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Generation of Sets of Sequences Suitable for Multicode Transmission in Quasi-Synchronous CDMA Systems

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SUMMARY In this letter, we present a method to generate sets of sequences suitable for multicode transmission in quasi-synchronous (QS) CDMA systems. We focus on Gold code but extension to orthogonal Gold code is straightforward. We show that by appropriate classification of sequences, it is possible to have sets whose cross correlation is small in QS situations.

key words: quasi-synchronous CDMA, Gold code, orthogonal Gold code, multicode transmission

1. Introduction

In synchronous CDMA systems, users try to transmit their signals synchronously at the aimed timing at which users should synchronize. Unfortunately, because of some disturbances, the base station receives those signals not at perfect synchronous timing but at quasi-synchronous (QS) timing. Such CDMA systems are referred to QS-CDMA systems or approximately synchronized CDMA systems [1], [2]. It is known that some spreading sequences have small cross correlation in those QS situation [1]–[5]. By using those sequences, multiple signals can be received with small amount of interference at the base station.

Let us extend the above notion to the case of multicode transmission. Suppose user 1 transmits his data using sequences $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Because of some disturbances, the base station receives those signals at QS timing (τ_1). Ordinarily, those sequences are orthogonal to each other. For user 2, he also transmits his data using two orthogonal sequences of $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ but they arrive at the base station at QS timing (τ_2). The situation is shown in Fig. 1 for two users in QS situation. Then, the question is whether we can make the set of sequences, $\{\boldsymbol{a}_1, \boldsymbol{a}_2\}$ and $\{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ for example, to have small or no cross correlation to each other in QS situation. In other words, between $\boldsymbol{a}_1$ and $\boldsymbol{b}_1$, as well as $\boldsymbol{a}_1$ and $\boldsymbol{b}_2$, $\boldsymbol{a}_2$ and $\boldsymbol{b}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{b}_2$, the cross correlations should be designed to be small or zero to each other in QS situation.

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In this study, we show that by appropriate classification of codes and sequences, it is possible to have sets, whose cross correlations are small in QS situations. Such classification is discussed for the case of Gold sequences, but extension to orthogonal Gold sequences is straightforward.

In Sects. 2 and 3, we present definitions and classification of Gold codes and Gold sequences are discussed. Gold code can be classified into two, one is suitable for a set of QS-sequences and the other is not. We further classified sequences in a desirable code into three groups. Some of those classified sequences can be used for multicode transmission. Extension to orthogonal Gold code is presented in Sect. 4, and concluding remark is in Sect. 5.

2. Definitions

To investigate the cross correlation property of Gold sequences in QS-CDMA, we define the following terms:

2.1 Sequences and Cross Correlation Function

In this study, we consider $\{+1, -1\}$ -valued binary sequences of period N given as

$$\boldsymbol{a}_i = \{a_{i,1}\ a_{i,2}\ \ldots\ a_{i,N}\} \tag{1}$$

for i-th sequence. And each element of a sequence $a_{i,j}$ is named chip. The discrete time periodic cross correlation function between sequences $\boldsymbol{a}_i$ and $\boldsymbol{a}_j$ is defined as [6],

$$R_{i,j}(\tau) = \sum_{n=1}^{N} a_{i,n} a_{j,(n+\tau-1) \bmod N + 1} \tag{2}$$

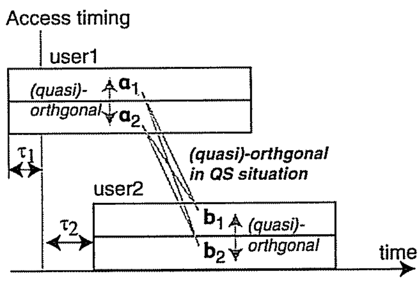

Fig. 1 Multicode transmission in QS-situation.

2.2 Gold Code

Gold code is generated by two m-sequences which are of a preferred-pair [6], [7]. We use the octal representation of shift-register polynomial to identify each m-sequence [7], [8]. Without loss of generality, we set the initial state of each sequence as $\{+1, +1, \ldots, +1, -1\}$.

Let $\mathbf{u}$ and $\mathbf{v}$ be m-sequences of a preferred-pair. The Gold code is generated as

$$\begin{aligned} G(\mathbf{u}, \mathbf{v}) &= \{\mathbf{u}, \mathbf{v}, \mathbf{u} \otimes \mathbf{v}, \mathbf{u} \otimes T\mathbf{v}, \ldots, \mathbf{u} \otimes T^{N-1}\mathbf{v}\} \\ &= \{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \ldots, \boldsymbol{a}_{N+2}\} \end{aligned} \quad (3)$$

and $\otimes$ is defined as:

$$\boldsymbol{a}_i \otimes \boldsymbol{a}_j = \{a_{i,1}a_{j,1} \;\; a_{i,2}a_{j,2} \;\; \ldots \;\; a_{i,N}a_{j,N}\}.$$

We denote T^s as an shift operator which cyclically shifts a sequence by s places, as follows,

$$\mathbf{v} = \{v_1 \; v_2 \; \ldots \; v_N\} \quad (4)$$
$$T^s\mathbf{v} = \{v_{s+1} \; v_{s+2} \; \ldots \; v_N \; v_1 \; \ldots \; v_s\} \quad (5)$$

From (3), the number of sequences in a Gold code is $N+2$. In this work, we denote the Gold code as G(45, 47), if m-sequences $\mathbf{u}$, $\mathbf{v}$ are generated by the shift-registers whose polynomials are 45 and 47.

Orthogonal Gold (OG) code is generated by adding one chip to a common phase of Gold sequences. When sequences in a Gold code is given as

$$\boldsymbol{a}_i = \{a_{i,1} \; a_{i,2} \; \ldots \; a_{i,N}\} \quad i = 1, 2, \ldots, N+2 \quad (6)$$

the OG code $\mathbf{og}$ that chip b is added between p-th chip and $p+1$-st chip of that Gold code, is shown as

$$\begin{aligned} \mathbf{og}_1 &= \{a_{1,1} \ldots a_{1,p} \; b \; a_{1,p+1} \ldots a_{1,N}\} & (7) \\ \mathbf{og}_2 &= \{a_{3,1} \ldots a_{3,p} \; b \; a_{3,p+1} \ldots a_{3,N}\} & (8) \\ &\vdots \\ \mathbf{og}_{N+1} &= \{a_{N+2,1} \ldots a_{N+2,p} \; b \; a_{N+2,p+1} \\ &\quad \ldots a_{N+2,N}\} & (9) \end{aligned}$$

where $b \in \{+1, -1\}$. Because the cross correlations between $\boldsymbol{a}_2$ and the other Gold sequences are not always -1 in $\tau = 0$, $\boldsymbol{a}_2$ is not used to generate OG code.

Each OG code is uniquely determined by the m-sequences which generate based Gold code, the phase p, and the chip b, therefore, we show the code (7)–(9) as OG($\mathbf{u}$, $\mathbf{v}$, p, b) if the Gold code (6) is generated by $\mathbf{u}$ and $\mathbf{v}$. An OG code contains $N+1$ sequences of period $N+1$†.

2.3 QS-CDMA

To denote the range of timing difference between any two sequences, in another expression, the degree of QS condition, we set the definition about QS condition. We define the r-QS conditions as the (discrete) timing difference between two sequences that succession of r chips around 0 (perfectly synchronous), as an index of degree of QS condition. For example, if $r = 3$, then r-QS condition means that the timings of succession of 3 chips, that is, the range $\tau = -1, 0, +1$.

2.4 Quasi-Orthogonal and Orthogonal Relation on QS-Conditions

According to the cross correlation property of two sequences, we define the following relation for a given range r of QS condition.

- Quasi-Orthogonal on r-QS condition 'QOQS(r)':

$$R_{i,j}(\tau) = -1 \;\text{ for }\; \tau = 0, \pm 1, \ldots, \pm\frac{r-1}{2} \quad (10)$$

- Orthogonal on r-QS condition 'OQS(r)':

$$R_{i,j}(\tau) = 0 \;\text{ for }\; \tau = 0, \pm 1, \ldots, \pm\frac{r-1}{2} \quad (11)$$

3. Classification of Gold Codes and Gold Sequences

3.1 Classification of Gold Codes

As described before, Gold code can be generated by two of preferred-pair m-sequences. Unfortunately, not all the combinations of m-sequences will generate the Gold code that a set of sequences in that code satisfying QOQS(r) relation for given r. In this work, we refer to the set of Gold codes which can generate sets of sequences having QOQS(r) relation as Class I codes. Complementary sets of Class I are referred to as Class II, but we only consider the sets belonging to Class I.

As an example, we show the classification of Gold codes for $N = 31$ and $r = 3$. There are 12 Gold codes identified by a combination of m-sequences. We examined all the pairs of sequences in each Gold code whether those sequences have QOQS(3) relation or not. The investigation results are as the following.

Class I : G(45, 47), G(45, 73), G(47, 51), G(47, 67)
G(51, 67), G(51, 75), G(67, 75)
Class II : G(45, 67), G(45, 75), G(47, 73), G(51, 73)
G(73,75)

3.2 Classification of Gold Sequences in a Class I Gold Code

Let us consider a sequence $\boldsymbol{a}_i$ belonging to a Class I

†In this study, we consider Gold codes of period $N = 2^m - 1$ for odd m, and generated OG codes based on those Gold codes, for avoiding the exceptions in following discussions.

Table 1 QOQS(3) relations of G(13,15).

a_i	A_i		group
1	7		(ii)
2			(iii)
3	9		(ii)
4	5	8	(i)
5	4	6	(i)
6	5	8	(i)
7	1		(ii)
8	4	6	(i)
9	3		(ii)

Gold code. There are some sequences $\boldsymbol{a}_j$ $(j \neq i)$ satisfying the relation of QOQS(r) for a given r. Let A_i be the set of sequences having QOQS(r) relation to the sequence $\boldsymbol{a}_i$. We note that A_i does not include $\boldsymbol{a}_i$. For the different sequence $\boldsymbol{a}_j$, we then have different set A_j having QOQS(r) relation to $\boldsymbol{a}_j$. These two sets, A_i and A_j can be classified as the following groups:

(i) For a given set A_i, there exists a set A_j $(j \neq i)$ whose elements (sequences) are the same as A_i.

(ii) For a given set A_i, there exists a set A_k $(k \neq i)$, whose elements are the same except for $\boldsymbol{a}_i$.

(iii) The size of set A_i is 0.

As an example, the QOQS(3) relation of G(13, 15) of period $N = 7$ is shown in Table 1. In the table, only sequence number i is shown for easy-to-read purpose, instead of showing as $\boldsymbol{a}_i$. According to the definitions, we can classify those 9 Gold sequences as, (i) $A_4 = A_6 = \{\boldsymbol{a}_4, \boldsymbol{a}_6\}$, $A_5 = A_8 = \{\boldsymbol{a}_5, \boldsymbol{a}_8\}$, and (ii) $A_1 = \{\boldsymbol{a}_7\}$ and $A_7 = \{\boldsymbol{a}_1\}$, $A_3 = \{\boldsymbol{a}_9\}$ and $A_9 = \{\boldsymbol{a}_3\}$, and (iii) $A_2 = \{\emptyset\}$. Since A_2 is null, we do not use $\boldsymbol{a}_2$ in this case.

3.3 Generation of Sets of Quasi-Synchronous Multicode

In the above example, we divide groups (i) and (ii) into further two sets of sequences, based on the set of QOQS(3) relation. From the Table 1, we confirm that $A_4 = A_6$ and $A_5 = A_8$, but $A_4, A_6 \neq A_5, A_8$. The relation follows that the set of $\{\boldsymbol{a}_4, \boldsymbol{a}_6\}$ are QOQS(3) to another set $\{\boldsymbol{a}_5, \boldsymbol{a}_8\}$. But $\boldsymbol{a}_4$ and $\boldsymbol{a}_6$ do not have QOQS(3) relation each other. Since $\boldsymbol{a}_4$ and $\boldsymbol{a}_6$ are quasi-orthogonal each other, it is better to use them as orthogonalization code for multicode transmission. $\boldsymbol{a}_5$ and $\boldsymbol{a}_8$ hold the same relation.

In this case, a user may transmit his data by using both $\boldsymbol{a}_4$ and $\boldsymbol{a}_6$ on 3-QS conditions. The other user uses both $\boldsymbol{a}_5$ and $\boldsymbol{a}_8$. Then each set of sequences would cause quite small amount of interference to each other. Because of the property of the sequences, those sets would be suitable for multicode transmission.

We define a pair of sets of sequences having the property above mentioned as a pair of Quasi-Synchronous MultiCode (QSMC) sets, and we denote each set of QSMC pair as QSMC set, and the sequences in the sets as QSMC sequences.

Table 2 QSMC and QS-sequence sets of G(45,73) on $r = 3, 5$ for $N = 31$. Sequence number i of $\boldsymbol{a}_i$ is shown.

$r=3$				$r=5$			
QSMC		QS-seq.		QSMC		QS-seq.	
A_8	A_5	Q_1	Q_4	A_{11}	A_9	Q_1	Q_4
3	8	1	4	9	11	1	4
9	11	12	5	20	22	12	5
13	16	17	6	23	24	17	10
20	22	18	7	32	26	19	15
23	24	19	10				
25	26	27	14				
28	29	30	15				
32	33	31	21				

On the other hand, for group (ii) sequences of G(13, 15), it seems that $\boldsymbol{a}_1$ and $\boldsymbol{a}_7$ have QOQS(3) relation each other, and $\boldsymbol{a}_3$ and $\boldsymbol{a}_9$ have the same relation. The relation is exactly what conventional QS-sequences have [3]. That is, a user is assigned $\boldsymbol{a}_1$ and another user is assigned $\boldsymbol{a}_7$, and they transmit those on 3-QS condition, the interference to the other sequence can be kept small by the QOQS(3) relation.

We define the set that any two sequences included in it having the QOQS(r) relation as QS-sequence set, and the sequences in a set as QS-sequences. In this case, there exists two QS-sequence sets $\{\boldsymbol{a}_1, \boldsymbol{a}_7\}$ and $\{\boldsymbol{a}_3, \boldsymbol{a}_9\}$, and the sizes of both sets are two.

Finally, we summarize the generation of QSMC and QS-sequence sets of this subsection, as follows,

1 The classification of sequences into group (i) or (ii)
2 The classification of sequences in each group into subgroups based on their QOQS(r) relations

As concrete examples of generation of QSMC and QS-sequence sets, the list of QSMC and QS-sequence sets is shown in Table 2 for $r = 3, 5$ of G(45, 73) for $N = 31$. In the table, sequence number i of $\boldsymbol{a}_i$ is shown. A_8 and A_5 are QSMC pairs for $r = 3$, while Q_1 and Q_4 are two sets of QS-sequences generated from Gold code. Here we denote Q_i, say Q_1 for example, as the set of QS-sequence $Q_1 = \{\boldsymbol{a}_1 \bigcup A_1\} = \{\boldsymbol{a}_{12} \bigcup A_{12}\} = \ldots = \{\boldsymbol{a}_{31} \bigcup A_{31}\}$, where the subscript of Q_i represents the smallest subscript of $\boldsymbol{a}_i$. For $r = 5$, A_{11} and A_9 make a pair of QSMC sets.

According to the table, a user can use the set A_8 for $r = 3$. For a single-code transmission he may choose a sequence from A_8. Since cross correlations of all 8 sequences are -1, the lowest of Gold code, he may choose at most 8 sequences for multicode transmission. The other user uses the set A_5. Since A_8 and A_5 are QSMC pair, those two users enjoy (multicode) transmission of their signals with low cross correlation (-1) even though the signals may arrive not at perfect timing, but QS timing (in this case $r = 3$). Since we only have one pair of QSMC for this case, total number of simultaneous users is two.

Table 3 QSMC and QS-sequence sets of G(203, 277) on $r = 3, 5$ for $N = 127$. Sequence number i of a_i is shown.

$r=3$				$r=5$				
QSMC		QS-seq.		QSMC				QS-seq.
A_4	A_3	Q_1	Q_9	A_{27}	A_3	A_{36}	A_9	Q_1
3	4	1	9	3	27	9	36	1
8	10	5	19	34	42	22	46	7
16	17	6	22	44	53	45	60	12
18	20	7	35	52	96	76	92	13
21	23	11	36	66	98	85	93	31
26	27	12	39	79	101	91	115	33
28	29	13	45	112	103	113	116	69
34	37	14	46	127	128	119	123	111
38	40	15	47	A_{28}	A_4	A_{18}	A_{10}	Q_{19}
41	42	24	56	4	28	10	18	19
44	48	25	59	17	41	20	21	35
52	53	30	60	29	58	23	38	39
55	57	31	71	63	62	37	73	56
58	61	32	76	67	90	48	75	59
62	63	33	80	74	97	57	95	71
66	67	43	81	89	106	109	102	80
70	72	49	85	105	118	117	104	107
73	74	50	91	A_6	A_5			Q_{24}
75	77	51	92	5	6			24
79	82	54	93	11	14			64
84	86	64	107	30	15			68
88	89	65	108	43	25			87
90	94	68	113	49	32			99
95	96	69	114	54	50			110
97	98	78	115	78	51			126
100	101	83	116	83	65			129
102	103	87	119	A_{40}	A_8			Q_{47}
104	105	99	120	8	40			47
106	109	110	121	16	61			81
112	117	111	122	26	72			108
118	125	126	123	55	77			114
127	128	129	124	70	82			120
				84	86			121
				88	94			122
				100	125			124

If all the users transmit their signals in a way of single-code transmission, then either sets of Q_1 or Q_4 can be used as CDMA code to distinguish each of users. In this case total number of simultaneous users are 8.

Table 3 shows QSMC and QS-sequence sets for $r = 3, 5$ of G(203, 277) for $N = 127$. For $r = 3$, A_4 and A_3 make a pair of QSMC sets. For $r = 5$, we have 6 QSMC pairs, $\{A_{27}, A_3\}$, $\{A_{28}, A_4\}$, $\{A_6, A_5\}$, $\{A_{40}, A_8\}$, $\{A_{36}, A_9\}$, $\{A_{18}, A_{10}\}$.

3.4 The Sizes of QSMC Set and QS-Sequence Set

We investigated the number of pairs of QSMC sets and the size of each set for $N = 7, 31, 127, 511$. We also investigated those of QS-sequence sets. Table 4 shows the sizes of QSMC sets and QS-sequence sets and the number of sequences in those sets for $3 \leq r \leq 9$.

When $r = 3$, $N+1$ sequences of a Gold code are classified into group (i) containing $\frac{N+1}{2}$ sequences and group (ii) containing $\frac{N+1}{2}$ sequences, and group (iii) containing one remainder sequence ($\boldsymbol{a}_2$ without exception). $\frac{N+1}{2}$ sequences of group (i), one pair of QSMC sets whose sizes are $\frac{N+1}{4}$, can be generated. And $\frac{N+1}{2}$ sequences of group (ii), two QS-sequence sets whose sizes are $\frac{N+1}{4}$, can be generated.

Table 4 The number of QSMC set pairs and QS-sequences sets and their sizes.

r	N	QSMC		QS-seq.	
		# of pairs	size	# of sets	size
3	7	1	2	2	2
	31	1	8	2	8
	127	1	32	2	32
	511	1	128	2	128
5	31	1	4	2	4
		6	2	4	2
	127	6	8	4	8
	511	6	32	4	32
7	127	6	4	4	4
		56	2	8	2
	511	28	8	8	8
9	127	1	4	2	4
		6	2	2	4
	511	28	4	8	4

Table 5 Orthogonal Gold codes which construct large set of QSMC and QS-sequences on $r = 3$ for $N = 32$.

OG(45, 47, 1, 18)	OG(45, 47, −1, 18)
OG(45, 73, 1, 18)	OG(45, 73, −1, 18)
OG(47, 51, 1, 12)	OG(47, 51, −1, 12)
OG(47, 67, 1, 12)	OG(47, 67, −1, 12)
OG(51, 67, 1, 17)	OG(51, 67, −1, 17)
OG(51, 75, 1, 17)	OG(51, 75, −1, 17)
OG(67, 75, 1, 27)	OG(67, 75, −1, 27)

From the table, we observe that an increase of r with 2, the sizes of QSMC and QS-sequence sets decrease to 1/4 as a global tendency for most of the case. However, for large value of range r and small sizes of sets, exceptions appear and the tendency becomes more complex.

4. Extension to Orthogonal Gold Code

The sets of sequences with zero cross correlation are interested in practical multicode CDMA systems. Because discussions about QOQS(r) relation for Gold codes can be replaced to OQS(r) relation for OG codes, we can generate the both QSMC and QS-sequence sets with orthogonal on QS conditions.

However, not all OG codes produce large sets of QSMC and QS-sequences. We have searched phases of adding the vector **1** or −**1**, by which the sizes of QSMC and QS-sequence sets is maximized. Table 5 shows the searched results of OG codes which contain 1 pair of QSMC sets with 8 sequences and 2 QS-sequence sets, in the case of period $N = 32, r = 3$. And an example of generation of the sets by OG(45, 47, 1, 18) on $r = 3$ is listed in Table 6.

5. Conclusions

We have investigated the cross correlation property of

Table 6 QSMC and QS-sequence sets of OG(45, 47, 1, 18) on $r = 3$ for $N = 32$. Sequence number i of $\mathbf{og}_i$ is shown.

QSMC		QS-seq.	
A_3	A_2	Q_1	Q_4
2	3	1	4
8	10	9	5
11	13	12	6
15	19	16	7
20	21	17	14
23	24	18	22
25	26	28	31
27	30	29	32

Gold sequences in QS conditions, and based on the results, we have introduced classification methods of Gold codes and Gold sequences and the generation methods of a pair of QSMC sets and QS-sequence sets. Some OG codes also have the same property of Gold code in QS conditions, so we can generate more practical set of sequences which are orthogonal even in the case of multicode transmission.

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