

## PAPER Special Section on Information Theory and Its Applications

## Queueing Analysis of CDMA Unslotted ALOHA Systems with Finite Buffers

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**SUMMARY** CDMA unslotted ALOHA system with finite size of queueing buffers is discussed in this paper. We introduce an analytical model in which the system is divided into two Markov chains; one is in the user part, and the other is in the channel part. In the user part, we can model the queueing behavior of the user station as an  $M/G/1/B$  queue. In the channel part, we can consider the number of simultaneously transmitted packets as an  $M_1 + M_2/D/\infty/K$  queue. We analyze the queueing system by using this analytical model, and evaluate the effect of buffer capacity in terms of the throughput, the rejection probability and the average delay. As a result, increase in the buffer size brings about an improvement in the grade of service in terms of higher throughput and lower rejection probability.

**key words:** CDMA unslotted ALOHA, finite buffer capacity, packet queueing, Markov chain

## 1. Introduction

Code-Division Multiple-Access (CDMA) ALOHA systems have drawn much attention for satellite and mobile communications because of the features such as random access capability, potentiality of high throughput performance and low peak power transmission. Moreover, CDMA unslotted ALOHA (CDMA U-ALOHA) systems have the advantage of no need to synchronize the packet transmissions so that they initiate at the beginning of a slot. Many works have been made so far aiming at improving the system performance [1]–[5].

If each user station is equipped with a certain size of queueing buffers, it brings about not only the reduction of the rejection of the packet transmission but also the possibility of the autonomous control of the packet transmission. Therefore, we can expect an improvement in the grade of service in terms of higher throughput and lower rejection probability for the packet transmission.

In this paper, we consider the CDMA U-ALOHA system equipped with a certain size of queueing buffers in order to improve the grade of service.

Incidentally, various approaches and approximations have been studied on conventional (unspread) slotted ALOHA systems [6]–[12]. On the CDMA slotted ALOHA (CDMA S-ALOHA) system, we have studied the queueing analysis [13]. In all of previous discussions, however, both CDMA and conventional unslot-

ted ALOHA systems have never been studied in consideration to the effect of packet queueing buffers. In the CDMA U-ALOHA system, a packet is transmitted asynchronously. Therefore, the number of simultaneously transmitted packets may change from moment to moment. This makes the analysis more complicated than that for the CDMA S-ALOHA system.

We analyze the performance of the CDMA U-ALOHA system with finite size of queueing buffers by using the Markov chain analysis. The packets arriving at the user station with empty buffer are transmitted immediately to reduce the delay time, while the other arriving packets and unsuccessful packets queue at each user's buffer and are transmitted from the top of the queue with a transmission rate  $p$ . In order to analyze the system performance, we introduce the analytical model in which the system is divided into two Markov chains; one is in user part, and the other is in channel part. We evaluate the system performance in terms of throughput, average delay and rejection probability, and clarify the effect of buffer capacity.

In Sect. 2, we discuss the system model. In Sect. 3, we analyze the performance of the CDMA U-ALOHA system with finite buffer capacity. In Sect. 4, we evaluate the system performance, and clarify the effect of buffer capacity. Finally a brief conclusion is described in Sect. 5.

## 2. System Model

Figure 1 shows the system model of the CDMA U-ALOHA with finite size of queueing buffers. The system consists of a single hub station and symmetric  $K$  user stations, each with a finite buffer capacity of  $B$  packets. Every user station transmits a packet to the hub station by one hop, and we consider only the packet access on the up-link. Each packet has a fixed length of a packet time duration  $T_p = L/R$ , where  $R$  is a data rate, and  $L$  [bit] is the length in bit.

The packet flow at each user station is shown in Fig. 2. Every user station generates a packet following the Poisson process with a birth rate  $\lambda$ . We define a busy station as a station with a nonempty buffer, and an idle station as a station with an empty buffer. If a packet arrives at a user station with a full buffer, this packet is rejected. Otherwise the packet arriving at a busy sta-

Manuscript received January 16, 1998.

Manuscript revised April 13, 1998.

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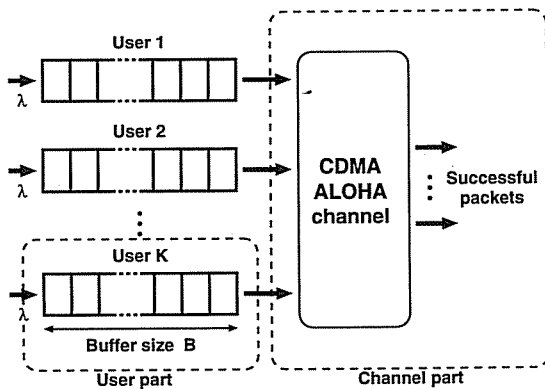


Fig. 1 System model of CDMA ALOHA system with finite size of queueing buffers.

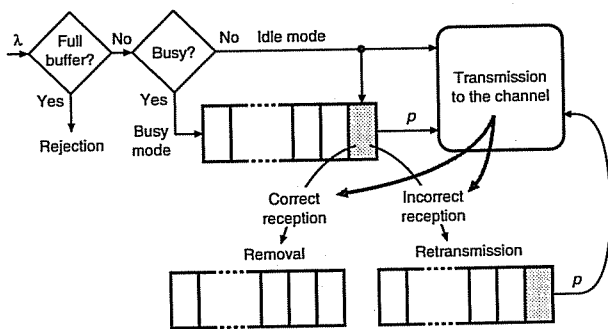


Fig. 2 Schematic of packet flow at each user station.

tion is stored at its buffer, while the packet arriving at an idle station is transmitted immediately to the channel to reduce the time delay and stored at its buffer as well. After successful transmission, the packet is removed from the buffer. Packets stored in a user station are served on a first-in-first-out (FIFO) rule. Busy user stations attempt to transmit the packet asynchronously at the head of queue with a packet transmission rate  $p$  (transmitting interval is exponentially distributed with average  $1/p$ ). The packet transmission rate is identical among the user stations. When the packet is transmitted to the channel and fails to be received correctly by the hub station, unsuccessful packet is retransmitted with a rate  $p$ .

The binary-phase-shift keying (BPSK) is assumed as the modulation scheme. Every packet is spectrum-spread with a uniquely assigned random signature sequence. The number of codes is assumed to be more than or equal to the number of user stations so as to simplify the analytical model. We expect that the analytical method proposed in this paper will be available even if the number of codes is less than the number of user stations. We assume that all packets are received with the equal power and all data bit errors are caused by the effect of multiple access interference (MAI) and additive white Gaussian noise (AWGN). The bit error

probability  $P_b(k)$  is expressed as [14],

$$P_b(k) \approx \frac{2}{3} Q \left[ \left( \frac{k}{3N} + \frac{N_0}{2E_b} \right)^{-0.5} \right] + \frac{1}{6} Q \left[ \left( \frac{k \cdot N/3 + \sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b} \right)^{-0.5} \right] + \frac{1}{6} Q \left[ \left( \frac{k \cdot N/3 - \sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b} \right)^{-0.5} \right], \quad (1)$$

with

$$\sigma^2 = k \left\{ N^2 \frac{23}{360} + N \left( \frac{1}{20} + \frac{k-1}{36} \right) - \frac{1}{20} - \frac{k-1}{36} \right\} \quad (2)$$

where  $N$  is the number of chip per bit,  $k$  is the number of interfering packets,  $E_b$  is the bit energy of the signal,  $N_0/2$  is two-sided spectral density of AWGN, and

$$Q[x] = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-u^2/2) du. \quad (3)$$

### 3. Queueing Analysis

#### 3.1 Two Markov Chains

The packet success probability depends on the birth rate for actually transmitted packets. This birth rate is determined by the ratio of the number of the idle user stations to the number of all users. In other words, it is determined by the steady state probability of having no packets at each user station. The behavior of the user station depends on the elapsed time from transmitting a packet for the first time until receiving the packet correctly at the hub station. This elapsed time is determined by the packet success probability and the packet transmission rate  $p$ .

Let  $P_j$  be the steady state probability of having  $j$  packets at a certain user station. We assume that the system condition changes slowly enough to regard the steady state probability  $P_j$  and the packet success probability  $Q_s$  as constant. This assumption allows us to construct the analytical model in which the system is divided into two parts; one is in the user part, and the other is in the channel part, as shown in Fig. 1.

In the user part, we can consider only a certain user's behavior because of symmetry of the system. In the user station, packets are generated according to the Poisson process, the number of outgoing channel is 1, and the number of stored packets which include a packet on service is  $B$ . Thus, we model the queueing behavior of the user station as an M/G/1/B queue taking account of the influence from other stations through the service time distribution.

In the channel part, birth interval is exponentially distributed with average  $1/\lambda$  at the idle user station or

with average  $1/p$  at the busy user station, service time (= packet length) is fixed, and the number of simultaneously transmitted packets is not limited. Thus, we can regard the number of simultaneously transmitted packets as an  $M_1 + M_2/D/\infty/K$  queue.

The discussion for each part is described in the following.

### 3.2 User Part

At first, we calculate the probability density function (pdf) of service time for transmitting the packets. The user station which is initially in the idle mode transmits the newly packet as soon as it arrives, while the user station which is initially in the busy mode transmits the stored packet with a birth rate  $p^\dagger$ . Thus, the pdf of service time for the idle user station is different from that of the busy user station, as shown in Fig. 3.

For the case of the user station which is initially in the idle mode, new arriving packet is transmitted immediately. Let  $d_I(t)$  be the pdf of service time for this case. If the packet is transmitted successfully at the first time, the elapsed time is  $T_p$  and its probability becomes  $Q_S$ . Otherwise user station retransmits the packet until the packet transmission succeeds. After being transmitted a total of  $m + 1$  times, the elapsed time is  $(m + 1)T_p$  plus sum of  $m$  exponentially distributed transmitting intervals, and its probability is  $(1 - Q_S)^m Q_S$ . The distribution of the sum of  $k$  exponentially distributions is  $k$ -Erlangian distribution. Therefore, the pdf of the service time is

$$\begin{aligned} d_I(t) &= Q_S \cdot \delta(t - T_p) \\ &+ \sum_{m=1}^{\infty} (1 - Q_S)^m Q_S \cdot E_m(t - (m + 1)T_p; p/m) \end{aligned} \quad (4)$$

where  $\delta(t)$  is a delta function,  $E_k(t; \nu)$  is the pdf of  $k$ -Erlangian distribution with an average  $1/\nu$ , defined as,

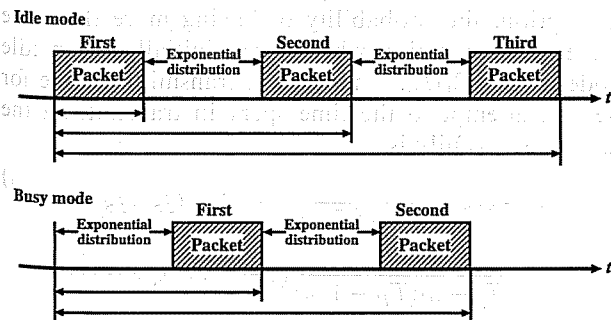


Fig. 3 Elapsed time between packets when the retransmission is required.

$$E_k(t; \nu) \equiv \begin{cases} \frac{(k\nu t)^{k-1}}{(k-1)!} k\nu e^{-k\nu t} & (t \geq 0) \\ 0 & (t < 0) \end{cases} \quad (5)$$

By averaging  $d_I(t)$ , the average service time  $1/\mu_I$  is derived as

$$1/\mu_I = T_p + \frac{T_p + 1/p}{Q_S} (1 - Q_S). \quad (6)$$

For the case of the user station which is initially in the busy mode, the packet is transmitted with a birth rate  $p$ . Let  $d_B(t)$  be the pdf of service time for this case. Each packet transmission occurs after exponentially distributed transmitting interval. After being transmitted a total of  $m$  times, the elapsed time is  $mT_p$  plus sum of  $m$  exponentially distributed transmitting intervals for  $m$  transmissions, and its probability is  $(1 - Q_S)^{m-1} Q_S$ . Thus, the pdf of the service time is

$$d_B(t) = \sum_{m=1}^{\infty} (1 - Q_S)^{m-1} Q_S \cdot E_m(t - mT_p; p/m) \quad (7)$$

and the average service time  $1/\mu_B$  is derived as

$$1/\mu_B = \frac{T_p + 1/p}{Q_S}. \quad (8)$$

To calculate the steady state probability of having  $j$  packets at the user station, we refer to the analytical method of a steady state probability for an  $M/G/1/B$  queue[16]. Let  $D_I(s)$  and  $D_B(s)$  be the Laplace-Stieljes Transform (LST) of the  $d_I(t)$  and  $d_B(t)$ , respectively. These are obtained as,

$$\begin{aligned} D_I(s) &\equiv \int_0^{\infty} e^{-st} d_I(t) dt \\ &= \sum_{m=0}^{\infty} (1 - Q_S)^m Q_S \cdot e^{-(m+1)T_p s} \left( \frac{p}{s+p} \right)^m \end{aligned} \quad (9)$$

$$D_B(s) = \sum_{m=1}^{\infty} (1 - Q_S)^{m-1} Q_S \cdot e^{-mT_p s} \left( \frac{p}{s+p} \right)^m \quad (10)$$

Let  $p_{Ij}$  be the probability that  $j$  packets are arriving at the user station which is initially in the idle mode during the service time. Because the packets are generated according to the Poisson process, this probability is derived as,

$$p_{Ij} = \int_0^{\infty} \frac{(\lambda t)^j}{j!} e^{-\lambda t} d_I(t) dt. \quad (11)$$

The probability generating function of  $p_{Ij}$  is

$$\begin{aligned} h(z) &\equiv \sum_{j=0}^{\infty} z^j p_{Ij} = \int_0^{\infty} \sum_{j=0}^{\infty} z^j \frac{(\lambda t)^j}{j!} e^{-\lambda t} d_I(t) dt \\ &= D_I([1 - z]\lambda). \end{aligned} \quad (12)$$

<sup>†</sup>In [13], both idle and busy user station transmit the packet with a birth rate  $p$  to simplify the analysis.

By the inversion formula,  $p_{Ij}$  is obtained as,

$$\begin{aligned} p_{Ij} &= \frac{1}{j!} \lim_{z \rightarrow 0} \frac{\partial^j}{\partial z^j} D_I([1-z]\lambda) \\ &= \sum_{m=0}^{\infty} \frac{1}{j!} \left(-\frac{\lambda}{p}\right)^j (1-Q_S)^m Q_S e^{-(m+1)T_p \lambda} \\ &\quad \cdot \sum_{n=0}^j \binom{j}{n} \frac{(-m)! \{-(m+1)T_p p\}^{j-n}}{(-m-n)!} \\ &\quad \cdot \left(\frac{p}{\lambda+p}\right)^{m+n} \end{aligned} \quad (13)$$

Similarly, the probability that  $j$  packets are arriving at the user station which is initially in the busy mode during the service time is

$$\begin{aligned} p_{Bj} &= \frac{1}{j!} \lim_{z \rightarrow 0} \frac{\partial^j}{\partial z^j} D_B([1-z]\lambda) \\ &= \sum_{m=1}^{\infty} \frac{1}{j!} \left(-\frac{\lambda}{p}\right)^j (1-Q_S)^{m-1} Q_S e^{-mT_p \lambda} \\ &\quad \cdot \sum_{n=0}^j \binom{j}{n} \frac{(-m)! (-mT_p p)^{j-n}}{(-m-n)!} \\ &\quad \cdot \left(\frac{p}{\lambda+p}\right)^{m+n} \end{aligned} \quad (14)$$

From (13) and (14), the probability of having  $j$  packets at a certain user station immediately after completion of the packet transmission is derived as the asymptotical equation:

$$\begin{aligned} \Pi_{j+1} &= (\Pi_j - p_{Ij}\Pi_0 - \sum_{m=1}^j p_{Bj-m+1}\Pi_m) p_{B0}^{-1} \\ &\quad (j = 0, 1, \dots, B-2) \end{aligned} \quad (15)$$

and the normalized condition:

$$\sum_{j=0}^{B-1} \Pi_j = 1. \quad (16)$$

In (15), we set  $C_j = \Pi_j/\Pi_0$ , and obtain the following asymptotical equations:

$$\begin{aligned} C_{j+1} &= (C_j - p_{Ij} - \sum_{m=1}^j p_{Bj-m+1}C_m) p_{B0}^{-1} \\ &\quad (j = 0, 1, \dots, B-2) \end{aligned} \quad (17a)$$

$$C_0 = 1 \quad (17b)$$

We also set

$$C = 1 + \sum_{j=1}^{B-1} C_j. \quad (18)$$

The steady state probability  $P_j$  is expressed in terms of  $\Pi_j$  [16] as the following equations:

$$P_j = \begin{cases} \frac{\Pi_j}{\Pi_0 + a} & (j = 0, 1, \dots, B-1) \\ 1 - \frac{1}{\Pi_0 + a} & (j = B) \end{cases} \quad (19)$$

where  $a$  is the traffic intensity at the user station, expressed as

$$a = \lambda/\mu \quad (20)$$

with

$$1/\mu = \Pi_0/\mu_I + (1 - \Pi_0)/\mu_B. \quad (21)$$

From (17)–(21), we obtain the following equations:

$$P_j = \begin{cases} \frac{C_j}{1 + aC} & (j = 0, 1, \dots, B-1) \\ 1 - \frac{C}{1 + aC} & (j = B) \end{cases} \quad (22)$$

where  $aC$  is expressed as

$$aC = \frac{\lambda}{\mu_I} + \frac{\lambda}{\mu_B} \cdot (C - 1). \quad (23)$$

By setting  $j = 0$  in (22), the ratio of the number of the idle user stations to the number of all users is

$$P_0 = \frac{C_0}{1 + aC} = \frac{1}{1 + aC}. \quad (24)$$

### 3.3 Channel Part

In the channel part, we can regard the number of simultaneously transmitted packets as an  $M_1 + M_2/D/\infty/K$  queue. The steady state probability of an  $M_1 + M_2/D/\infty/K$  queue is equal to that of an  $M/M/\infty/K$  queue [16]. We analyze the packet success probability by using the analytical method of CDMA U-ALOHA system with fixed packet length and finite population assumptions [5].

Let  $P_0^*$  be the ratio of the number of the users which transmit the packet with rate  $\lambda$  to the number of all users. We define the initially on-transmitting user as the user which is on transmitting the packet for the first time when it was initially in an idle mode. The user station which transmits the packet with rate  $\lambda$  is both idle user station and initially on-transmitting user station. The ratio of the number of the idle user stations is  $P_0$ . We assume that the probability of having  $j$  packets at the user station is independent of time. Under this assumption, the probability of having more than one packet at user station when it was initially in an idle mode is  $(1 - P_0)P_0$ . The ratio of transmitting time for the first attempt to the time spent in transmitting the packet successfully is

$$\begin{aligned} b &= \frac{T_p}{T_p} Q_S + \frac{T_p}{T_p + (T_p + 1/p)} (1 - Q_S) Q_S + \dots \\ &\quad + \frac{T_p}{T_p + m(T_p + 1/p)} (1 - Q_S)^m Q_S + \dots \\ &= \sum_{m=0}^{\infty} \frac{T_p}{T_p + m(T_p + 1/p)} (1 - Q_S)^m Q_S. \end{aligned} \quad (25)$$

The ratio of the number of the initially on-transmitting user stations is  $(1 - P_0)P_0 \cdot b$ . Thus,  $P_0^*$  is derived as

$$P_0^* = P_0 + (1 - P_0)P_0 \cdot b. \quad (26)$$

The birth rate for actually transmitted packets  $\lambda_c$  is expressed as

$$\lambda_c = \lambda P_0^* + p(1 - P_0^*). \quad (27)$$

Let  $G_{sys}$  be the average number of packets transmitted to the channel. From the steady state probability of an M/M/ $\infty$ // $K$  queue [15], we obtain  $G_{sys}$  as

$$G_{sys} = \sum_{m=0}^K m \cdot \frac{(\lambda_c/\mu_c)^m \binom{K}{m}}{(1 + \lambda_c/\mu_c)^K} = \frac{G_c}{1 + G_c/K} \quad (28)$$

where  $\mu_c = 1/T_p$  and  $G_c = K \cdot \lambda_c \cdot T_p$ .

In the CDMA U-ALOHA system, the number of simultaneously transmitted packets may change from moment to moment. This makes the analysis complicated, so we assume that the number of simultaneously transmitted packets is constant over a short period  $\Delta t$ , where we set  $\Delta t$  a bit interval. Moreover, because of exponentially distributed birth interval, two or more packets hardly arrive simultaneously. We assume that the number of simultaneously transmitted packets changes by  $\pm 1$  at most between adjacent  $\Delta t$ 's.

Let  $P_S(k, i)$  be the probability that the packet is transmitted successfully from the first bit to the  $(i-1)$ th bit, and the number of interfering packets on the  $i$ th bit is  $k$ .

Case  $i = 1$ ;

By using the steady state probability of an M/M/ $\infty$ // $K$  queue, we obtain  $P_S(k, i)$  as,

$$P_S(k, i) = \begin{cases} \frac{(\lambda_c/\mu_c)^k \binom{K-1}{k}}{(1 + \lambda_c/\mu_c)^{K-1}} & ; \text{if } k < K \\ 0 & ; \text{otherwise} \end{cases} \quad (29)$$

Case  $i > 1$ ;

We obtain  $P_S(k, i)$  as the following.

(a)  $k < K - 1$

$$\begin{aligned} P_S(k, i) &= P_S(k, i-1) \\ &\cdot \{1 - k\mu_c\Delta t - (K-1-k)\lambda_c\Delta t\} \cdot \{1 - P_b(k)\} \\ &+ P_S(k+1, i-1) \cdot (k+1)\mu_c\Delta t \cdot \{1 - P_b(k+1)\} \\ &+ P_S(k-1, i-1) \cdot (K-k)\lambda_c\Delta t \cdot \{1 - P_b(k-1)\} \end{aligned} \quad (30a)$$

(b)  $k = K - 1$

$$\begin{aligned} P_S(k, i) &= P_S(k, i-1) \cdot \{1 - k\mu_c\Delta t\} \cdot \{1 - P_b(k)\} \\ &+ P_S(k-1, i-1) \cdot (K-k)\lambda_c\Delta t \cdot \{1 - P_b(k-1)\} \end{aligned} \quad (30b)$$

(c)  $k > K - 1$

$$P_S(k, i) = 0 \quad (30c)$$

By using  $P_S(k, i)$ , the throughput, which is defined as the mean number of successful packets in a packet time duration, is derived as,

$$S = G_{sys} \sum_{k=0}^{\infty} P_S(k, L) \cdot (1 - P_b(k)). \quad (31)$$

Accordingly, the packet success probability  $Q_S$  is obtained as,

$$Q_S = S/G_c. \quad (32)$$

### 3.4 Combination of the User Part and the Channel Part

We solve the simultaneous equations (24) and (32) derived in the user part and channel part, respectively. If the packet success probability  $Q_S$  is 0, the steady state probability of having no packets at a user station  $P_0$  will become 0 because all packets are not transmitted successfully and not removed from each user's buffer. If  $Q_S$  is 1,  $P_0$  will come close to 1 because all packets are transmitted successfully and not stored at each user's buffer. Accordingly, Eqs. (24) and (32) must have one or more solutions. If the equations have more than one solution, we use the solution with the smallest value of the throughput by the analytical tool called Equilibrium Point Analysis (EPA) [17]. In such case, the system exhibits the bistable behavior [4] and the performance curves change discontinuously.

We substitute (24) into (32), and solve the equation by the appropriate algorithm, such as the bisection. By using this solution, we can obtain the system performance. The throughput is derived as (31). Rejection probability  $Q_R$ , which is the probability that a new packet arriving at a user station is rejected because its buffer is full, is expressed as

$$Q_R = P_B = 1 - \frac{C}{1 + aC}. \quad (33)$$

By Little's formula, the average delay, which is the elapsed time from generating a packet at the user station to receiving it correctly at the hub station, is derived as

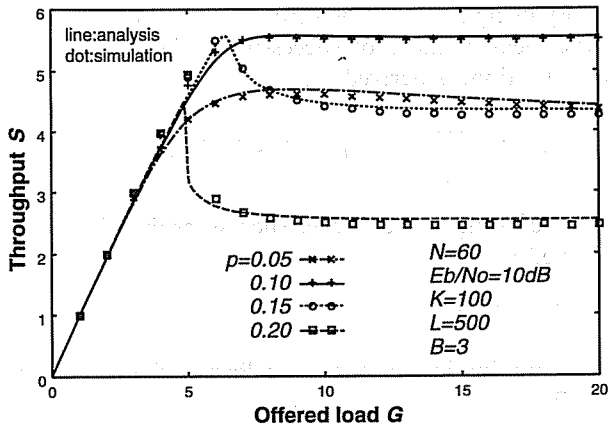
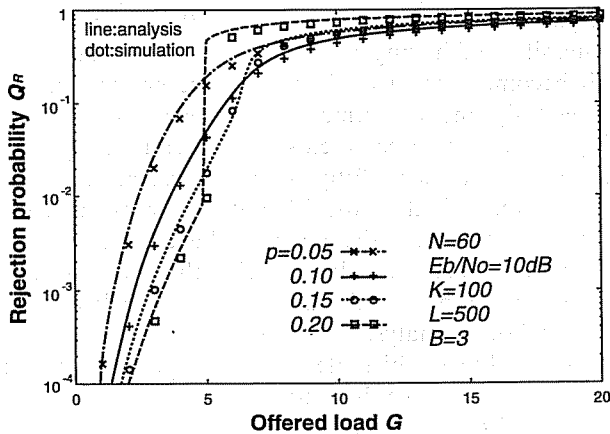
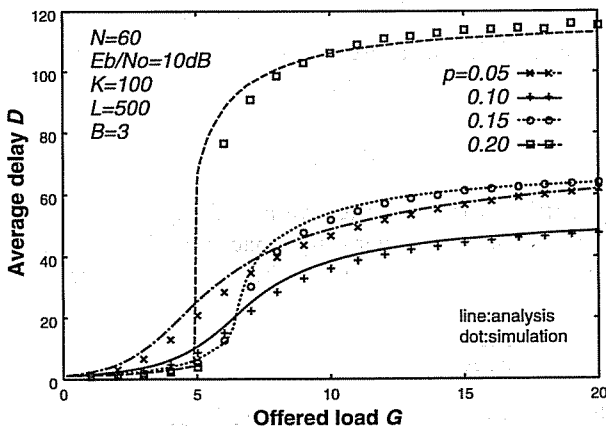
$$D = \frac{\bar{Q}}{S/K} \quad (34)$$

where  $\bar{Q}$  is the average queue length, derived as

$$\bar{Q} = \sum_{j=1}^B j \cdot P_j. \quad (35)$$

### 4. Numerical Examples

Figures 4–6 show the throughput, the rejection probability, and the average delay versus offered load with the parameters of  $N = 60$ ,  $E_b/N_0 = 10$  [dB],  $K = 100$ ,

Fig. 4 Throughput versus offered load curves for  $B = 3$ .Fig. 5 Rejection probability versus offered load curves for  $B = 3$ .Fig. 6 Average delay versus offered load curves for  $B = 3$ .

$L = 500$  [bits], and  $B = 3$ . Offered load  $G$  is defined as the average number of packets arriving in the system during one packet time duration, expressed as  $G = K \cdot \lambda \cdot T_p$ . The packet time duration  $T_p$  is normalized to 1. Simulated results are also plotted in these figures. We carry out the simulations to show the validity of our analysis. In the simulations, based on the

system model described in Sect. 2,  $K$  user stations generate the packets. Those are stored at their own buffers and transmitted to the channel. We can find that analytical results almost agree with simulated results.

For  $p \leq 0.1$ , the throughput, the rejection probability and the average delay gradually and monotonously increase. The larger the transmission rate  $p$  becomes, the better the performance is. But, for  $p > 0.1$ , the throughput curves are in the shape of a convex cap, and the rejection probability and the average delay are small for the region of small offered load and rapidly increased over the value of the offered load at which throughput takes the maximum value. These tend to be more remarkable for larger  $p$ . The reason is as follows. The throughput depends on the birth rate for actually transmitted packets  $\lambda_c$  in the channel part. The throughput curve as a function of  $\lambda_c$  is in the shape of a convex cap [5], i.e. there is the maximum value of the throughput  $S_{max}$ . In this condition,  $S_{max} = 5.6$  when  $\lambda_c = 0.096$ . From (27),  $\lambda_c$  varies from  $\lambda$  to  $p$  with  $P_0^*$  decreasing. We also expect that  $P_0^*$  will decrease from 1 to 0 with  $G$  increasing because large offered load brings about the degradation of the packet success probability. Thus,  $\lambda_c$  comes close to  $p$  with the offered load increasing. Within the region  $p \leq 0.096$ , packets are more rapidly processed for larger  $p$ , because  $\lambda_c$  will be less than 0.096 and the throughput will not be more than  $S_{max}$ . Within the region  $p > 0.096$ , however, because user stations attempt to transmit packets over the channel capacity, the number of retransmitted packets is increasing and system performance is rapidly degraded.

For the case of  $p = 0.20$  in Figs. 4–6, the performance caves change discontinuously at  $G \cong 5$  because of bistable behavior. The range of exhibiting the bistable behavior (i.e. having more than one solution) is  $5 < G < 6$ . The error of analytical results will be large within this range because the analysis does not consider the fluctuation between the solutions.

Let us see the throughput performance from the other point of view. Figure 7 shows the throughput as a function of packet transmission rate and offered load. We find that the packet transmission rate giving the maximum throughput depends on the offered load. This value is changing asymptotically from 1 to 0.096 by increasing the offered load.

Figures 8–10 show the throughput, the rejection probability and the average delay versus offered load with a buffer size of  $B = 1$ –5, and transmission rate  $p = 0.1$ . The larger buffer size a user station has, the more rapidly the throughput increases for the region of small offered load. For the case  $p = 0.1$ , if many user stations become in the busy mode,  $\lambda_c$  will come close to 0.1, and throughput will come near to the maximum value. The number of busy stations increases by increasing the buffer size, because each user station can keep more packets. We can, therefore, find the throughput curves as shown in Fig. 8. It can be seen from Fig. 9

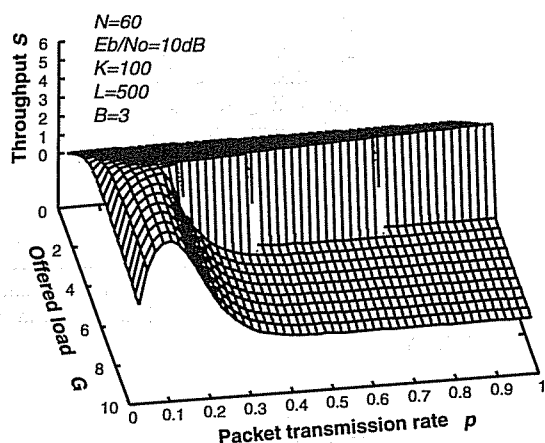


Fig. 7 Throughput performance as a function of packet transmission rate and offered load.

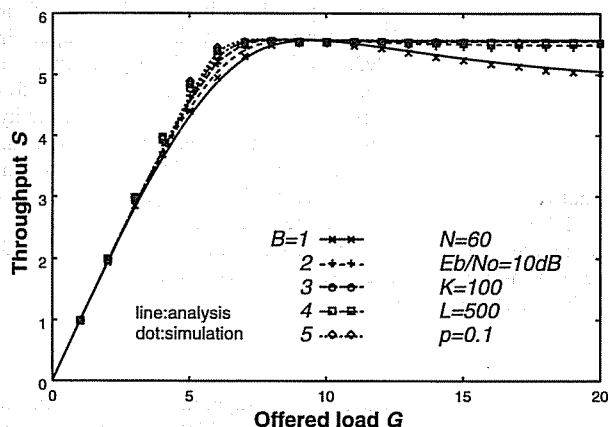


Fig. 8 Throughput versus offered load curves for  $p = 0.1$ .

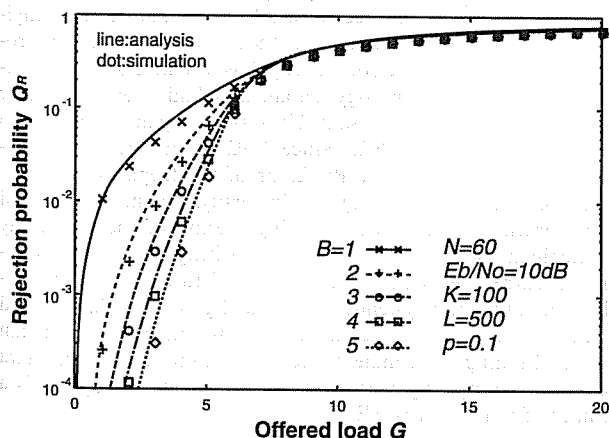


Fig. 9 Rejection probability versus offered load curves for  $p = 0.1$ .

that the rejection probability is improved by increasing the buffer size. From Fig. 10, large buffer size causes an increase of the average delay. It is, however, not serious because the average delay increases in compensation for reduction of the rejection probability of the packet transmission.

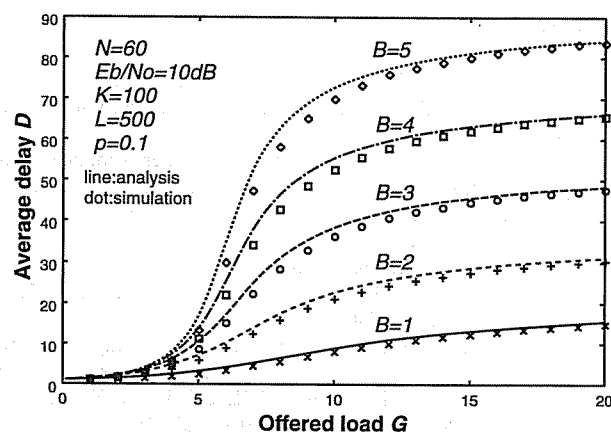


Fig. 10 Average delay versus offered load curves for  $p = 0.1$ .

## 5. Conclusions

CDMA U-ALOHA system with finite size of queueing buffers has been discussed. We introduced the analytical model in which the system was divided into two Markov chains and analyzed the system performance. Based on these results, we clarified the effect of buffer capacity in terms of the throughput, the average delay and the rejection probability. As compared with the simulated results, we found that analytical results almost agreed with simulated results.

As a result of the analysis, we found as the following. As the offered load becomes larger, the packet transmission rate at which the throughput takes the maximum value is asymptotically smaller. When we set the transmission rate so that the throughput takes the maximum value at large offered load, increase in the buffer size brings about an improvement in the grade of service in terms of higher throughput and lower rejection probability. Large buffer size causes an increase of the average delay. It is, however, not serious because the average delay increases in compensation for reduction of the rejection probability of the packet transmission.

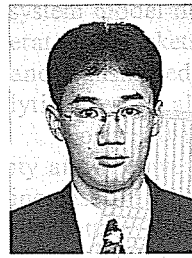
In this paper, the performance of the queueing system was analyzed by means of the Markov chain. This method is easy to implement because we can use the queueing theory that has ever been studied extensively [15], [16]. For example, the packetized voice and data traffic is modeled as the Markov modulated Poisson process (MMPP) [18]. In this case, we can model the user part as an MMPP/G/1 queue.

## Acknowledgment

This work was supported in part by Ministry of Education, Science and Culture under Grant-in-Aid for General Scientific Research, Uniden Corp., and Nippon Ericsson K.K.

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