes consisting of these waves. In a single-ion-species plasma, the autocorrelation function exhibits periodic behavior with the ion cyclotron period. On the other hand, in a multi-ion-species plasma, the autocorrelation function does not return to its initial value, because Bernstein waves are excited near the harmonics of many different ion cyclotron frequencies. This result indicates that in space plasmas such as the solar corona, the autocorrelation function will not be recovered.

In Sec. IV, we study the collective behavior of the ion Bernstein waves by means of a two-dimensional, electrostatic, particle code with full ion and electron dynamics. The simulations demonstrate that the autocorrelation function in a thermal-equilibrium, multi-ion-species plasma is not recovered. We then study the evolution of macroscopic disturbances that consist of the ion Bernstein waves. In a multi-ion-species plasma, a macroscopic disturbance is also initially damped and is not recovered to its initial value. The electric-field energy associated with this disturbance is thus significantly reduced and is transferred to the ions. As a result, the ion kinetic energy and the averaged ion temperature in a multi-ion-species plasma increase much more than those in a single-ion-species plasma, respectively. This indicates that the presence of multiple-ion species can enhance the energy dissipation.

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II. LINEAR THEORY

A. Dispersion relation

We briefly describe linear dispersion relations of ion Bernstein waves propagating perpendicular to a uniform magnetic field in a multi-ion-species plasma in thermal equilibrium with a temperature T. The dielectric function for perpendicular electrostatic waves is given by[15]

$$\varepsilon(k,\omega)=1+\sum_j \frac{k_{Dj}^2}{k^2}\left[1-\sum_{n'=-\infty}^{\infty}\frac{\omega\Gamma_{n'}(\mu_j)}{\omega-n'\Omega_j}\right], \tag{1}$$

where the subscript j refers to electrons (e) or ion species ($a,b,c,$ ...), Ω_j is the cyclotron frequency, and k_{Dj} is the Debye wave number. Also, $\Gamma_{n'}(\mu_j)=I_{n'}(\mu_j)\exp(-\mu_j)$, where $I_{n'}$ is the modified Bessel function of the n'th order, and μ_j is defined as $\mu_j=k^2\rho_j^2$; k is the perpendicular wave number and ρ_j is the gyroradius. We consider low-frequency waves with $\omega\ll|\Omega_e|$ and retain only the $n'=0$ term for electrons in Eq. (1). Using the relation $\Sigma_n\Gamma_n=1$, we then write Eq. (1) as

$$\varepsilon(k,\omega)=1+\frac{k_{De}^2}{k^2}[1-\Gamma_0(\mu_e)] -\sum_i \frac{k_{Di}^2}{k^2}\sum_{n'=-\infty}^{\infty}\frac{n'\Omega_i\Gamma_{n'}(\mu_i)}{\omega-n'\Omega_i}, \tag{2}$$

where the subscript i denotes ion species.

From $\epsilon=0$, we obtain dispersion relations of Bernstein waves for ion species "a" (in the following, we write these particles as a ions),

$$\omega_{an}=(n+\Delta_{an})\Omega_a, \tag{3}$$

with

$$\Delta_{an}=\frac{nk_{Da}^2\Gamma_n(\mu_a)}{k^2+k_{De}^2[1-\Gamma_0(\mu_e)]}. \tag{4}$$

Here, we retained the $n'=n$ term of a ions in Eq. (2), assuming that effects of the other ions are neglected. This assumption is valid when

$$\Delta_{an}\ll\frac{nk_{Da}^2\Gamma_n(\mu_a)|m\Omega_b-n\Omega_a|}{k_{Da}^2\Gamma_n(\mu_a)n\Omega_a+k_{Db}^2\Gamma_m(\mu_b)m\Omega_b}, \tag{5}$$

where we suppose that $m\Omega_b$ is the closest to ω_{an} among all the integer multiples of the other ion cyclotron frequencies.

For $\mu_j\gg1$, Γ_n is approximated as

$$\Gamma_n(\mu_j)=\frac{\exp(-n^2/2\mu_j)}{\sqrt{2\pi\mu_j}}. \tag{6}$$

As $k\to\infty$, Γ_n goes to zero, and the frequency ω_{an} approaches $n\Omega_a$. The frequencies of short-wavelength waves with $\mu_a\gg1$ can be thus approximated as

$$\omega_{an}\simeq n\Omega_a. \tag{7}$$

The condition, Eq. (5), is then written as

$$\frac{(k^2+k_{De}^2)k\rho_a|m\Omega_b-n\Omega_a|}{k_{Da}^2n\Omega_a+k_{Db}^2m\Omega_b(\rho_a/\rho_b)}\gg1. \tag{8}$$

When Eq. (5) is not satisfied, we cannot distinguish the nth harmonic a-ion wave and the mth harmonic b-ion wave. Their frequencies are given by

$$\omega_{an}=\omega_{bm}=n\Omega_a+\frac{[nk_{Da}^2\Gamma_n(\mu_a)+mk_{Db}^2\Gamma_m(\mu_b)]}{k+k_{De}^2[1-\Gamma_0(\mu_e)]}\Omega_a. \tag{9}$$

B. Fluctuation spectra of electrostatic fields

The fluctuation spectra of electrostatic fields in a spatially homogeneous, thermal equilibrium plasma are given by[16]

$$\frac{|E_{k,\omega}|^2}{8\pi}=\frac{\pi k_BT}{\omega}\frac{\mathrm{Im}\,\varepsilon}{|\varepsilon(k,\omega)|^2}. \tag{10}$$

For ion Bernstein waves propagating perpendicular to the magnetic field, Eq. (10) is reduced to

$$\frac{|E_{k,\omega}|^2}{8\pi}=\sum_i\sum_n P(\omega_{in})\delta(\omega-\omega_{in}) \tag{11}$$

with $P(\omega_{in})$

$$P(\omega_{in})=\frac{\pi k_BT}{\omega\frac{\partial}{\partial\omega}\varepsilon(k,\omega)|_{\omega=\omega_{in}}}, \tag{12}$$

where ω_{in} is the frequency of the nth harmonic Bernstein wave for ion species i.

With the aid of Eqs. (2) and (3), we set Eq. (12) into the form

$$P(\omega_{in})=\frac{\pi k_BTk^2k_{Di}^2\Gamma_n(\mu_i)}{\{k^2+k_{De}^2[1-\Gamma_0(\mu_e)]\}^2}. \tag{13}$$

For $\mu_i\gg1$, Eq. (13) is written as

$$P(\omega_{in})\propto\frac{n_iq_i^3}{\sqrt{m_i}}\exp\left(-\frac{n^2}{2\mu_i}\right), \tag{14}$$

where Eq. (6) is used. Equation (14) indicates that $P(\omega_{in})$ for ions with large q_i can be great.

When the nth harmonic a-ion wave and the mth harmonic b-ion wave have the same frequency, Eq. (9), the spectrum for this frequency is given by

$$P[\omega_{an}(=\omega_{bm})]=\frac{\pi k_BTk^2[k_{Da}^2\Gamma_n(\mu_a)+k_{Db}^2\Gamma_m(\mu_b)]}{\{k^2+k_{De}^2[1-\Gamma_0(\mu_e)]\}^2}. \tag{15}$$

C. Autocorrelation functions

For a given wave number k, there are many Bernstein waves with different frequencies. We consider a quasimode consisting of these waves and calculate its autocorrelation function $C_k(\tau)$. Although each Bernstein wave is undamped, $C_k(\tau)$ is initially damped owing to the phase mixing of many Bernstein waves. In a single-ion-species plasma with one ion cyclotron frequency Ω_a, however, $C_k(\tau)$ returns to its initial value at times $2n\pi/\Omega_a$. On the other hand, in a multi-ion-species plasma, the recurrence is incomplete, even if the abundances of heavy ions are small.

The autocorrelation function $C_k(\tau)$ of the quasimode is defined by

$$C_k(\tau)=\langle E_k(t)E_k(t+\tau)\rangle_t, \tag{16}$$

where $\langle\ \rangle_t$ represents a time average. This function is related to the fluctuation spectrum $|E_{k,\omega}|^2$ through the Fourier transformation in ω as

$$C_k(\tau)=\int_{-\infty}^{\infty}|E_{k,\omega}|^2\exp(-i\omega\tau)\,d\omega. \tag{17}$$

Substituting Eq. (11) into Eq. (17), we obtain[13]

$$C_k(\tau)=8\pi^2k_BT\sum_i\sum_n\frac{\exp(-i\omega_{in}\tau)}{\omega_{in}\frac{\partial}{\partial\omega}\varepsilon(k,\omega)\Big|_{\omega=\omega_{in}}}. \tag{18}$$

Using Eqs. (3), (13), and (11), we write $C_k(\tau)$ as

$$\frac{C_k(\tau)}{C_k(0)}=\frac{1}{k_{D\,\mathrm{sum}}^2}\sum_i\sum_n k_{Di}^2\Gamma_n(\mu_i)\exp(-in\Omega_i\tau)\times\exp(-i\Delta_{in}\tau), \tag{19}$$

where $k_{D\,\mathrm{sum}}^2$ is defined as

$$k_{D\,\mathrm{sum}}^2=\sum_i k_{Di}^2. \tag{20}$$

When $\Delta_{in}\tau\ll1$, Eq. (19) is approximated as

$$\frac{C_k(\tau)}{C_k(0)}=\frac{1}{k_{D\,\mathrm{sum}}^2}\sum_i\sum_n k_{Di}^2\Gamma_n(\mu_i)\exp(-in\Omega_i\tau). \tag{21}$$

This approximation is better for the short-wavelength waves $(\mu_i\gg1)$ than for the long-wavelength ones.

First, we discuss the initial behavior of $C_k(\tau)$, assuming that $\Omega_i\tau\ll1$. In the limit of $\Omega_i\tau\rightarrow0$, we may replace the summation over the harmonic numbers by an integral over a continuous variable ξ according to

$$n\Omega_i\tau\rightarrow\xi_i,\qquad\sum_{n=-\infty}^{\infty}\rightarrow\frac{1}{\Omega_i\tau}\int_{-\infty}^{\infty}d\xi_i. \tag{22}$$

We then obtain, with Eq. (6), the initial behavior as

$$\frac{C_k(\tau)}{C_k(0)}=\frac{1}{k_{D\,\mathrm{sum}}^2}\sum_i\frac{k_{Di}^2}{\sqrt{2\pi\mu_i}\Omega_i\tau}\int_{-\infty}^{\infty}d\xi_i\times\exp\left(-\frac{\xi_i^2}{2\mu_i\Omega_i^2\tau^2}\right)\exp(-i\xi_i)=\frac{1}{k_{D\,\mathrm{sum}}^2}\sum_i k_{Di}^2\exp(-k^2v_{Ti}^2\tau^2/2). \tag{23}$$

The initial $C_k(\tau)$ is a sum of decaying functions. The amplitude of each of them is proportional to $n_iq_i^2$. If the hydrogen is the dominant component, it would determine the behavior of $C_k(\tau)$ in the very initial phase. Later, then, $C_k(\tau)$ would decay more slowly because of the components with lower v_{Ti}, i.e., with larger masses.

Next, we consider its long-time behavior. For a single-ion-species plasma with one cyclotron frequency Ω_a, the autocorrelation function shows periodic behavior with the time period $2\pi/\Omega_a$, $C_k(\tau)\simeq C_k(\tau+2n\pi/\Omega_a)$, because $\omega_{an}\simeq n\Omega_a$. On the other hand, the recurrence is incomplete in a multi-ion-species plasma. At $\tau=2\pi/\Omega_a$, $C_k(\tau)$ takes the value

$$\frac{C_k(2\pi/\Omega_a)}{C_k(0)}\simeq\frac{1}{k_{D\,\mathrm{sum}}^2}\sum_i\sum_n k_{Di}^2\Gamma_n(\mu_i)\times\exp\left(-i\frac{2\pi n\omega_{in}}{\Omega_a}\right)=1-\sum_{i\neq a}\sum_n\frac{k_{Di}^2}{k_{D\,\mathrm{sum}}^2}2\Gamma_n(\mu_i)\sin^2\left(\frac{\pi n\Omega_i}{\Omega_a}\right). \tag{24}$$

This value decreases with increasing number of ion species.

In a multi-ion-species plasma, the autocorrelation function never returns to its initial value. This can be explained as follows. If the wave frequencies were exactly equal to the integral multiples of ion cyclotron frequencies, $\omega_{in}=n\Omega_i$, the autocorrelation function would return to its initial value at the time of the least common multiple of all the ion cyclotron periods. However, this time is extremely long in plasmas with numerous different ion cyclotron periods. On such a long time scale, the assumption that $\omega_{in}=n\Omega_i$ is not valid; Eq. (19) shows that the small frequency differences Δ_{in} become important for $\Delta_{in}\tau\sim1$. Accordingly, the recurrence peak is never recovered.

In the case of finite parallel wave number $k_\parallel$, recurrence peaks of the autocorrelation function could be damped owing to Landau (or cyclotron) resonance. In Ref. 13, it was shown that for electron Bernstein waves, the recurrence peaks are significantly reduced when $k_\parallel v_{Te}(2\pi/|\Omega_e|)\sim1$. We thus expect that for ion Bernstein waves in a plasma with a ions being major, the finite $k_\parallel$ effect would be negligible when $k_\parallel v_{T_a}(2\pi/\Omega_a)\lesssim1$.

III. NUMERICAL CALCULATION

We numerically calculate specific values of $P(\omega)$ and $C_k(\tau)$ for four different thermal equilibrium plasmas and compare them. Retaining terms from $n=-30$ to 30 for the ions and the $n=0$ term for the electrons in Eqs. (1) and (12), we calculate $P(\omega)$. We then have $C_k(\tau)$ from $P(\omega)$ through the Fourier transformation in ω.

We consider four different plasmas; single-ion (H), two-ion (H and He), three-ion (H, He, C), and six-ion (H, He, C, O, Si, and Fe) species plasmas. We take the hydrogen mass, charge and temperatures to be $m_\mathrm{H}/m_e=1836$, $q_\mathrm{H}/|q_e|=1$, and $T_\mathrm{H}/T_e=1.0$. We show in Table I the masses, charges, cyclotron frequencies, and densities of the heavy ions. These values are normalized to those of the H ions. The temperatures of the heavy ions are equal to that of the H ions. The magnetic field strength is set to be $|\Omega_e|/\omega_{pe}=4.0$.

Figure 1 shows power spectra of the electric fields in the four plasmas. The spectra $P(\omega)$ are normalized to πk_BT. In

TABLE I. Masses, charges, cyclotron frequencies, and densities of heavy ions.

Ion	m_i/m_H	q_i/q_H	Ω_i/Ω_H	n_i/n_H
He	4	2	0.5	0.1
C	12	5	0.42	0.01
O	16	6	0.38	0.01
Si	28	9	0.31	0.005
Fe	56	13	0.23	0.005

the single-ion-species plasma, the peaks are at $\omega \simeq n\Omega_H$, which correspond to the frequencies of H Bernstein waves. In the two-ion-species plasma, besides these waves, He Bernstein waves are excited at $\omega \simeq n\Omega_{He}$. The $2n$th harmonic He waves are identical to the nth harmonic H waves, and their amplitudes are given by Eq. (15). In the three-ion-species plasma, C Bernstein waves are also present. In the six-ion-species plasma, a great number of waves exist in the low frequency region $\omega/\Omega_H < 5$, because the Bernstein waves of O, Si, and Fe ions are also excited; some of their higher harmonic waves cannot be distinguished from the H or He waves (for example, the fourth harmonic O wave is identical to the third harmonic He wave). Although the abundances of the heavy ions are small, the power spectrum is significantly changed, as predicted by Eq. (14).

Figure 2 shows autocorrelation functions normalized to their initial values $C_k(0)$. In the single-ion-species plasma, periodic behavior with time period $2\pi/\Omega_H$ is observed; the amplitude is initially damped due to the phase mixing of many harmonic modes with $\omega \simeq n\Omega_H$, and it is recovered to its initial value because of phase coherence that is re-established at the times $\Omega_H\tau \simeq 2n\pi$. The recurrence peak values are almost the same as the initial one. In the two-ion-species plasma, the periodic behavior is also observed. In the three-ion-species plasma, however, the recurrence is incomplete, and the amplitude is not recovered to the initial value. This is because the phase coherence among many waves shown in the third panel of Fig. 1 has not been established by $\Omega_H t = 35$. In the six-ion-species plasma, the recurrence peaks are substantially reduced, as predicted by Eq. (24). Even if we observe $C_k(\tau)$ for a much longer time, its recurrence peak will not return to its initial value, because the small

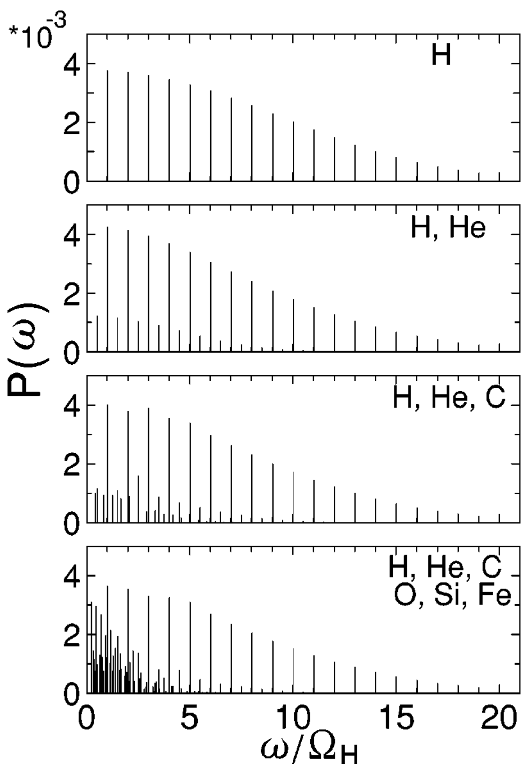

FIG. 1. Power spectra of electric field fluctuations with $k\rho_H = 8$ in four different plasmas.

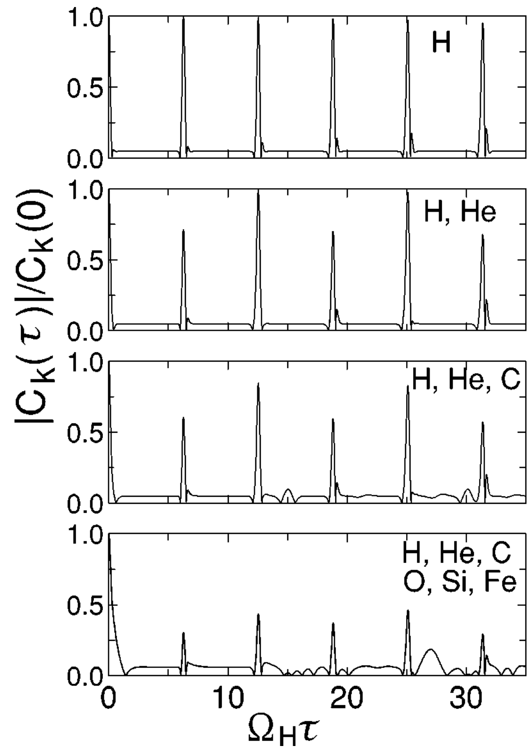

FIG. 2. Autocorrelation functions in the four different plasmas.

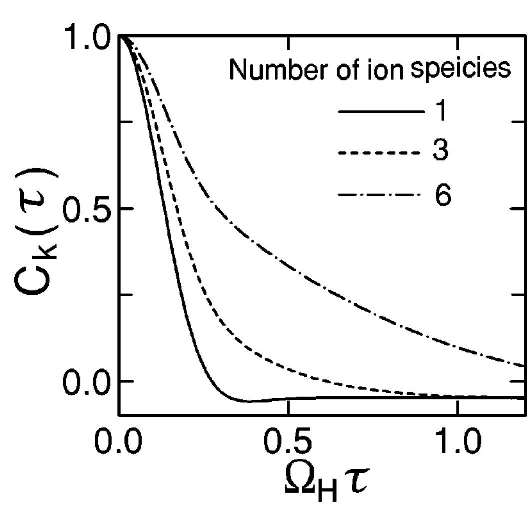

FIG. 3. Initial damping of autocorrelation functions in the single, three, and six-ion-species plasmas.

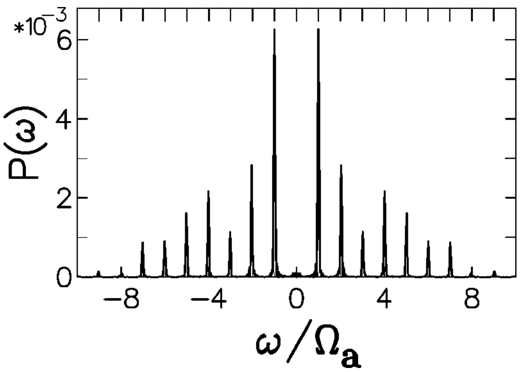

FIG. 4. Power spectrum of electrostatic-field fluctuations with $k\rho_a = 3.4$ in a single-ion-species plasma. The power is normalized to $m_e v_{Te}^2$.

frequency differences Δ_{in} become important, as described in the end of Sec. II.

We show in Fig. 3 the initial time variations of $C_k(\tau)$ in the single-, three-, and six-ion-species plasmas. As the number of ion species increases, the initial damping becomes slower, as predicted by Eq. (23); note that the value of $n_{Fe} q_{Fe}^2$ is comparable to $n_{\mathrm{H}} q_{\mathrm{H}}^2$ in the six-ion-species plasma.

IV. PARTICLE SIMULATION

A. Simulation method and parameters

By means of a two-dimensional (two space and three velocity components), electrostatic particle code with full ion and electron dynamics, we now study the collective behavior of ion Bernstein waves in a multi-ion-species plasma. The system size is $L_x \times L_y = 64\Delta_g \times 4\Delta_g$, where Δ_g is the grid spacing. We use periodic boundary conditions in both x and y directions. The external magnetic field is in the z direction, and its strength is $|\Omega_e|/\omega_{pe} = 4.0$. The electron Debye length is equal to the grid spacing Δ_g. The total number of electrons is $N_e = 122\,880$.

We simulate single-ion (a) and four-ion (a, b, c, and d) species plasmas. We choose the mass ratios as $m_a/m_e = 50$, $m_b/m_a = \sqrt{3}$, $m_c/m_a = \sqrt{5}$, and $m_d/m_a = \sqrt{10}$. In order to see the effect of multiple-ion species with a small number of ion species, we have taken the irrational ion mass rations. The charges are the same, $q_a = q_b = q_c = q_d = |q_e|$. The ion densities are set to be $n_b = n_c = n_d = 0.5 n_a$.

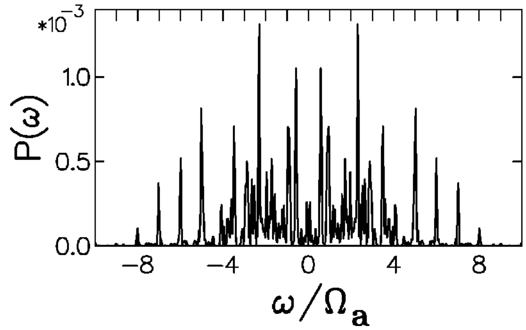

FIG. 6. Power spectrum of electrostatic field fluctuations with $k\rho_a = 3.4$ in a four-ion-species plasma.

In the following, we first study electric-field fluctuations in thermal equilibrium plasmas where all the particle species have equal temperatures. We then investigate evolution of a macroscopic disturbance and associated energy transport.

B. Simulation results

1. Thermal equilibrium plasmas

Figure 4 shows the power spectrum $P(\omega)$ of electrostatic-field fluctuations with $k\rho_a = 3.4$ propagating perpendicular to the magnetic field in a single-ion-species plasma. The Bernstein waves are excited at $\omega \simeq n\Omega_a$. Figure 5 shows the autocorrelation function $C_k(\tau)$. It is initially damped and is then almost recovered to its initial value at times $\Omega_a t = 2n\pi$.

We show in Figs. 6 and 7, $P(\omega)$ and $C_k(\tau)$ in the four-ion-species plasma, respectively. A great number of waves are excited. After the initial damping of $C_k(\tau)$, the recurrence is not clear. The peak values are significantly smaller than $C_k(0)$.

2. Damping of macroscopic disturbance and associated energy transport

Next, we study the macroscopic disturbance, setting the initial ion density perturbations to be

$$\delta n_i(x)/n_{i0} = 0.5\cos(k_0 x), \tag{25}$$

where n_{i0} is the average density and $k_0\rho_a = 2.8$. Along the y axis, δn_i is constant. The initial electron density is uniform in space. These density perturbations produce a macroscopic

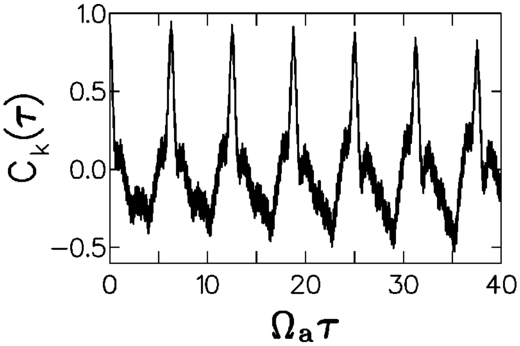

FIG. 5. Autocorrelation function in the single-ion-species plasma.

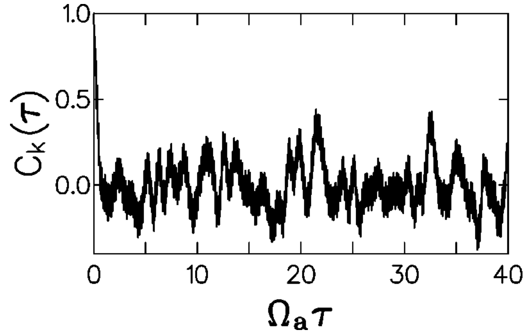

FIG. 7. Autocorrelation function in the four-ion-species plasma.

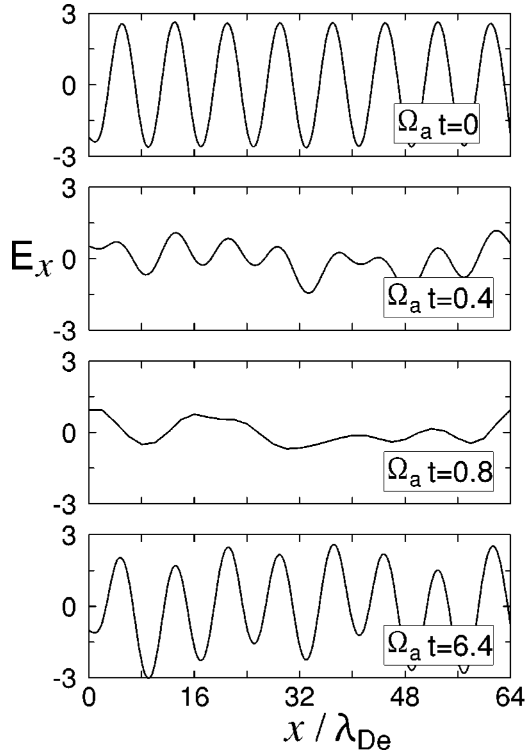

FIG. 8. Profiles of the electric field E_x in a single-ion-species plasma at the times $\Omega_a t = 0$, 0.4, 0.8, and 6.4. E_x is normalized to $m_e \omega_{pe}^2 \Delta_g / |q_e|$.

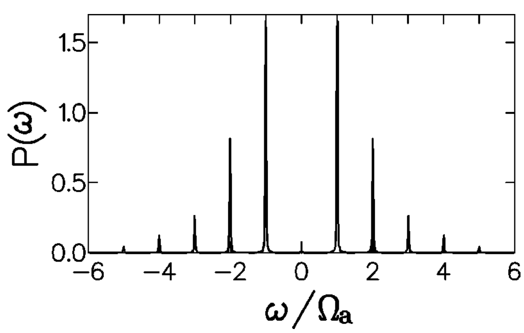

FIG. 9. Power spectrum of electric fields in a single-ion-species plasma. The normalization is the same as Fig. 4.

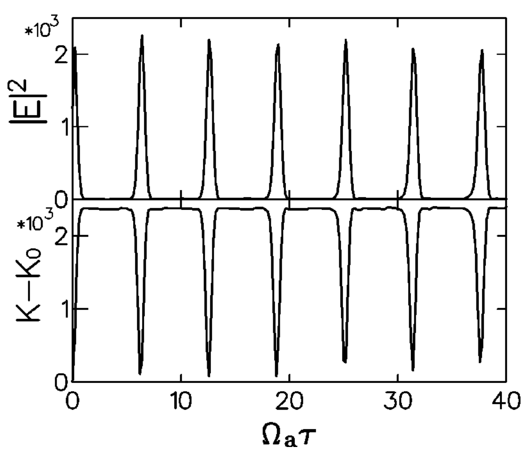

FIG. 10. Time variations of total electric-field energy and ion kinetic energy, $K - K_0$, in the single-ion-species plasma. The energies are normalized to $m_e v_{Te}^2$.

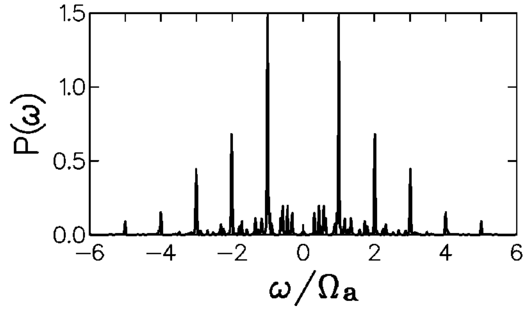

FIG. 11. Power spectrum of electric fields in a four-ion-species plasma.

electric field E_x with the wave number k_0. Initially, all the ion species have equal temperature; the electron-to-ion temperature ratio is chosen to be $T_i / T_e = 4$.

First, we present the result for a single-ion-species plasma. Figure 8 shows the profiles of E_x as a function x (along the line $y/\Delta_g = 1$) at four different times. In the early stage, the macroscopic electric field is rapidly damped. At $\Omega_a t \simeq 2\pi$, however, the wave profile and amplitude are almost recovered to the initial ones. Figure 9 shows the power spectrum, which was obtained from the data for the period from $\Omega_a t = 0$ to 160. The peaks are at $\omega \simeq n\Omega_a$. The macroscopic electric field is formed with these Bernstein waves. The initial damping of the electric field shown in Fig. 8 is due to the phase mixing of these waves. The recurrence is due to phase coherence established at $\Omega_a t \simeq 2\pi$.

Figure 10 shows time variations of total electric-field energy $|E|^2$ and ion kinetic energy $K - K_0$, where K is the total energy and K_0 is the initial one. At first, the electric-field energy decreases, and the ion kinetic energy increases. However, at $\Omega_a t \simeq 2n\pi$, both energies almost return to their initial values. Then, these processes repeat. The recurrence peaks are almost the same as the initial values.

We now present the results for the four-ion-species plasma. Figure 11 shows the power spectrum of electric fields. Besides the waves of a ions, numerous Bernstein waves of b, c, and d ions are excited.

Figure 12 shows time variations of total electric-field energy $|E|^2$ and ion kinetic energy $K - K_0$ (K is the sum of the kinetic energies of all the ion species, and K_0 is its initial

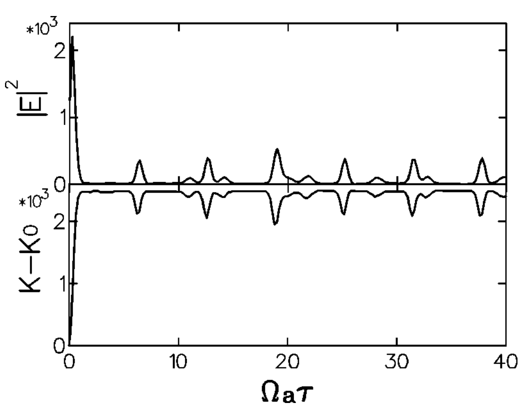

FIG. 12. Time variations of total electric-field energy and ion kinetic energy, $K - K_0$, in the four-ion-species plasma.

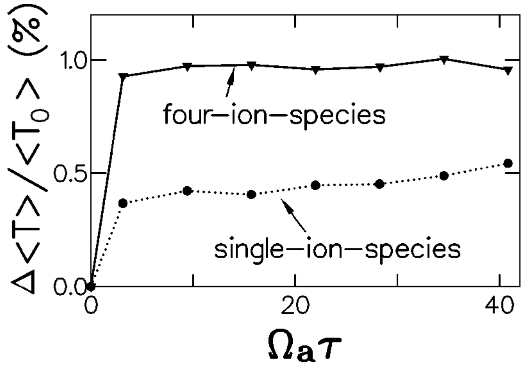

FIG. 13. Time variations of the average ion temperatures in the single- and four-ion-species plasmas.

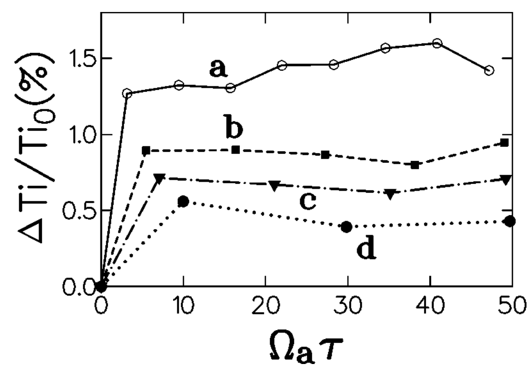

FIG. 14. Time variations of ion temperatures in the four-ion-species plasma. The values of ΔT_i are averaged over each ion cyclotron period.

value). After the initial damping, $|E|^2$ does not return to the initial value until the end of the simulation, although it slightly oscillates with the period $\Omega_a t = 2\pi$. On the other hand, the ion kinetic energy is significantly increased. The energy is transferred from the electric field to the ions.

We show in Fig. 13 time variations of average ion temperatures in the single- and four-ion-species plasmas, where $\Delta\langle T\rangle$ is $\langle T\rangle - \langle T_0\rangle$ with $\langle T_0\rangle$ the initial value. The temperature $\langle T\rangle$ is averaged over all the ions and is defined as

$$\langle k_B T\rangle = \frac{\Sigma_i \int d\mathbf{x} \int d\mathbf{v}\, m_i f_i(\mathbf{x},\mathbf{v})(\mathbf{v} - \langle \mathbf{v}_i(\mathbf{x})\rangle)^2}{\Sigma_i \int d\mathbf{x} \int d\mathbf{v}\, f_i(\mathbf{x},\mathbf{v})}, \tag{26}$$

where $\langle \mathbf{v}_i(\mathbf{x})\rangle$ is the fluid velocity for ion species i at the position, x,

$$\langle \mathbf{v}_i(\mathbf{x})\rangle = \frac{\int d\mathbf{v}\, f_i(\mathbf{x},\mathbf{v})\mathbf{v}}{\int d\mathbf{v}\, f_i(\mathbf{x},\mathbf{v})}. \tag{27}$$

The values of $\Delta\langle T\rangle$ are plotted in Fig. 13 are averaged over the time period $\Omega_a t = 2\pi$. This figure shows that the average ion temperature in the four-ion-species plasma becomes higher than that in the single-ion-species plasma. The presence of multiple-ion species can enhance the energy dissipation. Figure 14 shows temperature changes ΔT_i of four different ions. Lighter ions have greater temperature increase ΔT_i than heavier ions.

V. SUMMARY

We have theoretically and numerically studied the collective behavior of the ion Bernstein waves propagating perpendicular to an external magnetic field, giving special attention to the effect of the presence of multiple-ion species. We have calculated power spectra of electric fields due to these waves in thermal-equilibrium plasmas and autocorrelation functions of the quasimode consisting of these waves. In a single-ion-species plasma, the autocorrelation functions show periodic behavior with the ion cyclotron period, and the recurrence peaks almost return to their initial values. On the other hand, in a multi-ion-species plasma, the autocorrelation functions are not recovered, because a great number of Bernstein waves with different frequencies are excited.

We have also performed simulations of Bernstein waves, by means of a two-dimensional (two space and three velocity components), electrostatic, particle code with full ion and electron dynamics. It is demonstrated that the autocorrelation functions in a thermal-equilibrium, multi-ion-species plasma are not recovered, after the initial damping. Furthermore, we studied the evolution of a macroscopic disturbance. In a multi-ion-species plasma, the macroscopic disturbance is also damped and is not recovered to its initial amplitude. The electric-field energy is significantly reduced and is transferred to the ions. The ion temperature in a multi-ion-species becomes higher than that in a single-ion-species plasma.

Evidently, the presence of multiple-ion species can enhance the energy dissipation. Our results could be applicable to, for example, the solar corona where numerous different ion cyclotron frequencies exist, although ion Bernstein waves may be difficult to be observed there. It is desirable to extend this study to long wavelength fluctuations, which may enable us to discuss ion heating. Nonetheless, it is notable that the new damping phenomenon in a collisionless plasma has been presented.

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