# Low-frequency electromagnetic fluctuations in thermal-equilibrium, multi-ion-species plasmas

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Low-frequency electromagnetic thermal fluctuations propagating perpendicular to a magnetic field are theoretically studied with attention to the effect of multiple ion species. In the frequency regime lower than the lower hybrid frequency, there exist three types of modes; magnetosonic mode with $\omega \simeq kv_A$, ion cyclotron modes with $\omega \simeq n\Omega_i$, and heavy-ion cutoff modes with frequencies slightly higher than the ion-ion hybrid resonance frequencies. The power spectra of magnetic fluctuations due to these modes are obtained analytically and numerically. In a single-ion-species plasma, the magnetosonic mode is an overwhelmingly dominant mode. The autocorrelation function $C_k(\tau)$ is thus given by a cosine function with a constant amplitude. In a multi-ion-species plasma, however, the amplitudes of the heavy-ion cutoff modes can be comparable to that of the magnetosonic mode. Therefore, $C_k(\tau)$ is initially damped, and its recurrence time is extremely long. © *2005 American Institute of Physics*. [DOI: 10.1063/1.2089927]

## I. INTRODUCTION

Fusion and space plasmas usually contain multiple ion species. The presence of multiple ion species introduces many interesting issues to plasma physics, such as wave propagation,[1–9] heavy ion acceleration,[10,11] and minority heating.[12]

For example, in a plasma containing two ion species, the magnetosonic wave is split into two modes; high- and low-frequency modes.[1,4] The frequencies of the low-frequency mode propagating perpendicular to a magnetic field in a cold plasma are given by

$$\omega = kv_A, \tag{1}$$

in the long-wavelength limit. Here, $k$ is the wavenumber, and $v_A$ is the Alfvén speed,

$$v_A = B_0 \Big/ \left(4\pi \sum_i n_i m_i\right)^{1/2} \tag{2}$$

where $B_0$ is the strength of the external magnetic field, and $n_i$ and $m_i$ are the ion density and mass, respectively. The high-frequency mode has a finite cutoff frequency;[4,9] for a hydrogen-helium plasma, it is given as

$$\omega_{He0} = \left(\frac{\omega_{pH}^2}{\Omega_H^2} + \frac{\omega_{pHe}^2}{\Omega_{He}^2}\right)\frac{\Omega_H \Omega_{He} |\Omega_e|}{\omega_{pe}^2}, \tag{3}$$

where $\omega_{pj}$ and $\Omega_j$ are the plasma and cyclotron frequencies for particle species $j$ ($j$=H, He, or e), respectively. The cutoff frequency $\omega_{He0}$ is slightly greater than the ion-ion hybrid resonance frequency,[1]

$$\omega_{IH} = \left(\frac{\omega_{pH}^2\Omega_{He}^2 + \omega_{pHe}^2\Omega_H^2}{\omega_{pH}^2 + \omega_{pHe}^2}\right)^{1/2}. \tag{4}$$

Furthermore, it has been found with theory and three-fluid simulations that in a two-ion-species plasma, nonlinear magnetosonic pulses are damped even when they propagate perpendicular to a magnetic field.[6,7] The damping is due to the energy transfer from pulses to heavy ions.

Recently, studies have also been made of the effect of multiple ion species on collective behavior of ion Bernstein waves.[13] As is well known, a perpendicular sinusoidal wave is undamped in a collisionless plasma.[14] It was shown in Refs. 15 and 16, however, that the autocorrelation function of a quasimode consisiting of perpendicular electron Bernstein waves with $\omega \simeq n\Omega_e$, where $\Omega_e$ is the electron gyrofrequency and $n$ is an integer, exhibits quasi-periodic behavior with the electron gyroperiod $2\pi/|\Omega_e|$. It oscillates with frequencies much higher than $|\Omega_e|$. Even though it is initially damped owing to the phase mixing of higher harmonic waves, recurrence occurs; at $t=2\pi/|\Omega_e|$, the autocorrelation function has almost the same value as the initial one. This is also the case with perpendicular ion Bernstein waves in a single-ion-species plasma, for which the recurrence occurs with ion gyroperiod $2\pi/\Omega_i$. However, in a multi-ion-species plasma, each ion species has its fundamental ($n=1$) Bernstein wave and its harmonics. Hence, if a plasma contains a large number of ion species, the recurrence time of the autocorrelation function should be extremely long;[13] practically, the autocorrelation function will not be recovered to its initial value. It was also demonstrated with particle simulations that a macroscopic disturbance is damped in a multi-ion-species plasma, even if its profile is sinusoidal. The electric-field energy associated with this disturbance is transferred to the ions. The presence of multiple ion species affects the energy transport.

The above study[13] was on the electrostatic waves with short wavelengths, $k\rho_i \gg 1$, where $\rho_i$ is the ion gyroradius. In this paper, we investigate electromagnetic waves with long wavelengths, $k\rho_i \ll 1$, where magnetohydrodynamic perturbations could play an important role. We calculate power spectra $P_k(\omega)$ and autocorrelation functions $C_k(\tau)$ of magnetic fluctuations in thermal-equilibrium, multi-ion-species plas-

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mas. It is shown that modes with cutoff frequencies slightly higher than ion-ion hybrid resonance frequencies can have amplitudes comparable to those of the magnetosonic mode with $\omega \simeq kv_{\mathrm{A}}$, even if the heavy-ion abudances are quite small. The autocorrelation functions are initially damped, and their recurrence times are extremely long.

In Sec. II, we describe the dispersion relations of perpendicular electromagnetic waves with frequencies lower than the lower hybrid frequency $\omega_{\mathrm{LH}}$. In this frequency domain, we have three types of modes, which will be called the magnetosonic mode, ion cyclotron modes, and heavy-ion cutoff modes. The first one, which was called the low-frequency magnetosonic mode in Ref. 4, has frequencies $\omega \simeq kv_{\mathrm{A}}$ in the long-wavelength limit. The ion cyclotron modes have frequencies near $n\Omega_i$. Slightly above each ion-ion hybrid resonance frequency, which is close to the gyrofrequency of each heavy ion, there exists a heavy-ion cutoff mode. The power spectra of these three modes are also analytically obtained. In a thermal equilibrium state, the amplitudes of the ion cyclotron modes are quite small.

In Sec. III, we carry out numerical calculations for the power spectra $P_k(\omega)$ and autocorrelation functions $C_k(\tau)$ of magnetic fluctuations. In a single-ion-species plasma, $C_k(\tau)$ is not damped, because the magnetosonic mode is a dominant mode. In a multi-ion-species plasma, however, the amplitudes of the heavy-ion cutoff modes can be comparable to that of the magnetosonic mode, if $kv_{\mathrm{A}}$ is of the same order as the heavy-ion cutoff frequencies. Owing to the phase mixing of the magnetosonic and heavy-ion cutoff modes, $C_k(\tau)$ is initially damped. The recurrence time is quite long; practically, $C_k(\tau)$ is not recovered to the initial value.

## II. LINEAR THEORY

We study electromagnetic fluctuations in a thermal-equilibrium, multi-ion-species plasma in a uniform magnetic field. We describe the dispersion relations of low-frequency ($\omega \ll \omega_{\mathrm{LH}}$), perpendicular waves and then calculate power spectra of electromagnetic fluctuations.

### A. Dispersion relation

We consider low-frequency ($\omega \ll \omega_{\mathrm{LH}}$), perpendicular electromagnetic waves. In the fluid theory for a single-ion-species plasma, we have only the magnetosonic mode, $\omega \simeq kv_{\mathrm{A}}$, in this frequency domain. In the kinetic theory, however, there exist many modes,[17] as shown in Fig. 1. In the frequency region $0 < \omega < \Omega_{\mathrm{H}}$, we have the low-frequency magnetosonic mode, $\omega \simeq kv_{\mathrm{A}}$. The fundamental H cyclotron mode is in the region $\Omega_{\mathrm{H}} < \omega < 2\Omega_{\mathrm{H}}$. In the vicinity of the harmonics $\omega = n\Omega_{\mathrm{H}}$ ($n \geq 2$), we find two modes. The higher frequency one is the ordinary cyclotron mode. As $k$ goes to $\infty$, this becomes the electrostatic H Bernstein mode. The frequencies of the lower cyclotron mode are always quite close to $n\Omega_{\mathrm{H}}$.

Figure 2 shows schematic dispersion curves for an H-He plasma with $\Omega_{\mathrm{He}}/\Omega_{\mathrm{H}} = 0.5$. The dotted horizontal line indicates the ion-ion hybrid resonance frequency (4). In the region $\omega < \Omega_{\mathrm{H}}$, we now have three modes, i.e., the magnetosonic mode in $\omega < \Omega_{\mathrm{He}}$, fundamental He cyclotron mode in $\Omega_{\mathrm{He}} < \omega < \omega_{\mathrm{IH}}$, and a mode in $\omega_{\mathrm{He0}} < \omega < \Omega_{\mathrm{H}}$, which we will call the He cutoff mode. (We can view this as one of the two cyclotron modes near $\omega = 2\Omega_{\mathrm{He}}$.) In the region $\omega > \Omega_{\mathrm{H}}$, there exist fundamental H cyclotron and higher harmonic He and H cyclotron modes.

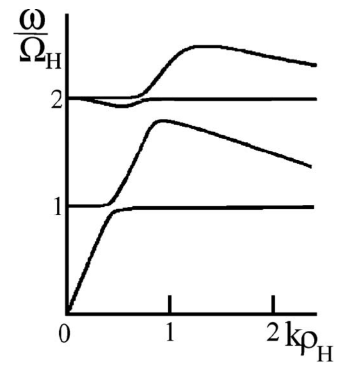


FIG. 1. Dispersion curves of low-frequency perpendicular waves in an H plasma. The beta value (the ratio of kinetic to magnetic energy densities) is taken to be $\beta = 0.2$.

As the number of ion species increases, the number of heavy-ion cyclotron modes and that of heavy-ion cutoff modes both increase. The number of ion-ion hybrid resonance frequencies also increases. We derive dispersion relations of these modes, assuming that the waves propagate in the $x$ direction in a magnetic field in the $z$ direction. By use of the dielectric tensor $\varepsilon$ (see Appendix A), the dispersion tensor for these waves is given by[18]

$$D = \begin{pmatrix} \varepsilon_{xx}, & \varepsilon_{xy} \\ -\varepsilon_{xy}, & \varepsilon_{yy} - c^2k^2/\omega^2 \end{pmatrix}. \quad (5)$$

The dispersion relations are obtained from the determinant of $D$:

$$|D| = \varepsilon_{xx}(\varepsilon_{yy} - c^2k^2/\omega^2) + \varepsilon_{xy}^2 = 0. \quad (6)$$

Because $|\omega| \ll \omega_{\mathrm{LH}}$ and $k\rho_{\mathrm{e}} \ll 1$, we can write $\varepsilon_{xx}$, $\varepsilon_{xy}$, and $\varepsilon_{yy}$ as

$$\varepsilon_{xx} = 1 + \frac{\omega_{\mathrm{pe}}^2}{\Omega_{\mathrm{e}}^2} - \sum_i \sum_{n=1}^{\infty} \frac{2\omega_{pi}^2}{(\omega^2 - n^2\Omega_i^2)} \frac{n^2}{\mu_i} \Gamma_n(\mu_i), \quad (7)$$

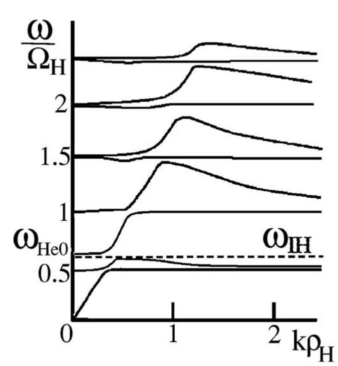


FIG. 2. Schematic dispersion curves in an H-He plasma. The dashed line indicates the ion-ion hybrid resonance frequency.

$$\varepsilon_{xy} = -i\frac{\omega_{\rm pe}^2}{\omega|\Omega_{\rm e}|} - i\sum_i \sum_{n=1}^{\infty} \frac{2\omega_{pi}^2}{\omega(\omega^2 - n^2\Omega_i^2)} n^2\Gamma_n'(\mu_i), \tag{8}$$

$$\varepsilon_{yy} = \varepsilon_{xx} + \sum_i \sum_{n=1}^{\infty} \frac{4\omega_{pi}^2}{(\omega^2 - n^2\Omega_i^2)} \mu_i \Gamma_n'(\mu_i) + \sum_i \frac{2\omega_{pi}^2}{\omega^2}\mu\Gamma_0'(\mu_i), \tag{9}$$

where the subscript $i$ refers to the ion species, $\mu_i = k^2\rho_i^2$, and $\Gamma_n(\mu_i) = I_n(\mu_i)\exp(-\mu_i)$ with $I_n$ the modified Bessel function of the $n$th order. For small $\mu_i$, $\Gamma_n(\mu_i)$ and $\Gamma_n'(\mu_i)$ are given as

$$\Gamma_n(\mu_i) = \frac{1}{n!}\left(\frac{\mu_i}{2}\right)^n (1-\mu_i) + \frac{2n+3}{(n+1)!}\left(\frac{\mu_i}{2}\right)^{n+2}, \tag{10}$$

$$\Gamma_n'(\mu_i) = \frac{n}{n!}\frac{\mu_i^{n-1}}{2^n} - \frac{n+1}{n!}\left(\frac{\mu_i}{2}\right)^n + \frac{(2n+3)(n+2)}{2(n+1)!}\left(\frac{\mu_i}{2}\right)^{n+1}. \tag{11}$$

We then find the dispersion relations of the ion cyclotron modes with $\mu_i \ll 1$ as

$$\omega = n\Omega_i^{(+)} \equiv n\Omega_i + \frac{\omega_{pi}^2}{\omega_{pe}^2}|\Omega_e| \frac{n+2}{(n+1)!}\left(\frac{\mu_i}{2}\right)^{n+1} \quad (n = 1,2,3,\ldots), \tag{12}$$

$$\omega = n\Omega_i^{(-)} \equiv n\Omega_i - \frac{\omega_{pi}^2}{\omega_{pe}^2}|\Omega_e| \frac{n}{(n-1)!}\left(\frac{\mu_i}{2}\right)^{n-1} \quad (n = 2,3,\ldots). \tag{13}$$

The heavy-ion cutoff modes are given as

$$\omega_{s0} = \Omega_s + \frac{\omega_{ps}^2}{\Omega_s} \frac{1}{\sum_{i\neq s} \omega_{pi}^2/\Omega_i/(\Omega_i - \Omega_s)}, \tag{14}$$

where the subscript $s$ denotes heavy-ion species. (For details of the calculations, see Appendix A.)

Slightly above the gyrofrequency of each heavy-ion, there exists an ion-ion hybrid resonance frequency. This is given as

$$\omega_{\rm sr} = \omega_s + \frac{\omega_{ps}^2}{2\Omega_s} \frac{1}{\sum_{i\neq s} \omega_{pi}^2/(\Omega_i^2 - \Omega_s^2)}. \tag{15}$$

The cutoff frequency $\omega_{s0}$ is slightly higher than the resonance frequency $\omega_{\rm sr}$. If the H density is much higher than heavy-ion densities, Eqs. (14) and (15) are reduced to

$$\omega_{s0} = \Omega_s + \frac{\omega_{ps}^2}{\omega_{\rm pH}^2} \frac{\Omega_{\rm H}(\Omega_{\rm H} - \Omega_s)}{\Omega_s}, \tag{16}$$

and

$$\omega_{\rm sr} = \Omega_s + \frac{\omega_{ps}^2}{2\omega_{\rm pH}^2} \frac{(\Omega_{\rm H}^2 - \Omega_s^2)}{\Omega_s}. \tag{17}$$

### B. Power spectra

We study power spectra of electromagnetic fluctuations in a thermal-equilibrium plasma with temperature $T$. The spectra are obtained from the tensor of electric fluctuations,

$$\frac{\boldsymbol{EE}^*(\boldsymbol{k},\omega)}{8\pi} = \frac{k_{\rm B}T}{\omega} Z \cdot \operatorname{Im}\varepsilon \cdot Z^\dagger, \tag{18}$$

where $Z$ is the inverse tensor of $D$ and $Z^\dagger$ is its Hermitian conjucate of $Z$. We find the spectra $|E_x(k,\omega)|^2$ of perpendicular extraordinary waves as

$$\frac{|E_x(k,\omega)|^2}{8\pi} = \frac{\pi k_B T}{\omega}\left(\varepsilon_{yy} - \frac{c^2k^2}{\omega^2}\right)\frac{\delta(\omega-\omega_n)}{\partial|D|/\partial\omega}, \tag{19}$$

where $\omega_n$ is a root of $|D| = 0$. (For details of the calculations, see Appendix B.) In the limit of $k \to \infty$, Eq. (19) gives perpendicular electrostatic fluctuations,[16]

$$\frac{|E_x(k,\omega)|^2}{8\pi} = \frac{\pi k_B T}{\omega}\frac{\delta(\omega-\omega_n)}{\partial|\varepsilon_{xx}|/\partial\omega}. \tag{20}$$

Also, with the aid of the relation, $D \cdot \boldsymbol{E} = 0$,

$$\varepsilon_{xx}E_x(k,\omega) + \varepsilon_{xy}E_y(k,\omega) = 0, \tag{21}$$

and Faraday's law,

$$kE_y(k,\omega) = (\omega/c)B_z(k,\omega), \tag{22}$$

we find magnetic fluctuations as

$$\frac{|B_z(k,\omega)|^2}{8\pi} = P_k(\omega)\delta(\omega-\omega_n) \tag{23}$$

with

$$P_k(\omega) = \frac{\pi k_B T}{\omega}\left(\frac{ck}{\omega}\right)^2 \frac{\varepsilon_{xx}}{\partial|D|/\partial\omega}. \tag{24}$$

We calculate $P_k(\omega)$ of the magnetosonic mode, ion cyclotron modes, and heavy-ion cutoff modes (see Appendix C). We assume that $kv_{\rm A}/\Omega_i \ll 1$ and $k\rho_i \ll 1$. Then, for the magnetosonic mode, we find

$$P_k(kv_{\rm A}) = \frac{\pi k_{\rm B}T/2}{1 + \sum_i (\omega_{pi}^2/\Omega_i^4)(v_{\rm A}^4/c^2)k^2}. \tag{25}$$

For the ion cyclotron modes, (12) and (13), we obtain

$$P_k[n\Omega_i^{(+)}] = \frac{\pi k_B T}{2}\frac{\omega_{pi}^2}{\Omega_i^2}\frac{\Omega_{\rm e}^2}{\omega_{\rm pe}^4}\frac{(n+2)}{n^2(n+1)!}\left(\frac{\mu_i}{2}\right)^{n+1} c^2k^2 \quad (n = 1,2,3,\ldots), \tag{26}$$

$$P_k[n\Omega_i^{(-)}] = \frac{\pi k_B T}{2}\frac{\omega_{pi}^2}{\Omega_i^2}\frac{\Omega_{\rm e}^2}{\omega_{\rm pe}^4}\frac{1}{n!}\left(\frac{\mu_i}{2}\right)^{n-1} c^2k^2 \quad (n = 2,3,\ldots). \tag{27}$$

Because $\mu_i \ll 1$, $P_k[\Omega_i^{(+)}]$ is much smaller than $P_k[2\Omega_i^{(-)}]$. In addition, both $P_k[n\Omega_i^{(+)}]$ and $P_k[n\Omega_i^{(-)}]$ rapidly decrease with increasing $n$. Therefore, if the H ions are major, the $\omega = 2\Omega_{\rm H}^{(-)}$ mode has the largest power among many ion cyclotron modes. The ratio of $P_k[2\Omega_{\rm H}^{(-)}]$ to $P_k(kv_{\rm A})$ is estimated as

$$\frac{P_k[2\Omega_{\rm H}^{(-)}]}{P_k(kv_{\rm A})} \sim \frac{k^2 v_{\rm A}^2}{\Omega_{\rm H}^2}\mu_{\rm H}, \tag{28}$$

indicating that the amplitudes of the ion cyclotron modes are much smaller than that of the magnetosonic mode.

We obtain the spectra of heavy-ion cutoff modes with $\omega \simeq \omega_{s0}$ as

$$P_k(\omega_{s0}) = \frac{\pi k_B T}{2}\frac{\omega_{ps}^2 \Omega_{\rm H}^2}{\omega_{\rm pH}^4 \Omega_s^4}(\Omega_{\rm H} - \Omega_s)^2 c^2 k^2. \tag{29}$$

If $\Omega_s \ll \Omega_{\rm H}$, $P_k(\omega_{s0})$ is written as

$$P_k(\omega_{s0}) \propto \frac{n_s q_{\rm H}^2 m_s^3}{n_{\rm H} q_s^2 m_{\rm H}^3}, \tag{30}$$

indicating that $P_k(\omega_{s0})$ for large $m_s$ can be great.

The ratio between $P_k(\omega_{s0})$ and $P_k[2\Omega_{\rm H}^{(-)}]$ is

$$\frac{P_k[2\Omega_{\rm H}^{(-)}]}{P_k(\omega_{s0})} \sim \frac{\omega_{\rm pH}^2 \Omega_s^4 \mu_{\rm H}}{4\omega_{\rm ps}^2 \Omega_{\rm H}^4}. \tag{31}$$

Hence, $P_k(\omega_{s0})$ is much larger than $P_k[2\Omega_{\rm H}^{(-)}]$ if $\mu_{\rm H} \ll \omega_{\rm ps}^2 \Omega_{\rm H}^4 / (\omega_{\rm pH}^2 \Omega_s^4)$. We also have the relation

$$\frac{P_k(\omega_{s0})}{P_k(kv_{\rm A})} \sim \frac{\omega_{ps}^2}{\omega_{\rm pH}^2}\frac{k^2 v_{\rm A}^2}{\Omega_s^2}. \tag{32}$$

If $kv_{\rm A} \ll \Omega_s$, $P_k(\omega_{s0})$ is much smaller than $P_k(kv_{\rm A})$. If $kv_{\rm A} \sim \Omega_s$, however, $P_k(\omega_{s0})$ can be comparable to $P_k(kv_{\rm A})$.

### C. Autocorrelation functions

The autocorrelation function of $B_k(t)$ is given by the Fourier transformation of $|B_z(k,\omega)|^2$; by use of Eq. (23), we have

$$C_k(\tau) = \sum_n P_k(\omega_n)\exp(-i\omega_n \tau). \tag{33}$$

In a single-ion-species plasma, the magnetosonic wave is the dominant mode. Therefore, $C_k(\tau)$ is a periodic function with a period $2\pi/kv_{\rm A}$,

$$C_k(\tau)/C_k(0) = \cos(kv_{\rm A}\tau). \tag{34}$$

In a multi-ion-species plasma, however, there are many heavy-ion cutoff modes as well as the magnetosonic mode. At $\tau = 2\pi n/(kv_{\rm A})$, $C_k(\tau)$ takes the value

$$\frac{C_k[2n\pi/(kv_{\rm A})]}{C_k(0)} \simeq 1 - \sum_s \frac{\omega_{\rm ps}^2 \Omega_{\rm H}^2}{\omega_{\rm pH}^4 \Omega_s^4}(\Omega_{\rm H} - \Omega_s)^2 \times c^2 k^2 \sin^2\left(\frac{\pi n \omega_{s0}}{kv_{\rm A}}\right). \tag{35}$$

This value would become unity, if $(n\omega_{s0}/kv_{\rm A})$'s are integers for all the heavy-ion species $s$. However, the ratio $\omega_{s0}/kv_{\rm A}$ can be an irrational number. Even if there exist $n$'s satisfying this condition, they must be extremely large. Generally, therefore, the recurrence time of $C_k(\tau)$ should be extremely long.

In a multi-ion-species plasma, the average values of $|C_k(\tau)|$,

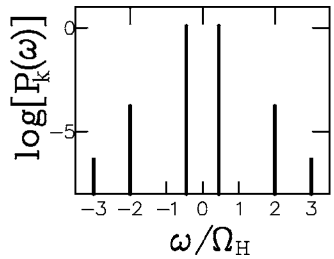


FIG. 3. Power spectrum of magnetic fluctuations for $k\rho_{\rm H}=0.1$ ($kv_{\rm A}=0.42\Omega_{\rm H}$) in a single-ion-species plasma.

$$\langle |C_k| \rangle \equiv \lim_{T\to\infty}\frac{1}{T}\int_0^T d\tau \frac{|C_k(\tau)|}{C_k(0)}, \tag{36}$$

can be small. In fact this is estimated as

$$\langle |C_k| \rangle \sim \frac{2}{\pi}\left(1 - \sum_s \frac{\omega_{\rm ps}^2 \Omega_{\rm H}^2}{\omega_{\rm pH}^4 \Omega_s^4}(\Omega_{\rm H} - \Omega_s)^2 c^2 k^2\right). \tag{37}$$

As the number of ion species increases, $\langle |C_k| \rangle$ decreases, indicating that initial memories are quickly lost.

## III. NUMERICAL STUDY

We numerically obtain $P_k(\omega)$ and $C_k(\tau)$ for four different plasmas: single-ion ($H^+$), two-ion ($H^+$ and $He^{+2}$), three-ion ($H^+$, $He^{+2}$, and $C^{+5}$), and six-ion ($H^+$, $He^{+2}$, $C^{+5}$, $O^{+6}$, $Si^{+9}$, and $Fe^{+13}$) species plasmas. These ions are the six most abundant species in the solar corona; we set the ion charge states assuming that the plasma temperature is $\sim 400$ eV. The gyrofrequencies of these ions normalized to $\Omega_{\rm H}$ are taken to be $\Omega_{\rm He}=0.5$, $\Omega_{\rm C}=0.417$, $\Omega_{\rm O}=0.375$, $\Omega_{\rm Si}=0.321$, and $\Omega_{\rm Fe}=0.232$. The densities of the ions normalized to $n_{\rm H}$ are $n_{\rm He}=0.1$, $n_{\rm C}=n_{\rm O}=0.01$, and $n_{\rm Si}=n_{\rm Fe}=0.005$.

The magnetic field strength is $|\Omega_{\rm e}|/\omega_{\rm pe}=1$. The plasma beta value (the ratio of kinetic to magnetic energy densities) is $\beta=0.0625$. In the calculation of the dielectric tensor (7)–(9), we retain the terms from $n=-10$ to 10 for the ions and the terms from $n=-1$ to 1 for the electrons.

Figure 3 shows the power spectrum $P_k(\omega)$ for $k\rho_{\rm H}=0.1$ in an $H^+$ plasma. The spectrum $P_k(\omega)$ is normalized to $\pi k_B T$. As predicted by the theory in Sec. II, the magnetosonic mode is dominant ($\omega=0.42\Omega_{\rm H} \simeq kv_{\rm A}$), and the amplitudes of the H cyclotron modes are quite small; $P_k[2\Omega_{\rm H}^{(-)}]/P_k(kv_{\rm A}) \sim 10^{-4}$.

Figure 4 shows the power spectra for $k\rho_{\rm H}=0.1$ for the four different plasmas; since $P_k(\omega)=P_k(-\omega)$, we have plotted the region $\omega \geqslant 0$. Here, the amplitudes of the ion cyclotron modes are extremely small. In the two-ion-species plasma, besides the magnetosonic mode ($\omega=0.4\Omega_{\rm H}$), the He cutoff mode appears; its frequency, $\omega \simeq 0.6\Omega_{\rm H}$, is given by Eq. (14). In the three-ion-species plasma, the C cutoff mode is also present at $\omega \simeq 0.45\Omega_{\rm H}$. The middle and right peaks are due to C and He cutoff modes, respectively. In the six-ion-species plasma, the cutoff modes of O, Si, and Fe ions also have amplitudes comparable to that of the magnetosonic mode, even though the abundances of the heavy ions are quite small. The six peaks in the bottom panel correspond to, from left to right, Fe, Si cutoff, magnetosonic, O, C, and He cutoff modes, respectively.

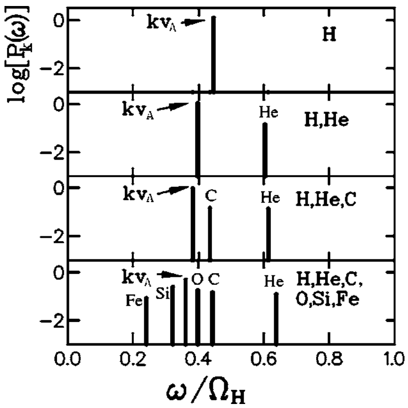


FIG. 4. Power spectra for $k\rho_{\mathrm{H}}=0.1$ for four different plasmas.

Figure 5 shows autocorrelation functions normalized to their initial values $C_k(0)$ for the same plasmas as in Fig. 4. In the single-ion-species plasma, $C_k(\tau)$ oscillates with the period $2\pi/(kv_{\mathrm{A}})$ and is undamped. In the two-ion-species plasma, $C_k(\tau)$ is initially damped for $\tau \lesssim 20/\Omega_{\mathrm{H}}$. Because the ratio $\omega_{\mathrm{He0}}/kv_{\mathrm{A}}$ happens to be very close to $3/2$, $C_k(\tau)$ almost returns to the initial value at $\tau \simeq 32/\Omega_{\mathrm{H}}$; this time is almost equal to $4\pi/kv_{\mathrm{A}}$ and $6\pi/\omega_{\mathrm{He0}}$. In the three- and six-ion-species plasmas, however, $C_{\mathrm{k}}(\tau)$'s do not recover until the end of the calculation. Also, we find that the damping rate increases with increasing number of ion species. The damping is due to the phase mixing of the magnetosonic mode and many heavy-ion cutoff modes. If we observe $C_k(\tau)$ for a much longer time, it would return to its initial value at the time of the least common multiple of all the wave periods. However, this time would be extremely long.

Figure 6 shows the values of $|C_k(\tau)|$ averaged over time from $\Omega_{\mathrm{H}}\tau=0$ to 200, $\langle|C_k(\tau)|\rangle$, for the single-, two-, and six-ion-species plasmas. The vertical dotted lines indicates the cutoff frequencies of Fe, Si, C, O, and He ions. In the single-ion-species plasma [denoted by the label (1)], the values of $\langle|C_k|\rangle$ depend little on $k$ and are larger than those in the two- and six-ion-species plasmas [denoted by (2) and (6), respectively]. In the two-ion-species plasma, $\langle|C_k|\rangle$'s are reduced for $\omega_{\mathrm{He0}} \lesssim kv_{\mathrm{A}} < \Omega_{\mathrm{H}}$, owing to the He cutoff mode. In the six-ion-species plasma, $|C_k|$'s are smaller for

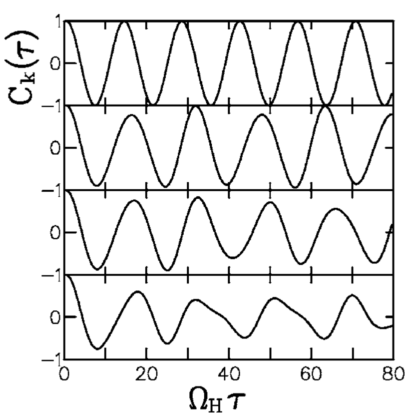


FIG. 5. Autocorrelation functions for $k\rho_{\mathrm{H}}=0.1$. The parameters are the same as those in Fig. 4.

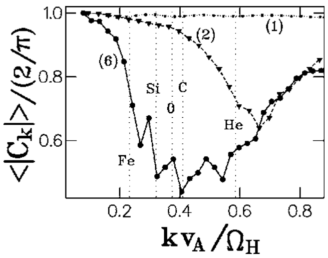


FIG. 6. Dependence of time-averaged $|C_k(\tau)|$ on the wavenumber $k$ and on the number of ion species. Here, $\langle|C_k|\rangle$ for three cases (single-, two-, and six-ion-species plasmas) are shown. The five dotted vertical lines indicate heavy-ion cutoff frequencies.

$\omega_{\mathrm{Fe0}} \lesssim kv_{\mathrm{A}} < \Omega_{\mathrm{H}}$. This is also caused by the heavy-ion cutoff modes. As the number of ion species increases, $\langle|C_k|\rangle$ becomes smaller, and the wavenumber range of small $\langle|C_k|\rangle$ increases.

## IV. SUMMARY AND DISCUSSION

We have theoretically studied low-frequency ($\omega \ll \omega_{\mathrm{LH}}$), long-wavelength ($k\rho_i \ll 1$), perpendicular electromagnetic fluctuations in thermal equilibrium plasmas, giving special attention to the effects of multiple ion species. In a plasma containing H and heavy ions, three types of modes exist; the magnetosonic mode with $\omega \simeq kv_{\mathrm{A}}$, ion cyclotron modes with $\omega \simeq n\Omega_i$, and heavy-ion cutoff modes with frequencies slightly higher than the ion-ion hybrid resonance frequencies. We have analytically obtained the dispersion relations and power spectra for these modes. The amplitudes of the ion cyclotron modes are quite small, while those of the magnetosonic mode and heavy-ion cutoff modes can be large.

Next, we have numerically calculated power spectra $P_k(\omega)$ and autocorrelation functions $C_k(\tau)$. In a single-ion-species plasma, $C_k(\tau)$ oscillates with period $2\pi/(kv_{\mathrm{A}})$ and is undamped, because the magnetosonic mode is an overwhelmingly dominant mode. In a multi-ion-species plasma, the amplitudes of the heavy-ion cutoff modes can be comparable to that of the magnetosonic mode, if $kv_{\mathrm{A}}$ is of the order of the heavy-ion cutoff frequencies. Owing to the presence of heavy-ion cutoff modes, $C_k(\tau)$ is initially damped. In a plasma containing many ion species, $C_k(\tau)$'s are quickly damped for a wide range of $k$. The recurrence time of $C_k(\tau)$ is extremely long; practically, it will not be recovered to its initial value.

We need a large device to do experiments of the long-wavelength perturbations. The device length perpendicular to the magnetic field should be several times as long as the ion skin depth $c/\omega_{\mathrm{pi}}$. Also, to test the present theory, we must observe magnetic perturbations for periods $\Omega_i t \sim 100$.

The long-wavelength magnetohydrodynamic perturbations should be important in plasma dynamics in, for instance, the solar corona, where the number of ion species is quite large and each ion species has many different ionic charge states, and thus there are numerous heavy-ion cutoff modes. Collisions or some nonlinear effects could destroy the initial memories before the recurrence of the autocorrelation function. This suggests that the initial damping due to multiple ion species can influence energy transport.

In a future paper, we will study with particle simulations the evolution of macroscopic magnetohydrodynamic perturbations in a multi-ion-species plasma. We will compare the power spectra and autocorrelation functions of the perturbations with those given in the present theory. The energy transport associated with these perturbations will also be investigated.

## APPENDIX A: DERIVATION OF DISPERSION RELATIONS

We consider electromagnetic waves in a spatially homogeneous plasma in an external magnetic field in the $z$ direction. The dielectric tensor for these waves is written as[18]

$$\varepsilon = \left(1 - \sum_j \frac{\omega_{pj}^2}{\omega^2}\right)\boldsymbol{I} + \sum_j \frac{\omega_{pj}^2}{\omega^2} \sum_{n=-\infty}^{\infty} \left[\int d\boldsymbol{v} \frac{(n\Omega_j/v_\perp)(\partial f_j/\partial v_\perp + k_\parallel \partial f_j/\partial v_\parallel)\Lambda_j}{\omega - k_\parallel v_\parallel - n\Omega_j + i\nu}\right], \tag{A1}$$

where $\boldsymbol{I}$ is the unit tensor, $f_j$ is the velocity distribution function, and the subscript $j$ refers to electrons (e) or ion species (H, He, C, ...). The small quantity $\nu(>0)$ is introduced to define the tensor $\varepsilon$ for all real $\omega$; eventually, we let $\nu \to +0$. The subscripts $\parallel$ and $\perp$ denote quantities parallel and perpendicular to the external magnetic field, respectively.

The dispersion tensor for the extraordinary waves propagating in the $x$ direction is given as Eq. (5). We have the components of dielectric tensor (7)–(9), for low-frequency ($\omega \ll \omega_{\mathrm{LH}}$) waves.

We derive the dispersion relations of ion cyclotron modes with $\mu_i$ ($\equiv k^2\rho_i^2$)$\ll 1$, assuming that their frequencies are written as

$$\omega = n\Omega_i(1+\Delta) \quad (|\Delta| \ll 1). \tag{A2}$$

The components of dielectric tensor become

$$\varepsilon_{xx} \cong -\frac{\omega_{pi}^2}{\Omega_i^2\Delta}\frac{\Gamma_n(\mu_i)}{\mu_i}, \tag{A3}$$

$$\varepsilon_{yy} \cong \varepsilon_{xx} + \frac{2\omega_{pi}^2}{n\Omega_i^2\Delta}\mu_i\Gamma_n'(\mu_i), \tag{A4}$$

$$\varepsilon_{xy} \cong -i\frac{\omega_{pe}^2}{n\Omega_i|\Omega_e|} - i\frac{\omega_{pi}^2}{n\Omega_i^2\Delta}\Gamma_n'(\mu_i). \tag{A5}$$

We will show later, below Eq. (A12), the behavior of these quantities near $\mu_i=0$. We also assume that $(ck/\omega)^2 \ll 1$. Then, the dispersion relations are given by

$$|D| = \varepsilon_{xx}(\varepsilon_{xx} + \delta\varepsilon_{yy}) + \varepsilon_{xy}^2 = 0, \tag{A6}$$

where $\delta\varepsilon_{yy}$ is

$$\delta\varepsilon_{yy} = \varepsilon_{yy} - \varepsilon_{xx}. \tag{A7}$$

One can show that

$$|\delta\varepsilon_{yy}| \ll |\varepsilon_{xx}|, |\varepsilon_{xy}|. \tag{A8}$$

It then follows from Eq. (A6) that

$$i\varepsilon_{xy} = \pm\varepsilon_{xx}\left(1 + \frac{\delta\varepsilon_{yy}}{2\varepsilon_{xx}}\right). \tag{A9}$$

Substituting Eqs. (A3), (A5), and (A7) in Eq. (A9) yields

$$\frac{\omega_{pe}^2}{n\Omega_i|\Omega_e|} = -\frac{\omega_{pi}^2}{n^2\Omega_i^2\Delta}\left[(n \mp \mu_i)\Gamma_n'(\mu_i) \pm \frac{n^2}{\mu_i}\Gamma_n(\mu_i)\right]. \tag{A10}$$

Equation (A10) gives two solutions for $\Delta$:

$$\Delta = \frac{\omega_{pi}^2|\Omega_e|}{\omega_{pe}^2\Omega_i}\frac{n+2}{n(n+1)!}\left(\frac{\mu_i}{2}\right)^{n+1} \quad (n=1,2,3,\ldots), \tag{A11}$$

$$\Delta = -\frac{\omega_{pi}^2|\Omega_e|}{\omega_{pe}^2\Omega_i}\frac{1}{(n-1)!}\left(\frac{\mu_i}{2}\right)^{n-1} \quad (n=2,3,\ldots), \tag{A12}$$

where Eqs. (10) and (11) have been used. We thus have the dispersion relations of the ion cyclotron modes, Eqs. (12) and (13). We note that $\Delta$'s go to zero as $\mu_i \to 0$. When $\Delta$ is given by Eq. (A12), the quantities $\Gamma_n/(\Delta\mu_i)$ and $\Gamma_n'/\Delta$ are proportional to $\mu_i^0$ for $\mu_i \sim 0$. Hence, the quantities $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\varepsilon_{xy}$ are finite when $\mu_i=0$. When $\Delta$ is given by Eq. (A11), $\Gamma_n/(\Delta\mu_i)$and $\Gamma_n'/\Delta$ are of the order of $\mu_i^{-2}$, while $\mu_i\Gamma_n'/\Delta \sim \mu_i^{-1}$. In Eq. (A6), however, these divergent terms cancel.

Retaining the $n=1$ terms of all the ions in Eqs. (7)–(9), we obtain the equation for the frequencies of heavy-ion cutoff modes,

$$-\sum_i \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2} = \frac{\omega_{pe}^2}{\omega|\Omega_e|} - \sum_i \frac{\omega_{pi}^2\Omega_i}{\omega(\omega^2 - \Omega_i^2)}. \tag{A13}$$

Assuming that the cutoff frequency $\omega_{s0}$ is close to $\Omega_s$ ($|\omega_{s0} - \Omega_s| \ll \Omega_s$), where the subscript $s$ denotes heavy ion species, we find $\omega_{s0}$ as Eq. (14).

The ion-ion hybrid resonance frequencies[1] are derived from $\varepsilon_{xx}=0$ with $\mu_i=0$:

$$\sum_i \frac{\omega_{pi^2}}{(\omega^2 - \Omega_i^2)} = 0. \tag{A14}$$

Assuming that the resonance frequency $\omega_{\mathrm{sr}}$ is close to $\Omega_s$, we obtain Eq. (15).

## APPENDIX B: DERIVATION OF ELECTRIC FLUCTUATIONS

The tensor for electric fluctuations in thermal equilibrium plasma in a uniform magnetic field is given by[19]

$$\frac{\boldsymbol{E}\boldsymbol{E}^*(\boldsymbol{k},\omega)}{8\pi} = \sum_j \frac{\omega_{pj}^2 m_j}{\omega^2} Z \cdot \left[\sum_n \int d\boldsymbol{v}\Lambda_j f_j(v_\perp, v_\parallel) \times \delta(\omega - k_\parallel v_\parallel - n\Omega_j + i\nu)\right] \cdot Z^\dagger. \tag{B1}$$

In an isothermal Maxwellian plasma with temperature $T$, Eq. (B1) is reduced to Eq. (18).

For extraordinary waves propagating in the $x$ direction, Eq. (18) gives

$$\frac{|E_x(k,\omega)|^2}{8\pi} = \frac{k_BT}{\omega}\sum_{\sigma=x,y}\sum_{\sigma'=x,y} Z_{x\sigma}\mathrm{Im}\,\varepsilon_{\sigma\sigma'}Z^{\dagger}_{\sigma' x}. \tag{B2}$$

We set $k_{\parallel}=0$ and take the limit of $\nu \to +0$ in Eq. (B2). We here write the real and imaginary parts of $\varepsilon$ as

$$\varepsilon^{(0)} \equiv \mathrm{Re}\,\varepsilon, \quad \varepsilon^{(1)} \equiv \mathrm{Im}\,\varepsilon. \tag{B3}$$

Assuming that $\varepsilon^{(1)}$ is small and that the second-order terms of $\varepsilon^{(1)}$ are negligible, we put the determinant of $D$ into the form

$$|D| = |D^{(0)}| + i|D^{(1)}|, \tag{B4}$$

where $|D^{(0)}|$ and $|D^{(1)}|$ are

$$|D^{(0)}| = \varepsilon_{xx}^{(0)}(\varepsilon_{yy}^{(0)} - c^2k^2/\omega^2) + \varepsilon_{xy}^{(0)2}, \tag{B5}$$

$$|D^{(1)}| = (\varepsilon_{yy}^{(0)} - c^2k^2/\omega^2)\varepsilon_{xx}^{(1)} + \varepsilon_{xx}^{(0)}\varepsilon_{yy}^{(1)} + 2\varepsilon_{xy}^{(0)}\varepsilon_{xy}^{(1)}. \tag{B6}$$

Using Eqs. (B3)–(B6), we write Eq. (B2) as

$$\frac{|E_x(k,\omega)|^2}{8\pi} = \frac{k_BT}{\omega}\frac{|D^{(1)}|}{(|D^{(0)}|^2 + |D^{(1)}|)^2}\left[\left(\varepsilon_{yy}^{(0)} - \frac{c^2k^2}{\omega_2}\right) - |D^{(0)}|\frac{\varepsilon_{yy}^{(1)}}{|D^{(1)}|}\right]. \tag{B7}$$

In the limit of $\nu \to +0$, each term of $\varepsilon^{(1)}$ goes to zero; hence, $\varepsilon \to \varepsilon^{(0)}$. With the aid of the relation

$$\lim_{\mu\to 0}\frac{\mu}{f(x)^2+\mu^2} = \pi\delta(f(x)) = \pi\frac{\delta(x-x_n)}{\partial f(x)/\partial x}, \tag{B8}$$

where $x_n$ is the root of $f(x)=0$, we obtain $|E_x(k,\omega)|^2$ of perpendicular extraordinary waves as Eq. (19).

## APPENDIX C: DERIVATION OF $P_k(\omega)$

We calculate the power spectra of magnetosonic mode, ion cyclotron modes, and heavy-ion cutoff modes from Eq. (24). For the magnetosonic mode, we retain the $n=1$ terms of all the ion species in Eqs. (7)–(9). Assuming that $\omega \ll \Omega_i$, we find that

$$\varepsilon_{xx} \simeq \varepsilon_{yy} \simeq c^2/v_{\mathrm{A}}^2, \tag{C1}$$

$$\varepsilon_{xy} = 0, \tag{C2}$$

$$\frac{\partial\varepsilon_{xx}}{\partial\omega} = \frac{\partial\varepsilon_{yy}}{\partial\omega} = \sum_i \frac{2\omega_{pi}^2}{\Omega_i^4}\omega. \tag{C3}$$

Substituting these equations into $\partial|D|/\partial\omega$ gives

$$\frac{\partial|D|}{\partial\omega} = \sum_i \frac{2\omega_{pi}^2}{\Omega_i^4}\omega\left(2\frac{c^2}{v_{\mathrm{A}}^2} - \frac{c^2k^2}{\omega^2}\right) + 2\frac{c^2}{v_{\mathrm{A}}^2}\frac{c^2k^2}{\omega^3}. \tag{C4}$$

Then, from Eqs. (24) and (C4), we obtain the spectrum of the magnetosonic mode as

$$P_k(kv_{\mathrm{A}}) = \frac{\pi k_BT/2}{1 + \sum_i (\omega_{pi}^2/\Omega_i^4)(v_{\mathrm{A}}^4/c^2)k^2}. \tag{C5}$$

For the ion cyclotron modes with $\omega \simeq n\Omega_i$ and $\mu_i \ll 1$, we have

$$\varepsilon_{xx} = \varepsilon_{yy} = -\frac{\omega_{pi}^2}{(\omega^2 - n\Omega_i^2)}\left(\frac{n^2}{n!}\right)\left(\frac{\mu_i}{2}\right)^{n-1}, \tag{C6}$$

$$\varepsilon_{xy} = -i\frac{\omega_{pe}^2}{\omega|\Omega_e|} - \frac{\omega_{pi}^2\Omega_i}{\omega(\omega^2 - n\Omega_i^2)}\left(\frac{n^3}{n!}\right)\left(\frac{\mu_i}{2}\right)^{n-1}, \tag{C7}$$

$$\frac{\partial\varepsilon_{xx}}{\partial\omega} = \frac{\partial\varepsilon_{yy}}{\partial\omega} = -\frac{2\omega_{pi}^2\omega}{(\omega^2 - n\Omega_i^2)^2}\left(\frac{n^2}{n!}\right)\left(\frac{\mu_i}{2}\right)^{n-1}, \tag{C8}$$

$$\frac{\partial\varepsilon_{xy}}{\partial\omega} = -\frac{\omega_{pi}^2(3\omega^2 - n^2\Omega_i^2)}{\omega^2(\omega^2 - n\Omega_i^2)^2}\left(\frac{n^3}{n!}\right)\left(\frac{\mu_i}{2}\right)^{n-1}. \tag{C9}$$

It then follows that

$$\frac{\partial|D|}{\partial\omega} = \frac{\omega_{pe}^2\omega_{pi}^2}{\Omega_i^4|\Omega_e|(\omega - n\Omega_i)^2}\frac{1}{n^2n!}\left(\frac{\mu_i}{2}\right)^{n-1} \tag{C10}$$

and

$$P_k[n\Omega_i(1+\Delta)] = \frac{\pi k_{\mathrm{B}}T}{2}\frac{c^2k^2|\Omega_e|}{\Omega_i\omega_{pe}^2}\frac{|\Delta|}{n}. \tag{C11}$$

Substituting Eqs. (A11) and (A12) into Eq. (C11), we obtain the spectra of the ion cyclotron modes as Eqs. (26) and (27).

For the heavy-ion cutoff modes with $\omega \simeq \omega_{s0}$, we have

$$\varepsilon_{xx} = \varepsilon_{yy} = -\frac{\omega_{pH}^2}{\omega^2 - \Omega_{\mathrm{H}}^2} - \frac{\omega_{ps}^2}{\omega^2 - \Omega_s^2}, \tag{C12}$$

$$\varepsilon_{xy} = -i\frac{\omega_{pe}^2}{\omega|\Omega_e|} - i\frac{\omega_{\mathrm{pH}}^2}{\omega(\omega^2 - \Omega_{\mathrm{H}}^2)} - \frac{\omega_{ps}^2}{\omega(\omega^2 - \Omega_s^2)}, \tag{C13}$$

where we retained the $n=1$ terms for H and heavy ion $s$ in Eqs. (7)–(9). If $\omega_{\mathrm{pH}} \gg \omega_{\mathrm{ps}}$, $\partial\varepsilon_{xx}/\partial\omega$ and $\partial\varepsilon_{xy}/\partial\omega$ are approximated as

$$\frac{\partial\varepsilon_{xx}}{\partial\omega} \simeq \frac{2\omega\omega_{ps}^2}{(\omega^2 - \Omega_s)^2}, \tag{C14}$$

$$\frac{\partial\varepsilon_{xy}}{\partial\omega} \simeq \frac{2\omega_s\omega_{ps}^2}{(\omega^2 - \Omega_s)^2}, \tag{C15}$$

which gives

$$\frac{\partial|D|}{\partial\omega} = 4\varepsilon_{xx}\frac{\omega_{ps}^2(\omega + \Omega_s)}{(\omega^2 - \Omega_s^2)^2}. \tag{C16}$$

We thus find the spectra of the heavy-ion cutoff modes as Eq. (29).

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