Vertex Corrections to the Anomalous Hall Effect in Spin-Polarized Two-Dimensional Electron Gases with a Rashba Spin-Orbit Interaction

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We study the effect of disorder on the intrinsic anomalous Hall conductivity in a magnetic two-dimensional electron gas with a Rashba-type spin-orbit interaction. We find that anomalous Hall conductivity vanishes unless the lifetime is spin-dependent, similar to the spin Hall conductivity in the nonmagnetic system. In addition, we find that the spin Hall conductivity does not vanish in the presence of magnetic scatterers.

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The spin-orbit interaction (SOI) in semiconductors allows optical and electrical control of spins and, because of this, has recently attracted much attention in the field of spintronics. The SOI gives rise to unusual Hall effects, such as the anomalous Hall (AH) effect [1] in ferromagnets and the spin Hall (SH) effect in normal conductors [2–5]. The detailed mechanisms of these effects are still controversial. For the AH effect, originally an intrinsic (i.e., band-structure-induced) mechanism originating from an effective magnetic field in momentum space was put forward [6], followed by extrinsic mechanisms, referred to as skew [7] and sidejump [8] scattering at impurities. Most experiments have been interpreted in terms of the extrinsic mechanisms, but the intrinsic AH effect has recently been shown to quantitatively explain experiments in ferromagnetic semiconductors [9–12]. Effects of disorder have recently been investigated [13,14]. While a current bias does not excite a Hall voltage in normal metals at zero external field, a Hall effect of spin currents should persist in the presence of an intrinsic or extrinsic SOI. This spin Hall effect was originally predicted assuming extrinsic scattering [2,3], but, analogous to the AH effect, an intrinsic SH effect is also possible. Murakami *et al.* [4] predicted that the effective magnetic field associated to the Berry phase in the valence band induce drift of up and down spin carriers towards opposite directions in *p*-doped zinc-blende-type semiconductors. Sinova *et al.* [5] found a universal spin Hall conductivity for the two-dimensional electron gas (2DEG) with a Rashba-type SOI.

Recently, two groups [15,16] reported optical detection of spin accumulation of opposite signs at the sample edges in current-biased nonmagnetic semiconductors. Such behavior can be caused by the extrinsic SH effect, as shown explicitly by Ref. [17] and is believed to be a signature of the intrinsic SH effect as well. However, the interpretation of the experimental results is not straightforward.

Current theories predict that the intrinsic SH effect can be strongly suppressed by disorder effects. Especially, the intrinsic SH current vanishes identically by disorder scattering of electrons in the bulk of a Rashba-split 2DEG [18–20]. On the other hand, the Rashba-split 2DEG is rather special in this respect, since in other systems the intrinsic SH effects survives disorder scattering. This has been demonstrated for 2DEGs with an SO interaction that is not directly proportional to the wave vector and for three-dimensional systems in the presence of the Dresselhaus SO interaction or a Luttinger type of SO interaction in the valence band [21–23].

Since one may view the SH effect as the zero-magnetization limit of the AH effect, the impurity-induced suppression of the intrinsic SH in a Rashba-split 2DEG raises the issue of whether the intrinsic AH effect is affected equally strongly by disorder scattering. And can, *vice versa*, the intrinsic SH effect survive in a disordered system when magnetic effects are added, e.g., carrier scattering by magnetic impurities? These are the questions we address in this Letter. We focus on a disordered 2DEG with the intrinsic Rashba-type SOI and investigate the effects of a constant exchange potential as well magnetic impurities on the AH and SH effects by generalizing the method we applied previously [18,24] to calculate the conductivity in the diffusive transport regime.

We will show that the intrinsic AH conductivity in a Rashba-split 2DEG with uniform exchange splitting vanishes unless the lifetime is spin-dependent, which is correct up to the second order of the SOI. This exemplifies the strong similarity between SH and AH effects. We also find that the SH conductivity is nonzero in the presence of magnetic impurity scattering. These results are not only relevant for a full understanding of the Hall effects [25] but should also apply to high *g*-factor, high-mobility narrow-gap magnetic semiconductors such as *n*-type

HgTe|HgCdTe quantum wells that exhibit many of the features needed for a clean observation of the intrinsic SOI-induced Hall effect in a transport experiment [26].

We start with the Hamiltonian

$$H = H_0 + V_m, \tag{1}$$

where H_0 is the unperturbed Hamiltonian for the 2DEG and V_m the random potential caused by impurities. For the nonmagnetic 2DEG, H_0 equals the Rashba Hamiltonian H_R. In Pauli spin and momentum space, we have

$$H_R = \frac{\hbar^2}{2m}k^2\mathbf{1} + \lambda\hbar(k_y\sigma_x - k_x\sigma_y), \tag{2}$$

where $k_\pm = k_x \pm ik_y$ with in-plane momentum vector $\mathbf{k} = (k_x, k_y)$, λ is the tunable strength of the spin-orbit coupling, and $\sigma_\alpha(\alpha = x, y, z)$ are the Pauli spin matrices. The eigenvalues of H_R are given as $E_{k\pm} = (\hbar^2k^2/2m) \pm \lambda\hbar k$, and the eigenfunctions are denoted as $|s = \pm\rangle$.

In order to deal with the AH and SH conductivities in the diffusive transport regime, we modify the Hamiltonian accordingly. In the calculation of the AH conductivity, the unperturbed part of the Hamiltonian is given as $H_0 = H_R + \Delta_{\rm ex}\sigma_z$, where the second part expresses an exchange potential. The random potential is supposed to be short-ranged and isotropic, but may be spin-dependent:

$$V_m = \sum_{i\sigma=\uparrow,\downarrow} V_\sigma\delta(\mathbf{r} - \mathbf{R}_i). \tag{3}$$

In the calculation of the SH conductivity, on the other hand, the unperturbed Hamiltonian is H_R, but we incorporate magnetic impurities that give rise to spin-flip scattering of electrons:

$$V_m = 2J\sum_i \mathbf{s}\cdot\mathbf{S}\delta(\mathbf{r} - \mathbf{R}_i), \tag{4}$$

where $\mathbf{s}$ and $\mathbf{S}$ are the spin operators of the conduction electrons and the localized magnetic impurities, respectively, and J is their exchange coupling. We consider the regime in which the Kondo effect is not important.

We compute the transport properties via the Kubo formula, using the charge and spin current operators

$$J_{x(y)} = e[(\hbar/m)k_{x(y)}\mathbf{1} - (+)\lambda\sigma_{y(x)}], \tag{5}$$

$$J_y^{\sigma_z} = (\hbar^2/2m)k_y\sigma_z, \tag{6}$$

respectively, where the latter is defined by $J_{x(y)}^{\sigma_\alpha} = \hbar\{v_{x(y)}, \sigma_\alpha\}$ [27]. In the diffusive regime, it is convenient to include the SOI in the eigenstates of the Hamiltonian and treat the impurity potentials as perturbation. We can then proceed to obtain the AH and SH conductivities as before by adopting the Born approximation for the self-energy and the ladder approximation for the current vertex that is determined self-consistently in order to satisfy the Ward identity.

After a lengthy but straightforward calculation along the lines of Refs. [18,24], we obtain the full expression (not shown) for the AH conductivity in terms of retarded and advanced Green functions in the Pauli spin space. The momentum integration of the products of retarded and advanced Green function can be done analytically by assuming $\lambda \ll \Delta_{\rm ex}$, keeping terms up to λ^2 and in the clean limit with lifetime broadening being smaller than the Rashba splitting of the bands.

By ignoring the real part of the self-energy $\Sigma_\sigma = (n/L^2)\sum_k |V_\sigma^2|g_{k\sigma}$, where n/L^2 is the impurity density, and with lifetime $\tau_\sigma = \hbar/2|\Sigma_\sigma|$, we obtain

$$\sigma_{yx}^{\rm AH} = \frac{4e^2\lambda^2 D_0}{4\Delta_{\rm ex}^2 + (|\Sigma_\uparrow| + |\Sigma_\downarrow|)^2}\bigg[-\epsilon_{F\uparrow}|\Sigma_\downarrow|\tau_\uparrow + \epsilon_{F\downarrow}|\Sigma_\uparrow|\tau_\downarrow + \frac{(\epsilon_{F\uparrow}\tau_\uparrow - \epsilon_{F\downarrow}\tau_\downarrow)^2}{(\tau_\uparrow\tau_\downarrow)^{1/2}}\frac{\Delta_{\rm ex}(|\Sigma_\uparrow| + |\Sigma_\downarrow| + C)}{4\Delta_{\rm ex}^2 + C^2}\bigg], \tag{7}$$

where $D_0 = m_e/2\pi\hbar^2$ is the 2DEG density of states and

$$\epsilon_{F\uparrow(\downarrow)} = \epsilon_F + (-)\Delta_{\rm ex}, \tag{8}$$

$$C = \pi n D_0(V_\uparrow - V_\downarrow)^2 = (\sqrt{|\Sigma_\uparrow|} - \sqrt{|\Sigma_\downarrow|})^2. \tag{9}$$

The first two terms inside the square brackets in Eq. (7) are the nonvertex (bubble) part, and the last one originates from the vertex correction.

We can simplify the expression for $\sigma_{yx}^{\rm AH}$ further, assuming that $\Delta_{\rm ex} \gg |\Sigma_\uparrow|, |\Sigma_\downarrow|, \tau_{\uparrow(\downarrow)} = \tau(1 + (-)\delta)$, with $\delta \ll 1$, neglecting the self-energy part in the denominator. Expanding up to δ^2

$$\sigma_{yx}^{\rm AH} \sim \sigma_{xx}^{\rm SO}\left(\frac{\epsilon_F}{\Delta_{\rm ex}}\right)^2\frac{|\Sigma|}{\Delta_{\rm ex}}\delta^2, \tag{10}$$

where $\sigma_{xx}^{\rm SO} = 2e^2\tau D_0\lambda^2$ is the correction to the longitudinal conductivity by the SOI in the presence of nonmagnetic impurities, and $\tau = \hbar/2|\Sigma|$ [24].

This is our main result for the AH conductivity. We can draw several conclusions from the above expression. First, we very generally find that $\sigma_{yx}^{\rm AH} = 0$ when the carrier lifetime is spin-independent. This follows already from inspection of Eq. (7), and we verified that this statement is valid even when the real part of the self-energy is retained. We note the similarity with the SH conductivity which vanishes by introducing electron scattering by nonmagnetic impurities. The result emphasizes the role of the ferromagnetism in the AH effect as a filter that converts an intrinsic spin current into a charge current. When, as in the Rashba 2DEG, the SH current vanishes, the AH effect naturally vanishes as well. Note, however, again in analogy with the SH effect, the AH effect does not have to vanish for ferromagnets with different band structures. Also, whereas the AH effect vanishes, the exchange interaction does induce a finite SH current in a Rashba 2DEG.

Second, the AH conductivity is proportional to λ^2 and (like $\tau|\Sigma|$) independent of the lifetime τ, very similar to the extrinsic AH conductivity based on the sidejump mechanism [28]. Our expression is not identical to that for sidejump scattering, however; for example, Crepieux and Bruno [29] obtained a nonzero result even in the limit $\tau_\uparrow = \tau_\downarrow$ for a finite magnetization. Skew scattering does not occur in our model since the SOI is intrinsic and homogenous, and carrier scattering is caused by a normal impurity potential.

Finally, the AH conductivity is odd with respect to the magnetization M, since $\Delta_{\rm ex} \propto M$. Therefore, the AH conductivity changes sign when the magnetization is reversed, as expected. We see in Eq. (12) $\sigma_{yx}^{\rm AH}$ increases with decreasing M. Although Eq. (12) may not be strictly valid for $M \to 0$ because we assumed $\Delta \gg |\Sigma_\uparrow|, |\Sigma_\downarrow|$, $\sigma_{yx}^{\rm AH}$ has to vanish in this limit. Thus, we expect a nonmonotonic dependence of σ_{yx} on M.

We now turn to magnetic impurity effects on the SH conductivity. To this end, we will introduce spin-dependent random potentials into a Rashba-split 2DEG. The random potentials of V_σ are not suitable for the present purpose, since $\langle V_\sigma \rangle$ is spin-dependent which gives rise to a finite exchange potential (and SH effect) in the lowest order approximation. We therefore adopt an s-d type interaction $\delta(\mathbf{r} - \mathbf{R}_i)\mathbf{s} \cdot \mathbf{S}_i$ between conduction electron $\mathbf{s}$ and localized impurity spins $\mathbf{S}_i$ at sites $\mathbf{R}_i$. This type of interaction is isotropic, and no first order correction appears. This isotropy persists in the presence of the Rashba SOI, so the self-energy and Green function are diagonal in the $|s = \pm\rangle$ space which is a convenient basis. The self-energy in the Born approximation is proportional to the thermal average $\langle S^2 \rangle$, which consists of three contributions, one from spin-conserving scattering $\langle \sigma_z S_z \sigma_z S_z \rangle$ and two from spin-flip scattering $\langle \sigma_+ S_- \sigma_- S_+ \rangle$ and $\langle \sigma_- S_+ \sigma_+ S_- \rangle$. Accordingly, the self-energy

$$|\Sigma| \simeq \frac{\hbar}{2}\left(\frac{1}{\tau_0} + \frac{1}{\tau_{sf}}\right) \tag{11}$$

is governed by two scattering rates. $1/\tau_0$ and $1/\tau_{sf}$ correspond to the $\langle S_z^2 \rangle$ and $(1/2)\langle S_+ S_- + S_- S_+ \rangle$ contributions, respectively. In our case, $1/\tau_{sf} = 2/\tau_0$, since the potentials are isotropic.

After evaluating the vertex corrections self-consistently, we may define an effective current vertex as

$$\tilde{\mathbf{J}}_x^{kk} = e\left[\frac{\hbar}{m} k_x \mathbf{1} + \frac{\lambda + \lambda'}{k}(k_x \sigma_z - k_y \sigma_y)\right], \tag{12}$$

where λ' is an effective SOI originating from the vertex corrections that is the solution of

$$\lambda' = -\langle v_m^2 \rangle [A + (\lambda + \lambda')(B + D)], \tag{13}$$

with

$$\langle v_m^2 \rangle = \frac{n_m J^2 \langle S_z^2 \rangle}{4L^2}, \tag{14}$$

where n_m/L^2 is the density of magnetic impurities. The expression of the current vertex λ' is apparently [18,24] the same for magnetic and nonmagnetic impurities except for the definition of $\langle v_m^2 \rangle$. A, B, and D are momentum integrals over products of retarded and advanced Green functions that can be integrated analytically, such that

$$\lambda' = \lambda \Delta_0 / (8 + 7\Delta_0), \tag{15}$$

with $\Delta_0 = (2\tau_0 \lambda k_F)^2$ for magnetic impurities. Here $\langle v_m^2 \rangle$ has been inverted to τ_0 using the relation $\langle S_z^2 \rangle = \langle S^2 \rangle / 3$. Note that $\lambda' = -\lambda$ for nonmagnetic impurities [18]. The final result of the SH conductivity is the same for magnetic and nonmagnetic impurities

$$\sigma_{yx}^{\sigma_z} = e(\lambda + \lambda')/4\pi\hbar\lambda. \tag{16}$$

We therefore find that the SH conductivity, which has been found before to vanish for nonmagnetic impurities, becomes finite in the presence of magnetic impurities. This is our main result concerning the SH effect.

There are two main reasons for the different SH conductivities for magnetic and nonmagnetic impurities. The matrix elements for the magnetic potential include terms $\langle s = \pm | \sigma_z | s = \pm \rangle$, while in the nonmagnetic case the corresponding matrix elements are $\langle s = \pm | \mathbf{1} | s = \pm \rangle$ which have the opposite sign. The other reason is $\langle v_m^2 \rangle$, which is proportional to $\langle S_z^2 \rangle$ not $\langle S^2 \rangle$ as in the self-energy, because the $\langle S_+ S_- + S_- S_+ \rangle$ term couples with the linear terms of k_x or k_y and vanishes in the momentum integral. Therefore, the vertex function includes only τ_0, while the self-energy basically includes both τ_0 and τ_{sf}.

The present results modify the expressions we obtained previously [18,24] for the longitudinal charge conductivity and the in-plane spin accumulation, as follows:

$$\sigma_{xx} = 2e^2\tau_0\left[\frac{n_0}{m} + D_0\lambda^2 - \lambda(\lambda + \lambda')\frac{D_0}{2}\frac{\Delta_0}{1 + \Delta_0}\right], \tag{17}$$

$$\langle s_y \rangle = 4\pi e D_0 \tau_0 \left[\lambda - (\lambda + \lambda')\frac{2 + \Delta_0}{1 + \Delta_0}\right]E, \tag{18}$$

where n_0 is the carrier density, and E is the applied electric field. Since these observables do not vanish under the vertex correction even for nonmagnetic impurities, the effect of introducing magnetic impurities is small.

Finally, let us consider the coexistence of magnetic and nonmagnetic impurities. Since there is no mixing between magnetic and nonmagnetic scattering, the current vertex is still defined by Eq. (12), but now we have $\lambda' = \lambda'_m - \lambda'_n$, where λ'_m and λ'_n are effective SOIs caused by the magnetic and nonmagnetic impurities, respectively. Both satisfy Eq. (13) with $\langle v_m^2 \rangle$ for λ'_m and with $\langle v_n^2 \rangle = n_n \langle V^2 \rangle / 4L^2$ for λ'_n. Here n_n/L^2 and V are the density and potential of the nonmagnetic impurities, respectively. Then λ' also satisfies the relation (13) with $\delta\langle v^2 \rangle = \langle v_m^2 \rangle - \langle v_n^2 \rangle$, which determines λ' self-consistently.

The self-energy, on the other hand, is given by the sum of the contributions from magnetic and nonmagnetic impurities $|\Sigma| = |\Sigma_m + \Sigma_n| \equiv \hbar/2\tau$, with

$$\frac{1}{\tau} = [n_m 4J^2\langle S^2\rangle + n_n\langle V^2\rangle]\frac{m}{\hbar^3}. \tag{19}$$

Consider varying n_m and n_n such that the lifetime τ is kept constant. Noting that $\langle S_z^2\rangle = \langle S^2\rangle/3$ in the paramagnetic state, the effective spin-orbit interaction is then given as

$$\lambda' = \left[n_m J^2 \frac{4}{3}\langle S^2\rangle - \frac{\hbar^3}{\tau m}\right] \times \frac{m\tau}{2\hbar^3}\lambda \frac{\Delta_0}{1+\Delta_0+[n_m J^2\frac{4}{3}\langle S^2\rangle - \frac{\hbar^3}{\tau m}]\frac{m\tau}{2\hbar^3}(2+\Delta_0)}. \tag{20}$$

This expression reproduces the results $\lambda' = -\lambda$ and $\lambda' = \lambda\Delta_0/(8+7\Delta_0)$ for $n_m = 0$ and $n_n = 0$, respectively. At an intermediate value of n_m, $\delta\langle v^2\rangle$ can be zero. In this case, the vertex correction itself vanishes, and the universal spin Hall conductivity is realized even in the presence of impurity scattering. The condition $\delta\langle v^2\rangle \sim 0$ means that normal impurities limit the conductance by the same amount as magnetic impurities. Since the mobility of HgTe|HgCdTe quantum wells can be very high, and the scattering potential of magnetic impurities is quite strong, very small amount of magnetic impurities should be sufficient to achieve complete recovery of the bulk spin Hall effect.

These results shed some doubt on Rashba's general argument for a generally vanishing SH conductivity [20], since he introduced a vector potential representing an external magnetic field but neglected the Zeeman term. Indeed, the latter gives rise to a nonvanishing spin Hall current even in a Rashba-split 2DEG [30]. It should also be noted that the present results have been obtained for an infinite system. Finite size systems may show a nonvanishing SH conductance [31] or SH accumulations at source-drain contacts [32] and Hall edges [30,33].

In conclusion, we studied the intrinsic AH effect in the diffusive transport regime for an exchange-split 2DEG with a Rashba-type SOI. We found that the AH conductivity vanishes unless the lifetime is spin-dependent, indicating a strong similarity between the intrinsic AH and SH effects. Inclusion of a spin-dependent lifetime yields a nonvanishing AH conductivity, the expression of which is similar to that obtained for the sidejump mechanism. The SH conductivity for the Rashba 2DEG with magnetic scattering of electrons has been found to be finite even in the diffusive transport regime. The SH conductivity in the presence of both magnetic and nonmagnetic impurities shows that the SH conductivity may be controlled by the amount of magnetic impurities.

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