

CDMA Slotted ALOHA System with Finite Buffers

Hiraku OKADA[†], Student Member, Takaya YAMAZATO[†], Masaaki KATAYAMA[†],
and Akira OGAWA[†], Members

SUMMARY In this paper, CDMA slotted ALOHA system with finite size of buffers is proposed. To analyze the system performance, we use the linear approximate solution based on restricted occupancy urn models. We evaluate the system performance in terms of throughput, average delay, and rejection probability and clarify the effect of buffer capacity.

key words: CDMA slotted ALOHA, finite buffer capacity, finite population, throughput, average delay, rejection probability

1. Introduction

Because of its features such as random access capability, potentiality of high throughput performance and low peak power transmission, CDMA ALOHA systems draw much attention for satellite and mobile communications. Many works have been made so far aiming at improving the system performance [1]–[5].

In all of previous discussions, CDMA ALOHA systems have been analyzed without consideration of packet queueing at user stations so as to simplify the system model. If each user station is equipped with a certain size of queueing buffers, we can expect the improvement in the grade of service in terms of higher throughput and lower rejection of the packet transmission. Incidentally, various approaches and approximations have been studied on conventional (unspread) slotted ALOHA systems [6]–[12]. In conventional slotted ALOHA systems, it is well-known that two or more packets cannot be transmitted simultaneously. On the other hand, CDMA slotted ALOHA (CDMA S-ALOHA) system has an advantage that the bit error probability tends to degrade gracefully as the number of interfering signals increases, if we are allowed to employ a wideband signal. Hence the system performance should be evaluated in consideration of this advantage.

In this paper, we propose the CDMA S-ALOHA system with finite size of buffers so as to improve the grade of service, and evaluate the system performance. Every packet queues at each user's buffer, and is transmitted from the top of the queue with probability p . Unsuccessful transmission is due to the multiple access interference (MAI) and the additive white Gaussian noise (AWGN). If a packet is received incorrectly by the hub

station, it is retransmitted. To analyze the system performance, we use the linear approximate solution based on the tool called the restricted occupancy urn models [10], [11] because it can be applied to the multiple transmission. Considering aggregate queue sizes and assuming network symmetry property, this approach allows us to assume a Markov chain with the number of linear equations being equal to the number of buffers plus one. The analytical results of the system performance are derived by solving these linear equations. In order to apply this analytical method to the CDMA S-ALOHA system with finite size of buffers, we have only to calculate the probability of having a certain number of successful transmissions in a slot given the number of user stations with nonempty buffer in consideration of the advantage of CDMA signals. We evaluate the system performance in terms of throughput, average delay and rejection probability, and clarify the effect of buffer capacity.

In Sect. 2, we discuss the system model. In Sect. 3, we analyze the CDMA S-ALOHA with finite buffer and evaluate the system performance in Sect. 4. Finally a brief conclusion is described in Sect. 5.

2. System Model

Figure 1 shows the system model of the CDMA S-ALOHA with a finite buffer size and finite population of users. The system consists of a single hub station and K user stations, each with a finite buffer capacity of B packets. Every user station transmits a packet to

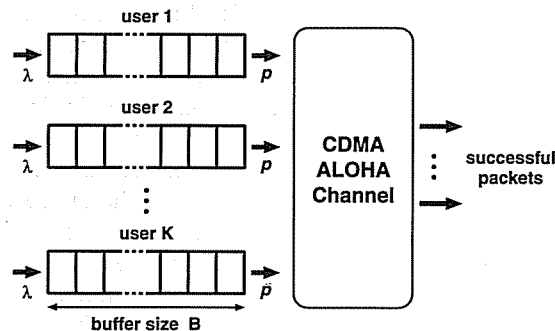


Fig. 1 System model of CDMA ALOHA system with finite buffers and finite population.

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[†]The authors are with the School of Engineering, Nagoya University, Nagoya-shi, 464-8603 Japan.

the hub station by one hop, and we consider only the packet access on the up-link. Each packet has a fixed length of L [bits]. The time axis is divided into slots, each has a duration equals to one packet length. All of user stations synchronize their transmissions so that they initiate at the beginning of each slot.

The binary phase-shift keying (BPSK) is assumed as the modulation scheme for the up link of the CDMA S-ALOHA system. Each packet is spectrum-spread with a uniquely assigned Gold sequence. Transmitter for the user station and receiver for the hub station are shown in Fig. 2. Signals are assumed to be band-limited by a root cosine roll-off filter. All packets are received by the hub station with an equal power. Unsuccessful transmissions are due to MAI and AWGN. Since the effect of an odd cross-correlation can be ignored for the case of slotted system with assigned Gold sequences [13], we can use the bit error probability $P_b(k)$ calculated in [14]. It is expressed as

$$P_b(k) = \frac{1}{2} \operatorname{erfc}(\sqrt{SNIR/2}), \quad (1)$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function given by

$$\operatorname{erfc}(x) = \frac{2}{\pi} \int_x^\infty e^{-y^2} dy, \quad (2)$$

$SNIR$ is the signal-to-noise plus interference ratio derived as

$$SNIR \cong \left\{ \frac{k-1}{2N} \left(1 - \frac{\alpha}{4} \right) + \frac{N_0}{2E_b} \right\}^{-1}, \quad (3)$$

N is the number of chips per bit, k is the number of simultaneously transmitted packets, E_b is the bit energy of the signal, $N_0/2$ is two-sided spectral density of AWGN, and α is the roll-off factor. We assume that the hub station returns a message whether the packet transmission from the user station succeeds or not to the user station without error and elapsed time.

The buffer model proposed in this paper is described as follows. The packet flow at each user station is shown in Fig. 3. Every user station generates a packet in each slot with birth probability λ . If a packet arrives at a user station with a full buffer, it is rejected. Otherwise it is stored at the buffer. Whether the rejected packet is regenerated or not is up to the upper layer. Stored packets are served on a first-in-first-out (FIFO) discipline. We define a busy station as a station with a nonempty buffer. Busy user stations attempt the transmission of a packet from the top of the queue with probability p , which can be set at each user station. The packet transmission probability is identical among the user stations. When the packet is transmitted to the channel and fails to be received correctly by the hub station due to MAI and AWGN, the unsuccessful packet is retransmitted with a probability p . We assume that a

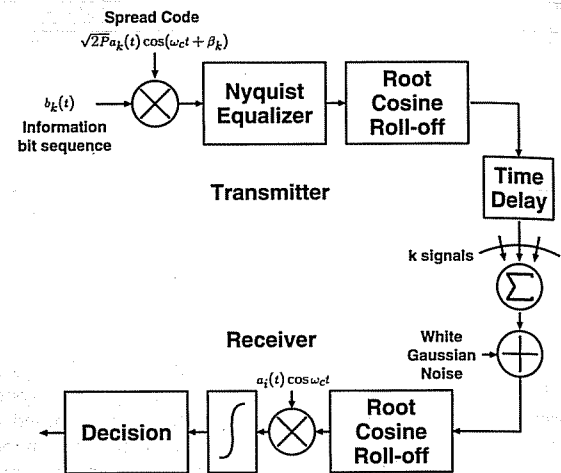


Fig. 2 Transmitter and receiver for CDMA S-ALOHA system with BPSK band-limited by root cosine roll-off filter.

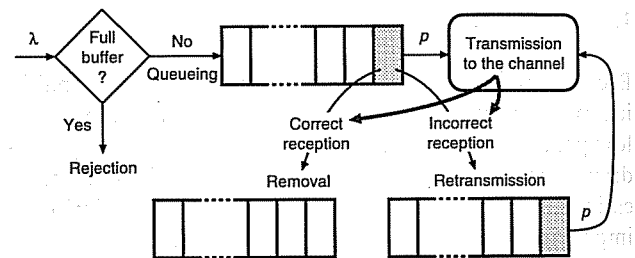


Fig. 3 Schematic of packet flow at each user station.

packet is received correctly only if all bits in a packet are received successfully. The retransmission process is repeated until the packet is received correctly. After successful transmission, the packet is removed from the buffer and the next packet is served if there exists.

3. Queueing Analysis by Using Linear Approximate Solution

A linear approximate solution has been proposed in [10]. In this solution, a symmetric system is assumed, i.e., all the users have uniform input traffic and contribute equally to the number of successful transmissions. Considering the aggregate number of packets in the system and assuming a symmetric system allow us to construct a Markov chain with $(B \cdot K + 1)$ linear equations. Restricted occupancy urn models are used for computing the transition probabilities [15]. By solving these linear equations, we can obtain the steady-state probability π_n that there are n packets in the system.

The system proposed in this paper is also assumed as a symmetric system. Therefore, we can analyze the system performance by using the linear approximate solution if only we calculate the $s(i, k)$, which is defined as the probability of k successful transmissions in a time slot given there were i busy stations. We should calculate $s(i, k)$ in consideration of the advantage of

CDMA signal that is the graceful degradation of the bit error probability as the number of interfering signals increases. This probability is derived as

$$s(i, k) = \sum_{l=k}^i b(i, l) \cdot d(l, k), \quad (4)$$

where $b(i, l)$ is the probability that l packets are transmitted, given there were i busy stations, and $d(l, k)$ is the probability that exactly k packets are received successfully, given there were l transmissions. Since each busy user station transmits a packet with probability p , the probability $b(i, l)$ is clearly obtained as

$$b(i, l) = \binom{i}{l} p^l (1-p)^{i-l}. \quad (5)$$

Taking account of the advantage of CDMA signals, the probability $d(l, k)$ is

$$d(l, k) = \binom{l}{k} Q_S^k(l) (1 - Q_S(l))^{l-k}, \quad (6)$$

where $Q_S(l)$ is the packet successful probability given l simultaneously transmitted packets, derived as

$$Q_S(l) = (1 - P_b(l))^L. \quad (7)$$

4. Numerical Examples and Discussion

In this section, we evaluate the system performance in terms of throughput, average delay, and rejection probability based on analytical results. We define the throughput S as the average number of successful packets during a time slot. The throughput indicates the channel efficiency. The average delay time D is the elapsed slot time from generating a packet at the user station until receiving it correctly at the hub station. The rejection probability Q_R is the probability that a new packet arriving at a user station is rejected because its buffer is full. The average delay and rejection probability are important measures for evaluating the quality of service.

Numerical examples in the case of Gold sequence of $N = 63$ are considered. Because a sequence is uniquely assigned to user stations, we set $K = 65$. In consideration of the band-limitation and AWGN, we set $\alpha = 0.5$ and $E_b/N_o = 10$ [dB]. The packet length is set as $L = 500$ [bits] (for example, ATM system has a fixed packet length of 53 [bytes]).

Figures 4–6 show the analytical results of normalized throughput, average delay, and rejection probability, respectively, with the parameters of $p = 0.05, 0.1, 0.12, 0.15, 0.2$, and $B = 2$. The offered load G is defined as the average number of packets arriving in the system during one slot time, obtained as $G = K \cdot \lambda$. The offered load and throughput are normalized by N . For $p \leq 0.12$, throughput, average delay and rejection probability are gradually and monotonously increased

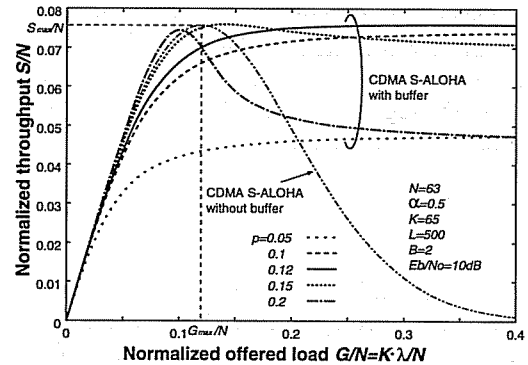


Fig. 4 Throughput versus offered load curves for $B = 2$.

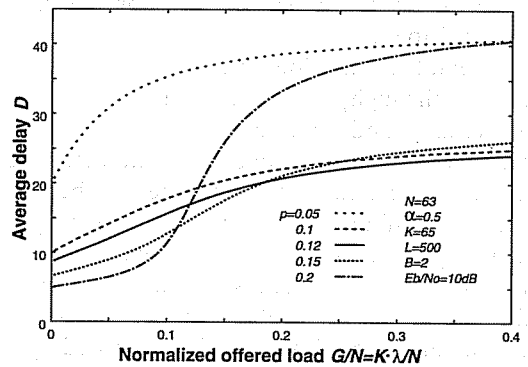


Fig. 5 Average delay versus offered load curves for $B = 2$.

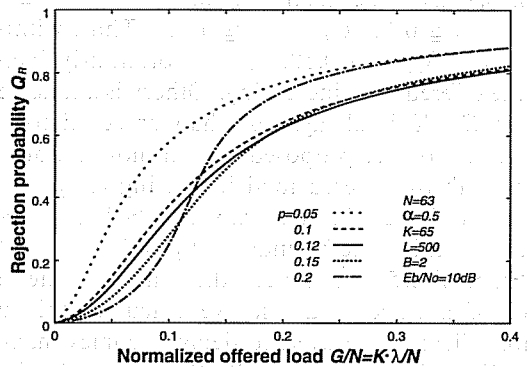


Fig. 6 Rejection probability versus offered load curves for $B = 2$.

with the offered load increasing. In this range, the performance for the case of large p is better than that for the case of small p . But, for $p > 0.12$, the throughput curve is in the shape of a convex cap, and the average delay and the rejection of a probability are small for the region of small offered load and rapidly increased over the value of offered load at which throughput takes the maximum value, where the maximum throughput implies the highest value of the throughput. These tend to be more remarkable for larger p . The reason for those is as follows; packets are more rapidly served for larger transmission probability p . But, for $p > 0.12$, the packet success probability is degraded because more and

more packets are transmitted simultaneously. Thus the number of retransmitted packets is increased and the system performance is rapidly degraded. Figure 4 also shows the throughput of the CDMA S-ALOHA without buffer. It is derived as

$$S_C = \sum_{k=1}^K k \cdot \binom{K}{k} \lambda^k (1-\lambda)^{K-k} \cdot Q_S(k), \quad (8)$$

because K user stations transmit the packet with a probability λ . The throughput of the CDMA S-ALOHA without buffer is degraded for the region of large offered load. On the other hand, if we select appropriate p in the case of the CDMA S-ALOHA with buffer, higher throughput can be obtained for the region of large offered load.

Incidentally, why does the boundary at $p \cong 0.12$ exist? The throughput depends on the average number of actually and simultaneously transmitted packets, which we call actual load G_a . The actual load is derived as the product of p and the number of busy user stations. If all user stations are in busy mode, the throughput for the proposed system is equal to S_C when $\lambda = G_a/K = p$ because all user stations transmit the packet with a probability p . If not, we can expect that the throughput for the proposed system is similar to S_C when $\lambda = G_a/K < p$, because the actual load is the same between these two cases. From Fig. 4, S_C takes the maximum value $S_{max}/N \cong 0.076$ when $\lambda_{max} \cong 0.12$ ($G_{max}/N \cong 0.12$). Thus, within the region $\lambda < \lambda_{max} \cong 0.12$, S_C monotonously increases with the offered load increasing. Otherwise, it decreases. Because G_a/K is always less than or equal to p , the throughput for the proposed system monotonously increases with the offered load increasing for $p < \lambda_{max}$, or is in the shape of convex cap for $p > \lambda_{max}$. Accordingly, there exists the boundary at $p = \lambda_{max} \cong 0.12$.

From Fig. 5, we expect that there is the upper bound of average delay, so we calculate this upper bound. Clearly, the average delay becomes maximum when all user's buffers are full. If all user's buffers are full, all users are in busy mode. The upper bound of the average delay, therefore, is obtained by

$$D \leq \frac{B \cdot K}{S_C|_{\lambda=p}}. \quad (9)$$

If we set p so as to maximize $S_C|_{\lambda=p}$ (that is $p = \lambda_{max}$), this upper bound takes minimum and increases in proportion to the buffer size, as listed in Table 1.

Figures 7-9 show the normalized throughput, the average delay and the rejection probability versus normalized offered load, respectively, with a buffer size of $B = 1 - 5$, and the transmission probability $p = 0.12$.

The larger buffer size a user station has, the more rapidly the throughput increases for the region of small offered load. If many user stations become in the busy mode for the case of $p = 0.12$, the actual load will come

close to G_{max} , and the throughput will take the maximum value. The number of busy user stations increases as increasing the buffer size, because a lot of packets are stored at the user's buffer. We can, therefore, find

Table 1 Upper bound of average delay for $N = 63$, $\alpha = 0.5$, $K = 65$, $L = 500$, $E_b/N_0 = 10$ [dB], and $p = 0.12$.

Buffer size	Upper bound of average delay
1	13.6
2	27.1
3	40.7
4	54.3
5	67.8

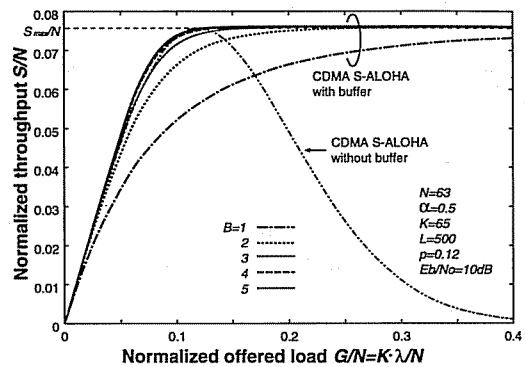


Fig. 7 Throughput versus offered load curves for $p = 0.12$.

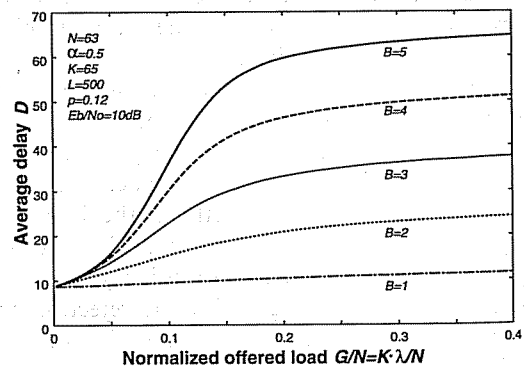


Fig. 8 Average delay versus offered load curves for $p = 0.12$.

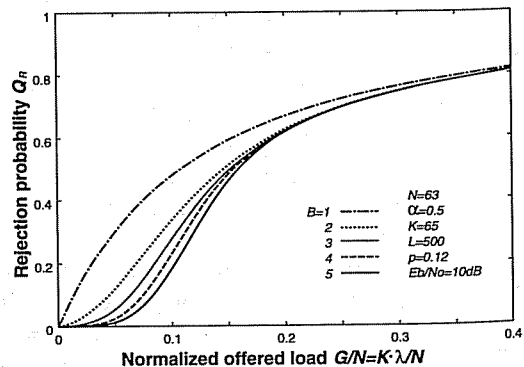


Fig. 9 Rejection probability versus offered load curves for $p = 0.12$.

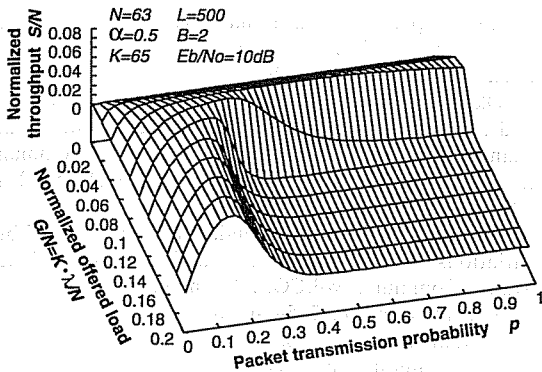


Fig. 10 Throughput performance as a function of packet transmission probability and offered load.

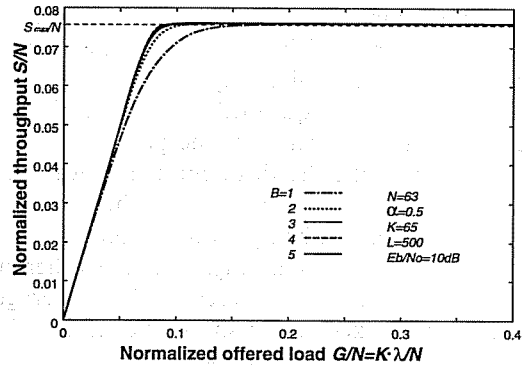


Fig. 11 Throughput performance for the optimum case of the packet transmission probability for $B = 1 - 5$.

the throughput curves as shown in Fig. 7. Moreover, the throughput for $B \geq 4$ is always better than that of the CDMA S-ALOHA without buffer. Thus, if each user station is equipped with a certain size of queuing buffers, higher throughput can be obtained. Large buffer size causes an increase in the average delay as shown in Fig. 8. But, it is not so serious because the packets which should be rejected in the case of small buffer size can be served as increasing the buffer size. The rejection probability is improved as increasing the buffer size for the region of small offered load. On the other hand, since user stations generate packets over the acceptable number of simultaneously transmitting packets for the region of large offered load, buffers of almost all users will become full. It is for this reason that there is little effect of buffer size on the rejection probability for the region of large offered load.

Figure 10 shows the throughput performance as a function of the packet transmission probability and the offered load. In this figure, we find that the packet transmission probability giving the maximum throughput depends on the offered load. This value is changed asymptotically from 1 to 0.12 as increasing the offered load. If the packet transmission probability is adjusted so as to obtain the maximum throughput for each value of the offered load, the system performance will become better.

Figures 11–13 show analytical results for the optimum case where the packet transmission is adjusted so as to obtain the maximum throughput for each value of the offered load. It goes without saying that the performance for the optimum case with $B = 2$ is better than that for the case of Figs. 4–6. Incidentally, the upper bound of the throughput, which is the throughput when all packets are transmitted successfully, is obviously a straight line which linearly increases at a slope of 1. From Fig. 11, we find that the throughput curves approach the upper bound asymptotically for the region of small offered load as increasing the buffer size. For the region of large offered load, the throughput curves keep the maximum value S_{max} . The average delay and

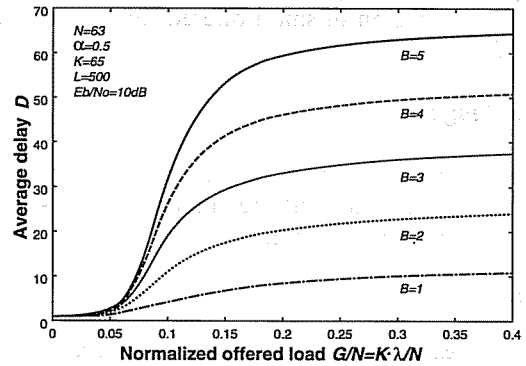


Fig. 12 Average delay for the optimum case of the packet transmission probability for $B = 1 - 5$.

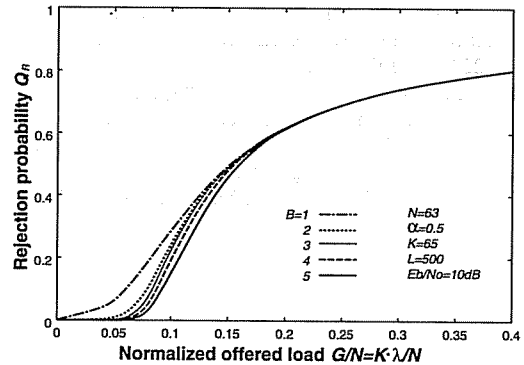


Fig. 13 Rejection probability for the optimum case of the packet transmission probability for $B = 1 - 5$.

the rejection probability are improved remarkably for the region of small offered load. Moreover, the region where the rejection probability is almost zero becomes wider as increasing the buffer size.

5. Conclusions

The CDMA S-ALOHA with finite size of buffers has been proposed. By using the linear approximate solution based on restricted occupancy urn models, we have analyzed the CDMA S-ALOHA system under fi-

nite buffer assumption. We have evaluated the system performance in terms of the throughput, the average delay and the rejection probability.

If user station is equipped with a certain size of queueing buffers and transmits the packet with an appropriate probability, the throughput is always higher than that of the CDMA S-ALOHA without buffer. Increase in the buffer size brings about higher throughput and lower rejection probability. On the other hand, large buffer size causes an increase of the average delay because the packets which should be rejected for the case of small buffer size can be served. When we adjust the packet transmission probability in order to take the maximum throughput for each value of the offered load, throughput curves approach the upper bound asymptotically for the region of small offered load as increasing the buffer size.

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