

# City Structure, Search and Workers' Job Acceptance Behavior\*

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## **Abstract**

This paper develops a stochastic search model having a monocentric city structure and investigates how city structure affects workers' job acceptance behavior and a labor market. In the model, workers reside in a city and commute to the Central Business District (CBD) to work when employed and to be interviewed when unemployed. When a job searcher contacts a firm having a vacant job, he/she observes the level of training costs necessary for employment and decides whether to accept the job. It is shown that there exists a unique market equilibrium in which the employed live close to the CBD and the unemployed reside far away from the CBD. Analysis shows that 1) improvement of commuting technology induces job searchers to accept more costly jobs and lowers the unemployment rate in the city, 2) growth of the city makes job searchers less tolerant of training and raises the unemployment rate, and 3) as job searchers search more intensively, they become choosier if commuting costs are sufficiently small. Efficiency properties of the equilibrium are also explored.

**Keywords:** City structure; Unemployment; Stochastic job search; Job acceptance behavior

**JEL classification:** D83, J64, R10, R20

# 1 Introduction

In the modern economy, many people reside and work in urban areas. In fact, World Urbanization Prospect [16] reports that in 2000, 76 percent of the populations of developed countries live and work in urbanized areas, while this figure is 39 percent in developing countries. In light of this high degree of urbanization, urban economists have investigated the features of urban labor markets. Among the vast volume of studies on urban labor markets, there is a strand of studies that has focused on the relationship between cities and unemployment.

Recently, search and matching models with urban structures have been developed in order to analyze this relationship. For example, Wasmer and Zenou [14] showed that when workers' search intensity is negatively affected by access to jobs, different configurations emerge in equilibrium: one is such that unemployed workers live close to jobs and the other is such that unemployed workers reside far away from jobs. The latter is consistent with the spatial mismatch hypothesis.

It is worth referring to this hypothesis because the urban configuration obtained in the model described in this paper is also consistent with it. The hypothesis was first stated by Kain [5] and claims that the job decentralization to the suburbs not combined with residential movement of African Americans has created the high unemployment rate and low wages in inner-city neighborhoods where African Americans are concentrated. Since then, a large number of empirical studies have focused on this hypothesis and they have offered much evidence for its support (see Preston and McLafferty [8] for a survey of recent empirical studies on this subject).

Coulson, Laing, and Wang [2] constructed a search model involving two districts and showed that a configuration consistent with the hypothesis emerges as an inter-district equilibrium.<sup>1</sup>

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<sup>1</sup>There are, of course, studies that explain the hypothesis using other types of model. For instance, Zenou [17], using a efficiency wage model with urban structure, showed that firms do not recruit workers who live too far away from them because their productivity is lower than workers residing closer, and a configuration consistent with the hypothesis emerges.

Smith and Zenou [13] showed that when search intensity is endogenously determined, another type of configuration is obtained in equilibrium: unemployed workers live either close to or far away from jobs and employed workers live in between the unemployed. Crampton [3] analyzed the worker's decision regarding job search area. Rouwendal [9] demonstrated the possibility of excess commuting due to the information problem in the job search process. Sato [10] investigated the relationship between wage and unemployment differences between different cities in inter-urban equilibrium. Sato [11] and Wheeler [15] showed the link between agglomeration economies and the worker-firm matching process. Especially, the former provided the conditions of the matching process for the existence of agglomeration economies.

This paper aims to contribute this research area by constructing a stochastic search and matching model having a monocentric city structure and investigating how city structure affects a labor market *via* workers' job acceptance behavior. To the best of the author's knowledge, no existing study has fully analyzed this issue. Labor economists have shown that worker's job acceptance behavior is one of the most important factors that influence a labor market (see Pissarides [7]). It would then be meaningful to shed light on the relationship between city structure and worker's job acceptance behavior, if any. In doing so, this paper extends the model developed in Wasmer and Zenou [14] to include the worker's job acceptance behavior.

In the model described in this paper, workers reside in a city in which there are no relocation costs. While employed workers commute to the Central Business District (CBD) to work, unemployed workers commute to the CBD to be interviewed. It is assumed that employed workers commute more frequently than do unemployed workers. When an unemployed worker gets employed by a firm, the worker must bear the training costs. The level of training costs is determined stochastically and is not observable for each worker until he/she contacts a firm. Contacting a firm having a vacant job, each job searcher observes the level of training costs and decides whether to accept the job. Under these assumptions, a unique market equilibrium

is shown to exist in which both land and labor markets are solved simultaneously. In the equilibrium, employed workers live close to the CBD and unemployed workers reside far away from the CBD.

The main results are the following. First, improvement of commuting technology induces job searchers to accept more costly jobs and lowers the unemployment rate in the city. Because the employed commute more frequently than do the unemployed, the former benefits more than the latter from improvement of commuting technology. This enhances the attraction of being employed and workers become more tolerant of training for employment given the likelihood of job interviews. Therefore, both the possibility that each job interview will bear fruit and the expected revenue of a firm having a vacant position rise. Hence, the number of vacancies increases to lower the unemployment rate.

Second, growth of the city makes job searchers less tolerant of training and raises the unemployment rate. Growth of the city adds to each worker's cost of living by enlarging the commuting area. Since this effect is also stronger for the employed than for the unemployed, workers come to accept only jobs that require less training given the possibility of job interviews. This reduces both the probability that each job interview succeeds and the expected revenue of a firm having a vacant job. Hence, the number of vacancies decreases to raise the unemployment rate.

Third, as job searchers search more intensively, they become choosier if commuting costs are sufficiently small. A rise in search intensity has two effects. One is such that it reduces the difference in the cost of living between the employed and the unemployed, which raises the attraction of being employed and makes workers more tolerant of training. The other is such that it raises the worker's likelihood of job contact, which induces workers to be more selective. When the commuting cost is small, the latter effect dominates the former.

Finally, it is shown that the equilibrium is not optimal since under-acceptance takes place

in the workers' job acceptance decision. An adequate urban policy such as a subvention for commuting is shown to solve this problem.

This paper is structured as follows. In section 2, we introduce the basic structure of the model. Section 3 shows the existence of the unique equilibrium. In section 4, we explore the influence of changes in city structure on the equilibrium. Welfare issues are addressed in section 5 and conclusions are given in Section 6.

## 2 Model

In this section, we introduce the basic structure of the model.

### 2.1 City structure

Consider a city that is closed and linear. The city is monocentric and it has one Central Business District (CBD) that is approximated by a point and all firms are assumed to be exogenously located in the CBD.<sup>2</sup> Its land is owned by absentee landlords. There is a continuum of risk-neutral workers of size  $n$  in the city.  $u$  of  $n$  are unemployed workers and  $n - u$  are employed workers. Workers live infinitely and they reside, occupying the same amount of land (normalized to 1), outside the CBD. We assume that the density of land is 1. These assumptions imply that there are  $x$  units of housing within a distance  $x$  of the CBD. Employed workers commute to the CBD once per each unit time. Unemployed workers go to the CBD to be interviewed  $s$  times per unit time, where we assume that  $s$  is exogenous and that  $0 < s < 1$ .<sup>3</sup> Because of this assumption that the unemployed commute less frequently than do the employed, as we see in section 3, the

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<sup>2</sup>This is a model of a centralized city. We can easily modify this model to describe a decentralized city. In that case, all firms are located in the suburban business district that is located at the one end of the linear city. This modification does not alter any results of this study.

<sup>3</sup>For a discussion of the determinant of search intensity, see Smith and Zenou [13] and Wasmer and Zenou [14].

equilibrium urban configuration is such that employed workers reside close to the CBD and unemployed workers live far away from the CBD. This is consistent with the spatial mismatch hypothesis. Larger  $s$  indicates that unemployed workers search for jobs more intensively.

Let  $tx$  denote the commuting cost for the employed who reside at a location that is  $x$  distant from the CBD, with  $t > 0$ . The commuting cost for the unemployed at location  $x$  is then  $stx$ . The cost of living in the city is the sum of the residential land rent  $R(x)$  and the commuting cost:  $R(x) + tx$  for an employee and  $R(x) + stx$  for a job searcher.

## 2.2 Matching framework

There are  $v$  firms that have vacant positions searching for workers. We assume, for simplicity, that each firm can employ only one worker.

Job contacts are generated by a Poisson process with the aggregate rate of  $M = \mu(su, v)$ .  $\mu(su, v)$  is defined on  $\mathbf{R}_+ \times \mathbf{R}_+$  and strictly increasing in both its arguments. Note that more job contacts take place when workers search more intensively. Furthermore,  $\mu(su, v)$  is assumed to be twice differentiable, strictly concave, homogeneous of degree one, and to satisfy  $0 \leq \mu(su, v) \leq \min[su, v]$  and  $\mu(su, 0) = \mu(0, v) = 0$ . The function  $\mu(su, v)$  is called the “technology of search.”

For each unemployed worker, such job contacts arrive at the rate of  $p(\theta) = M/u = \mu(s, \theta)$ , and for each vacant firm at the rate of  $q(\theta) = M/v = \mu(s/\theta, 1)$ , where  $\theta$  is the measure of labor market tightness and is defined as  $\theta = v/u$ . Note that  $p(\theta)u = q(\theta)v$ ,  $dp/d\theta > 0$  and  $dq/d\theta < 0$  for any  $\theta \in (0, +\infty)$ . From this, we can see that if the number of unemployed workers  $u$  rises with the number of firms having vacancies  $v$  constant, the contact rate for each unemployed worker  $p$  declines ( $\partial p/\partial u < 0$ ) and that for each firm  $q$  rises ( $\partial q/\partial u > 0$ ). In contrast,  $p$  rises and  $q$  declines if  $v$  rises with  $u$  fixed ( $\partial p/\partial v > 0$  and  $\partial q/\partial v < 0$ ). Finally, we assume that  $p(\theta)$  satisfies the Inada condition with respect to  $\theta$  ( $\lim_{\theta \rightarrow 0} dp/d\theta = \infty$  and  $\lim_{\theta \rightarrow \infty} dp/d\theta = 0$ ).

Just when an unemployed worker becomes employed by a firm, the worker must train him-

self/herself and bear the costs for training. This assumption considers that because workers differ in their preference and ability to have heterogeneous skills, it is often the case that the skill of a particular worker does not meet the skill requirement of a particular firm and the worker needs to train himself/herself to adjust his/her skill in order to meet the skill requirement of the firm when becoming employed by it. The level of training costs is assumed to be match-specific and to be determined stochastically. A worker does not know the level of training costs until he/she contacts a firm. When a job searcher contacts a firm having a vacant job and perceives the level of training costs necessary to become employed by the firm, he/she decides whether or not to accept the job. If the searcher accepts it, he/she carries out training and gets employed by the firm. Otherwise, the searcher starts searching again. Thus, flows out of unemployment are given by acceptable job matches from job contacts.

Once a match is made, the wage is determined by Nash bargaining between a worker and a firm. We assume that separation of a job and a worker is generated by a Poisson process with the exogenous aggregate rate of  $\delta > 0$ .

[Figure 1 here]

### 2.3 Workers

An unemployed worker, bearing the cost of living  $R(x) + stx$ , goes to the CBD to be interviewed and receives unemployment benefits  $b$  ( $b$  is assumed to be a positive constant). Just when an unemployed worker becomes employed by a firm, the worker must bear training costs. The level of training costs  $c$  is assumed to be match-specific and to be determined stochastically. More concretely,  $c$  is a random variable whose cumulative distribution function  $F(c)$  is defined on  $[0, \bar{c}]$  and is continuously differentiable, where  $\bar{c}$  is a positive constant. A worker does not know  $c$  until

he/she contacts a firm. When a job searcher contacts a firm having a vacant job and observes the level of training costs, he/she decides whether or not to accept the job. If the searcher accepts it, he/she trains himself/herself and get employed by the firm. Otherwise, he/she starts searching again.

It is assumed that after the training, every worker-firm pair attains the same level of productivity, which implies that every employed worker receives the same wage. An employed worker, bearing the cost of living  $R(x) + tx$ , goes to the CBD to work and receives a wage  $w$ .

We assume that workers can change their location without cost. Let  $W(x)$  and  $U(x)$  be the discounted expected incomes (asset values) of the employed and the unemployed, respectively. Let  $\widehat{W}(x, c)$  represent the worker's value of contacting a vacant job that requires training costs of level  $c$ . Furthermore, we define  $W_{\max}$  and  $U_{\max}$  as the maximized values of  $W(x)$  and  $U(x)$  with respect to  $x$  (i.e.,  $W_{\max} = \max_x W(x)$  and  $U_{\max} = \max_x U(x)$ ). These incomes are given by

$$rW(x) = w - R(x) - tx + \delta \{U_{\max} - W(x)\}, \quad (1)$$

$$rU(x) = b - R(x) - stx + p(\theta) \left\{ E_c \left( \widehat{W}(x, c) \right) - U(x) \right\}, \quad (2)$$

$$\widehat{W}(x, c) = \max [W_{\max} - c, U(x)],$$

where  $r$  is the discount rate ( $r > 0$ ) and  $E_c \left( \widehat{W}(x, c) \right)$  is the expectation of  $\widehat{W}(x, c)$  with respect to  $c$ . An employed worker, bearing the cost of living  $R(x) + tx$ , commutes to the CBD and earns the wage  $w$  until the match is destroyed.  $U_{\max} - W(x)$  indicates the change in income when the worker becomes unemployed and relocates optimally. An unemployed worker, bearing the cost of living  $R(x) + stx$ , commutes to the CBD to search for a firm having a vacant job while earning unemployment income  $b$ . When he/she encounters a vacant job, he/she decides whether or not to accept the job.  $E_c \left( \widehat{W}(x, c) \right) - U(x)$  represents the change in expected income when an unemployed worker comes into contact with a vacant job.

## 2.4 Firms

Let  $J$  and  $V$  be the discounted expected incomes of a firm employing a worker and a firm having a vacant position, respectively. We assume that apart from the foregone profit that a firm suffers when it has a vacancy, it must bear also costs  $k$  per unit time ( $k > 0$ ).  $k$  is the fixed cost of machines that has to be borne regardless of whether jobs are filled or not, and any other labor recruitment costs. In order to simplify the exposition,  $k$  is assumed to be a flow. Output  $y$  is thought as being net of the part of  $k$  that has to be borne when the machine is occupied.  $y$  is assumed to be larger than the unemployment income  $b$ . The discounted expected incomes are given by

$$rJ = y - w + \delta(V - J), \quad (3)$$

$$rV = -k + q(\theta) \Pr(\text{accept})(J - V), \quad (4)$$

where  $\Pr(\text{accept})$  is the probability that the match is accepted by a worker when a firm contacts the worker. A firm with a vacancy searches for an unemployed worker and bears the vacancy costs  $k$  per unit time. When it meets an unemployed worker and the worker accepts the job, the firm employs the worker. A firm that employs a worker obtains the profit  $y - w$  per unit time until the match is destroyed.

## 3 Equilibrium

In order to determine the equilibrium, we will proceed as follows. We first derive conditions that determine the equilibrium urban configurations. We then explore conditions that generate the labor market outcomes. We refer to the former as the spatial conditions and the latter as the labor market conditions. The equilibrium is a set of variables that satisfies the spatial conditions and the labor market conditions simultaneously.

### 3.1 Spatial conditions

Because we assume that workers can change their location without cost, no worker has an incentive to relocate in equilibrium. Therefore, in equilibrium, all the employed workers enjoy the same level of income ( $W(x) = W_{\max} = \bar{W}$ ) and all the unemployed workers enjoy the same level of income ( $U(x) = U_{\max} = \bar{U}$ ). This implies that the worker's value of contacting a vacancy  $\widehat{W}(x, c)$  does not depend on a location ( $\widehat{W}(x, c) = \widehat{W}(c)$ ). In order to determine the equilibrium location of workers, we use the concept of bid rents. They are defined as the maximum land rent at location  $x$  which each type of worker is willing to pay in order to reach his/her respective equilibrium utility (in this paper, income) level. From the definition of bid rents, (1) and (2) give the bid rents of the employed and unemployed, respectively:

$$\Omega_e(x, \bar{W}, \bar{U}) = w - tx + \delta\bar{U} - (r + \delta)\bar{W}, \quad (5)$$

$$\Omega_u(x, \bar{W}, \bar{U}) = b - stx + p(\theta)E_c(\widehat{W}(c)) - \{r + p(\theta)\}\bar{U}. \quad (6)$$

Differentiating these gives the bid rent slopes of each type of worker:

$$\frac{\partial\Omega_e(x, \bar{W}, \bar{U})}{\partial x} = -t < 0, \quad \frac{\partial\Omega_u(x, \bar{W}, \bar{U})}{\partial x} = -st < 0. \quad (7)$$

Since  $0 < s < 1$ , the bid rent of the employed is steeper than that of the unemployed. The residential land rent  $R(x)$  is the upper envelope of all workers' bid rents and of the agricultural land rent that is assumed to be zero:

$$R(x) = \max[\Omega_e(x, \bar{W}, \bar{U}), \Omega_u(x, \bar{W}, \bar{U}), 0] \quad (8)$$

for each  $x \in [0, \bar{x}]$ , where  $\bar{x}$  represents the edge of the city. The fact that the bid rent of an employed worker is steeper than that of an unemployed worker implies that the possible equilibrium urban configuration is such that employed workers reside near the CBD and unemployed workers live on the outskirts of the city. With this configuration, the edge of the city  $\bar{x}$  is determined such

that the bid rent of the unemployed is equal to the agricultural rent:  $R(\bar{x}) = \Omega_u(\bar{x}, \bar{W}, \bar{U}) = 0$  (the city edge condition). Let  $x_0$  denote the location of the boundary between the employed and unemployed. Then, at location  $x_0$ , the bid rent of the employed is equal to that of the unemployed:  $R(x_0) = \Omega_e(x_0, \bar{W}, \bar{U}) = \Omega_u(x_0, \bar{W}, \bar{U})$  (the boundary condition). Since each worker occupies one unit of land, it must be the case that  $\bar{x} = n/2$  and  $x_0 = (n - u)/2$ :

$$R(n/2) = \Omega_u(n/2, \bar{W}, \bar{U}) = 0, \quad (9)$$

$$R((n - u)/2) = \Omega_e((n - u)/2, \bar{W}, \bar{U}) = \Omega_u((n - u)/2, \bar{W}, \bar{U}). \quad (10)$$

The equilibrium urban configuration is determined by the above two conditions, which we refer to as the spatial conditions.

[Figure 2 here]

### 3.2 Labor market conditions

When an unemployed worker contacts a firm having a vacant job and observes the level of necessary training costs, he/she will accept the job if the expected employed income minus the training costs  $\bar{W} - c$  is higher than the expected unemployed income  $\bar{U}$ . If  $\bar{W} - c$  is lower than  $\bar{U}$ , he/she will reject the job. Therefore, if the level of training costs  $c$  is lower (higher) than  $\bar{W} - \bar{U}$ , then the worker accepts (rejects) the job. This optimization behavior of a worker represents the labor supply condition. We define  $c^*$  as  $c^* = \bar{W} - \bar{U}$ , and we call  $c^*$  the reservation training level of a worker. From this, we can see that the probability that the match is accepted by a worker when an encounter between a worker and a firm takes place  $\Pr(\text{accept})$  and the expectation of the worker's value of contact  $E_c(\widehat{W}(c))$  are given by<sup>4</sup>

$$\Pr(\text{accept}) = \int_0^{c^*} f(c)dc = F(c^*), \quad (11)$$

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<sup>4</sup>Note that  $\widehat{W}$  no longer depends on  $x$  as was explained in the beginning of the previous section.

$$\begin{aligned}
E_c(\widehat{W}(c)) &= \int_0^{c^*} (\overline{W} - c)f(c)dc + \int_{c^*}^{\overline{c}} \overline{U}f(c)dc \\
&= F(c^*)c^* - \int_0^{c^*} cf(c)dc + \overline{U}.
\end{aligned} \tag{12}$$

We assume free entry of firms. Therefore, vacant jobs are created until the expected income of a vacancy  $V$  is equal to zero:

$$V = 0. \tag{13}$$

This represents the labor demand condition.

The total surplus from a job match is the sum of the surplus of a worker  $\overline{W} - \overline{U}$  and that of a firm  $J - V$ . The wage is determined by decentralized Nash bargaining between a firm and a worker, which assumes that the total surplus is shared in such a way that the worker receives a fraction  $\beta$  of it, and the firm receives the remaining fraction  $1 - \beta$ .<sup>5</sup>  $\beta$  indicates the bargaining power of workers. Therefore, the wage  $w$  is determined by

$$\overline{W} - \overline{U} = \beta(\overline{W} + J - \overline{U} - V). \tag{14}$$

Finally, we assume the steady state condition. This requires that the number of unemployed workers  $u$  is unchanged given the number of workers  $n$ :

$$F(c^*)\mu(u, v) = \delta(n - u). \tag{15}$$

$F(c^*)\mu(u, v)$  represents the flow per unit time out of the pool of unemployed workers, and  $\delta(n - u)$  the flow per unit time into the pool of them. The number of unemployed workers is unchanged when the inflow is equal to the outflow. We refer to the above four conditions (the labor supply condition  $c^* = \overline{W} - \overline{U}$ , the labor demand condition (13), the Nash bargaining condition (14), and the steady state condition (15)) as the labor market conditions.

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<sup>5</sup>This wage determination rule is commonly used in the search theoretic models. See Pissarides [7], for example.

### 3.3 Market Equilibrium

We are now ready to explore the equilibrium of the model. Remaining variables to be determined are the incomes of the employed and the unemployed ( $\bar{U}$  and  $\bar{W}$ ), the reservation training level ( $c^*$ ), the measure of labor market tightness ( $\theta$ ), the wage ( $w$ ) and the number of unemployed workers ( $u$ ). These variables are determined by the spatial conditions (the city edge condition (9) and the boundary condition (10)) and the labor market conditions (the labor supply condition ( $c^* = \bar{W} - \bar{U}$ ), the labor demand condition (13), the Nash bargaining condition (14), and the steady state condition (15)).

First, we derive  $\bar{U}$  and  $\bar{W}$  as functions of  $c^*$ ,  $\theta$ ,  $w$ , and  $u$ . From the city edge condition (9), (6) and (12) give the income of the unemployed  $\bar{U}$ :

$$r\bar{U} = b - \frac{stn}{2} + p(\theta) \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\}. \quad (16)$$

Substituting this into (6) and using (8), we have the cost of living for the unemployed (and hence the bid rent of the unemployed) at location  $x \in [(n-u)/2, n/2]$ :

$$R(x) + stx = \frac{stn}{2}. \quad (17)$$

This equation describes the agglomeration diseconomies in this model. The more workers that reside in the city, the more distant from the CBD the edge of the city is, which generates the pressure of raising the cost of living.

From (5), (8), and (16), the cost of living for the employed at location  $x \in [0, (n-u)/2]$  is given by

$$\begin{aligned} R(x) + tx &= w - \delta c^* - r\bar{W} & (18) \\ &= w - \delta c^* - r(\bar{U} + c^*) \\ &= w - \{r + \delta + p(\theta)F(c^*)\} c^* - \left\{ b - \frac{stn}{2} - p(\theta) \int_0^{c^*} cf(c)dc \right\}. \end{aligned}$$

The boundary condition (10) is rearranged as (from (5), (6) and the labor supply condition  $c^* = \bar{W} - \bar{U}$ ):

$$\{r + \delta + p(\theta)F(c^*)\}c^* = w - b - \frac{(1-s)t(n-u)}{2} + p(\theta) \int_0^{c^*} cf(c)dc. \quad (19)$$

Substituting this into (18) gives

$$R(x) + tx = \frac{t\{n - (1-s)u\}}{2}. \quad (20)$$

Note that while the cost of living is the same within each group of workers, the cost of living for the employed is different from that for the unemployed and the former is larger than the latter. This difference comes from the fact that the unemployed commutes less intensively than the employed ( $0 < s < 1$ ). (20), combined with the first equality of (18), generates the income of the employed:

$$r\bar{W} = w - \frac{t\{n - (1-s)u\}}{2} - \delta c^*. \quad (21)$$

Next, we derive the wage  $w$  as a function of  $c^*$ ,  $\theta$  and  $u$ . From (16) and (21), we have

$$\bar{W} - \bar{U} = \frac{1}{r + \delta} \left[ w - b - \frac{(1-s)t(n-u)}{2} - p(\theta) \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\} \right]. \quad (22)$$

Since the labor demand condition (13) requires  $V = 0$ , (3) yields

$$J - V = J = \frac{y - w}{r + \delta}. \quad (23)$$

Substituting (22) and (23) into the Nash bargaining condition (14) and rearranging it give

$$w = \beta y + (1 - \beta) \left[ b + \frac{(1-s)t(n-u)}{2} + p(\theta) \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\} \right]. \quad (24)$$

Because the output level  $y$  is larger than the unemployment income  $b$  by assumption, the wage  $w$  is larger than  $b$ . Thus, while the employed bear higher costs of living than do the unemployed, the former earn more than the latter does.

Furthermore, the steady state condition ((7), i.e.,  $F(c^*)\mu(u, v) = \delta(n - u)$ ) determines the number of unemployed workers  $u$  as a function of  $\theta$  and  $c^*$ :

$$u = \frac{\delta n}{\delta + p(\theta)F(c^*)}. \quad (25)$$

With the results above, we can see that all the other variables are determined adequately if  $c^*$  and  $\theta$  are determined. By substituting the above results into the labor supply condition ( $c^* = \bar{W} - \bar{U}$ ) and the labor demand condition ((13), i.e.,  $V = 0$ ), we can obtain two equations that consist of  $c^*$  and  $\theta$ . Therefore, the equilibrium is summarized by a tuple  $(c^*, \theta)$  that is determined by these two equations. In order to show the existence and uniqueness of the equilibrium, let us work with these two equations.

First, the labor supply condition. Substituting (22), (24) and (25) into  $c^* = \bar{W} - \bar{U}$ , we obtain the reduced-form labor supply condition:

$$\beta \left[ y - b - \frac{(1-s)tnp(\theta)F(c^*)}{2\{\delta + p(\theta)F(c^*)\}} - p(\theta) \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\} \right] - (r + \delta)c^* = 0. \quad (26)$$

The following lemma describes the properties of the labor supply condition (26).

**Lemma 1** *If  $\bar{c} \geq \beta(y - b)/(r + \delta)$ , then the following hold. For any  $\theta \in [0, +\infty)$ , there exists a unique  $c^* \in (0, \bar{c}]$  that satisfies (26). Treat such  $c^*$  as a function of  $\theta$  defined on  $[0, +\infty)$  and describe it as  $c^* = c_s(\theta)$ . Then  $c_s(\theta)$  is a strictly decreasing and continuously differentiable function. Furthermore,  $c_s(0) = \beta(y - b)/(r + \delta)$  and  $\lim_{\theta \rightarrow +\infty} c_s(\theta) = \tilde{c} \in (0, \beta(y - b)/(r + \delta))$ .*

**Proof:** See Appendix A.

If  $\bar{c} \geq \beta(y - b)/(r + \delta)$ , the labor supply condition (26) is represented by the labor supply function  $c^* = c_s(\theta)$  and by the negatively sloped labor supply curve (**LS**) in the  $\theta - c^*$  plane as described in Figure 3. Even if this inequality fails to hold, we can still show the existence

and uniqueness of an equilibrium. However, in such a case,  $c^*$  may not be strictly decreasing in  $\theta$ , that is,  $c^*$  that satisfies (26) may be  $\bar{c}$  and constant for a certain range of  $\theta$ . This implies the possibility of an equilibrium with a corner solution. When the measure of market tightness  $\theta$  is high, there are many vacant jobs per job searcher. This enables job searchers to contact firms easily and allows them selectivity in rejecting jobs that require high training costs, which implies low  $c^*$ .

[Figure 3 here]

Next, the labor demand condition (13), i.e.,  $V = 0$ . The condition  $V = 0$ , combined with (3) and (4), yields

$$\frac{y - w}{r + \delta} = \frac{k}{q(\theta)F(c^*)}.$$

From  $p(\theta)u = q(\theta)v$ , (24), (25), and (26), this can be rewritten as

$$\frac{\beta k}{1 - \beta} = \frac{p(\theta)F(c^*)c^*}{\theta}. \quad (27)$$

The following lemma describes the properties of the labor demand condition (27).

**Lemma 2** *For any  $\theta \in (0, \tilde{\theta}]$ , there exists a unique  $c^* \in (0, \bar{c}]$  that satisfies (27). Here,  $\tilde{\theta}$  is  $\theta \in (0, +\infty)$  that is determined uniquely by an equation  $\beta k / (1 - \beta) = p(\theta)\bar{c} / \theta$ .<sup>6</sup> Treat such  $c^*$  as a function of  $\theta$  defined on  $(0, \tilde{\theta}]$  and describe it as  $c^* = c_d(\theta)$ . Then  $c_d(\theta)$  is a strictly increasing and continuously differentiable function. Furthermore,  $\lim_{\theta \rightarrow 0} c_d(\theta) = 0$  and  $c_d(\tilde{\theta}) = \bar{c}$ .*

**Proof:** See Appendix B.

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<sup>6</sup>The uniqueness of  $\tilde{\theta}$  is guaranteed by the assumptions on  $p(\theta)$  and the matching technology  $\mu(su, v)$ .

The labor demand condition (27) is represented by the labor demand function  $c^* = c_d(\theta)$  and by the positively sloped labor demand curve (**LD**) in the  $\theta - c^*$  plane as described in Figure 3. This is interpreted as follows. When job searchers are very selective and  $c^*$  is low, firms having vacant positions have difficulty in filling their vacancies. This indicates that the expected revenue of a firm having a vacant job is small while the cost of maintaining a vacancy is irrelevant to workers' behavior. Therefore, there is not much incentive for firms to open vacancies and the measure of market tightness  $\theta$  is low.

Lemmas 1 and 2 imply that the **LS** ( $c^* = c_s(\theta)$ ) and **LD** ( $c^* = c_d(\theta)$ ) intersect once at  $(c_e^*, \theta_e)$  in the  $\theta - c^*$  plane (see Figure 3) and we can see that the equilibrium exists and is unique. The relevant variables of the equilibrium are marked with the subscript  $e$ . The following proposition summarizes the above arguments.

**Proposition 1** *If  $\bar{c} \geq \beta(y - b)/(r + \delta)$ , the equilibrium exists and is unique.*

Hereafter, in order to focus on the equilibrium with interior solutions, we assume the inequality  $\bar{c} \geq \beta(y - b)/(r + \delta)$ . This guarantees a possibility of sufficiently high training costs and excludes the case in which for some  $\theta$ , workers accept any jobs.

In the equilibrium of this model, the urban configuration is consistent with the spatial mismatch hypothesis that states while employed workers live close to jobs and unemployed workers reside far away from jobs. Note that if we assume that the unemployed and the employed commute in the same way ( $s = 1$ ) as in several previous studies (Sato [11], for example), no segregation takes place in the equilibrium.

## 4 Influence of changes in city structure on the labor market

In this section, we investigate the effects of changes in the city's structural parameters on the equilibrium. In doing so, we focus on the effects on the reservation training level of a worker

$c_e^*$ , on the measure of market tightness  $\theta_e$  and on the unemployment rate  $u_e/n$ . From (25), the unemployment rate  $u/n$  is given by

$$\frac{u}{n} = \frac{\delta}{\delta + p(\theta)F(c^*)}.$$

For a given  $\theta$ , this is represented by the negatively sloped unemployment rate curve (UR) in the  $c^* - u/n$  plane. The equilibrium unemployment rate  $u_e/n$  is described in Figure 4.

[Figure 4 here]

#### 4.1 Commuting cost

First, we consider a change in commuting cost  $t$ . While a rise in  $t$  does not affect the labor demand condition (27), it does affect the labor supply condition (26) and shifts the **LS** downward as described in Figure 5 (**LS** to **LS'**). The intuition behind this is that a rise in the commuting cost increases the burden from the commuting of the employed more than that of the unemployed and lowers the attractiveness of being employed, which makes workers less tolerant of training given the likelihood of job interviews.

[Figure 5 here]

From Figure 5, we can see that a rise in the commuting cost decreases both  $\theta_e$  and  $c_e^*$ . Therefore, the **UR** move upward (**UR** to **UR'**) and  $u_e/n$  increases.

**Proposition 2** *A rise in the commuting cost  $t$  decreases both the reservation training level of a worker  $c_e^*$  and the measure of market tightness  $\theta_e$ , and increases the unemployment rate  $u_e/n$ .*

As stated above, larger commuting costs makes workers more selective, which reduces the probability that each job interview bears fruit and lowers the expected revenue of a firm having a vacant position. Hence, the number of firms having vacant position decreases to lower the measure of market tightness and raise the unemployment rate. This proposition claims that commuting technology can affect the labor market, and that the improvement of commuting technology, *via* a change in workers' job acceptance behavior, lowers the unemployment rate in the city.

## 4.2 City size

Second, we explore the effect of a change in city size  $n$ . An increase in  $n$  does not affect the labor demand condition (27) but does affect the labor supply condition (26) and shifts the **LS** downward. The city's growth adds to the cost of living of each worker by enlarging the commuting area. As in the previous section, this effect is stronger for the employed than for the unemployed, which makes workers accept only jobs that require less training. Then, again, Figure 5 describes the change: the **LS** moves to **LS'** and the **UR** moves to **UR'**, which decreases both  $\theta_e$  and  $c_e^*$  and increases  $u_e/n$ .

**Proposition 3** *An increase in the city size  $n$  decreases both the reservation training level of a worker  $c_e^*$  and the measure of market tightness  $\theta_e$ , and increases the unemployment rate  $u_e/n$ .*

Sato [11] showed that with scale diseconomies in the matching technology, the city's growth raises the unemployment rate. This paper shows that even without scale diseconomies in the matching technology, the difference in the cost of living between the employed and the unemployed generates the negative effect of an increase in the city size on the unemployment rate.

### 4.3 Search intensity

Finally, we investigate the effect of a change in job search intensity  $s$ . A rise in  $s$  shifts the **LD** downward. It moves the **LS** upward when the commuting cost  $t$  is large and downward when  $t$  is small. As job searchers search more intensively, a firm's possibility of contacting a worker becomes higher ( $\partial q/\partial s > 0$ ). This induces more firms to open vacancies for a given reservation training level of a worker and the **LD** moves downward. Compared with this, a rise in  $s$  has two effects on the **LS**. One is such that it reduces the difference in the cost of living between the employed and the unemployed, which has the effect of raising the attraction of being employed and making workers more tolerant of training given the likelihood of job interviews. The other is that it raises the worker's possibility of contacting a firm ( $\partial p/\partial s > 0$ ), which has the effect of making workers more selective. When  $t$  is small, the latter effect dominates the former and the **LS** shifts downward. When  $t$  is large, the former dominates the latter and the **LS** moves upward. These are illustrated in Figure 6.

[Figure 6 here]

From Figure 6, we can understand the effect of an increase in  $s$  on  $\theta_e$  and  $c_e^*$ . The effect on  $u_e/n$  is ambiguous.

**Proposition 4** *When the commuting cost  $t$  is large, an increase in the job search intensity  $s$  adds to the measure of market tightness  $\theta_e$ . When  $t$  is small, it decreases the reservation training level of a worker  $c_e^*$ .*

## 5 Welfare Analysis

In this section, we explore the efficiency properties of the equilibrium. The efficiency criterion is the same as that used in Pissarides [7], the output of the city. The output of the city  $\Pi$  is defined as

$$\begin{aligned} \Pi = & \int_0^\infty e^{-r\tau} \left\{ (n-u)y + ub - u\theta k - up(\theta) \int_0^{c^*} cf(c)dc \right. \\ & \left. - 2 \int_0^{(n-u)/2} txdx - 2 \int_{(n-u)/2}^{n/2} stxdx \right\} d\tau. \end{aligned}$$

This is the sum of output from job matches and unemployment benefits minus the vacancy costs, the training costs, and the commuting costs. Because land rents are pure transfers, they are excluded in the output of the city. The social planner is subject to the same matching constraints as workers and firms. Therefore, the evolution of unemployment that constrains social choices is the same as the one that constrains private choices:

$$\dot{u} = \delta(n-u) - p(\theta)F(c^*)u.$$

In this paper, we evaluate the optimal path in the steady state. The corresponding optimality conditions are the following <sup>7</sup>:

$$y - b + k\theta - \frac{(1-s)tnp(\theta)F(c^*)}{2\{\delta + p(\theta)F(c^*)\}} + p(\theta) \left\{ \int_0^{c^*} cf(c)dc - F(c^*)c^* \right\} - (r + \delta)c^* = 0, \quad (28)$$

$$k = \frac{dp(\theta)}{d\theta} \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\}. \quad (29)$$

Hereafter, the optimum values carry the subscript  $o$ . The optimality condition (28) is concerned with the reservation training level  $c^*$  and (29) is concerned with the measure of market tightness  $\theta$ .

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<sup>7</sup>Derivation of the optimality conditions is given in Appendix C.

Let  $\theta = \theta_d(c^*)$  be the inverse function of the labor demand function  $c^* = c_d(\theta)$  and let  $c^* = c_o(\theta)$  and  $\theta = \theta_o(c^*)$  be the functions that represent the optimality conditions (28) and (29), respectively. Comparing (26) and (27) with (28) and (29), we obtain the following proposition.

**Proposition 5** *The labor supply condition (26) generates too little acceptance for a given measure of market tightness ( $c_s(\theta) < c_o(\theta)$ ). The labor demand condition (27) gives the optimal measure of market tightness for a given reservation training level ( $\theta_d(c^*) = \theta_o(c^*)$ ) if and only if the bargaining power of workers  $\beta$  is equal to  $1 / \{1 + \xi(c^*)\eta(\theta_d)\}$ , where  $\xi(c^*) = 1 - \int_0^{c^*} cf(c)dc / (F(c^*)c^*)$  and  $\eta(\theta_d) = dp(\theta_d(c^*)) / d\theta$ . Therefore, the equilibrium is not optimal.*

**Proof:** See Appendix D

In a search and matching environment, there inevitably exist externalities: given the number of firms, adding one more worker in the matching process makes it less possible for each worker to secure a job and more possible for each firm to hire a worker. Similarly, with the number of workers constant, as the number of firms seeking workers increases, it becomes more difficult for a firm to fill its vacancy and easier for a worker to land a job. These externalities generally cause distortions. However, in many search and matching models, when the matching technology (or matching function) exhibits constant returns to scale and the bargaining power of workers takes some particular value (that is usually the elasticity of the matching technology (or matching function) with respect to unemployment), the private return of an entering firm coincides with its social return.<sup>8</sup> And the equilibrium condition with respect to the measure of market tightness generates its optimal level given other variables (See Acemoglu and Shimer [1], Hosios [4], and Sato and Sugiura [12], for example.).

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<sup>8</sup>In the model described in this paper, the matching function corresponds with the matching technology multiplied by the probability of acceptance.

The condition with respect to the bargaining power is called the Hosios condition. The Hosios condition is also relevant to our model, though it must be slightly modified because of the spatial dimension and the worker's training decision.

Under the labor supply condition, some of the socially beneficial jobs are rejected. This is because while workers bear all the training costs, the revenues from production are divided through the bargaining between firms and workers. Similar problems are often observed in many search and matching models having investment decisions of economic agents, that is, under-investment takes place (See Acemoglu and Shimer [1], Moen [6], and Sato and Sugiura [12], for example.). Proposition 2 and 5 provide us one policy implication: an adequate transportation policy can solve the under-acceptance problem. More concretely, consider a subvention for commuting whose rate is  $\sigma$ . Then, the commuting cost per distance for workers becomes  $(1-\sigma)t$ . Let  $c^* = c_{s\sigma}(\theta)$  be the labor supply function under the subvention whose rate is  $\sigma$ . Here, we obtain the following proposition.

**Proposition 6** *For any  $\theta$ , there exists a particular subvention rate  $\sigma^* > 0$  that eliminates the under-acceptance problem; that is,  $c_{s\sigma^*}(\theta) = c_o(\theta)$ .*

**Proof:** See Appendix E

The subvention reduces the commuting cost for both the unemployed and the employed, whose effect is stronger for the employed than for the unemployed. This raises the attractiveness of being employed and increases the reservation training to a higher level. Thus, the transportation policy can solve the under-acceptance problem.

## 6 Conclusions

This paper developed a stochastic search model having a monocentric city structure and investigated how urban structure affects workers' job acceptance behavior and a labor market. The unique equilibrium in which employed workers live close to the CBD and unemployed workers reside far away from the CBD was shown to exist. We showed that 1) improvement of transportation technology induces workers to accept jobs that require more training costs and lowers the unemployment rate in a city, 2) an increase in city size makes workers more selective and raises the unemployment rate, and 3) as workers search for jobs more intensively, they become more selective if commuting costs are sufficiently small. It was also shown that the equilibrium is not optimal since under-acceptance occurs in the workers' job acceptance decision and that an adequate subvention for commuting can solve this problem.

These results indicate that urban structure has significant influence on a labor market *via* workers' job acceptance behavior. It is worth recognizing them in discussing urban policies.

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# Appendix

## Appendix A: Proof of lemma 1.

**Lemma 1** *If  $\bar{c} \geq \beta(y - b)/(r + \delta)$ , then the following hold. For any  $\theta \in [0, +\infty)$ , there exists a unique  $c^* \in (0, \bar{c}]$  that satisfies (26). Treat such  $c^*$  as a function of  $\theta$  defined on  $[0, +\infty)$  and describe it as  $c^* = c_s(\theta)$ . Then  $c_s(\theta)$  is a strictly decreasing and continuously differentiable function. Furthermore,  $c_s(0) = \beta(y - b)/(r + \delta)$  and  $\lim_{\theta \rightarrow +\infty} c_s(\theta) = \tilde{c} \in (0, \beta(y - b)/(r + \delta))$ .*

**Proof:** Integrating  $F(c)$  by parts, we can see that for any  $c^* \in [0, \bar{c}]$ ,

$$\begin{aligned} F(c^*)c^* - \int_0^{c^*} cf(c)dc &= \int_0^{c^*} F(c)dc \geq 0, \\ \frac{d}{dx} \left\{ F(x)x - \int_0^x cf(c)dc \right\} \Big|_{x=c^*} &= F(c^*) \geq 0, \end{aligned}$$

where equality holds if and only if  $c^* = 0$ . Let us describe the left-hand side of (26) as  $\Lambda(\theta, c^*)$ .  $\Lambda(\theta, c^*)$  is strictly decreasing and continuously differentiable in both its arguments. Fix  $\theta$  to some value in  $[0, +\infty)$ . We can see that

$$\begin{aligned} \Lambda(\theta, 0) &= \beta(y - b) > 0, \\ \lim_{c^* \rightarrow \bar{c}} \Lambda(\theta, c^*) &= \beta \left[ y - b - \frac{(1-s)tnp(\theta)}{2\{\delta + p(\theta)\}} - p(\theta) \left\{ \bar{c} - \int_0^{\bar{c}} cf(c)dc \right\} \right] - (r + \delta)\bar{c}. \end{aligned}$$

If  $\bar{c} \geq \beta(y - b)/(r + \delta)$ , it must be the case that  $\lim_{c^* \rightarrow \bar{c}} \Lambda(\theta, c^*) \leq 0$ . Because  $\Lambda(\theta, c^*)$  is strictly decreasing and continuously differentiable in  $c^*$ , there exists a unique  $c^* \in (0, \bar{c}]$  that satisfies  $\Lambda(\theta, c^*) = 0$ . Since  $\theta$  can be chosen arbitrarily and  $\partial\Lambda(\theta, c^*)/\partial c^* \neq 0$ , we can apply the implicit function theorem to see that there exists a unique continuously differentiable function  $c^* = c_s(\theta)$  that is defined on  $[0, +\infty)$  and satisfies  $\Lambda(\theta, c_s(\theta)) = 0$ . Because  $\Lambda(\theta, c^*)$  is strictly decreasing in both its arguments,  $c_s(\theta)$  is strictly decreasing in  $\theta$ . Simple calculation shows that  $c_s(0) = \beta(y - b)/(r + \delta)$ . Consider a sequence  $\{\theta_n\}$  such that  $\theta_n \rightarrow +\infty$  as  $n \rightarrow +\infty$

and a sequence  $\{c_n\}$  such that  $c_n = c_s(\theta_n)$ . Then,  $\{c_n\}$  is a monotone decreasing sequence and is bounded since  $c_n \in (0, \bar{c}]$  for any  $n$ . Therefore, the sequence  $\{c_n\}$  converges to some value  $\tilde{c} \in (0, \beta(y-b)/(r+\delta))$  as  $n \rightarrow +\infty$ , which implies that  $\lim_{\theta \rightarrow +\infty} c_s(\theta) = \tilde{c}$ .

## Appendix B: Proof of lemma 2.

**Lemma 2** *For any  $\theta \in (0, \tilde{\theta}]$ , there exists a unique  $c^* \in (0, \bar{c}]$  that satisfies (27). Here,  $\tilde{\theta}$  is  $\theta \in (0, +\infty)$  that is determined uniquely by an equation  $\beta k/(1-\beta) = p(\theta)\bar{c}/\theta$ .<sup>9</sup> Treat such  $c^*$  as a function of  $\theta$  defined on  $(0, \tilde{\theta}]$  and describe it as  $c^* = c_d(\theta)$ . Then  $c_d(\theta)$  is a strictly increasing and continuously differentiable function. Furthermore,  $\lim_{\theta \rightarrow 0} c_d(\theta) = 0$  and  $c_d(\tilde{\theta}) = \bar{c}$ .*

**Proof:** Fix  $c^*$  to some value in  $(0, \bar{c}]$ . The assumptions regarding  $p(\theta)$  and the matching technology  $\mu(su, v)$  implies that there exists a unique  $\theta \in [0, +\infty)$  that satisfies (27). (Note that for  $c^* = 0$ , there is no such  $\theta \in [0, +\infty)$ ) Let us define  $\Upsilon(\theta, c^*)$  as the right-hand side of (27) minus the left-hand side of (27) that is equal to  $p(\theta)F(c^*)c^*/\theta - \beta k/(1-\beta)$ .  $\Upsilon(\theta, c^*)$  is continuously differentiable in both its arguments, strictly increasing in  $c^*$  and strictly decreasing in  $\theta$ . Since  $c^*$  can be chosen arbitrarily and  $\partial\Upsilon(\theta, c^*)/\partial\theta \neq 0$ , we can apply the implicit function theorem to see that there exists a unique continuously differentiable function  $\theta = \theta_d(c^*)$  that is defined on  $(0, \bar{c}]$  and satisfies  $\Upsilon(\theta_d(c^*), c^*) = 0$ . From the fact that  $\Upsilon(\theta, c^*)$  is strictly increasing in  $c^*$  and strictly decreasing in  $\theta$ , we can prove that  $\theta_d(c^*)$  is strictly increasing in  $c^*$ . Furthermore, we can easily show that  $\lim_{c^* \rightarrow 0} \theta_d(c^*) = 0$ . Therefore, we can consider an inverse function  $c^* = c_d(\theta) = \theta_d^{-1}(\theta)$  defined on  $(0, \tilde{\theta}]$  that is strictly increasing and continuously differentiable and satisfies  $\Upsilon(\theta, c_d(\theta)) = 0$ ,  $\lim_{\theta \rightarrow 0} c_d(\theta) = 0$ , and  $c_d(\tilde{\theta}) = \bar{c}$ .

## Appendix C: Derivation of the optimality conditions.

<sup>9</sup>The uniqueness of  $\tilde{\theta}$  is guaranteed by the assumptions on  $p(\theta)$  and the matching technology  $\mu(su, v)$ .

Define the following present value Hamiltonian:

$$H = e^{-r\tau} \left\{ (n-u)y + ub - u\theta k - up(\theta) \int_0^{c^*} cf(c)dc - 2 \int_0^{(n-u)/2} txdx - 2 \int_{(n-u)/2}^{n/2} stxdx \right\} + \mu \{ \delta(n-u) - p(\theta)F(c^*)u \}.$$

Note that the state variable is  $u$ , and that the instrumental variables are  $c^*$  and  $\theta$ . The corresponding first order conditions are

$$e^{-r\tau} \left\{ b - y - \theta k - p(\theta) \int_0^{c^*} cf(c)dc + \frac{(1-s)t(n-u)}{2} \right\} - \mu \{ \delta + p(\theta)F(c^*) \} + \dot{\mu} = 0, \quad (C1)$$

$$e^{-r\tau} up(\theta)c^*f(c^*) + \mu up(\theta)f(c^*) = 0, \quad (C2)$$

$$e^{-r\tau} \left\{ uk + u \frac{dp(\theta)}{d\theta} \int_0^{c^*} cf(c)dc \right\} + \mu u \frac{dp(\theta)}{d\theta} F(c^*) = 0. \quad (C3)$$

(C1), (C2), and (C3) are concerned with  $u$ ,  $c^*$ , and  $\theta$ , respectively. Solving the differential equation (C1), we obtain

$$\mu = \frac{e^{-r\tau} \left\{ b - y - \theta k - p(\theta) \int_0^{c^*} cf(c)dc + (1-s)t(n-u)/2 \right\}}{r + \delta + p(\theta)F(c^*)}. \quad (C4)$$

Substituting (C4) into (C2), we have

$$y - b + k\theta - \frac{(1-s)t(n-u)}{2} + p(\theta) \left\{ \int_0^{c^*} cf(c)dc - F(c^*)c^* \right\} - (r + \delta)c^* = 0. \quad (C5)$$

From the steady state condition (25), (C5) yields (28). Substituting (C5) into (C4) gives

$$\mu = -\frac{e^{-r\tau} \{ p(\theta)F(c^*)c^* + (r + \delta)c^* \}}{r + \delta + p(\theta)F(c^*)} = e^{-r\tau} c^*.$$

Plugging this into (C3), we obtain (29).

## Appendix D: Proof of Proposition 5.

**Proposition 5** *The labor supply condition (26) generates too little acceptance for a given measure of market tightness ( $c_s(\theta) < c_o(\theta)$ ). The labor demand condition (27) gives the optimal measure of market tightness for a given reservation training level ( $\theta_d(c^*) = \theta_o(c^*)$ ) if and only if the bargaining power of workers  $\beta$  is equal to  $1/\{1 + \xi(c^*)\eta(\theta_d)\}$ , where  $\xi(c^*) = 1 - \int_0^{c^*} cf(c)dc / (F(c^*)c^*)$  and  $\eta(\theta_d) = dp(\theta_d)/d\theta$ . Therefore, the equilibrium is not optimal.*

**Proof:** Define  $\rho(\theta, c^*)$  as

$$\rho(\theta, c^*) = y - b - \frac{(1-s)tnp(\theta)F(c^*)}{2\{\delta + p(\theta)F(c^*)\}} - p(\theta) \left\{ F(c^*)c^* - \int_0^{c^*} cf(c)dc \right\},$$

where  $\partial\rho(\theta, c^*)/\partial\theta < 0$  and  $\partial\rho(\theta, c^*)/\partial c^* < 0$ . Then, the labor supply condition (26) and the optimality condition (28) are rewritten as

$$\beta\rho(\theta, c^*) = (r + \delta)c^*, \quad (D1)$$

$$\rho(\theta, c^*) + k\theta = (r + \delta)c^*. \quad (D2)$$

Since  $\beta\rho(\theta, c^*)$  is smaller than  $\rho(\theta, c^*) + k\theta$  for given  $\theta \in [0, +\infty)$  and  $c^* \in [0, \bar{c}]$ , the fact that  $\partial\rho(\theta, c^*)/\partial c^* < 0$  proves that  $c^*$  that satisfies (D1) is smaller than  $c^*$  that satisfies (D2) for a given  $\theta$ . This implies that  $c_s(\theta) < c_o(\theta)$ . Simply comparing (27) with (29) gives that  $\theta_d(c^*) = \theta_o(c^*)$  if and only if  $\beta$  is equal to  $1/\{1 + \xi(c^*)\eta(\theta_d)\}$ . The above arguments show that the equilibrium is not optimal.

## Appendix E: Proof of Proposition 6.

**Proposition 6** *For any  $\theta$ , there exists a particular subvention rate  $\sigma^* > 0$  that eliminates the under-acceptance problem; that is,  $c_{s\sigma^*}(\theta) = c_o(\theta)$ .*

**Proof:** Replacing  $t$  with  $(1 - \sigma)t$  in the labor supply condition (26) and comparing it with the optimality condition (28) gives

$$\sigma^* = \frac{2\{\delta + p(\theta)F(c^*)\}}{\beta(1-s)tnp(\theta)F(c^*)} \{(1 - \beta)\rho(\theta, c^*) + k\theta\} > 0,$$

where  $\rho(\theta, c^*)$  is defined in Appendix D.

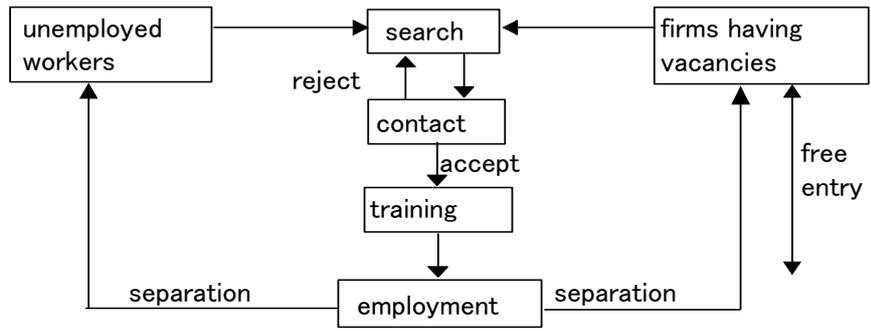


Figure 1: The outline of the model

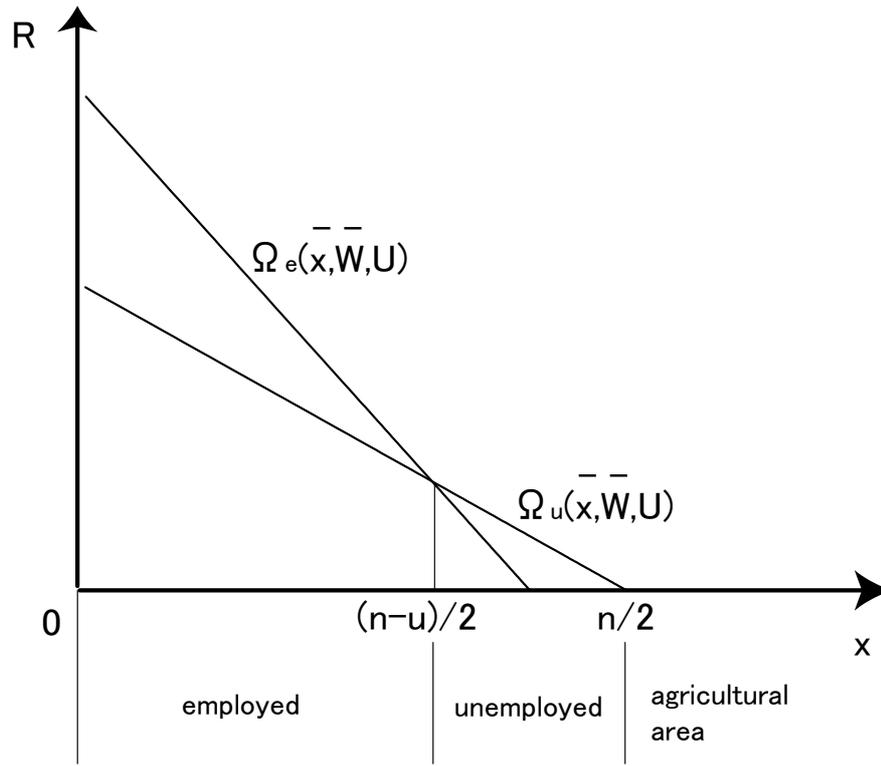


Figure 2: The urban configuration

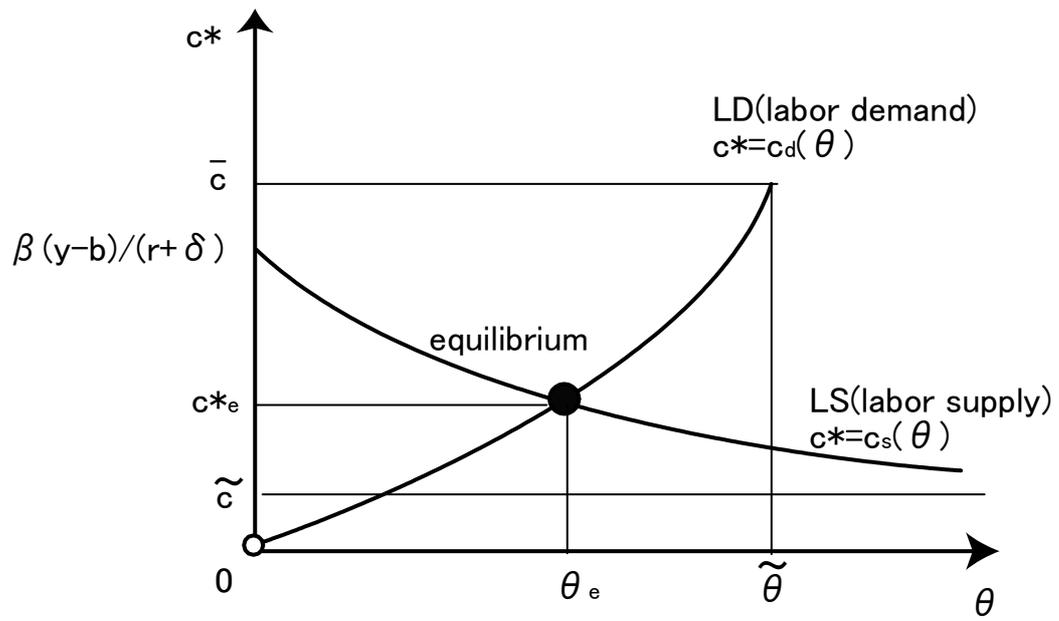


Figure 3: The equilibrium

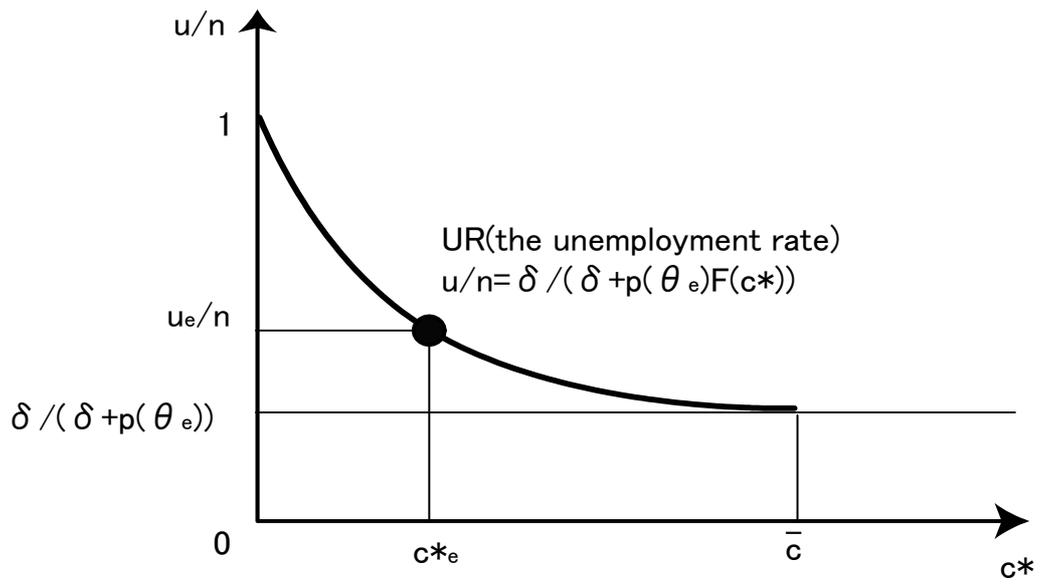


Figure 4: The unemployment rate

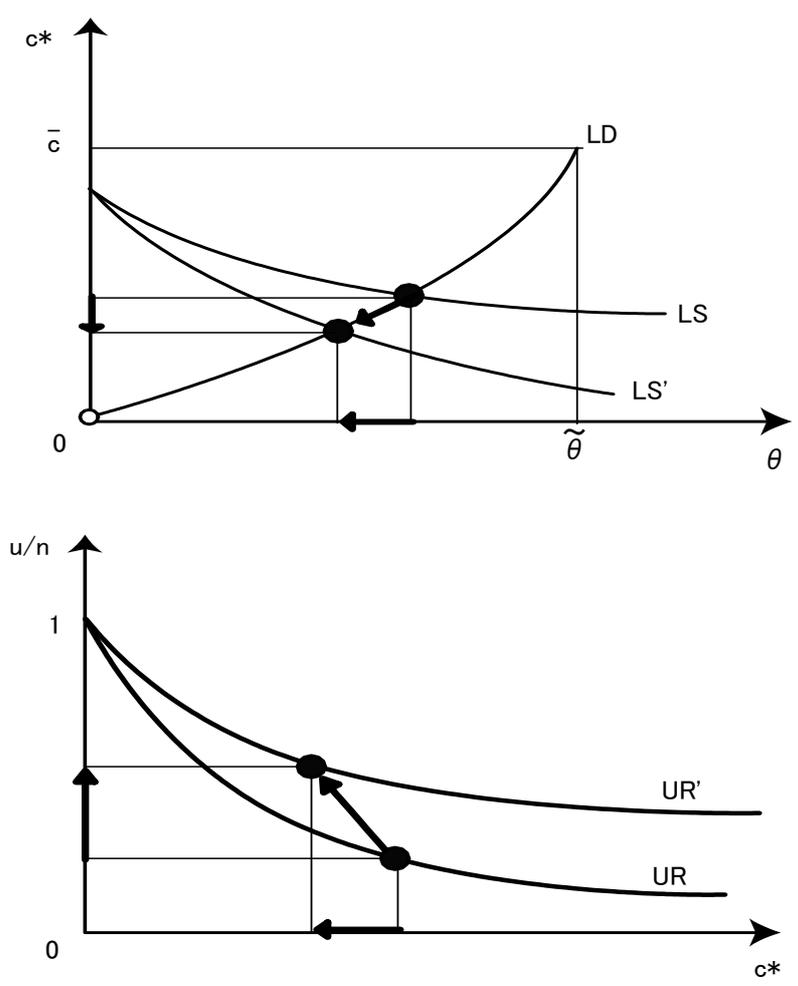


Figure 5: A change in commuting cost or in city size

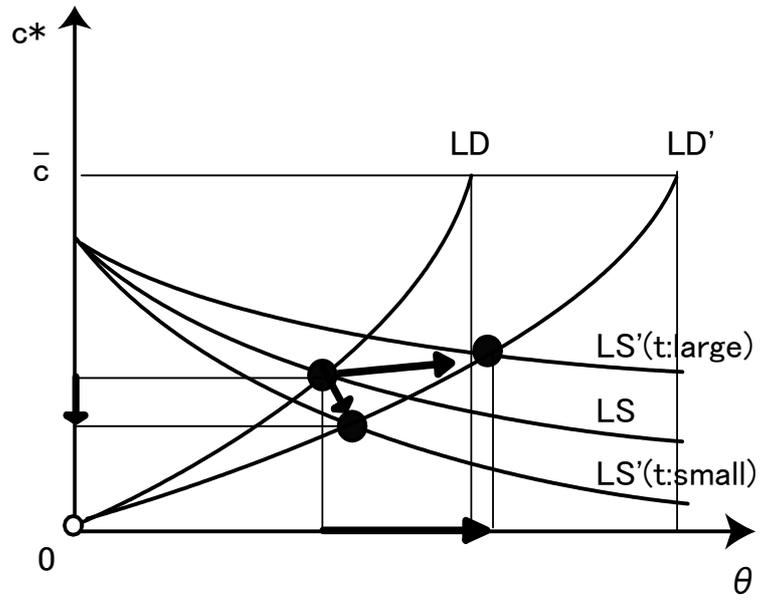


Figure 6: A change in search intensity