Hadronic Light-by-Light Scattering Effect on Muon $g - 2$

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(Received 29 March 1995)

The hadronic light-by-light scattering contribution to muon $g - 2$ is examined using low energy effective theories of QCD, the Nambu–Jona-Lasinio model, and hidden local chiral symmetry, as guides. Our result is -36×10^{-11} with an uncertainty of $\pm 16 \times 10^{-11}$, which includes our best estimate of model dependence. This is within the expected measurement uncertainty of 40×10^{-11} in the forthcoming experiment at Brookhaven National Laboratory. Our result removes one of the main theoretical obstacles in verifying the existence of the weak contribution to the muon $g - 2$.

PACS numbers: 13.40.Em, 11.15.Pg, 12.39.Fe, 14.60.Ef

The anomalous magnetic moment of the muon $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$ is one of the basic physical observables that can be measured with high precision. The most recent results of experiment and theory are as follows [1,2]:

$$a_\mu(\text{exp}) = 1165\,923(8.5) \times 10^{-9},$$
$$a_\mu(\text{th}) = 116\,591\,877(176) \times 10^{-11}. \quad (1)$$

The uncertainty in the measurement of $a_\mu(\text{exp})$ will be reduced in the new experiment at the Brookhaven National Laboratory to $\sim 40 \times 10^{-11}$ [3]. This is about one-fifth of the one-loop weak correction [4]

$$a_\mu(\text{weak-1}) = 195(1) \times 10^{-11}, \quad (2)$$

and of the same order of magnitude as the leading logarithmic term of the two-loop electroweak correction [5], $a_\mu(\text{weak-2}) = -42 \times 10^{-11}$, offering an exciting opportunity to test the quantum effect of the electroweak theory.

Before comparing theory with the forthcoming measurement, however, it is necessary to reduce further the uncertainties in the theoretical prediction for the hadronic contribution. The largest uncertainty comes from the hadronic vacuum-polarization contribution, $a_\mu(\text{had.v.p.})$ [6]. Fortunately, this contribution can be expressed as a convolution of a known function with the experimentally measurable quantity R, the ratio of the hadron production cross section to the $\mu^+\mu^-$ production cross section in e^+e^- collisions. Recent measurements of R at VEPP-2M will improve this estimate significantly [7]. Together with future measurements at DAΦNE, BEPC, etc., the error in $a_\mu(\text{had.v.p.})$ will be reduced to the level of the upcoming experimental limit.

The contribution of the hadronic light-by-light scattering diagram shown in Fig. 1 is potentially a source of more serious difficulty because it cannot be expressed in terms of experimentally accessible observables and hence must be evaluated by purely theoretical consideration. Previous crude estimates, one based on a quark loop model which is a simple adaptation of the corresponding leptonic one [8] and the other based on charged pion loop and low energy resonances, gave contributions of several $\times 10^{-10}$ [9]. Recently, some doubts have been raised [10,11] about the reliability of these estimates. In view of its importance in interpreting the experiment and in drawing inferences about potential "new physics," we have decided to reexamine its theoretical basis.

The bulk of hadronic light-by-light scattering contribution to a_μ is determined by the dynamics around the muon mass $m_\mu \simeq 106$ MeV, which is right in the midst of the nonperturbative regime of QCD. What we need is a reliable evaluation of an off-shell four-point function at these energies. In view of the current status of QCD, however, it is not an easy job to carry out such a calculation from first principles.

Fortunately, this energy region is populated mostly by pions, and considerable information is available about low-energy pion dynamics. Chiral symmetry governs most of it. However, higher energy regions may also

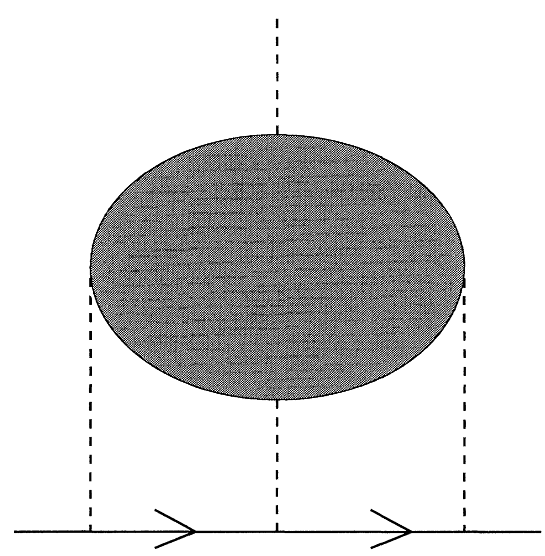

FIG. 1. Hadronic light-by-light scattering contribution (shown by the shaded blob) to the muon anomaly. Solid line and dashed line represent muon and photon, respectively.

 0031-9007/95/75(5)/790(4)$06.00

contribute significantly to a_μ. Momentum-expanded amplitudes obtained in a systematic chiral expansion, however, cannot be introduced directly into Feynman graphs for muon anomaly since it leads to divergent integrals. To get around this difficulty, Ref. [12] used the Nambu–Jona-Lasinio (NJL) model. We chose to rely on the hidden local symmetry (HLS) approach [13]. Since the HLS Lagrangian can be derived from the extended NJL (ENJL) model [14], these two approaches are equivalent as far as application to our problem is concerned.

(a) Relevant diagrams: The NJL model and the $1/N_c$ expansion suggest three distinct contributions to the light-by-light scattering amplitude at low energies [12]. Their contributions to a_μ are shown in Fig. 2. Figure 2(a) shows the charged pseudoscalar meson loop contribution. It is $\mathcal{O}(1)$ in $1/N_c$ expansion and $\mathcal{O}(p^4)$ in chiral perturbation. Figure 2(b) shows a one pion pole diagram which is $\mathcal{O}(N_c)$ and $\mathcal{O}(p^6)$. Figure 2(c) shows a quark loop diagram which is $\mathcal{O}(N_c)$ and starts at $\mathcal{O}(p^8)$ in chiral expansion.

From the viewpoint of QCD, the single-quark-loop diagram Fig. 2(c) represents the averaged hadronic continuum effect in a certain energy region. At low energies, there should be higher-order QCD corrections, which can be approximated by a pion loop diagram Fig. 2(a) and the Nambu-Goldstone boson pole diagram Fig. 2(b). Note that the latter two exclude a single-quark-loop contribution since the pion loop requires at least two quark loops and the pion pole starts from a diagram in which at least two gluons propagate between a quark and an antiquark forming the pion.

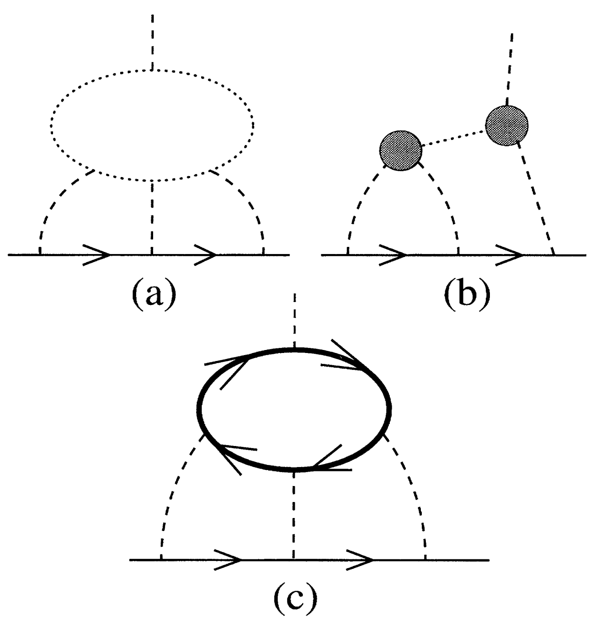

FIG. 2. Diagrams which dominate the hadronic light-by-light effect on a_μ at low energies. (a) Charged pseudoscalar diagram in which the dotted line corresponds to $\pi^\pm$, etc. (b) One of the π^0 pole graphs, in which the dotted line corresponds to π^0 and the blob represents the $\pi\gamma\gamma$ vertex. (c) Quark loop contribution, where the quark is denoted by the bold line.

Contributions involving more loops will be suppressed by a factor $m_\mu/4\pi f_\pi \sim 1/11$. Diagrams in which the pion loop in Fig. 2(a), π^0 pole in Fig. 2(b), and u, d quarks in Fig. 2(c) are replaced by kaon loop, η pole, and s quark, respectively, may also be non-negligible.

(b) Imbedding the light-by-light scattering amplitude: When the light-by-light scattering amplitudes are included in the Feynman graphs for $g-2$, photons must be taken off-shell. The validity of the low energy approximation to these amplitudes will then be affected by the convergence of photon momentum integration.

In order to see what kinds of problems might be in store, let us begin by comparing two previous treatments of the contribution to a_μ due to the charged pion loop diagram of Fig. 2(a) [9]. One deals with the pointlike $\gamma\pi\pi$ and $\gamma\gamma\pi\pi$ couplings (namely, the scalar QED), which yields

$$a_\mu(\text{sQED}) = -0.035\,58(18)\left(\frac{\alpha}{\pi}\right)^3 = -44.57(23)\times 10^{-11}. \quad (3)$$

(This is a slight improvement in statistics over that of Ref. [9].) The second approach attempts to improve (3) based on the vector meson dominance (VMD) model in which photons couple to charged pions through the ρ meson. In Ref. [9], this was achieved by modifying the photon propagator as

$$\frac{i}{q^2} \to \frac{i}{q^2}\frac{m_\rho^2}{m_\rho^2 - q^2} = \frac{i}{q^2} - \frac{i}{q^2 - m_\rho^2}. \quad (4)$$

This leads to

$$a_\mu(\text{nVMD}) = -0.0125(19)\left(\frac{\alpha}{\pi}\right)^3 = -15.7(2.4)\times 10^{-11}. \quad (5)$$

Comparing (3) and (5) we see that the introduction of the ρ meson makes a big difference. This looks alarming. After all, vector mesons hardly contribute to $\pi\pi$ scattering near threshold since the chiral symmetry demands that their contribution vanishes in the soft pion limit. Why should they make such a difference then? Actually, this question is not posed correctly since the role of the ρ meson in (5) is primarily that of modifying the photon propagator and not directly related to the π-π interaction. What is more important is to check whether the vector meson contribution is properly included in (5).

It was pointed out in Ref. [11] that the prescription given by (4) does not respect the Ward identities for the coupling of photons to charged pions, even though it maintains gauge invariance. Upon closer examination, we found further that the substitution (4) is not consistent with chiral symmetry.

In this Letter, we solved these problems based on the HLS approach [13]. This is a convenient way to introduce the dynamics of pions and vector mesons preserving chiral symmetry and gauge invariance. It should be noted that this approach reproduces all current algebra results, such as the KSRF relation [15], as low energy theorems. In this sense, it is the leading candidate for the extension of chiral dynamics of pions to include vector mesons. Actually, insofar as only the low energy dynamics is relevant for the computation of a_μ, any model consistent with chiral symmetry should yield a similar contribution to a_μ.

Before presenting our result, some comments relevant for computing a_μ are in order.

(1) The most important feature of the HLS Lagrangian, in computing a_μ(a) which corresponds to Fig. 2(a), is that it does not have the $\rho^0\rho^0\pi^+\pi^-$ coupling. The naive substitution (4) assumes the presence of this coupling in the Lagrangian. Indeed we found this to be the source of the problem pointed out in [11]. The Ward-Takahashi identity is satisfied once this is corrected.

(2) In computing a_μ(b), the naive VMD model adopted in Ref. [9] did not have a strong theoretical basis beyond that it provided an effective UV cutoff. It has been shown recently, however, that it is justifiable within the HLS approach [16], at least as far as going off shell with respect to photon momenta is concerned. This is also realized in the ENJL model, in which the π^0 pole diagram contains two triangle loops of constituent quarks and the ρ meson is allowed to propagate before the quark couples to the photon.

(3) To compute a_μ(c), it is necessary to know how the quark couples to the photon and how vector mesons come into the picture. In this respect we are guided again by the ENJL model in which a quark loop couples to a photon through a vector meson (see Fig. 4 of Ref. [12]). As in the work of Ref. [9], the constituent quark masses (0.3, 0.3, 0.5, and 1.5 GeV for u, d, s, and c, respectively) are used in Fig. 2(c). Note that the theoretical evaluation of the hadronic vacuum-polarization contribution [12,17] seems to prefer the set of quark masses given above [18].

(c) Numerical results: An extensive numerical evaluation of the contribution of Fig. 2 to a_μ, within the framework of HLS, has yielded

$$\begin{aligned} a_\mu(\mathrm{a}) &= -0.003\,55(12)\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{b}) &= -0.026\,94(5)\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{c}) &= 0.007\,72(31)\left(\frac{\alpha}{\pi}\right)^3. \end{aligned} \tag{6}$$

The errors quoted above are those of numerical integration only and do not include estimates of model dependence. The result a_μ(a) is much smaller than (3). Similarly, a_μ(c) is considerably smaller than the corresponding results without vector mesons. a_μ(b) has a sign opposite to that of Ref. [9], which had a sign error in some terms. Thus, the difference between (3) and (5), which worried us a great deal, does not go away in spite of the improved theory which preserves chiral symmetry and the Ward-Takahashi identity. This forced us to examine which regions of momentum space dominate in a_μ. We explored this problem by varying masses instead of examining momentum dependence directly.

(d) Mass dependence: The dependence of a_μ on the mass m_π or m_q of internal loop in the light-by-light scattering amplitude were found to be as follows:

$$\begin{aligned} a_\mu(\mathrm{a})\,(xm_\pi, M_\rho) &\sim 2.81 \times 10^{-2}x^{-2}\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{b})\,(xm_\pi, M_\rho) &\sim -9.57 \times 10^{-2}x^{-2}\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{c})\,(xm_q, M_\rho) &\sim 1.14 \times 10^{-4}x^{-3.7}\left(\frac{\alpha}{\pi}\right)^3, \end{aligned} \tag{7}$$

for $x > 3$, where x is a scale factor of pion mass in a_μ(a) and a_μ(b), or of quark masses in a_μ(c). The first result in (7) shows that the contribution of pion loop momenta drops off as x^{-2} as x increases. For instance, the contribution of pion momenta higher than 800 MeV accounts for only 7% of (3). From these results, we conclude that the hadronic light-by-light scattering amplitude, even when it is inserted in Feynman graphs for muon anomaly, can be described reasonably well by the graphs we have studied.

We find the behavior $a_\mu(\mathrm{c}) \sim x^{-4}$ quite encouraging. The fact that Fig. 2(c) contributes at all, as is seen from (6), implies that the energy scale of $\mathcal{O}(1\ \mathrm{GeV})$ is important. On the other hand, the steep x dependence of a_μ(c) found in (7) is consistent with the fact that only the physical degree of freedom (mainly pions) is important at low energies [19].

We have also studied the dependence of a_μ on the vector meson mass:

$$\begin{aligned} a_\mu(\mathrm{a})\,(m_\pi, M) &\sim \left[-0.0356 + 0.23\left(\frac{m_\mu}{M}\right)\right]\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{b})\,(m_\pi, M) &\sim \left[-0.0693 + 0.31\left(\frac{m_\mu}{M}\right)\right]\left(\frac{\alpha}{\pi}\right)^3, \\ a_\mu(\mathrm{c})\,(m_\pi, M) &\sim \left[+0.0440 - 0.43\left(\frac{m_\mu}{M}\right)\right]\left(\frac{\alpha}{\pi}\right)^3, \end{aligned} \tag{8}$$

for $M > M_\rho$. This shows that these functions decrease very slowly for large M. Such an M^{-1} (instead of M^{-2}) behavior seems to cast some doubt on the effectiveness of our approach since it implies that an appreciable contribution to a_μ comes from photon momenta far off mass shell. Actually, this is entirely compatible with the dominance of low energy states in the light-by-light scattering amplitude. The naively expected M^{-2} behavior cannot be justified unless the $M_\rho \to \infty$ limit and subintegrations in the Feynman diagram commute. As is clear from power counting, this is not the case. Also, the results (8) show that there are strong cancellations between the first and second terms. In particular, the cancellation in a_μ(a) for $M = M_\rho$ is almost complete.

(e) Error estimates: As stated above, the three diagrams shown in Fig. 2 seem indeed to dominate the light-by-light scattering amplitude in the Feynman graphs for the muon anomaly. The contributions of kaon loop to a_μ(a), η pole to a_μ(b), and heavier quarks to a_μ(c) can be readily included. We expect the error coming from further additional diagrams, as well as from the double counting possibility, to be less than the error caused by the approximate treatment of photon propagators in the HLS approach. By using HLS, we have not taken account, for example, of the continuum states above the vector mesons [20]. As long as we can restrict ourselves to pseudoscalar and quark loop diagrams, however, it is hard to imagine that these continuum states are relevant. Indeed the pion form factor is saturated with ρ meson.

Taking these considerations into account, we estimate that the model-dependent errors from the terms of (6) are well within 20% of the M_ρ-dependent second term. Including the contributions of the kaon loop coupled to ϕ in a_μ(a), the η pole to a_μ(b), and the strange and charm quarks to a_μ(c), we are thus lead to

$$\begin{aligned} a_\mu(\mathrm{a}) &= -0.0036(64)\left(\frac{\alpha}{\pi}\right)^3 \\ &= -4.5(8.1) \times 10^{-11}, \\ a_\mu(\mathrm{b}) &= -0.0328(66)\left(\frac{\alpha}{\pi}\right)^3 \\ &= -41.1(8.3) \times 10^{-11}, \\ a_\mu(\mathrm{c}) &= 0.0077(88)\left(\frac{\alpha}{\pi}\right)^3 \\ &= 9.7(11.1) \times 10^{-11}. \end{aligned} \tag{9}$$

In summary, using chiral symmetry, the $1/N_c$ expansion, and hidden local symmetry as guides we have found that the hadronic light-by-light scattering amplitude can be represented reasonably well as the sum of diagrams containing a charged pion loop, diagrams with a π^0 pole, and diagrams with a quark loop at energies below 1 GeV. Based on this observation we have computed the hadronic light-by-light scattering correction to the muon $g - 2$ due to three diagrams of Fig. 2. The integration over the photon momenta receives considerable contribution from the region where the photons are far off shell. Estimating that these high mass contributions should be well within 20% of the vector meson contribution, we have obtained the result (9) which leads to

$$a_\mu(\text{light-by-light}) = -36(16) \times 10^{-11}. \tag{10}$$

This is within the error expected in the upcoming experiment. Based on our analysis, and in view of the progress in the measurement of R [7], we are quite hopeful that the next round of experiments will indeed verify the weak interaction correction to the muon anomaly.

T. K. thanks S. Tanabashi and H. Kawai for useful discussions. He appreciates the hospitality of the National Laboratory for High Energy Physics (KEK), Japan, where he was visiting on sabbatical leave from Cornell University. His work is supported in part by the U. S. National Science Foundation.

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[20] We also expect, as in the ENJL model, that a q^2-dependent function $M_V^2(q^2)$, where q is the ρ momentum, appears in place of the constant ρ meson mass. As is seen from the explicit form of $M_V^2(q^2)$ given in [12], the q^2 dependence in the relevant range ($\lesssim M_\rho$) is relatively mild so that the correction to this approximation seems to be small.