# The maximum energy of $^3$He ions accelerated by current-driven instabilities

Mieko Toida[a)] and Hayato Okumura
*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*

(Received 28 July 2003; accepted 30 December 2003; published online 1 April 2004)

Nonlinear development of strong current-driven instabilities can cause selective acceleration of $^3$He ions through H cyclotron waves with frequencies $\omega \simeq 2\Omega_{^3\mathrm{He}}$ ($\Omega_{^3\mathrm{He}}$ is the cyclotron frequency of $^3$He). Energies of these ions are studied theoretically and numerically. Theoretical expressions for the wave numbers, $k_\parallel$ and $k_\perp$, of the H cyclotron waves with $\omega \simeq 2\Omega_{^3\mathrm{He}}$ and for the maximum energy of $^3$He ions are presented. As initial electron drift speed $v_d$ increases, the value of $k_\perp$ decreases; which leads to the increase in the maximum $^3$He energy. Simulation results for the wave numbers and for the maximum $^3$He energy obtained by a two-dimensional, electrostatic, particle code are quantitatively in good agreement with the theory. It is also shown that when the initial electron drift energy is of the order of 10 keV, many $^3$He ions can be accelerated to energies of the order of MeV/$n$. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1650846]

## I. INTRODUCTION

During solar flares, high-energy particles are produced. The compositions of these energetic particles are different between gradual and impulsive flares; the two categories of solar flares are commonly divided on the basis of duration of soft x-ray emission.[1]

Energetic particles from the gradual flares are dominated by protons, and the elemental compositions of the heavy ions are, on average, similar to those of the background solar coronal plasma.[2,3] It has been experimentally realized that the energetic ions in these events are produced by shock waves (Ref. 4 and references therein). Theoretically, a large-amplitude magnetosonic shock wave accelerates all the heavy ions to nearly the same speed,[5,6] which would explain the observed compositions of the energetic heavy ions.

On the other hand, in impulsive flares, the abundance of energetic $^3$He ions is extremely enhanced.[7–10] The abundance ratio of $^3$He/$^4$He in energetic particles sometimes exceeds unity, although this ratio is usually $\sim 10^{-4}$ in the solar corona. In these $^3$He rich events, the abundances of Fe, Si, Mg, etc., also tend to increase. Very high charged states were reported for Fe ions;[11,12] this suggests that the electron temperature is considerably beyond 1 keV in the acceleration region.

The $^3$He rich events are associated with enhancement of 1–100 keV electrons[13,14] and possess impulsive hard x-ray emission in the early phases of the flares (Ref. 9 and references therein). A standard interpretation of this hard x-ray emission is that the electrons are initially accelerated to energies $\gtrsim 10$ keV, and then stream along the magnetic field toward and into the chromosphere, producing bremsstrahlung via interaction with ambient protons. The number of energetic electrons can be comparable to the electron content of the whole flare region ($\sim 10^{38}$) (e.g., Ref. 15). The hard x-ray emission is observed to be followed, with a little time delay, by a soft x-ray emission from heated plasma.[16,17] This indicates that the electrons exist initially as an energetic beam with relatively cold temperature and are heated later. The density and velocity distribution of the electrons at the original acceleration place are unclear, and the electron acceleration mechanism has been unresolved.[18] However, it seems plausible that, near the original place, the density of the energetic electrons is so high that the total electron velocity distribution (including background electrons with zero stream speed) has a positive slope.

As a mechanism for selective energization of $^3$He ions, cyclotron resonances with waves are believed to be important, and several theories have been developed over the years (Refs. 19–24 and references therein). There are, however, some conflicts between these models and observations. For example, Fisk[19] proposed that the resonant heating of $^3$He ions is due to electrostatic $^4$He cyclotron waves with frequencies $\omega \simeq \Omega_{^3\mathrm{He}}$ ($\Omega_{^3\mathrm{He}}$ is the cyclotron frequency of $^3$He ions). There, as a wave generation mechanism, instabilities due to electron currents parallel to the magnetic field were studied based on the linear theory; the entire electrons were assumed to have a shifted Maxwellian distribution. Fisk's model requires a high $^4$He-to-H density ratio, $n_{^4\mathrm{He}}/n_\mathrm{H} \gtrsim 0.2$. In Ref. 23, it was shown that H cyclotron waves with $\omega \simeq 2\Omega_{^3\mathrm{He}}$ are destabilized by electron currents if the electron temperature $T_e$ is sufficiently higher than the H temperature $T_\mathrm{H}$. This condition agrees with the observations because the electron temperature in the acceleration region tends to be considerably beyond 1 keV (Refs. 11 and 12). However, the acceleration of $^3$He ions to energies of the order of MeV/$n$ cannot be explained.

Furthermore, these previous models assumed that the instabilities are driven by very weak currents with $v_d < v_{Te}$; here, $v_d$ is the initial electron drift speed along the magnetic field, and $v_{Te}$ is the initial electron thermal speed. However, as mentioned above, energetic electron beams with a relatively cold temperature ($v_d > v_{Te}$) can exist in the early phases of the flares. On the basis of the linear stability theory, it was expected that such electrons would destabilize many kinds of waves with a wide range of frequencies and

[a)]Electronic mail: toida@plab.phys.nagoya-u.ac.jp

 

that selective acceleration of $^3$He ions would be difficult.[23]

Another mechanism for the enhancement of energetic $^3$He ions was presented by Temerin and Roth,[25] who stated that the selective acceleration of $^3$He ions is due to electromagnetic H cyclotron waves with $\omega \simeq \Omega_{^3\mathrm{He}}$. Motivated by the Earth's auroral observations of keV electron beams and electromagnetic H cyclotron waves, they considered instabilities via electron beams in the background electrons and ions. Miller and Viñas[26] later studied their model in detail and predicted that an electron beam with several keV (the beam velocity is assumed to be larger than its thermal velocity, $v_d > v_{Te}$) could produce $^3$He ions with energies of the order of MeV/$n$ via the electromagnetic H cyclotron waves. In these studies, however, the effects of two-stream and electron-plasma waves were neglected, although these waves have much greater growth rates than electromagnetic H cyclotron waves. The fast growing waves would change the electron velocity distribution significantly and could prevent the electromagnetic H cyclotron waves from growing.

Recently, Paesold *et al.*[27,28] presented that parallel propagating, left-hand polarized electromagnetic waves driven by electron firehose instabilities, can selectively accelerate $^3$He ions. These electron firehose instabilities are excited by the electron temperature anisotropy ($T_{e\parallel} > T_{e\perp}$) that is expected to occur in the course of the energization of electrons during the impulsive phase of flares; here the subscripts $\parallel$ and $\perp$ denote quantities parallel and perpendicular to the magnetic field, respectively. It was suggested that these instabilities could account for the acceleration of ions up to MeV/$n$. However, the wave amplitude was assumed to be determined by the linear growth rates, although the wave saturation is a nonlinear process.

All the above models were thus based on the linear theory of plasma instabilities. In order to study the associated energy transport to ions, nonlinear effects of the instabilities should be investigated in a self-consistent manner. Recently, current-driven instabilities in a multi-ion-species plasma have been investigated by means of a two-dimensional (two space and three velocity components), electrostatic particle simulation code with full ion and electron dynamics[29–31] where all the electrons were assumed to be initially drifting. It has been found that the nonlinear effects of the instabilities strongly influence energy transfer from electrons to heavy ions. (These nonlinear effects were not shown in Ref. 32 where the authors were motivated by observations of ion heating in the Earth's ionosphere and studied electrostatic ion cyclotron instabilities, which were driven by electrons with a shifted Maxwellian distribution.) Electron currents with $v_d > v_{Te}$ destabilize Buneman waves at first,[33] as predicted by the linear theory. However, the Buneman waves quickly saturate owing to electron trapping.[34–36] The electron trapping drastically changes the shape of electron velocity distribution function, $f_e(v_\parallel)$, and significantly broadens its width. Because of this broadening of $f_e(v_\parallel)$, the effective electron temperature $T_{e\parallel}$ rises significantly. The saturation level of $T_{e\parallel}$ is estimated as

$$T_{e\parallel} \sim m_e v_d^2/2. \tag{1}$$

On the other hand, the ion temperature rises little. Thus, a plasma with $T_{e\parallel}$ higher than $T_\mathrm{H}$ is produced. The change in $f_e(v_\parallel)$ then destabilizes the electrostatic H cyclotron waves which were marginal in the initial state; the phase velocities of these waves are

$$\omega/k_\parallel \simeq v_d. \tag{2}$$

These waves eventually grow to the largest amplitudes and strongly influence energy transport to heavy ions.

In Ref. 37, particle simulations were carried out for a plasma containing H, $^4$He, and $^3$He ions, and it was demonstrated that nonlinear development of current-driven instabilities with $v_d > v_{Te}$ can cause selective acceleration of $^3$He ions. This acceleration is caused by H cyclotron waves with frequencies near $2\Omega_{^3\mathrm{He}}$. The dispersion relation was theoretically and numerically analyzed in order to show the condition where these waves become dominant. The growth rates of these waves can be written as

$$\gamma = \alpha_n \left. \frac{\partial g(v_\parallel)}{\partial v_\parallel} \right|_{v_\parallel = \omega/k_\parallel}, \tag{3}$$

where $g(v_\parallel)$ is the electron parallel distribution function (the expression for $\alpha_n$ will be presented in Sec. II). It was found that $\alpha_n$ have the largest values at $\omega \simeq 2\Omega_{^3\mathrm{He}}$, if $T_{e\parallel} \gtrsim 10 T_\mathrm{H}$. (The frequencies and growth rates are almost independent of the values of the magnetic field and the plasma density.) These results indicate that if an electron beam with its initial drift energy higher than 1 keV exists, and the effective electron temperature becomes $T_{e\parallel} \gtrsim 1$ keV, in the solar corona where $T_\mathrm{H} = 100$ eV, the H cyclotron waves with $\omega \simeq 2\Omega_{^3\mathrm{He}}$ would eventually grow to large amplitudes and would preferentially accelerate $^3$He ions. This prediction agrees with the observation that $^3$He rich events are almost accompanied by energetic electrons with 1–100 keV,[9,13,14] and the electron temperature in the acceleration region is high.[11,12]

In the simulation, the entire electrons were assumed to have a shifted Maxwellian distribution at the time $t=0$, and background electrons with $v_d = 0$ were not included. However, the theory, expressed by Eqs. (1), (2), and (3), can be applied to a situation where background electrons exist. (The initial growing waves are electron plasma waves when the background temperature is relatively cold. The electron plasma waves change the electron velocity distribution function, which destabilize H cyclotron waves with $\omega/k_\parallel \sim v_d$.) The theory gives a good estimate of when the density of the energetic electrons is high.

In these studies, however, the maximum energy of $^3$He ions was not given. In this paper, we have obtained the maximum energy using theory and simulation. We then show that if the initial electron energy is of the order of 10 keV (and the electron temperature becomes $T_{e\parallel} \sim 10$ keV), many $^3$He ions with energies of the order of MeV/$n$ would be produced. These results explain the observations that, in $^3$He rich events, many $^3$He ions are accelerated to energies of the order of MeV/$n$.[9,10]

In Sec. II, we give theoretical expressions for the wave numbers, $k_\parallel$ and $k_\perp$, of the H cyclotron waves with $\omega$

$\simeq 2\Omega_{^3\mathrm{He}}$, assuming that the wave phase velocities are $\omega/k_\parallel \sim v_d$ and that the effective electron temperature is $T_{e\parallel} \sim m_e v_d^2$. We will also give a theoretical estimate for the maximum energy of $^3$He ions accelerated by these waves. It is then found that as the initial electron drift speed $v_d$ increases, the value of $k_\perp$ decreases, and the maximum $^3$He energy thus increases. In Sec. III, we present results obtained by a two-dimensional, electrostatic, particle code with full ion and electron dynamics. Setting electrons to have a shifted Maxwellian distribution at $t=0$, we study how the wave numbers of the H cyclotron waves and the maximum $^3$He energy depend on $v_d$. The observed wave numbers and the observed maximum $^3$He energy are both in agreement with the theory. It is also shown that as the $^3$He density decreases, the fraction of high-energy $^3$He ions increases. In Sec. IV, we compare the theory with the observations. Assuming that the initial electron drift energy is of the order of 10 keV and that the electron temperature also becomes this value, we estimate $k_\parallel$ and $k_\perp$ of the H cyclotron wave with $\omega = 2\Omega_{^3\mathrm{He}}$. We then show that the corresponding maximum $^3$He energy is of the order of MeV/$n$. Furthermore, by numerically calculating the orbits of many test particles in the wave, we demonstrate that many $^3$He ions can be accelerated to these energies. In Sec. V, we summarize our work.

In our theoretical model, we assume that the original density of the energetic electrons is high and that a positive slope exists in electron velocity distribution function. It is deduced from the spectrum of the bremsstrahlung in the solar flares that the velocity distribution of the energetic electrons has no positive slope.[18] This discrepancy can be explained as follows: We consider an energetic electron beam produced in the initial stage of the flare and assume that its beam density is high and its stream velocity parallel magnetic field is larger than its thermal velocity. This situation is very unstable, and relaxation to the distribution without the positive slope can occur in a short time (according to the simulation results, it can be of the order of, at most, the ion cyclotron period). The ratio of its relaxation time to the slowing-down time of the electrons through collisions with background ions can be estimated as $\sim 10^{-2}$, where the magnetic field, ion density, and electron energy are assumed to be $B=100$ G, $n_\mathrm{H}=10^{10}$ cm$^{-3}$, and $m_e v_d^2 = 10$ keV, respectively. Therefore, before the bremsstrahlung occurs, the density of the energetic electrons would be significantly reduced and the positive slope in the distribution would be relaxed.

## II. THEORY

We consider instabilities by energetic electrons that are expected to be produced in the initial phases of flares and to then stream along the magnetic field. We assume that the electron stream velocity $v_d$ is larger than its thermal speed $v_{Te}$, and the beam density is so high that the total electron velocity distribution has a positive slope. We also assume that after development of the first growing waves, effective electron temperature becomes $T_{e\parallel} \sim m_e v_d^2$, and then electrostatic H cyclotron waves with phase velocity $\omega/k_\parallel \simeq v_d$ are destabilized. We estimate the wave numbers of the H cyclotron waves and the maximum energy of $^3$He ions accelerated by these waves. Inhomogeneities of the background plasma and the magnetic field are neglected.

### A. Wave number of H cyclotron waves

We give theoretical expressions for the wave numbers of H cyclotron waves with $\omega \simeq 2\Omega_{^3\mathrm{He}}$ in a plasma with $T_{e\parallel} > T_{e\perp}$, $T_\mathrm{H}$. In the case of strong current-driven instabilities, the electron temperature is related to the initial electron drift speed as $T_{e\parallel} \sim m_e v_d^2$. It was theoretically and numerically shown that the waves with $\omega \simeq 2\Omega_{^3\mathrm{He}}$ and $\omega/k_\parallel \sim v_d$ grow to the largest amplitudes if $T_{e\parallel} \gtrsim 10 T_\mathrm{H}$.[37]

We assume that the electron distribution function is written as

$$f(v_\parallel, v_\perp) = \frac{1}{(2\pi)^{3/2} v_{Te\parallel} v_{Te\perp}^2} \exp\left( -\frac{v_\perp^2}{2 v_{Te\perp}^2} \right) g(v_\parallel). \quad (4)$$

We consider waves whose wave numbers and frequencies are

$$|\omega - n\Omega_i|/(k_\parallel v_{Ti}) \gg 1, \quad |\omega - n\Omega_e|/(k_\parallel v_{Te\parallel}) \ll 1,$$
$$k_\perp \rho_e \ll 1, \quad (5)$$

where $v_{Ti}$ is the thermal velocity of ions, and $\rho_e$ is the electron Larmor radius. We keep ion thermal effects in the perpendicular direction and electron thermal effects in the parallel direction.

We shortly describe the growth rates of the H cyclotron waves, which are given by

$$\gamma_n = \alpha_n \left. \frac{\partial g(v_\parallel)}{\partial v_\parallel} \right|_{v_\parallel = \omega/k_\parallel} \quad (6)$$

with $\alpha_n$

$$\alpha_n = \left[ \frac{4\pi n_\mathrm{H} q_\mathrm{H}^2}{T_\mathrm{H}} \sum_{n'=-\infty}^{\infty} \frac{n' \Omega_\mathrm{H} \Gamma_{n'}(\mu_\mathrm{H})}{(\omega_r - n'\Omega_\mathrm{H})^2} + \frac{4\pi n_\mathrm{He} q_\mathrm{He}^2}{T_\mathrm{He}} \sum_{n'=-\infty}^{\infty} \frac{n' \Omega_\mathrm{He} \Gamma_{n'}(\mu_\mathrm{He})}{(\omega_r - n'\Omega_{^4\mathrm{He}})^2} \right]^{-1}, \quad (7)$$

where $\Gamma_n(\mu_j) = I_n(\mu_j)\exp(-\mu_j)$ with $I_n$ being the modified Bessel function of the $n$th order. The quantity $\mu_j$ is defined as $\mu_j = k_\perp^2 \rho_j^2$ with $\rho_j$ being the gyroradius, $n_j$ is the density, $q_j$ is the charge, and $T_j$ is the temperature. The effects of $^3$He ions on the waves are neglected, because the abundance of $^3$He ions is much smaller than those of H and $^4$He ions in the solar corona. If $T_e \gtrsim 10 T_\mathrm{H}$, $\alpha_n$ is the greatest at $\omega \simeq 2\Omega_{^3\mathrm{He}}$. (Strictly, the frequency differences from $2\Omega_{^3\mathrm{He}}$ are of the order of $T_\mathrm{H}/T_{e\parallel}$.[37] We here neglect these differences.)

In order to obtain the theoretical expressions for $\omega_r$, we approximate $g(v_\parallel)$ as a Maxwellian distribution with thermal velocity $v_{Te}$. Retaining the $n$th and $(n+1)$th terms of H ions and the 0th term of $^4$He ions, we write dispersion relation of the $n$th harmonic H cyclotron waves as

$$\frac{\omega}{\Omega_{\mathrm{H}}}=1+\frac{n_{\mathrm{H}}T_{e\parallel}}{n_e T_{\mathrm{H}}}n\Gamma_n(\mu_{\mathrm{H}})\Biggl\{1+\frac{n_{\mathrm{H}}T_{e\parallel}}{n_e T_{\mathrm{H}}}\Biggl[1-\Gamma_0(\mu_{\mathrm{H}}) -\Gamma_n(\mu_{\mathrm{H}})+n\Gamma_{n+1}(\mu_{\mathrm{H}})+\frac{m_e}{m_{\mathrm{H}}}\frac{\Omega_e^2}{\omega_{pe}^2}\mu_{\mathrm{H}}\Biggr] +\frac{q_{\mathrm{He}}^2 n_{\mathrm{He}}T_{\mathrm{He}}}{e^2 n_e T_e}[1-\Gamma_0(\mu_{\mathrm{He}})]\Biggr\}^{-1}. \tag{8}$$

In a plasma with $T_{e\parallel}>T_{\mathrm{H}}$, the fundamental $(n=1)$ waves with small $k_\perp$,

$$k_\perp \rho_{\mathrm{H}}<1, \tag{9}$$

become important.[37] For these waves, we set Eq. (8) into the form

$$\omega \simeq \left(1+\frac{n_{\mathrm{H}}T_{e\parallel}}{2n_e T_{\mathrm{H}}}k_\perp^2\rho_{\mathrm{H}}^2\right)\Omega_{\mathrm{H}}. \tag{10}$$

Substituting $T_{e\parallel}\sim m_e v_d^2$ into Eq. (10), we obtain $k_\perp$ of the $\omega \simeq 2\Omega_{^3\mathrm{He}}$ waves as

$$k_\perp\rho_{\mathrm{H}}\sim\left[\frac{n_e m_{\mathrm{H}} v_{T\mathrm{H}}^2}{n_{\mathrm{H}} m_e v_d^2}\left(\frac{2\Omega_{^3\mathrm{He}}-\Omega_{\mathrm{H}}}{\Omega_{\mathrm{H}}}\right)\right]. \tag{11}$$

As the initial electron drift speed $v_d$ increases, $k_\perp$ decreases. We estimate $k_\parallel$ from Eq. (2) as

$$k_\parallel \simeq 2\Omega_{^3\mathrm{He}}/v_d. \tag{12}$$

The ratio of $k_\perp$ to $k_\parallel$ is written as

$$\frac{k_\perp}{k_\parallel}\simeq\left[\frac{m_{\mathrm{H}}n_e}{m_e n_{\mathrm{H}}}\left(\frac{2\Omega_{^3\mathrm{He}}-\Omega_{\mathrm{H}}}{\Omega_{^3\mathrm{He}}}\right)\right]^{1/2}\gg 1. \tag{13}$$

Hence, the waves propagate nearly perpendicular to the ambient magnetic field, and the perpendicular electric field $|E_\perp|$ is much greater than the parallel one $|E_\parallel|$.

Electromagnetic components in these waves are negligible. In the extremely long wavelength region, electrostatic H cyclotron waves would be connected to magnetosonic waves.[38] Hence, we can write the condition where the electrostatic approximation is valid as

$$\omega/k_\perp < v_A, \tag{14}$$

where $v_A$ is the Alfvén speed. For H cyclotron waves with $\omega\sim\Omega_{\mathrm{H}}$, Eq. (14) is written as

$$k_\perp\rho_{\mathrm{H}}>v_{T_{\mathrm{H}}}/v_A. \tag{15}$$

In coronal magnetic tubes, $v_{T_{\mathrm{H}}}/v_A$ is extremely small. In fact, for $B\sim 10^2$ G, $n_{\mathrm{H}}\sim 10^8\ \mathrm{cm}^{-3}$, and $T_{\mathrm{H}}\sim 100$ eV, we have $v_{T_{\mathrm{H}}}/v_A \simeq 6\times 10^{-3}$. Thus, if $v_{T_{\mathrm{H}}}/v_A<k_\perp\rho_{\mathrm{H}}<1$ is satisfied, the electromagnetic components are negligible.

We do not consider electromagnetic waves with much longer wavelengths, although a part of the electron free energy may be transferred to those waves. When $T_{e\parallel}=T_{\mathrm{H}}$, the electromagnetic waves might be important, because the electrostatic H cyclotron waves have only small growth rates. In a plasma with $T_{e\parallel}>T_{\mathrm{H}}$, however, the growth rates of the electrostatic waves become much greater. From Eq. (7), values of $\alpha_1$ can be approximated as

$$\alpha_1\sim\lambda_{\mathrm{DH}}^2\Omega_{\mathrm{H}}\Delta^2/(k_\perp^2\rho_{\mathrm{H}}^2), \tag{16}$$

where $\lambda_{\mathrm{DH}}$ is the H Debye length, and $\Delta$ is $\Delta=\omega-\Omega_{\mathrm{H}}$. As discussed in the previous paper,[37] we can roughly estimate as $\Delta\simeq 0.1$ and $k_\perp\rho_{\mathrm{H}}\sim 1$ for $T_{e\parallel}/T_{\mathrm{H}}=1$. As $T_{e\parallel}/T_{\mathrm{H}}$ increases, $\Delta$ increases and $k_\perp\rho_{\mathrm{H}}$ decreases; which leads to the increase in the growth rates.

### B. Basic equations for accelerated ions

We consider particle orbits in an electrostatic H cyclotron wave under three assumptions; (1) the wave amplitude does not change in time, (2) its frequency is near $n\Omega$ ($n$ is integer and $\Omega$ is the cyclotron frequency of the particle), (3) the amplitude is so small that the stochastic acceleration[39,40] does not occur. The particles satisfying the resonance condition, $\omega-n\Omega-k_\parallel v_\parallel\simeq 0$, will then be accelerated, and their Larmor radii will increase. However, the acceleration will not continue infinitely, and the upper limit of the Larmor radius exists. We give the maximum Larmor radius $\rho_{\mathrm{Max}}$.

Suppose that $k_\parallel$ is much smaller than $k_\perp$, as shown by Eq. (13). Then, for the resonant particles with $\omega-n\Omega-k_\parallel v_\parallel\simeq 0$, we have a constant of motion, $J_{n0}(k_\perp\rho)\cos\xi$ (Refs. 40 and 41 and references therein). Here, the magnetic field is set to be in the $z$ direction, and the wave is assumed to propagate in the $(x,z)$ plane. The quantity $\xi$ is defined as

$$\xi\equiv n\theta-(\omega-k_\parallel v_\parallel)t+k_\perp x_{g0}+k_\parallel z_0, \tag{17}$$

where $\theta$ and $x_{g0}$ are the gyration phase and the initial position of the gyration center, respectively. Using the initial conditions, we write the constant of motion as

$$J_n(k_\perp\rho)\cos\xi=J_n(k_\perp\rho_0)\cos\xi_0. \tag{18}$$

Equation (18) shows that $\rho$ becomes maximum at $\cos\xi=\pm 1$, and its value $\rho_{\mathrm{Max}}$ satisfies

$$|J_n(k_\perp\rho_{\mathrm{Max}})|=|J_n(k_\perp\rho_0)\cos\xi_0|. \tag{19}$$

The zero points of the Bessel function $J_n$ gives the maximum energy of $^3$He ions. When the initial Larmor radius $\rho_0$ is much smaller than the perpendicular wavelength,

$$k_\perp\rho_0\ll 1, \tag{20}$$

the left-hand side of Eq. (19) is extremely small. Then, $\rho_{\mathrm{Max}}$ can be estimated from

$$J_n(k_\perp\rho_{\mathrm{Max}})\simeq 0. \tag{21}$$

That is, the value of $k_\perp\rho_{\mathrm{Max}}$ corresponds to the first zero point of the Bessel function $J_n$ and is written as

$$k_\perp\rho_{\mathrm{Max}}\sim\frac{\pi}{2}\left(n+\frac{3}{2}\right). \tag{22}$$

The ratio of the maximum Larmor radius to the initial one is given by

$$\frac{\rho_{\mathrm{Max}}}{\rho_0}\sim\frac{\pi}{2}\left(n+\frac{3}{2}\right)\frac{1}{k_\perp\rho_0}. \tag{23}$$

If $k_\perp \rho_0 \ll 1$, $\rho_{\rm Max}$ is much greater than $\rho_0$. We also note that $\rho_{\rm Max}$ is almost independent of $\xi_0$. The particles can have the maximum Larmor radius given by Eq. (21), almost independent of the initial gyration phase $\theta_0$ or the initial position, $x_{g0}$ and $z_0$.

### C. Maximum energy of $^3$He ions

As described in Sec. II A, $^3$He ions are accelerated by the H cyclotron waves with $\omega \simeq 2\Omega_{^3{\rm He}}$. These waves have long wavelengths, $k_\perp \rho_{\rm H} \ll 1$. Suppose that the initial $^3$He temperature is equal to the H temperature. Then, most of the $^3$He ions would satisfy Eq. (20). From Eq. (22) with $n=2$, we estimate the maximum Larmor radius $\rho_{\rm Max}$ of the $^3$He ions as

$$\rho_{\rm max} \simeq 5/k_\perp . \tag{24}$$

The ratio of the maximum energy to the initial thermal energy is written as

$$\frac{v_{\rm Max}^2}{v_{T^3{\rm He}}^2} \simeq \frac{25}{(k_\perp \rho_{\rm H})^2} \frac{v_{T_{\rm H}}^2}{v_{T^3{\rm He}}^2} \frac{\Omega_{^3{\rm He}}^2}{\Omega_{\rm H}^2}, \tag{25}$$

where $v_{T_{\rm H}}$ and $v_{T^3{\rm He}}$ are the initial thermal velocities of H and $^3$He ions, respectively. Equation (25) shows that in the wave with $k_\perp \rho_{\rm H} \ll 1$, the maximum $^3$He energy is much greater than the initial one. As $k_\perp$ decreases, the maximum $^3$He energy increases.

Using Eq. (11), we approximately write Eq. (25) as

$$\frac{v_{\rm Max}^2}{v_{T^3{\rm He}}^2} \sim 25 \frac{m_e v_d^2}{T_{^3{\rm He}}} \frac{\Omega_{^3{\rm He}}}{(2\Omega_{^3{\rm He}} - \Omega_{\rm H})} \sim 50 \frac{m_e v_d^2}{T_{^3{\rm He}}}, \tag{26}$$

where $T_{^3{\rm He}}$ is the initial $^3$He temperature. The maximum $^3$He energy is proportional to the initial electron drift energy $m_e v_d^2$.

## III. SIMULATION

### A. Method and parameters

By means of a two-dimensional (two space and three velocity components) electrostatic particle code with full ion and electron dynamics, we study the evolution of the energies of $^3$He ions accelerated by nonlinear development of strong current-driven instabilities. Since the simulation method was described in detail in the pervious paper,[37] we only briefly mention it. We consider a plasma consisting of electrons, H, $^4$He, and $^3$He ions. Their total numbers in the simulation are $N_e = 16\,777\,216$, $N_{\rm H} = 12\,783\,616$, $N_{^4{\rm He}} = 1\,597\,440$, and $N_{^3{\rm He}} = 399\,360$. The mass ratios are $m_{\rm H}/m_e = 100$, $m_{^4{\rm He}}/m_{\rm H} = 4$, and $m_{^3{\rm He}}/m_{\rm H} = 3$; the charge ratios are $q_{\rm H}/|q_e| = 1$, $q_{^4{\rm He}}/q_{\rm H} = 2$, and $q_{^3{\rm He}}/q_{\rm H} = 2$. The system size is $L_x \times L_y = 512\Delta_g \times 1024\Delta_g$, where $\Delta_g$ is the grid spacing. We use periodic boundary conditions in both the $x$ and $y$ directions. The external magnetic field is in the $y$ direction. The electron cyclotron frequency is set to be $|\Omega_e|/\omega_{pe} = 4$, where $\omega_{pe}$ is the electron plasma frequency. The time step is $\omega_{pe}\Delta t = 0.1$.

We assume that the ions have isotropic Maxwellian velocity distribution functions at $t=0$, and that the entire electrons have a shifted Maxwellian distribution with the initial drift speed $v_d$ along the magnetic field. The initial electron temperature is equal to the ion temperature, $T_{e0} = T_{i0}$, where $i$ denotes H, $^4$He, or $^3$He.

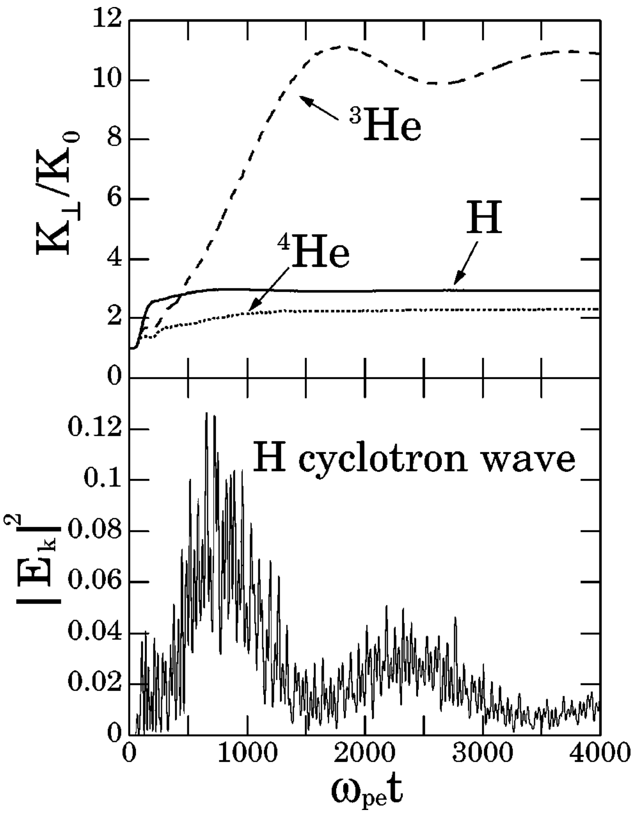


FIG. 1. Time variations of the total perpendicular energies of H, $^4$He, and $^3$He ions, and time variation of $|E_k|^2$ of the H cyclotron wave with $(k_\| \rho_{\rm H}, k_\perp \rho_{\rm H}) = (0.031, 0.43)$. The ion energies are normalized to their initial values, and the wave amplitude $|E_k|^2$ is normalized to $m_e v_{Te}^2$.

Although the initial electron distribution used in the simulation may be hardly formed in solar flares, this simulation system enables us to qualitatively understand energy transfer from electrons to $^3$He ions through nonlinear processes of the instabilities. We here pay attention to how the wave numbers of H cyclotron waves and the maximum energy of $^3$He ions depend on $v_d$. We also compare simulation results with the theory that can be applied to cases where the initial electron distribution is not a shifted Maxwellian distribution.

### B. Wave properties and $^3$He energy

According to the linear theory based on these initial conditions, Buneman waves are unstable, while H cyclotron waves are almost stable. Certainly, as shown in Refs. 31 and 37, the Buneman waves grow at first. However, they quickly saturate owing to electron trapping. The electron trapping drastically changes the shape of the electron velocity distribution function $f_e(v_\|)$; which leads to the rise in the electron temperature. The H cyclotron waves are then destabilized by the change in $f_e(v_\|)$. The waves with $\omega \simeq 2\Omega_{^3{\rm He}}$ and $\omega/k_\| \simeq v_d$ become dominant and preferentially accelerate $^3$He ions.

Figure 1 shows time variations of the total perpendicular

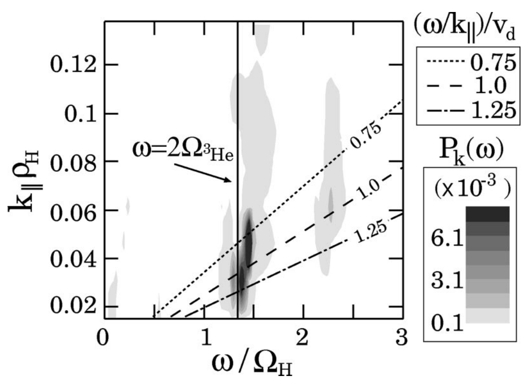


FIG. 2. Power spectrum of H cyclotron waves and contour lines of phase velocity $\omega/k_\parallel$ in the $(\omega,k_\parallel)$ plane. Vertical line represents the frequency $\omega = 2\Omega_{^3He} = (4/3)\Omega_H$.

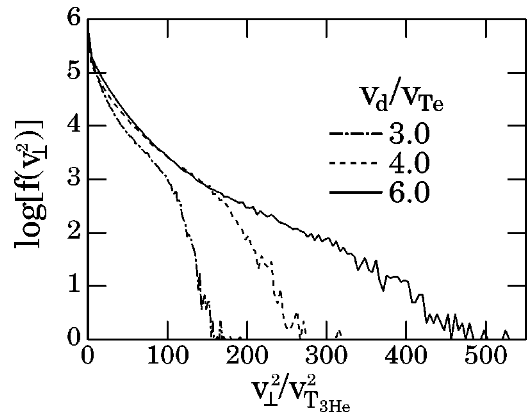


FIG. 4. Energy distributions of $^3$He ions for $v_d/v_{Te}=3$, 4, and 6. The energies of the $^3$He ions are normalized to their initial thermal energies.

energies of H, $^4$He, and $^3$He ions and of the H cyclotron wave that grows to the largest amplitude. (Here, the initial electron drift speed is set to be $v_d = 4_{Te}$.) We see that the $^3$He ions are selectively accelerated. The H cyclotron wave is destabilized at $\omega_{pe}t \simeq 200$ and saturates at $\omega_{pe}t \simeq 800$; the saturation is caused by the flattening of $f_e(v_\parallel)$ around the phase velocity of the wave.[31] After the time $\omega_{pe}t \simeq 800$, the wave is damped because of the energy transfer to the $^3$He ions. At $\omega_{pe}t \simeq 1700$, the wave amplitude is significantly decreased, and the $^3$He energy saturates.

Only a small part of the initial electron free energy (about 0.03) is converted to the energies of the H cyclotron waves. Most of their wave energies are then transferred to the $^3$He ions. As shown by Eq. (1), a large part of the initial free energy is converted to the electron thermal energy.

Figure 2 shows a contour map of the power spectrum for many H cyclotron waves in the $(\omega,k_\parallel)$ plane; the data from $\omega_{pe}t=0$ to 4000 is used. Contour lines of phase velocity $\omega/k_\parallel$ are also plotted. The waves with $\omega \simeq 2\Omega_{^3He}$ and $\omega/k_\parallel \simeq v_d$ eventually become dominant. The H cyclotron wave shown in the bottom panel of Fig. 1 is one of these waves.

Now, we compare simulation results with the theory. According to the theory, the values of $k_\perp$ of the H cyclotron waves with $\omega \simeq 2\Omega_{^3He}$ depend on $v_d$. We have performed simulations with various values of $v_d$ and obtained Fig. 3, where the value of $k_\perp$ of the largest amplitude wave is shown as a function of $v_d$. The dots are simulation results, and the solid line represents the theoretical prediction, Eq. (11). The simulation results are in good agreement with the theory. As $v_d$ increases, $k_\perp$ decreases.

Figure 4 shows energy distributions of the $^3$He ions for $v_d/v_{Te}=3$, 4, and 6, at the time $\omega_{pe}t=2000$. The maximum $^3$He energy increases with $v_d$, as Eq. (26) predicts. Figure 5 shows the maximum $^3$He energy as a function of $k_\perp$ of the largest amplitude wave. The solid line represents the theoretical value estimated from Eq. (25). The simulation results also agree with the theory.

### C. Fraction of high-energy $^3$He ions

Equation (25) indicates that the maximum $^3$He energy is independent of the $^3$He density $n_{^3He}$. However, the fraction of high-energy $^3$He ions may depend on $n_{^3He}$. In order to study this, we have performed simulations assuming that $n_{^3He}q_{^3He}/(n_{^4He}q_{^4He}) = 1/4$, 1/16, and 1/400. In the case of 1/400, we use the fine particle method[42] for the $^3$He ions; $q_{^3He}=0.02q_H$, $m_{^3He}=0.03m_H$, $N_{^3He}=399\,360$, and the initial thermal velocity is $v_{T_{^3He}}=0.026v_{Te}$.

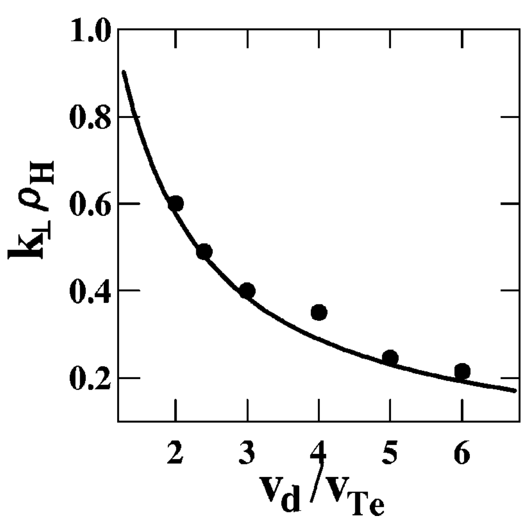


FIG. 3. Wave number $k_\perp$ of the largest amplitude wave as a function of $v_d$.

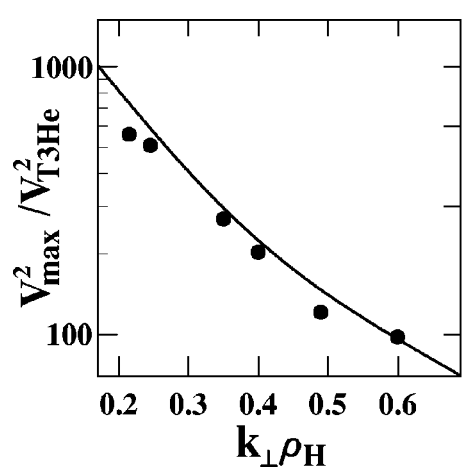


FIG. 5. The maximum energy of $^3$He ions as a function of $k_\perp$ of the largest amplitude wave.

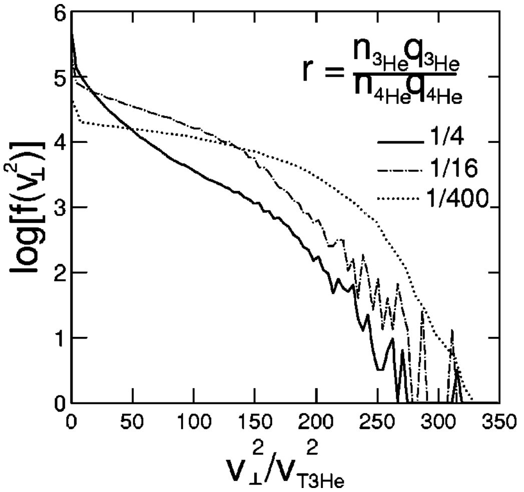


FIG. 6. Energy distributions of $^3$He ions in the three plasmas with $n_{^3\mathrm{He}}q_{^3\mathrm{He}}/(n_{^4\mathrm{He}}q_{^4\mathrm{He}})=1/4$, 1/16, and 1/400. The initial electron drift speed $v_d$ is set to be $v_d/v_{Te}=4$.

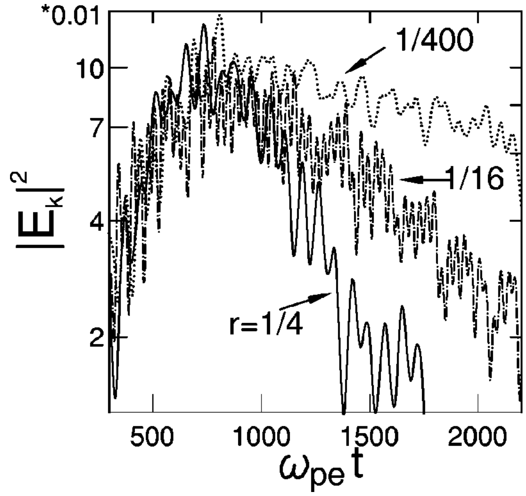


FIG. 8. Time variations of the amplitudes of the H cyclotron waves with $\omega\simeq 2\Omega_{^3\mathrm{He}}$ and $\omega/k_\parallel \simeq v_d$, in the three plasmas where $r \equiv n_{^3\mathrm{He}}q_{^3\mathrm{He}}/(n_{^4\mathrm{He}}q_{^4\mathrm{He}})=1/4$, 1/16, and 1/400.

Figure 6 shows the energy distributions of the $^3$He ions at $\omega_{pe}t=4000$ for $n_{^3\mathrm{He}}q_{^3\mathrm{He}}/(n_{^4\mathrm{He}}q_{^4\mathrm{He}})=1/4$, 1/16, and 1/400 (the initial electron drift speed $v_d$ is set to be $v_d/v_{Te}=4$). The fraction of energetic particles such that $v_\perp^2/v_{T_{^3\mathrm{He}}}^2>50$ increases as $n_{^3\mathrm{He}}$ decreases.

Figure 7 shows time variations of the total energies of $^3$He ions. As $n_{^3\mathrm{He}}$ decreases, the saturation level of $K/K_0$ increases, where $K_0$ is the initial energy. This is because in a plasma with smaller $n_{^3\mathrm{He}}$, a larger part of the $^3$He ions are accelerated to high energies, as shown in Fig. 6.

Figure 8 shows evolution of the H cyclotron waves with $\omega\simeq 2\Omega_{^3\mathrm{He}}$ and $\omega/k_\parallel \simeq v_d$. In a plasma with smaller $n_{^3\mathrm{He}}$, the wave keeps a large amplitude for a longer time, because the damping due to the $^3$He ions is weaker. Thus, we expect that in the solar corona with $n_{^3\mathrm{He}}/n_{^4\mathrm{He}}\sim 10^{-4}$, the H cyclotron waves with $\omega\simeq 2\Omega_{^3\mathrm{He}}$ would keep large amplitudes for a much longer time, and the fraction of energetic $^3$He ions would be much greater.

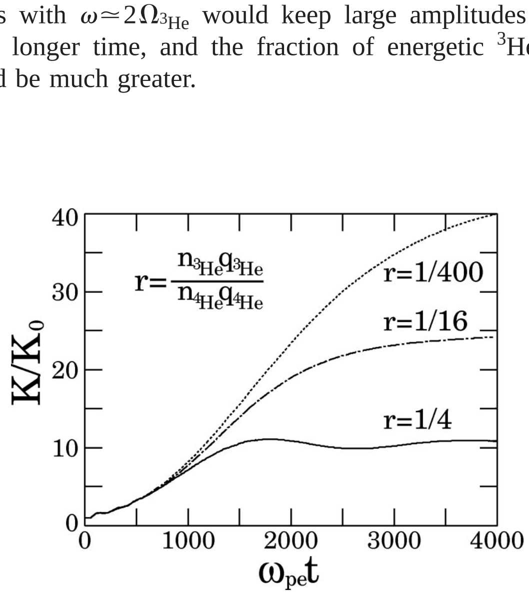


FIG. 7. Time variations of the total perpendicular energies of $^3$He ions for $r\equiv n_{^3\mathrm{He}}q_{^3\mathrm{He}}/(n_{^4\mathrm{He}}q_{^4\mathrm{He}})=1/4$, 1/16, and 1/400. The energies are normalized to their initial values.

## IV. COMPARISON WITH OBSERVATIONS

It has been reported that $^3$He rich events are almost always accompanied by energetic electrons with 1–100 keV.[9,14] In this section, we suppose that electron drift energy is of the order of 10 keV. We then estimate the wave number of an electrostatic H cyclotron wave that would be destabilized by these electrons. Energies of $^3$He ions accelerated by this wave are studied theoretically and numerically.

Although the simulation is started with a situation where all the electrons are drifting, the discussion in this section can be applied to situations where background electrons exist; the total electron velocity distribution must have a positive slope near $v_\parallel = v_d$. The theory gives a good estimate of when the beam density is high.

From Eqs. (11) and (12), we obtain $(k_\parallel, k_\perp)$ of the wave with $\omega = 2\Omega_{^3\mathrm{He}}$ and $\omega/k_\parallel = v_d$ as

$$(k_\parallel \rho_\mathrm{H}, k_\perp \rho_\mathrm{H}) \sim (0.0011, 0.045). \tag{27}$$

Here, we assume that the initial electron drift energy is $m_e v_d^2/2=40$ keV, and that the temperature of H ions is $T_\mathrm{H}=100$ eV. The H density is set to be equal to the electron density; the effects of $^3$He and $^4$He ions on the wave are neglected. As described in Sec. II, the electromagnetic effects are negligible.

Suppose that the magnetic field is 100 G, the parallel and perpendicular wavelengths of the H cyclotron waves are of the order of $10^5$ cm and $10^4$ cm, respectively. They are much smaller than the typical flare size, $10^9$ cm. Hence, we neglect effects of inhomogeneities of the background plasma on the waves.

Substituting Eq. (27) into Eq. (25), we estimate the ratio of the maximum $^3$He energy to the initial thermal energy as

$$\frac{v_{\max}^2}{v_{T_{^3\mathrm{He}}}^2} \sim 10^4, \tag{28}$$

where the initial $^3$He temperature is assumed to be equal to the H temperature, 100 eV. That is, the maximum $^3$He energy

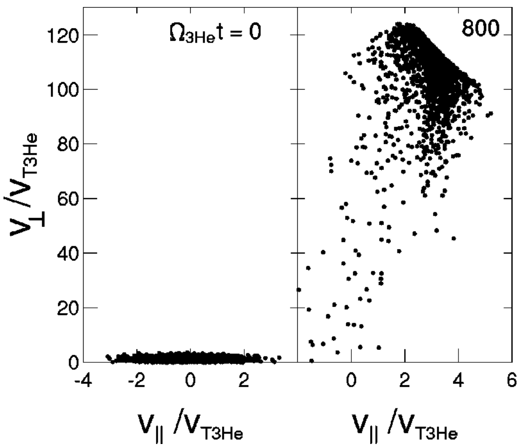


FIG. 9. Phase space plots $(v_\parallel, v_\perp)$ of the test $^3$He particles in the H cyclotron wave with $(k_\parallel \rho_{\rm H}, k_\perp \rho_{\rm H}) = (0.0011, 0.045)$ and $\omega = 2\Omega_{^3{\rm He}}$ at the times $\Omega_{^3{\rm He}} t = 0$ and 800.

is of the order of 1 MeV/$n$, which is the same as the observed energies of $^3$He ions in the $^3$He rich events.[9,13]

*Test particle simulation*: Now, in order to study energies of many $^3$He ions, we numerically calculate test particle orbits in a stationary monochromatic H cyclotron wave with the wave number given by Eq. (27). Initially, particles are distributed uniformly in space with a Maxwellian velocity distribution. The total number of the test particles is $N_{^3{\rm He}} = 8192$. The wave amplitude is set to be $\hat{E}_0 = 1.0$, where $\hat{E}_0$ is the electric field normalized to $v_{T{\rm H}} B_0 / c$. The electron-to-ion mass ratio is set to be $m_{\rm H}/m_e = 1{,}836$. Numerical integration of the equation of motion is performed with Adams–Bashforth–Moulton method.[43]

Figure 9 shows phase space plots $(v_\parallel, v_\perp)$ of the test $^3$He particles at $\Omega_{^3{\rm He}} t = 0$ and 800. The particles are accelerated perpendicularly to the magnetic field. At $\Omega_{^3{\rm He}} t = 800$, most of the particles have large velocities, $v_\perp / v_{T_{^3{\rm He}}} \sim 100$, almost independent of the initial gyration phase or the initial position, as predicted by Eq. (21). Evidently, many $^3$He ions can be accelerated to the energy of the order of MeV/$n$.

Almost all the particles resonate with the wave. In Fig. 9, the maximum value of the parallel velocity at $\Omega_{\rm H} = 800$ is about 4 $v_{T_{^3{\rm He}}}$. Even for the particles with this parallel velocity, the resonance condition is well satisfied because $\omega - 2\Omega_{^3{\rm He}} - k_\parallel v_\parallel \simeq 2 \times 10^{-3} \Omega_{^3{\rm He}}$.

The $^3$He ions are accelerated to the energy of the order of MeV/$n$ in a very short time. Assuming $B = 100$ G, we estimate its acceleration time as $\sim 10^{-3}$ s. The $^3$He ions then travel only a short distance ($\sim 10^4$ cm) along the magnetic field during this period; this distance is much smaller than the characteristic scale length of solar coronal tubes ($10^9$ cm). Therefore, inhomogeneities of the magnetic field do not limit the acceleration of $^3$He ions.

We compare the wave energy with the initial electron free energy. Suppose that the H cyclotron wave with the electric-field amplitude $E_0$ is excited in a plasma. Then, the total energy of the wave is written as[44]

$$W_{k,\omega} = \epsilon |E_0|^2 / (16\pi), \tag{29}$$

with

$$\epsilon = 1 + \frac{\omega_{pe}^2}{k^2 v_{Te}^2} + \sum_n \frac{\omega_{p{\rm H}}^2 \Omega_{\rm H} \Gamma_n (k_\perp^2 \rho_{\rm H}^2)}{k^2 v_{T{\rm H}}^2 (\omega - n\Omega_{\rm H})^2}. \tag{30}$$

The ratio of the wave energy to the initial electron free energy is written as

$$\frac{W_{k,\omega}}{n_e m_e v_d^2} = \frac{\epsilon |\hat{E}_0|^2}{4} \frac{v_{T{\rm H}}^2}{v_d^2} \frac{|\Omega_e|^2}{\omega_{pe}^2}. \tag{31}$$

Assuming that $|\hat{E}_0| = 1.0$, $\omega = 2\Omega_{^3{\rm He}}$, and $\Omega_e / \omega_{pe} = 3$ ($B \simeq 100$ G, $n_e \simeq 10^8$ cm$^{-3}$), we estimate this ratio as $\sim 10^{-1}$. The wave energy is smaller than the initial free energy.

We now calculate the ratio of the $^3$He total energy to the wave energy, which is given by

$$\frac{n_{^3{\rm He}} m_{^3{\rm He}} v_{\rm Max}^2}{W_{k,\omega}} = \frac{4 \times 10^4}{\epsilon |\hat{E}_0|^2} \frac{n_{^3{\rm He}} m_{^3{\rm He}}}{n_{\rm H} m_e} \frac{\omega_{pe}^2}{|\Omega_e|^2}, \tag{32}$$

where all the $^3$He ions are assumed to be accelerated to the energy given by Eq. (28). This ratio is of the order of 0.1. Thus, the damping due to the $^3$He ions is small, even if all the $^3$He ions are accelerated to the energy $10^4$ times as high as the initial one.

## V. SUMMARY AND DISCUSSIONS

We have studied energies of $^3$He ions accelerated by strong current-driven instabilities. The acceleration is via the H cyclotron waves with frequencies $\omega \simeq 2\Omega_{^3{\rm He}}$ and phase velocities $\omega / k_\parallel \sim v_d$. We have given a theoretical expression for the wave numbers, $k_\parallel$ and $k_\perp$, of these waves as a function of the initial electron drift speed $v_d$. We have also theoretically estimated the maximum energy of $^3$He ions accelerated by these waves. As $v_d$ increases, the value of $k_\perp$ decreases, and the corresponding maximum $^3$He energy increases. By means of a two-dimensional (two space and three velocity components), electrostatic, particle code with full ion and electron dynamics, we have performed simulations. The observed wave number of the H cyclotron waves and the observed maximum $^3$He energy are both in good agreement with the theory. It is also demonstrated that the fraction of energetic $^3$He ions increases as the $^3$He density decreases.

Based on the theory given in this paper, we have estimated $k_\parallel$ and $k_\perp$ of the H cyclotron waves that would be destabilized by electrons with initial drift energies of the order of 10 keV. We then showed that the corresponding maximum $^3$He energy is of the order of MeV/$n$. Furthermore, the test particle simulation demonstrated that many $^3$He ions can be accelerated to such high energies. These results can explain the observations that the $^3$He rich events are almost always accompanied by energetic electrons with 1–100 keV, and that many $^3$He ions are accelerated to the energy of the order of MeV/$n$.

In this paper, we have considered electrostatic H cyclotron waves whose group velocities are in the region

$$v_{g\parallel} \ll v_{g\perp} \ll v_A, \tag{33}$$

where $v_{g\parallel}$ and $v_{g\perp}$ are parallel and perpendicular group velocities, respectively. Because the group velocities are small, these waves are important near the place where the electrons are originally accelerated in coronal magnetic tubes. For the $^3$He acceleration over a long distance, electromagnetic waves with larger group velocities may be important.

## ACKNOWLEDGMENTS

The authors are grateful to Professor Y. Ohsawa for a critical reading of this paper. This work was carried out by the joint research program of the Solar-Terrestrial Environment Laboratory, Nagoya University.

[1] R. Pallavicini, S. Serio, and G. S. Vaiana, Astrophys. J. **216**, 108 (1977).
[2] J. P. Meyer, Astrophys. J., Suppl. Ser. **57**, 151 (1985).
[3] J. P. Meyer, Astrophys. J., Suppl. Ser. **57**, 173 (1985).
[4] D. V. Reames, L. M. Barbier, and C. K. Ng, Astrophys. J. **466**, 473 (1996).
[5] M. Toida and Y. Ohsawa, J. Phys. Soc. Jpn. **64**, 2036 (1995).
[6] M. Toida and Y. Ohsawa, Sol. Phys. **171**, 161 (1997).
[7] O. A. Shaeffer and J. Zähringer, Phys. Rev. Lett. **8**, 389 (1962).
[8] J. D. Anglin, Astrophys. J. **198**, 733 (1975).
[9] D. V. Reames, Astrophys. J. **73**, 235 (1990).
[10] D. V. Reames, J. P. Meyer, and T. T. von Rosenvinge, Astrophys. J., Suppl. Ser. **90**, 649 (1994).
[11] B. Klecker, D. Hovestadt, G. Gloeckler, F. M. Ipavich, M. Scholer, C. Y. Fan, and L. A. Fisk, Astrophys. J. **281**, 458 (1984).
[12] A. Luhn, B. Klecker, D. Hovestadt, and E. Möbius, Astrophys. J. **317**, 951 (1987).
[13] D. V. Reames, T. T. von Rosenvinge, and R. P. Lin, Astrophys. J. **292**, 716 (1985).
[14] G. C. Ho, E. C. Roelof, S. E. Hawkins III, R. E. Gold, G. M. Mason, J. R. Dwyer, and J. E. Mazur, Astrophys. J. **552**, 863 (2001).
[15] A. O. Benz, *Plasma Astrophysics, Kinetic Processes in Solar and Stellar Coronae*, 2nd ed. (Kluwer Academic, Berlin, 2002).
[16] D. V. Reames and R. G. Stone, Astrophys. J. **308**, 902 (1986).
[17] D. V. Reames, B. R. Dennis, R. G. Stone, and R. P. Lin, Astrophys. J. **327**, 998 (1988).
[18] J. A. Miller, P. J. Cargill, A. G. Emslie *et al.*, J. Geophys. Res., [Planets] **102**, 14631 (1997).
[19] L. A. Fisk, Astrophys. J. **224**, 1048 (1978).
[20] L. G. Kocharov and G. E. Kocharov, Space Sci. Rev. **38**, 89 (1984).
[21] S. Riyopoulos, Astrophys. J. **381**, 578 (1991).
[22] T. X. Zhang, M. Toida, and Y. Ohsawa, J. Phys. Soc. Jpn. **62**, 2545 (1993).
[23] T. X. Zhang and Y. Ohsawa, Sol. Phys. **158**, 115 (1995).
[24] S. Nakazawa, T. X. Zhang, and Y. Ohsawa, Sol. Phys. **166**, 159 (1996).
[25] M. Temerin and I. Roth, Astrophys. J. Lett. **391**, L105 (1992).
[26] J. A. Miller and A. F. Viñas, Astrophys. J. **412**, 386 (1993).
[27] G. Paesold, R. Kallenbach, and A. O. Benz, Astrophys. J. **582**, 495 (2003).
[28] G. Paesold and A. O. Benz, Astron. Astrophys. **401**, 711 (2003).
[29] M. Toida, T. Maeda, I. Shiiba, A. Sugishima, and Y. Ohsawa, Phys. Plasmas **7**, 4882 (2000).
[30] M. Toida and A. Sugishima, J. Phys. Soc. Jpn. **70**, 3285 (2001).
[31] M. Toida, A. Sugishima, and Y. Ohsawa, Phys. Plasmas **9**, 2541 (2002).
[32] H. Okuda and M. Ashour-Abdalla, J. Geophys. Res., [Space Phys.] **88**, 899 (1983).
[33] O. Buneman, Phys. Rev. **115**, 503 (1959).
[34] T. O'Neil, Phys. Fluids **8**, 2255 (1965).
[35] O. Ishihara, A. Hirose, and A. B. Langdon, Phys. Rev. Lett. **44**, 1404 (1980).
[36] O. Ishihara and A. Hirose, Phys. Fluids **24**, 452 (1981).
[37] M. Toida and H. Okumura, J. Phys. Soc. Jpn. **72**, 1106 (2003).
[38] T. D. Kaladze, D. G. Lominadze, and K. N. Stepanov, Sov. Phys. JETP **17**, 196 (1972).
[39] G. R. Smith and A. N. Kaufman, Phys. Rev. Lett. **34**, 1613 (1975).
[40] A. Fukuyama, H. Momota, and R. Itatani, Phys. Rev. Lett. **38**, 701 (1977).
[41] M. Brambilla, *Kinetic Theory of Plasma Waves* (Clarendon, Oxford, 1998), Chap. 9.
[42] Y. Ohsawa and J. M. Dawson, Phys. Fluids **27**, 1491 (1984).
[43] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in Fortran* (Cambridge University Press, New York, 1992), Chap. 6.
[44] T. H. Stix, *Waves in Plasmas* (AIP, New York, 1992).