

# Migration, Frictional Unemployment, and Welfare Improving Labor Policies

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## Abstract

Studies have suggested that there exists job search and recruiting friction in urban areas. This paper constructs a two-sector (rural and urban) model involving this factor and investigates how it affects migration and what the optimal policies should be. An analysis shows that frictional urban unemployment brings about inter-sector wage differentials and that an economy almost always has distortion in the absence of government intervention. Tax and subsidy policies that remove the distortion are explored. Setting urban wages appropriately is also shown to attain the optimum. Finally, we explore the criterion to judge whether changing urban wages as a policy such as the minimum wage law enhances social welfare.

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# 1 INTRODUCTION

The phenomenon of urbanization has been observed across countries, regardless of their levels of development. In fact, World Urbanization Prospect (2001) reports that in 2000, 76 percent of the populations of developed countries live in urbanized areas, while this figure is 39 percent in developing countries. The common causes of this phenomenon are industrialization and commercialization that shift the composition of output away from primary production toward secondary and tertiary production, which tends to concentrate spatially and form cities (Bairoch, 1988). This shift in production brings about large migration from rural areas to cities and has attracted much attention from economists of various fields.

Harris and Todaro (1970) and related studies have provided a series of models that constitute the received theory of rural-urban migration.<sup>1</sup> (Hereafter, we refer to these models as H-T related models.) H-T related models treat migration primarily as an economic phenomenon. These models presume that in cities, wages are set institutionally or for some other reason above the market-clearing level and unemployment exists, while rural workers are assumed to have jobs at all times. Workers are assumed to compare expected incomes in cities with agricultural wages and migrate if the former exceeds the latter. Migration is the equilibrating force which equates the two. An equilibrium is attained when they are equalized and there is no migration.

In these models, the imposed high wages in cities are the sources of unemployment. However, looking into a labor market in urban areas more closely, we can recognize an alternative story: inevitable frictional unemployment in urban areas causes high urban wages. Our aim is to formalize this story. Urban economists have recognized diversity in urban economic activities and that an urban labor market is characterized by heterogeneity and spatial concentration of economic agents (Fujita and Thisse, 1996). Though this is often pointed out in regard to developed countries, as Behrman (1999) describes, it is also the case with developing countries. In such a labor market, there exist unmatched agents in both parties composing the matching process (that is, unemployed workers and vacant jobs)

at the same time, due to information imperfections regarding potential trading partners, heterogeneity, congestion due to large numbers, and other similar factors. Traditionally, labor economists have used the term “friction” as the generic term for these factors and showed its significant influence on a labor market theoretically (Pissarides, 2000) using the framework of search theory. With respect to developed countries, many empirical studies have supported this view (Ridder and van den Berg, 1997). Recently, Rama (1998) showed empirically that there are a significant number of vacancies as well as unemployed workers in the Tunisian formal sector and provided evidence that this view can be applied to the formal sector that in developing countries is mostly urban.

With the presence of friction, it is difficult for a particular worker to secure a job even if there are many vacancies. Similarly, even if there are many workers, a particular firm has some difficulty in finding an appropriate worker to employ, and a certain number of jobs that are profitable if matched with appropriate workers remain vacant. Consequently, unemployment and vacancies exist at the same time. Moreover, with friction, matches are random and the matching process is akin to two-sided musical chairs.<sup>2</sup> Therefore, it inevitably has externalities: given the number of firms, adding one more worker in the matching process makes it less possible for each worker to secure a job and more possible for each firm to hire a worker. Similarly, with the number of workers constant, as the number of firms seeking workers increases, it becomes more difficult for a firm to fill its vacancy and easier for a worker to land a job.

This paper constructs a two-sector model having a rural sector that has full employment and an urban sector with friction in forming job-worker pairs, and explores the implication of friction on rural-urban migration. A closely related model is that developed in Ortega (2000).<sup>3</sup> Ortega (2000) constructed a dynamic model having two sectors, either of which produces the same good and has a labor market with friction. The two sectors are different in separation rate, at which a worker-firm match breaks up. Multiple steady state equilibria were shown to exist: one no-migration equilibrium and two migration equilibrium. Because of the difference in separation rate, an equilibrium with

migration from the high separation sector to the low separation sector dominates a no-migration equilibrium in the Pareto sense. The differences between the model described in this paper and Ortega's model are the following: while only one sector has frictional unemployment in the former, both sectors have it in the latter. Furthermore, though the former deals with two goods and is static, the latter has only one good and is dynamic. These differences imply that this study and Ortega (2000) complement each other. Interestingly, these two studies suggest the possibility that a migration promoting policy improves social welfare: in Ortega (2000), a policy that promotes migration from a high separation rate sector to a low separation rate sector may improve welfare by leading the economy from a no-migration equilibrium to a migration equilibrium. In this paper, as we see later in detail, a policy that enhances rural to urban migration increases welfare when the urban sector falls into labor scarcity.

The main results are the following. First, frictional urban unemployment causes an inter-sector wage disparity, which is larger as the friction becomes more extreme. Second, because of the search and recruiting externalities, an economy almost always has distortion, and an appropriate combination of a production tax or subsidy to the rural sector and a entry tax or subsidy to the urban sector can eliminate this distortion. The key parameter is the labor share in cities: the urban sector has too few workers and too many firms when the labor share is low enough, while it has too many workers and too few firms when the labor share is high enough. In previous studies, when wages are for some reason higher than the market clearing rate, and so generate unemployment in the urban sector, the optimal policies are those that attract firms to this sector to increase employment and repel workers from this sector. In contrast, this paper shows that when frictional unemployment generates high wages in the urban sector, there is a case in which the optimal policies are those that repel firms from cities and attract workers to cities even if unemployment exists there. Finally, we consider a situation in which a government can set urban wages and show that with frictional unemployment, an appropriate wage setting policy can attain the optimum. Furthermore, we derive a criterion to examine whether

changing urban wages as a policy (e.g., raising urban wages by the minimum wage law) can improve social welfare.

The paper is structured as follows. Section 2 describes the model and characterizes its equilibrium. Section 3 explores the efficiency properties of an equilibrium. The effects of policies that set or change urban wages are investigated in Section 4. Section 5 concludes the paper.

## 2 MODEL

In this section, we introduce the basic structure of the model and characterize its equilibrium. In this paper, we develop a static model. Though many of the existing search and matching models are dynamic, as Diamond (1982) showed, we can describe the essence of job search and recruiting externalities using a static model.<sup>4</sup> In order to make analysis as simple as possible, we adopt a static framework.

### *Workers/Households*

Consider a country divided into two sectors: rural (traditional) and urban (modern). There is a continuum of workers of size  $L$  in this country, where  $L$  is assumed to be exogenous.  $L - l$  of  $L$  workers are in the rural sector and  $l$  workers are in the urban sector. Rural workers are self-employed and engage in the production of traditional good  $X$ . In the urban sector, there is search and recruiting friction and workers are either employed or unemployed. If employed, they engage in the production of modern good  $Y$ . In order to model the friction, we apply the “matching approach”, which summarizes the complicated exchange process into a well-behaved function that gives the number of productive matches formed in terms of the number of workers looking for jobs and the number of firms looking for workers (Pissarides, 2000). Such a function is called the matching function. It is a modeling device similar to a production function or other aggregate functions.<sup>5</sup>

There is a continuum of firms of size  $f$  in the urban sector. We assume, for simplicity, that each

firm can employ only one worker. The number of successful matches in this sector  $M$  is determined by the matching function  $m(l, f)$  that is defined on  $\mathbb{R}_+ \times \mathbb{R}_+$  and  $0 \leq m(l, f) \leq \min[l, f]$ .

$$M = m(l, f).$$

Furthermore,  $m(l, f)$  is assumed to be strictly increasing in both its arguments, twice differentiable, concave, homogeneous of degree one, and  $m(l, 0) = m(0, f) = 0$ .<sup>6</sup> We assume that securing a job is equally likely for all workers in the urban sector. Then, the probability that each worker finds a job is represented as

$$\phi = \phi(\theta) = \frac{M}{l} = m(1, \theta),$$

where  $\theta = f/l$ . Similarly, finding a worker is equally likely for all urban firms and the probability of a successful match for each firm is

$$(1) \quad \psi = \psi(\theta) = \frac{M}{f} = m\left(\frac{1}{\theta}, 1\right).$$

For given  $l$  and  $f$ , small  $M$  indicates small  $\phi$  and  $\psi$ , and represents the presence of much friction.

In the urban sector,  $\phi l$  of  $l$  workers are employed and  $(1 - \phi)l$  workers are unemployed.  $\psi f$  of  $f$  firms fill their vacancies and  $(1 - \psi)f$  firms remain with vacancies. Note that  $\phi l = \psi f$ ,  $d\phi/d\theta > 0$  and  $d\psi/d\theta < 0$ . From this, we can see that if the number of workers  $l$  rises with the number of firms  $f$  constant, the probability of a successful match for each worker  $\phi$  declines ( $\partial\phi/\partial l < 0$ ) and that for each firm  $\psi$  rises ( $\partial\psi/\partial l > 0$ ). In contrast,  $\phi$  rises and  $\psi$  declines if  $f$  rises with  $l$  fixed ( $\partial\phi/\partial f > 0$  and  $\partial\psi/\partial f < 0$ ). Thus, the matching approach successfully expresses the externalities of the matching process with friction.

Let  $c_x$  and  $c_y$  denote the consumption of traditional good  $X$  and modern good  $Y$ , respectively. Workers/households in this model are assumed to have an identical utility function of the Cobb-Douglas form:<sup>7</sup>

$$(2) \quad U = a c_x^\alpha c_y^{1-\alpha}, \alpha \in (0, 1).$$

We treat traditional good  $X$  as a numeraire and let  $p$  denote the relative price of modern good  $Y$ . From the first-order conditions of the maximization of (2), we obtain the uncompensated demand for good  $X$  and that for good  $Y$ , and the indirect utility function:

$$\begin{aligned} c_{xi} &= \alpha I_i, \quad c_{yi} = \frac{(1-\alpha)I_i}{p}, \\ V_i &= Ap^{\alpha-1}I_i, \quad i = r, e, u. \end{aligned}$$

$I_i$  represents the income of a worker and  $A$  is defined as:  $A = a\alpha^\alpha(1-\alpha)^{1-\alpha}$ . Hereafter, the relevant variables of the rural sector are marked with the subscript  $r$  and those of the urban sector carry the subscript  $e$  or  $u$  depending on whether workers are employed or unemployed.

Each worker is endowed with one unit of labor and supplies it when he/she is employed and obtains the wage  $w_i$ . Let  $\Pi$  denote the sum of profits of all firms in the urban sector. Workers own firms and each worker obtains a share  $\mu \in [0, 1]$  of  $\Pi$ . We assume that  $\mu$  is common to all workers and  $\mu = 1/L$ .<sup>8</sup> Then,  $I_i = w_i + \mu\Pi$ .

As in Harris and Todaro (1970) and many other studies, we assume that workers are perfectly mobile between sectors and that migration occurs so as to equate the expected indirect utility between sectors. It is also assumed that an unemployed worker does not obtain any wage. Then, we obtain

$$V_r = \phi V_e + (1-\phi)V_u.$$

This condition is reduced to

$$(3) \quad w_r = \phi w_e.$$

We refer to this condition as the no-migration condition.

### *Firms and Wages*

Workers in the rural sector are self-employed. Production of traditional good  $X$  needs only labor as input and is under constant returns to scale. The technology is assumed to require one unit of labor

for  $x$  units of output. Without loss of generality, we normalize  $x$  to one. Because good  $X$  is considered to be a numeraire, each worker earns 1 by engaging in traditional production and  $w_r = 1$ .<sup>9</sup>

In the urban sector, firms post vacancies and search for workers to employ. We suppose that each firm can employ only one worker. A firm must bear costs  $c$  when it opens a vacancy.  $c$  is the fixed cost of labor recruitment, which is represented in terms of the numeraire good.  $\psi f$  of  $f$  firms can form productive matches and engage in production of modern good  $Y$ . We assume that each productive match produces  $y$  units of output, where  $y$  is assumed to be exogenous. Again, without loss of generality, we normalize  $y$  to one. These indicate that the production of good  $Y$  needs labor and capital (machines) as input and is under constant returns to scale. Analogously to Helsley and Strange (1990) and Monfort and Ottaviano (2000), before paying the cost of posting a vacancy, a firm is not sure to be matched with a worker. Consequently, it forms expectation regarding its profit under (1). The expected profit of each firm is

$$\pi = \psi(p - w_e) - c.$$

We assume free entry of firms.<sup>10</sup> Therefore, vacant jobs are created until the expected profit of posting a vacancy  $\pi$  is equal to zero:

$$(4) \quad \psi(p - w_e) = c.$$

We refer to this condition as the free-entry condition.

The wage in the urban sector  $w_e$  is determined by the decentralized Nash bargain, which imposes a particular division of matching surplus between the two parties involved in the bargaining process according to the relative bargaining power between them.<sup>11</sup> For a worker, the matching surplus is the difference between the indirect utility when employed and that when unemployed:  $V_e - V_u = Ap^{\alpha-1}w_e$ . For a firm, the matching surplus is the difference between the profit when it fills a vacancy and that when it remains with a vacancy:  $p - w_e - c - (-c) = p - w_e$ .  $w_e$  is determined as

$$w_e = \arg \max (Ap^{\alpha-1}w_e)^\beta (p - w_e)^{1-\beta},$$

where  $\beta \in [0, 1]$  measures the bargaining power of workers.<sup>12</sup> Simple calculation yields

$$(5) \quad w_e = \beta p.$$

This implies that  $\beta$  represents the labor share.

### Prices

Because traditional good  $X$  is a numeraire, we need only to determine the price of modern good  $Y$ .

Let  $\Xi$  be the total income of workers:  $\Xi = (L-l)I_r + \phi l I_e + (1-\phi)l I_u$ . The total demand for traditional good  $X$  and that for modern good  $Y$  are

$$\begin{aligned} cf + (L-l)c_{xr} + \phi l c_{xe} + (1-\phi)l c_{xu} &= cf + \alpha \Xi, \\ (L-l)c_{yr} + \phi l c_{ye} + (1-\phi)l c_{yu} &= \frac{(1-\alpha)\Xi}{p}. \end{aligned}$$

The total supply for good  $X$  is  $L-l$  and that for good  $Y$  is  $\phi l$ . We assume that there is no friction in good markets. The material balance conditions for good  $X$  and  $Y$  require that the total demands,  $cf + \alpha \Xi$  and  $(1-\alpha)\Xi/p$ , be equal to the total supplies,  $L-l$  and  $\phi l$ . These yield the price of good  $Y$ :

$$(6) \quad p = \frac{1-\alpha}{\alpha} \frac{L-l-cf}{\phi l}.$$

### Equilibrium

We define *an equilibrium of the model* as a tuple  $(l^*, f^*, w_r^*, w_e^*, p^*)$  that satisfies the following conditions: [C1] the no-migration condition (3), [C2] the free-entry condition (4), [C3] the rural wage equation  $w_r = 1$ , [C4] the urban wage equation (5), and [C5] the price equation (6).

From (5), (6), and  $\phi l = \psi f$ , (4) yields the equilibrium number of firms given the number of workers:

$$(7) \quad f^* = \frac{(1-\alpha)(1-\beta)(L-l)}{1-\beta(1-\alpha)} \frac{1}{c}.$$

Substituting  $w_r = 1$ , (5), (6), and (7) into (3) gives the equilibrium number of urban workers:

$$(8) \quad l^* = \beta(1-\alpha)L.$$

From (7) and (8), we obtain

$$(9) \quad f^* = (1 - \alpha)(1 - \beta) \frac{L}{c}.$$

As the labor share in the urban sector  $\beta$  increases, the number of workers in this sector increases and firms that open vacancies decrease. We define  $\theta^*$  and  $\phi^*$  as:  $\theta^* = f^*/l^* = (1 - \beta)/\beta c$  and  $\phi^* = \phi(\theta^*)$ .

The equilibrium price  $p^*$  and urban wage  $w_e^*$  are

$$(10) \quad \begin{aligned} p^* &= \frac{1}{\phi^* \beta}, \\ w_e^* &= \beta p^* = \frac{1}{\phi^*} = \frac{w_r}{\phi^*}. \end{aligned}$$

Note that in an equilibrium, the urban wage is higher than the rural wage. This wage disparity is more serious as the friction becomes more severe; that is,  $\phi^*$  is smaller. In the former H-T related models, wages in cities are determined to be higher than rural wages due to some institutional reason or some wage determination process, and these high wages generate urban unemployment. In contrast, in the model described in this paper, the causality is reversed: there is inevitable frictional unemployment and this generates high wages in the urban sector. (7) to (10) prove that an equilibrium exists and is uniquely determined by (8), (9),  $w_r = 1$ , and (10).

Proposition 1 summarizes the above arguments.

**Proposition 1** *The model has the unique equilibrium, which has an inter-sector wage disparity due to friction in forming urban jobs. With a larger labor share in the urban sector  $\beta$ , there are more workers, and fewer firms open vacancies in this sector.*

### 3 WELFARE CONSIDERATIONS I - AN ALLOCATION OF AGENTS

In this and the next section, we investigate the welfare implication of friction. This section explores whether the no-migration condition and the free-entry condition allocate agents optimally between the two sectors. We focus on a constrained optimum, with certain behaviors of the actors taken as

given. The social planner is able to fix how many workers are in the rural and urban areas, and how many firms operate. However, the planner is assumed to take the wage and price equations as given and to be unable to directly alter payments to individuals. Note also that the planner is subject to the same matching constraints as workers and firms.

The efficiency criterion in this paper is the Benthamite social welfare function:<sup>13</sup>

$$(11) \quad \begin{aligned} W &= \phi l V_e + (1 - \phi) l V_u + (L - l) V_r, \\ &= A p^{\alpha-1} \{ \phi l w_e + (L - l) w_r + \Pi \}. \end{aligned}$$

We define *the optimal allocation of agents* as a tuple  $(l^{**}, f^{**}, w_r^{**}, w_e^{**}, p^{**})$  that maximizes  $W$  with respect to  $l$  and  $f$  under the wage equations  $w_r = 1$  and (5), and the price equation (6).

Substituting the wage equation  $w_r = 1$ , the price equation (6),  $\Pi = f\pi$  and  $\phi l = \psi f$  into (11) gives

$$(12) \quad W = \frac{A}{\alpha} \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha-1} (\phi l)^{1-\alpha} (L - l - cf)^{\alpha}.$$

The first-order conditions for maximizing  $W$  with respect to  $l$  and  $f$  are<sup>14</sup>

$$\begin{aligned} (1 - \alpha) \frac{1 - \eta}{l} &= \frac{\alpha}{L - l - cf}, \\ (1 - \alpha) \frac{\eta}{f} &= \frac{\alpha c}{L - l - cf}. \end{aligned}$$

$\eta$  is the elasticity of the worker's probability of securing a job with respect to  $\theta$ :  $\eta = \eta(\theta) = (d\phi/d\theta)\theta/\phi$ .

Note that  $\forall \theta$ ,  $0 < \eta < 1$  since the matching function  $m(l, f)$  is homogeneous of degree one. Solving these equations together gives the optimal number of workers and firms ( $l^{**}$  and  $f^{**}$ ):

$$(13) \quad \begin{aligned} l^{**} &= (1 - \eta^{**})(1 - \alpha)L, \\ f^{**} &= \frac{\eta^{**}(1 - \alpha)}{\eta^{**}(1 - \alpha) + \alpha} \frac{L - l^{**}}{c}, \end{aligned}$$

where  $\eta^{**} = \eta(\theta^{**})$  and  $\theta^{**} = f^{**}/l^{**}$ .

Proposition 2 states the basic efficiency property of the model.

**Proposition 2** *When  $\beta \neq 1 - \eta^{**}$ , the equilibrium does not attain the optimal allocation of agents.*

*When  $\beta = 1 - \eta^{**}$ , the equilibrium attains the optimal allocation.*

**Proof.** Comparing (8) and (13), the equilibrium number of workers  $l^*$  coincides with the optimal number of workers  $l^{**}$  if and only if

$$(14) \quad \beta = 1 - \eta^{**}.$$

When (14) holds, (7) becomes

$$f^* = \frac{\eta^{**}(1 - \alpha)}{\eta^{**}(1 - \alpha) + \alpha} \frac{L - l^{**}}{c},$$

which implies that  $l^* = l^{**}$  and  $f^* = f^{**}$ . If  $\beta \neq 1 - \eta^{**}$ , then it is true that  $l^* \neq l^{**}$ . ■

Thus, the equilibrium is not optimal except for a point of measure zero. In order to understand the efficiency properties of the model intuitively, we explore the no-migration condition and the free-entry condition separately. Note here that  $\beta$  represents the urban labor share: an urban worker receives  $\beta$  percent of the value of a match and an urban firm receives  $1 - \beta$  percent of it. First, we examine the no-migration condition. This condition gives the number of workers as in (8). Note that this does not depend on the number of firms. Therefore, when  $\beta$  is large enough (and is larger than the threshold level  $1 - \eta^{**}$ ), there are strong incentives for workers to flow into the urban sector and the no-migration condition gives too many workers in this sector ( $l^* > l^{**}$ ). Conversely, when  $\beta$  is small enough (and is smaller than the threshold level  $1 - \eta^{**}$ ), there are only weak incentives for workers to be in this sector and there are too few workers ( $l^* < l^{**}$ ). Distortion in the number of workers does not occur if and only if  $\beta = 1 - \eta^{**}$ . Second, we explore the free-entry condition. This condition gives the number of firms given the number of workers as in (7). Evaluating (7) at  $l = l^{**}$  and comparing it with  $f^{**}$ , we can observe that  $f^*|_{l=l^{**}}$  coincides with  $f^{**}$  if and only if  $\beta = 1 - \eta^{**}$ . If  $\beta > (<)1 - \eta^{**}$ , then  $f^*|_{l=l^{**}} < (>)f^{**}$ ; that is, when  $\beta > 1 - \eta^{**}$ , potential firms have weak incentives to open vacancies and the free-entry condition gives too few firms. With  $\beta < 1 - \eta^{**}$ , potential firms have strong incentives to post vacancies and there are too many firms. Again, distortion in the number of firms does not

occur if and only if  $\beta = 1 - \eta^{**}$ . Figure 1 summarizes the above arguments.

**[Please insert Figure 1 here]**

The result that the equilibrium is optimal when the urban labor share  $\beta$  takes a certain value  $1 - \eta^{**}$  and is not when  $\beta \neq 1 - \eta^{**}$  is obtained in standard search models. The condition  $\beta = 1 - \eta^{**}$  is called the Hosios condition (Hosios, 1990). This result implies that even if we add the rural-urban migration behavior into a search model, the Hosios condition is still relevant.

Next, we show that appropriate taxes and subsidies can lead the equilibrium to the optimal allocation. Let  $\tau$  and  $s$  denote a production tax or subsidy to the rural sector and a entry tax or subsidy to firms in the urban sector, respectively.<sup>15</sup> The no-migration condition (3) and the free-entry condition (4) change, respectively to

$$(15) \quad (1 + \tau)w_r = \phi w_e,$$

$$(16) \quad \psi(p - w_e) = (1 - s)c.$$

Negative  $\tau$  and  $s$  represent taxes and positive  $\tau$  and  $s$  represent subsidies. Now we define *an equilibrium with taxes and subsidies* as a tuple  $(l^{*'}, f^{*'}, w_r^{*'}, w_e^{*'}, p^{*'})$  that satisfies the following conditions: [C'1] the no-migration condition (15), [C'2] the free-entry condition (16), [C'3] the rural wage equation  $w_r = 1$ , [C'4] the urban wage equation (5), and [C'5] the price equation (6). Combined with the wage equations and the price equation, (15) and (16) yield the equilibrium number of workers and firms:

$$(17) \quad \begin{aligned} l^{*'} &= \beta \left\{ \beta + (1 + \tau) \left( \frac{\alpha}{1 - \alpha} + \frac{1 - \beta}{1 - s} \right) \right\}^{-1} L, \\ f^{*'} &= \left( 1 + \frac{\alpha}{1 - \alpha} \frac{1 - s}{1 - \beta} \right)^{-1} \frac{L - l^{*'}}{c}. \end{aligned}$$

Comparing (17) with (13), we can determine the optimal level of  $\tau$  and  $s$ :

$$\tau^{**} = \frac{\beta - 1 + \eta^{**}}{1 - \eta^{**}}, \quad s^{**} = \frac{\beta - 1 + \eta^{**}}{\eta^{**}},$$

which leads to Proposition 3.

**Proposition 3** *The equilibrium with taxes or subsidies set at  $\tau^{**} = (\beta - 1 + \eta^{**})/(1 - \eta^{**})$ ,  $s^{**} = (\beta - 1 + \eta^{**})/\eta^{**}$  attains the optimal allocation of agents.*

Figure 2 describes whether  $\tau^{**}$  and  $s^{**}$  are taxes or subsidies.

**[Please insert Figure 2 here]**

When too many workers are in the urban sector,  $\tau^{**}$  is a subsidy to the traditional good production to remove workers from cities to rural areas. When there are too few urban firms,  $s^{**}$  is a subsidy to them to induce new firms to enter. When the reverse conditions are true,  $\tau^{**}$  and  $s^{**}$  are taxes. With  $\beta < 1 - \eta^{**}$ , a production tax to the rural sector and a entry tax to urban firms attain the optimal allocation ( $\tau^{**} < 0$  and  $s^{**} < 0$ ). With  $\beta = 1 - \eta^{**}$ , no tax or subsidy is needed. With  $\beta > 1 - \eta^{**}$ , production and entry subsidies generate the optimum ( $\tau^{**} > 0$  and  $s^{**} > 0$ ).

Basically, in the model described in this paper, two instruments (i.e., two taxes/subsidies) must be applied in order to assure that the competitive economy achieves the optimal allocation. This is because we have two margins in which efficiency must be obtained, and wages cannot adjust to assure that either operates efficiently. The first margin is the labor allocation between urban and rural areas. This responds to the difference between rural wages and the expected urban wages. The second margin is in the firm-worker ratio in urban areas, where neither workers nor firms are given the correct price signals to induce their participation. Even though we can assure the appropriate urban-rural labor allocation by a rural subsidy/tax, this does not assure the right number of firms in the urban area. The urban area subsidy must be used to assure that the ratio of urban to rural workers will be optimal.

We compare these welfare maximizing policies with those determined by some of the existing H-T related models. In the original Harris and Todaro model (Harris and Todaro, 1970) in which exogenous rigid wages yield unemployment in cities, the optimal allocation of agents is attained by 1) a uniform

wage subsidy, regardless of the sector of employment, or 2) a wage subsidy to the urban sector plus a production subsidy to the rural sector (Bhagwati and Srinivasan, 1974; Basu, 1980). Calvo (1978) and Quibria (1988) constructed models in which urban wages are determined endogenously and not competitively. In Calvo's model, they depend on rural wages and in order to attain the optimum, a wage subsidy to the urban sector and migration barriers are necessary. In Quibria's model, there are formal and informal sectors in cities. Wages of the formal sector depend on wages of the informal sector and a combination of a wage tax and an employment subsidy to the formal sector can attain the optimum.<sup>16</sup> The common feature of the optimal policies in these models is that when wages are for some reason higher than the market-clearing rate to generate unemployment in the urban sector, it is always necessary to implement policies that attract firms to the urban sector in order to increase employment and remove workers from this sector to the rural sector. In contrast, in the model described in this paper in which frictional unemployment generates high urban wages, there is a case in which the urban sector has too many firms and too few workers, and policies that repel firms from this sector and attract workers to this sector can lead the equilibrium to the optimal allocation ( $\beta < 1 - \eta^{**}$ ).

## 4 WELFARE CONSIDERATIONS II - OPTIMAL WAGES

The minimum wage law has been put in place in many countries and has been an important issue in policy debate. In the standard model of rural-urban migration, raising the minimum wage reduces both urban employment and social welfare. How would such a wage changing policy, if possible, affect the equilibrium of the model described in this paper? This section explores the government's potential role in setting or changing urban wages. Here, the social planner is able to fix not only the allocation of agents but also the urban wage rate  $w_e$ . The efficiency criterion is again given by (11). This time, we define *the optimal allocation of agents* as a tuple  $(l^{**}, f^{**}, w_r^{**}, w_e^{**}, p^{**})$  that maximizes  $W$  with respect to  $l$ ,  $f$ , and  $w_e$  under the rural wage equation  $w_r = 1$  and the price equation (6). Using

$w_r = 1$  and (6), (11) is reduced to (12). Note that (12) does not depend on  $w_e$ , and that the optimal allocation is again given by (13).

We replace the urban wage equation in the definition of an equilibrium of the model (5) to  $w_e = \bar{w}_e$  and define a *fixed-wage equilibrium with a given urban wage*  $\bar{w}_e$  as a tuple  $(\bar{l}^*, \bar{f}^*, \bar{w}_r^*, \bar{w}_e^*, \bar{p}^*)$  that satisfies the following conditions: [C"1] the no-migration condition (3), [C"2] the free-entry condition (4), [C"3] the rural wage equation  $w_r = 1$ , [C"4] the urban wage equation  $w_e = \bar{w}_e$ , and [C"5] the price equation (6).

Substituting the wage equations, the price equation, and  $\phi l = \psi f$  into (3) and (4), we obtain equations that determine the fixed-wage equilibrium number of workers and firms.

$$(18) \quad \begin{aligned} \bar{\phi}^* \bar{w}_e &= 1, \\ \frac{\bar{\phi}^* \bar{l}^*}{\bar{f}^*} \left\{ \frac{1 - \alpha}{\alpha} \frac{L - \bar{l}^* - c \bar{f}^*}{\bar{\phi}^* \bar{l}^*} - \bar{w}_e \right\} &= c, \end{aligned}$$

where  $\bar{\phi}^* = \phi(\bar{\theta}^*)$  and  $\bar{\theta}^* = \bar{f}^* / \bar{l}^*$ .

**Proposition 4** *A fixed-wage equilibrium with  $\bar{w}_e^{**} = 1/\phi(\eta^{**}/\{(1 - \eta^{**})c\})$  attains the optimal allocation of agents.*

**Proof.** Eliminating  $w_e$  from (18) gives

$$(19) \quad \bar{f}^* = \frac{1}{c} \left\{ (1 - \alpha)L - \bar{l}^* \right\}.$$

Meanwhile, the optimal number of workers and firms (13) can be solved as

$$(20) \quad \begin{aligned} l^{**} &= (1 - \eta^{**})(1 - \alpha)L, \\ f^{**} &= \frac{1}{c} \eta^{**}(1 - \alpha)L. \end{aligned}$$

From (19) and (20), we can see that  $\bar{f}^*|_{\bar{l}^* = l^{**}} = f^{**}$ , which implies that the number of firms is optimal if the number of urban workers is optimal.  $\bar{\phi}^* \bar{w}_e = 1$  (in (18)) and (19) give

$$\bar{l}^* = \frac{(1 - \alpha)L/c}{1/c + \phi^{-1}(1/\bar{w}_e)}.$$

Comparing this with (20), we can prove that the number of urban workers is optimal if and only if

$$\frac{1/c}{1/c + \phi^{-1}(1/\bar{w}_e)} = 1 - \eta^{**},$$

which yields

$$\bar{w}_e^{**} = \frac{1}{\phi(\eta^{**}/\{(1 - \eta^{**})c\})}.$$

■

This proposition describes the government's potential role in setting wages. If the government has the ability to influence the urban wage that firms and workers settle on, the government can obtain the optimum ((13) or equivalently, (20)). The important feature here is that efficiency can be obtained with a single instrument, and without any need for compensating taxes. A wage level different from  $\bar{w}_e^{**}$  results in a loss in welfare.

Even if we have no information regarding the optimal allocation, and thus regarding  $\bar{w}_e^{**}$ , we can see whether or not the wage hike or reduction improves welfare. Since the fixed-wage equilibrium with  $\bar{w}_e$  coincides with the equilibrium of the model if we set  $\bar{w}_e = w_e^*$ , we can see the effects of changing urban wages as a policy on the equilibrium by examining the effects of changing  $\bar{w}_e$  on the fixed-wage equilibrium and evaluating them at  $\bar{w}_e = w_e^*$ .

**Proposition 5** *At the equilibrium of the model  $(l^*, f^*, w_r^*, w_e^*, p^*)$ , raising (lowering) the urban wage as a policy increases urban employment if and only if  $1 - \eta^* > (<)\beta$ .*

**Proof.** Comparative statics on the fixed-wage equilibrium show that

$$(21) \quad \begin{aligned} \frac{\partial \bar{l}^*}{\partial \bar{w}_e} &= \frac{1}{|A|} c \bar{\phi}^* > 0, & \frac{\partial \bar{f}^*}{\partial \bar{w}_e} &= -\frac{1}{|A|} \bar{\phi}^* < 0, \\ |A| &= \frac{\bar{w}_e d\phi/d\theta|_{\theta=\bar{\theta}^*}}{\bar{l}^{*2}} (c\bar{f}^* + \bar{l}^*) > 0. \end{aligned}$$

From (21), we can see the effect of an increase in  $\bar{w}_e$  on employment  $\bar{M}^* = m(\bar{l}^*, \bar{f}^*)$ :

$$\frac{\partial \bar{M}^*}{\partial \bar{w}_e} = \frac{\partial m(\bar{l}^*, \bar{f}^*)}{\partial l} \frac{\partial \bar{l}^*}{\partial \bar{w}_e} + \frac{\partial m(\bar{l}^*, \bar{f}^*)}{\partial f} \frac{\partial \bar{f}^*}{\partial \bar{w}_e}.$$

Evaluating this at  $\bar{w}_e = w_e^*$  and using (8) and (9), we obtain

$$(22) \quad \begin{aligned} \operatorname{sgn} \left( \left. \frac{\partial \bar{M}^*}{\partial \bar{w}_e} \right|_{\bar{w}_e = w_e^*} \right) &= \operatorname{sgn} \left( c(1 - \eta^*) - \eta^* \frac{l^*}{f^*} \right) \\ &= \operatorname{sgn} [1 - \eta^* - \beta], \end{aligned}$$

where  $\eta^* = \eta(\theta^*)$ . Since the fixed-wage equilibrium coincides with the equilibrium of the model when  $\bar{w}_e = w_e^*$ , (22) shows the effect of raising the urban wage on employment at the equilibrium of the model. ■

The social welfare level for a fixed-wage equilibrium is derived by substituting the five fixed-wage equilibrium conditions ((3), (4),  $w_r = 1$ ,  $w_e = \bar{w}_e$ , and (6)) into (11).

$$(23) \quad \bar{W} = AL \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha-1} \left( \frac{L - \bar{l}^* - c\bar{f}^*}{\bar{\phi}^* \bar{l}^*} \right)^{\alpha-1}.$$

Differentiating (23) with respect to  $\bar{w}_e$  and evaluating it at  $\bar{w}_e = w_e^*$  (and using (8) and (9)) yield

$$\begin{aligned} \operatorname{sgn} \left( \left. \frac{\partial \bar{W}}{\partial \bar{w}_e} \right|_{\bar{w}_e = w_e^*} \right) &= \operatorname{sgn} \left( c(1 - \eta^*) - \eta^* \frac{l^*}{f^*} \right) \\ &= \operatorname{sgn} (1 - \eta^* - \beta). \end{aligned}$$

**Proposition 6** *At the equilibrium of the model  $(l^*, f^*, w_r^*, w_e^*, p^*)$ , raising (lowering) the urban wage as a policy increases social welfare if and only if  $1 - \eta^* > (<) \beta$ .*

In typical H-T related models, whenever urban unemployment exists, an execution or an amendment of the minimum wage law that increases wages in cities definitely reduces urban employment and social welfare. In the model of Harris and Todaro (1970), it is the primary cause of unemployment and distortion. In models in which they are caused by some other reason, it worsens the situation (Calvo, 1978; Moene, 1988; Quibria, 1988).<sup>17</sup> Stiglitz (1974) and Carter (1998) provided exceptions. In the model of Stiglitz (1974), labor turnover is costly for firms, and for each firm, the quit rate, i.e., the percentage of the labor force quitting, declines as (i) the firm raises its wage relative to the average urban wage and (ii) to rural wages, and as (iii) the urban unemployment rate rises. If the marginal effect of (iii) is great enough, the competitive wage is too low. Though each firm takes the

unemployment rate as given, an increase in all firms' wage rates increases the unemployment rate and hence lowers the turnover costs. This causes each firm to underestimate the marginal effect of raising its wage. Carter (1998) revised the original Harris-Todaro model using efficiency wages and showed that making urban wages higher than the market-clearing rate decreases employment in cities but increases urban productivity and so improves social welfare.<sup>18</sup> The model described in this paper provides another possibility of welfare-improving minimum wage law. Moreover, the result here indicates the possibility of welfare-improving maximum wage law.

From Propositions 4 and 5, we can see the strong relationship between the effects of a change in wages on social welfare and on employment.

**Proposition 7** *At the equilibrium of the model  $(l^*, f^*, w_r^*, w_e^*, p^*)$ , the necessary and sufficient condition for changing the urban wage as a policy to increase the social welfare is that it increases urban employment.*

Whenever a wage raising policy increases employment, it improves welfare. Similarly, whenever a wage lowering policy increases employment, it improves welfare. Thus, the policy's effect on employment can be regarded as a criterion for examining the effect on social welfare. Because the effect on employment can be examined through empirical studies, this result is useful to assess a wage changing policy such as the execution or an amendment of the minimum wage law that hikes wages in cities.

## 5 CONCLUDING REMARKS

This paper explored the implications of urban job search and recruiting friction in the rural-urban migration theory. The analysis showed that frictional urban unemployment generates an inter-sector wage disparity, that an economy almost always has distortion, and that an appropriate combination of a production tax (or subsidy) to the rural sector and a entry tax (or subsidy) to the urban sector can eliminate this distortion. The distinct feature of the results here is that there is a case that policies

that repel firms from cities and attract workers to cities can attain the optimum even if unemployment is present there. The effects of a policy that sets or changes urban wages were also investigated. It was shown that efficiency can be obtained if the urban wage level is appropriately set. Furthermore, contrary to the typical H-T related models, the present model suggested that raising urban wages by the minimum wage law can increase urban employment and that in such a case, it also improves social welfare. These results would be valid in a situation in which the shortfall of workers in the urban sector makes it difficult for urban firms to fill their vacancies with appropriate workers.

In the previous literature regarding this subject, welfare analysis has hitherto been performed using models in which high wages generate unemployment in the urban sector. The results here indicate what we could say if the causality is reversed and given frictional unemployment. Such an analysis is meaningful because friction is thought to be one of the main features of the modernized urban labor market.

Of course, this paper is a first attempt to analyze the effect of friction on rural-urban migration and there are a number of dimensions into which the current analysis could be extended. First, including the urban informal sector in the model as discussed in Bhattacharya (1998) is significant. With the informal sector, matching externalities would have effects on the movement of workers between the formal and informal sectors, and cause another distortion. Second, though the matching process is scale neutral in the model described in this paper, we should also take the scale effects of the process into consideration. This is because existing empirical studies show that the process sometimes has the scale effects (Petrongolo and Pissarides, 2001), which combined with heterogeneity in the skills of workers, can bring about agglomeration economies or diseconomies (Monfort and Ottaviano, 2000; Sato, 2001), and Shukla and Stark (1990) and Chao and Yu (1994) showed that such agglomeration economies or diseconomies affect migration. Third, introducing the spatial structure of a city would affect the efficiency properties of the model described in this paper. Wasmer and Zenou (2002) showed that the matching process influences the spatial structure, and Brueckner and Zenou (1999) constructed

a Harris-Todaro model having a monocentric city structure and showed that a city structure has significant effects on rural-urban migration. Fourth, introduction of capital mobility as in Corden and Findlay (1975) may change the results of this paper. Fifth, it will be interesting to consider distribution issues such as how the results would change if urban firms are foreign-owned. Finally, making the model a dynamic one is also an essential extension. These are all important topics for future investigation.

## Notes

<sup>1</sup>For a survey of this type of model, see Khan (1987) and Bhattacharya (1993).

<sup>2</sup>This view is called *the matching approach* in the literature of search theory and is most intensively adopted in recent labor market analysis (Pissarides, 2000).

<sup>3</sup>Coulson, Laing and Wang (2001) also constructed a two-sector search model, in which either sector has a labor market with friction. However, welfare properties with respect to the case in which workers can migrate between sectors were not characterized.

<sup>4</sup>Examples of static search and matching models include those described in Snower (1996) and Monfort and Ottaviano (2000).

<sup>5</sup>The details of the microfoundations and the empirical validity of the matching function are summarized in Petrongolo and Pissarides (2001).

<sup>6</sup>Whether or not the matching function exhibits constant returns to scale is an empirical issue. In fact, as the empirical studies enumerated in Petrongolo and Pissarides (2001) show, the empirical matching functions have variety in their returns to scale. With decreasing or increasing returns, the matching function has scale economy or diseconomy as well as externalities. With vertical or horizontal heterogeneity in skills of workers, scale economy and diseconomy in the matching process lead to agglomeration economies and diseconomies, respectively (Monfort and Ottaviano, 2000; Sato, 2001). Since in this paper, we aim to analyze the effects of externalities and the analysis of scale economy and diseconomy is beyond the scope of this paper, we adopt a constant returns to scale matching function.

<sup>7</sup>Even if we use a CES utility function, the main results of this paper are unaltered though we cannot solve the model analytically. Simulation analysis is given in Appendix.

However, if the two kinds of goods were perfect substitutes in consumption (i.e., if utility could be represented as linear in the two goods), the model has only an equilibrium in which all workers concentrate in one sector, or the allocation of workers is undetermined. Hence, one cannot analyze rural-urban migration unless some other mechanism is assumed to determine the allocation of work-

ers between sectors. This is beyond the scope of this paper and we leave it as a subject of future investigation.

<sup>8</sup>Even if we assume that workers have different  $\mu$ , or that only urban workers obtain a share, the results of this paper are unaltered. Furthermore, considering owners of firms separately from workers does not change any results. However, if we assume that the owner's utility function is different from the worker's utility function, the results would change. How the results would change depends on the form of the owner's utility function, and analysis of it is beyond the scope of this paper. Hence, we leave it as a subject of future investigation.

<sup>9</sup> $x (= 1)$  may either be the marginal product of the rural sector, or the average productivity, depending on social and organizational conditions of the rural areas.

<sup>10</sup>In the matching approach, it is common to focus on the long-run zero-profit situation. See Pissarides (2000), for example.

<sup>11</sup>This wage determination rule is commonly used in models of the matching approach. See Pissarides (2000), for example.

<sup>12</sup>In this paper, in order to make the analysis as simple as possible, bargaining power is assumed to be relevant only to the determination of wage level. Therefore, a rise in the bargaining power of workers raises the attractiveness of the urban sector and induces more workers to be in it. If bargaining power relates also to the determination of employment, the opposite effect will emerge. For example, consider that a rise in bargaining power implies that it becomes more difficult for firms to dismiss workers. This will make firms hesitate to hire new workers and lower the probability that new migrants from rural areas find jobs in urban areas. Such an insider/outsider argument is an important issue of future research.

<sup>13</sup>This criterion is equivalent to the ex-ante expected indirect utility of a worker.

<sup>14</sup>We assume that the second order conditions are satisfied. For example, if we assume that the matching function has constant elasticity  $\eta$ , as in a Cobb-Douglas function, the second order conditions

are satisfied.

<sup>15</sup>Imposing an income tax to workers in both sectors can cover a source of revenue for the subsidies: the necessary tax rate  $t$  is  $t = \max[0, [\tau(L - l) + scf]/(1 + \tau)L]$ .

<sup>16</sup>The relationship between Calvo's and Quibria's models is analyzed in Chau and Khan (2001).

<sup>17</sup>The effects of the minimum wage law on an economy according to standard theory are enumerated in Brown (1999).

<sup>18</sup>The model of Jones and O'Neill (1994) may be another exception. They showed that raising urban wages increases urban unemployment but decreases rural deforestation. If people place much value on rural forest, this may raise the social welfare, though they did not investigated this possibility in their paper.

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## **APPENDIX:** *Analysis using a CES utility function.*

Consider a CES utility function:

$$U = D \left\{ \delta c_x^{-\rho} + (1 - \delta) c_y^{-\rho} \right\}^{-1/\rho},$$

where  $D$ ,  $\delta$ , and  $\rho$  are constants satisfying  $D > 0$ ,  $0 < \delta < 1$ , and  $\rho \geq -1$ , respectively. In order to simulate the model numerically, we specify the matching function as a Cobb-Douglas form, which is supported by many empirical studies (Petrongolo and Pissarides, 2001):

$$M = Kl^\varepsilon f^{1-\varepsilon},$$

where  $K$  and  $\varepsilon$  are positive constants, and  $\varepsilon$  satisfies  $0 < \varepsilon < 1$ .

The values of the parameters are as follows: the utility parameters are  $D = 1$ ,  $\delta = 0.5$ , and  $\rho = 1$ ; the matching parameters are  $K = 0.5$  and  $\varepsilon = 0.5$ ; the firm's cost of labor recruitment is  $c = 0.5$ ; the total number of workers is  $L = 10$ . In order to see the relationship between the urban labor share and efficiency properties of the equilibrium, we change the labor share  $\beta$  from 0.1 to 0.9. Figure 3, similar to Figure 1, show this relationship.

**[Please insert Figure 3 here]**

Figure 3 proves that the basic efficiency properties are the same with those in section 3.

Figure 4 describes how the social welfare level changes in the fixed-wage equilibrium as the fixed wage level changes. This figure proves that the results of section 4 can also be obtained using a CES utility function: the optimal wage level exists, and changes in wages as established by policy may improve the welfare.

**[Please insert Figure 4 here]**