

Bargaining Power of Landlords and Underdevelopment in a System of Cities

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Abstract

We consider two models of a system of cities in which landlords have some power in negotiating rental prices at which lands are rented to develop cities. Analysis shows that the equilibrium city size is smaller than the optimal city size in a model in which city sizes are controlled by development companies or by local governments. Furthermore, it is shown that the equilibrium city size tends to be either smaller or larger than the optimal city size in a model in which city sizes are not controlled. These results point out the possibility of underdevelopment of cities.

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Running title: Bargaining and Underdevelopment

Keywords: Urban System; Bargaining; Underdevelopment;

JEL classification: R13; C78

1 Introduction

Urban economists have investigated whether cities are too large or too small based on models of a system of cities. Basically, two types of models have been presented in previous studies. One is a model in which development companies or local governments control city size in order to maximize their profit or to maximize the utility level of city residents, respectively. Here, we call this type of model a controlled city model. The other is a model in which development companies are absent, and local governments, even if they exist, do not control city size. We call this type of model an uncontrolled city model. The basic results of earlier studies were that the equilibrium city size in a controlled city model is optimal and that in an uncontrolled city model tends to be too large (see Henderson (1974) and Kanemoto (1980), among others).

In models described in earlier studies, it is assumed implicitly or explicitly that any amount of land is rented by development companies or by local governments from landlords at the opportunity land rent, which is usually the agricultural land rent. This assumption implies that landlords have no bargaining power in determining rental prices of lands. This is, of course, a good simplification and it has made it possible for us to investigate the basic features of a system of cities (see Abdel-Rahman (2000) and Henderson (1987)).¹

However, in reality, landlords often have some bargaining power in determining rental prices of lands.

¹Abdel-Rahman (2000) enumerated studies that examined what factors lead to the formation of cities, what determines the types and sizes of cities, what the role of trade is in a system of cities, and what determines income disparities among different types of households. Henderson (1987) provided an overview of studies that investigated how and why cities vary in economic functions, sizes and price levels, what determines the size distribution of cities, what are the impacts of natural resources on clustering of cities, and how governmental policies affect a system of cities.

This holds true when the number of landlords is finite or when landlords form coalitions. We incorporate this factor into a model of a system of cities and investigate whether the basic efficiency results with respect to a city size would change.

We do this by letting the rental land price be determined by the Nash bargaining between development companies (or local governments) and landlords. The reason we adopt the Nash bargaining is that it is one of the most simple ways to describe the situation in which landlords have bargaining power, and it can also describe, as a limiting case, the case in which landlords have no bargaining power and the rental land price is set to be the opportunity land rent as in models constructed in previous studies.²

The bargaining power of landlords is thought to change according to the economic environment. For example, Eckart (1985) showed that when a development project is indivisible and each landlord negotiates with a development company independently, the landlord's bargaining power is large when his/her land share is small. As each landlord's land share gets smaller, the effect of raising his/her asking price on the possibility that the development company will give up the project gets smaller. This induces the landlord to raise his/her asking price and makes his/her bargaining power greater.

Our main findings are as follows. First, the equilibrium city size is smaller than the optimal city size in a controlled city model. Because of the Nash bargaining between development companies and landlords, a portion of the profits from development companies' city development belongs to landlords. Therefore, there exists a discrepancy between social and private benefits from the city development. This results in underdevelopment. When landlords have no bargaining power, which is the case investigated in earlier studies, underdevelopment does not take place. Second, the equilibrium city size tends to be either too small or too large in an uncontrolled city model. Again, when landlords have no bargaining power, the equilibrium city size tends to be too large. The distinctive feature of these results is the possibility of underdevelopment, which, to the best of my knowledge, has not been pointed out in earlier studies.

This paper is structured as follows. In Section 2, we analyze a controlled city model. Section 3 deals with an uncontrolled city model. Conclusions are given in Section 4.

2 Controlled City Model

In a controlled city model, development companies or local governments determine city sizes in order to maximize their profits or to maximize the utility level of city residents. Since either set of specifications generates the same results, we adopt the former specification; i.e., a controlled city model with development companies.

2.1 Model

Consumers and Urban Structure

Consider a flat and featureless plane and a closed economy consisting of a system of cities that spreads over it. Here, we assume that the plane is large but is not unlimited. There are N consumers/workers in the economy. Each consumer resides in one of the cities that are created by development companies. Consumers are assumed to have an identical utility function of the following form:

$$U = ay,$$

where a is a positive constant and y is the consumption of the composite good. The composite good is treated as a numeraire. Each consumer in a given city can reside at only one location consuming one unit of land. The total amount of lands is assumed to be large enough for all the consumers to live on (that is, it is larger than N). We assume full employment and that each consumer in a given city commutes to the central business district (CBD) to supply one unit of labor inelastically. We further assume that the wage rate is an increasing function of the population of a city n and takes the form n^α where α is a

²Alternatively, we can adopt the solution of the Rubinstein's bilateral bargaining game and obtain the same results as those described in this paper. In that case, the bargaining power of landlords reflects their time discount rate.

positive constant satisfying $0 < \alpha < 1$. This assumption represents the agglomeration economies in the model described in this paper.³ The utility level of a consumer consuming his/her endowments is normalized to be zero. Let t and R denote the commuting cost per distance and the unit land rent, respectively.

The budget constraint for a consumer residing at distance x from the CBD in a given city is

$$y + tx + R = n^\alpha + s + \mu, \quad (1)$$

where s represents a subsidy from a development company of a given city to a consumer locating in that city. Development companies are owned by consumers in the economy, who, as shareholders, collect dividends. μ is the income from share-holding that is assumed to be the same across consumers, and is described as

$$\mu = \frac{1}{N}(\text{aggregate profits of development companies (AP)}). \quad (2)$$

Since each consumer is assumed to consume one unit of land, the population of a given city n is

$$n = \int_0^{\bar{x}} 2\pi x dx = \pi \bar{x}^2, \quad (3)$$

where \bar{x} denotes the distance between the CBD and the edge of the city. Equilibrium requires that no consumer has an incentive to relocate within the city. This implies that the cost of living (the commuting cost plus the land rent) is the same everywhere in the city:

$$R + tx = t\bar{x} + R_x, 0 < x < \bar{x}. \quad (4)$$

R_x represents the land rent at the edge of the city. At the edge of the city, the land rent R_x is equal to the agricultural land rent that is normalized to zero.⁴ This and (4) give the market land rent $R(x, \bar{x})$:

$$R(x, \bar{x}) = \begin{cases} t(\bar{x} - x) & \text{if } 0 < x < \bar{x} \\ 0 & \text{if } x \geq \bar{x} \end{cases}. \quad (5)$$

City Formation

Lands are owned by absentee landlords whose number is assumed to be finite and exogenously determined. Development companies rent lands from these absentee landlords and sublet them to consumers and form cities so as to maximize their profits. We assume that each development company creates one city. Since the number of landlords is finite, they have some bargaining power: each development company must negotiate with landlords about the unit rental price of lands in renting lands. We refer to this negotiated rental price as the land lease price. In subletting lands to consumers, each development company acts as a monopoly and collects the land rent described in (5) from each unit of lands.

We assume that the land lease price r at location x is determined through the generalized Nash bargaining between a development company that want to rent lands at location x and an absentee landlord who owns lands at location x . In this paper, for the expositional simplicity, we only use the Nash bargaining solution and do not describe the bargaining game that attains the solution.⁵ The Nash bargaining solution imposes a particular division of the total surplus between the two parties involved in

³We do not specify causes of agglomeration economies in this paper. See Fujita and Thisse (1996) for the discussion on causes of agglomeration economies.

⁴Allowing a positive value of agricultural land rent does not alter any results of this paper.

⁵Using the Nash bargaining solution without describing the bargaining game that attains the solution is common in many studies. See Hart (1995), Pissarides (2000) and studies cited therein, for example. For a detailed discussion on the Nash bargaining solution and the bargaining game that attains the solution, see Osborne and Rubinstein (1994, Chapter 15).

the bargaining process according to the relative bargaining power between them. In this model, the total surplus to be divided is the market land rent $R(x, \bar{x})$, and the land lease price $r(x, \bar{x})$ is determined as

$$r = \operatorname{argmax} \{ r^\beta [R(x, \bar{x}) - r]^{1-\beta} \},$$

where β is a positive constant satisfying $0 < \beta < 1$. β represents the bargaining strength of an absentee landlord, which is assumed to be common to all landlords.⁶ In the Nash bargaining, the threshold level for an absentee landlord is the agricultural land rent (that is zero by assumption), and that for a development company is the profit from not developing (that is also zero). Solving the corresponding maximization problem, we obtain the land lease price $r(x, \bar{x})$:⁷

$$\begin{aligned} r(x, \bar{x}) &= \beta R(x, \bar{x}) \\ &= \begin{cases} \beta t(\bar{x} - x) & \text{if } 0 < x < \bar{x} \\ 0 & \text{if } x \geq \bar{x} \end{cases}. \end{aligned} \quad (6)$$

When β converges to zero, the land lease price r converges to the agricultural land rent. Thus, this model can express a model with no bargaining power of landlords as a limiting case.

Renting lands from absentee landlords at the land lease price, each development company sublets the lands to consumers at the market land rent and forms a city. In doing so, the development company offers subsidies in order to attract consumers to its city. From (3) and (5), the total land rent (TLR) in a given city is

$$\begin{aligned} TLR &= \int_0^{\bar{x}} 2\pi R(x, \bar{x}) x dx \\ &= \frac{\pi t}{3} \left(\frac{n}{\pi} \right)^{3/2}. \end{aligned} \quad (7)$$

Similarly, from (3) and (6), the total expenditure (TE) for a given development company to rent lands to

⁶In this paper, we do not specify how many units of land each landlord owns, but assume for simplicity that each landlord owns the same amount of land, otherwise the bargaining power among landlords will not be identical. Also, we assume that these landlords organize one agent to bargain with the developer of local government on their behalf, otherwise the developer would face many landlords in the bargaining process. Under certain circumstances, the smaller the land share of each landlord is, the larger his/her bargaining strength would be. Eckart (1985) showed that when a development project is indivisible and each landlord negotiates with a development company independently, the landlord's bargaining power is large when his/her land share is small. This is because, as each landlord's land share gets smaller, the effect of raising his/her asking price on the possibility that the development company will give up the project gets smaller. This eases the landlord to raise his/her asking price and makes his/her bargaining power greater.

⁷Even if we adopt the solution of the Rubinstein's bilateral bargaining game instead of the Nash bargaining solution, we obtain the same land lease price. Consider the fictitious time in which the infinite horizon Rubinstein's bilateral bargaining game is played. In the game, a development company that want to rent land at location x and an absentee landlord who owns land at location \bar{x} negotiate the land lease price to divide the total surplus from land development. The total surplus to be divided is the market land rent $R(x, \bar{x})$. Assume that the development company makes an offer in the first period. Let $\delta \in (0, 1)$ denote the time discount rate. Then, in the subgame perfect Nash equilibrium, the company makes an offer that gives $(1/(1+\delta))R(x, \bar{x})$ to the company and $(\delta/(1+\delta))R(x, \bar{x})$ to the landlord in the first period, and the landlord accepts it. This implies that the land lease price is $(\delta/(1+\delta))R(x, \bar{x})$, which is equivalent to (6) if we define $\beta = \delta/(1+\delta)$. For a detailed derivation of the subgame perfect Nash equilibrium of the Rubinstein's bilateral bargaining game, see Gibbons (1992, Chapter 2), for example.

form a city is

$$\begin{aligned} TE &= \int_0^{\bar{x}} 2\pi r(x, \bar{x}) x dx \\ &= \frac{\beta\pi t}{3} \left(\frac{n}{\pi}\right)^{3/2}. \end{aligned} \quad (8)$$

Let s denote per consumer subsidies. The development company's profit Ψ is given as

$$\begin{aligned} \Psi &= TLR - TE - sn \\ &= (1 - \beta) \frac{\pi t}{3} \left(\frac{n}{\pi}\right)^{3/2} - sn. \end{aligned} \quad (9)$$

Consumers are assumed to be freely mobile among cities. Therefore, each consumer attains the same utility level in every city, which we refer to as the national utility level. Each development company maximizes Ψ with respect to s and n , guaranteeing consumers the national utility level \bar{U}_c . We assume that the number of cities in the economy is large and that each city to be formed is so small compared to the national economy that each development company takes the national utility level \bar{U}_c and the income from share-holding μ as given. In order to guarantee \bar{U}_c , each development company must offer per consumer subsidies s such that $\bar{U}_c = ay = a(n^\alpha + s + \mu - tx - R)$. From (3) and (4), it must be the case that $s = \bar{U}_c / a + t(n/\pi)^{1/2} - n^\alpha - \mu$. Substituting this into (9), we have

$$\Psi = (1 - \beta) \frac{\pi t}{3} \left(\frac{n}{\pi}\right)^{3/2} - n \left[\frac{\bar{U}_c}{a} + t \left(\frac{n}{\pi}\right)^{1/2} - n^\alpha - \mu \right]. \quad (10)$$

The first order condition for the maximization of Ψ with respect to n is

$$(1 - \beta) \frac{t}{2} \left(\frac{n}{\pi}\right)^{1/2} = \frac{\bar{U}_c}{a} + \frac{3t}{2} \left(\frac{n}{\pi}\right)^{1/2} - (1 + \alpha)n^\alpha - \mu. \quad (11)$$

The corresponding second order condition is satisfied in an equilibrium of the model when $1/2 > \alpha$.⁸ The left hand side of (11) represents the marginal revenue from enlarging a city and the right hand side of (11) represents the expenditure for subsidies that is necessary to attract one more worker to the city.

Equilibrium

We assume the free entry of development companies. Thus, development companies enter the city-formation business until profits in (10) are driven to zero:

$$(1 - \beta) \frac{\pi t}{3} \left(\frac{n}{\pi}\right)^{3/2} = n \left[\frac{\bar{U}_c}{a} + t \left(\frac{n}{\pi}\right)^{1/2} - n^\alpha - \mu \right].$$

Solving this for \bar{U}_c and substituting it into (11) give

$$\alpha n^\alpha = \frac{t}{3} \left(1 + \frac{\beta}{2}\right) \left(\frac{n}{\pi}\right)^{1/2}. \quad (12)$$

This equation generates the equilibrium city size in the controlled city model n_c^* :

$$n_c^* = \left[\frac{3\alpha\pi^{1/2}}{t(1 + \beta/2)} \right]^{2/(1-2\alpha)}. \quad (13)$$

(13) indicates that as the landlord's bargaining strength β increases, the equilibrium city size n_c^*

⁸See Appendix A.

shrinks. As landlords gain more bargaining power, the land lease price rises. This reduces the development companies' profits from development and induces development companies to form smaller cities. In this paper, we assume symmetric cities, which implies that the number of cities is N/n_c^* . The equilibrium utility level is

$$U_c^* = \bar{U}_c = a(1-2\alpha) \left[\frac{3\alpha\pi^{1/2}}{t(1+\beta/2)} \right]^{2\alpha/(1-2\alpha)}. \quad (14)$$

Because the utility level of a consumer consuming his/her endowments is normalized to zero, in order for cities to be formed, it must be the case that

$$(1-2\alpha) \left[\frac{3\alpha\pi^{1/2}}{t(1+\beta/2)} \right]^{2\alpha/(1-2\alpha)} > 0,$$

which is satisfied if $1/2 > \alpha$. The following proposition summarizes the above results:

Proposition 1 *A unique equilibrium of the controlled city model exists if $1/2 > \alpha$. The equilibrium city size n_c^* is given by (13) and the equilibrium number of cities is N/n_c^* . The equilibrium utility level U_c^* is given by (14).*

Hereafter, we assume that the inequality $1/2 > \alpha$ holds. This assumption requires that the agglomeration economies are not too strong.⁹

2.2 Efficiency of the Equilibrium

In this paper, the optimal allocation is summarized by a combination of s and n that maximizes the utility level of a consumer maintaining the total revenue of absentee landlords at the equilibrium level. Here, as in the previous section, we assume symmetric cities. In the equilibrium, the total revenue of absentee landlords is $(N/n_c^*)TE|_{n=n_c^*}$. Therefore, from (7) and (8), when we fix the total revenue of absentee landlords at the equilibrium level, the aggregate profits (AP) of development companies from city development is

$$\begin{aligned} AP &= (N/n)TLR - (N/n_c^*)TE|_{n=n_c^*} - sN \\ &= N \left[\frac{t}{3} \left(\frac{n}{\pi} \right)^{1/2} - \frac{\beta t}{3} \left(\frac{n_c^*}{\pi} \right)^{1/2} - s \right]. \end{aligned} \quad (15)$$

From (2) and (15), the utility level of each consumer is

$$\begin{aligned} U &= a(n^\alpha + s - \mu - t\bar{x}) \\ &= a \left[n^\alpha - \frac{2t}{3} \left(\frac{n}{\pi} \right)^{1/2} - \frac{\beta t}{3} \left(\frac{n_c^*}{\pi} \right)^{1/2} \right]. \end{aligned} \quad (16)$$

The first order condition for maximizing U with respect to n gives the optimal city size n^{**} ¹⁰:

$$n^{**} = \left(\frac{3\alpha\pi^{1/2}}{t} \right)^{2/(1-2\alpha)}. \quad (17)$$

By comparing this with (13), we obtain the following proposition.

⁹Without this assumption, each developer enlarges its city as much as possible.

¹⁰The second order condition is satisfied. See Appendix B.

Proposition 2 *The equilibrium city size n_c^* is smaller than the optimal city size n^{**} . When the bargaining strength of an absentee landlord β converges to zero, n_c^* converges to n^{**} .*

As a result of the Nash bargaining between development companies and absentee landlords, a portion of the profit from each development company's city development belongs to the absentee landlords. This generates a discrepancy between social and private benefits from the city development, and leads to underdevelopment.¹¹ This result is in contrast with that of previous controlled city models, in which the equilibrium city size is optimal (see Henderson (1974) and Kanemoto (1980), for example). When the landlord's bargaining strength decreases to zero, underdevelopment does not take place. This result indicates that the model described in this paper generates the result of the previous models as a limiting case of no bargaining power of absentee landlords. It is also shown that as the bargaining power of absentee landlords rises, the discrepancy between the equilibrium and the optimum becomes larger.

3 Uncontrolled City Model

If there are no development companies operating in national land markets, or if local governments do not determine the population of cities, city sizes are not controlled. In this section, we construct an uncontrolled city model in which local governments, though they exist, do not control city sizes.

3.1 Model

Consumers, Urban Structure and City Formation

Suppose that local governments rent lands from absentee landlords and sublet them to consumers. In doing so, each local government collects the land rent described in (5) from consumers, and pays absentee landlords for lands at the land lease price described in (6). The local government then redistributes the total land rent (TLR in (7)) minus the total expenditure to rent lands (TE in (8)) to consumers in the corresponding city as subsidies. The budget constraint (1) for a consumer residing at distance x from the CBD in a given city changes to

$$y + tx + R = n^\alpha + s.$$

In this model, per consumer subsidies s are given by $s = (TLR - TE)/n = (1 - \beta)(t/3)(n/\pi)^{1/2}$.

Then, the utility level of a consumer is

$$\begin{aligned} U &= a(n^\alpha + s - t\bar{x}) \\ &= a\left[n^\alpha - \frac{2t}{3}\left(1 + \frac{\beta}{2}\right)\left(\frac{n}{\pi}\right)^{1/2}\right]. \end{aligned}$$

Stable Equilibrium

Consumers are assumed to be freely mobile among cities. This assumption implies that all the consumers attain the same utility level, which we again refer to as the national utility level. As was the case before, symmetric cities are assumed. Because local governments do not control city sizes, this configuration represents an equilibrium if the national utility level \bar{U}_u is larger than the utility level of a consumer consuming his/her endowments that is normalized to zero. Therefore, in an equilibrium in which each city has n consumers, it is required that

$$\bar{U}_u = a\left[n^\alpha - \frac{2t}{3}\left(1 + \frac{\beta}{2}\right)\left(\frac{n}{\pi}\right)^{1/2}\right] > 0,$$

¹¹This mechanism is similar to that of the problem of holdups in a firm's physical capital investment. See Hart (1995), for example.

which implies

$$n^\alpha > \frac{2t}{3} \left(1 + \frac{\beta}{2}\right) \left(\frac{n}{\pi}\right)^{1/2} = \Theta(n).$$

Here, we further impose a stability condition as was the case in previous studies. An equilibrium is stable when a migrant obtains no more income than he/she obtained before moving. Therefore, the following condition must hold in a stable equilibrium:

$$\frac{dU}{dn} < 0,$$

which implies

$$\alpha n^\alpha < \frac{t}{3} \left(1 + \frac{\beta}{2}\right) \left(\frac{n}{\pi}\right)^{1/2} = \Omega(n).$$

Let n_{\max} denote the solution to the equation $n^\alpha = \Theta(n)$ for $n > 0$. It is shown that

$$n_{\max} = \left[\frac{3\pi^{1/2}}{2t(1 + \beta/2)} \right]^{2/(1-2\alpha)}.$$

Notice that the equilibrium city size in the controlled city model n_c^* is the solution to the equation $\alpha n^\alpha = \Omega(n)$ for $n > 0$ (see (12) and (13)).

Proposition 3 *A continuum of stable equilibrium of the uncontrolled city model exists. (Any $n \in (n_c^*, n_{\max})$ can be a stable equilibrium city size.)*

Proof: See Appendix C.

3.2 Efficiency of the Stable Equilibrium

As in Section 2.2, it can be shown that the optimal city size in the uncontrolled city model is also given by (17). Simple calculations show that n_c^* is smaller than n^{**} and that n_c^* converges to n^{**} as the landlord's bargaining strength β converges to zero. Thus, we obtain the following proposition.

Proposition 4 *There is a continuum of equilibrium in which the equilibrium city size is smaller than the optimal city size n^{**} . If $(1 + \beta/2)\alpha$ is smaller than $1/2$, n_{\max} is larger than n^{**} . In this case, there is a continuum of equilibrium in which the equilibrium city size is larger than n^{**} and there is an equilibrium in which the equilibrium city size is equal to n^{**} .*

This proposition implies that the equilibrium city size is almost always inefficient. The key feature here is that it is sometimes too small, which never occurred in the previous uncontrolled city models, in which cities tend to be too large (see Henderson (1974) and Kanemoto (1980)). The intuition behind this result is that a portion of profits of consumers agglomerating in a given city belongs to landlords, which generates the possibility that too few consumers agglomerate in the city. When β converges to zero, the equilibrium city size is too large except at a point of measure zero. Thus, again, the model described in this paper generates the result of previous models as a limiting case.

4 Conclusions

This paper constructed a model of a system of cities in which landlords have bargaining power in

determining rental prices of lands. Analysis showed that when the bargaining power of landlords exists, the equilibrium city size is smaller than the optimal city size in a controlled city model. Furthermore, it was shown that the equilibrium city size tends to be either smaller or larger than the optimal city size in an uncontrolled city model. Apropos of both models, the result that the equilibrium city size can be too small was not obtained in any previous studies. This paper supplements the literature on a system of cities with insight into the role played by the bargaining power of landlords in determining rental prices of lands in city formation.

Appendix A: *The second order condition for the maximization of (10) with respect to n*

$$\frac{d^2\Psi}{dn^2} = -\frac{t}{2\pi} \left(1 + \frac{\beta}{2}\right) \left(\frac{n}{\pi}\right)^{-1/2} + \alpha(1+\alpha)n^{\alpha-1}. \quad (\text{A1})$$

Substituting (12) into (A1) gives

$$\left. \frac{d^2\Psi}{dn^2} \right|_{n=n_c^*} = \alpha \left(\alpha - \frac{1}{2} \right) n_c^{*\alpha-1},$$

which is smaller than zero if $1/2 > \alpha$.

Appendix B: *The second order condition for the maximization of (16) with respect to n*

$$\frac{d^2U}{dn^2} = a \left[\alpha(\alpha-1)n^{\alpha-2} + \frac{t}{6\pi^2} \left(\frac{n}{\pi}\right)^{-3/2} \right]. \quad (\text{B1})$$

(B1) at the first order condition is given by

$$\left. \frac{d^2U}{dn^2} \right|_{n=n_c^{**}} = \alpha \left(\alpha - \frac{1}{2} \right) n_c^{**\alpha-2} < 0.$$

Appendix C: *Proof of proposition 3*

• **Proposition 3** *A continuum of stable equilibrium of the uncontrolled city model exists. (Any $n \in (n_c^*, n_{\max})$ can be a stable equilibrium city size.)*

Proof.

Because n_c^* is larger than zero, a continuum of equilibrium of the uncontrolled city model exists if and only if n_{\max} is larger than n_c^* . This holds if and only if

$$\left[\frac{3\pi^{1/2}}{2t(1+\beta/2)} \right]^{1/(1/2-\alpha)} > \left[\frac{3\alpha\pi^{1/2}}{t(1+\beta/2)} \right]^{1/(1/2-\alpha)}.$$

This inequality holds if $1/2 > \alpha$, which is satisfied by assumption.

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