New Analytical Model for the TCP Throughput in Wireless Environment

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Abstract

In this paper, we propose a new analytical model of different versions of the TCP, viz., Tahoe, Reno and New Reno, where TCP mechanisms such as slow start, fast retransmit, fast recovery and timeout are modeled as a Markov chain. In our proposed model, we consider the exponential increase of the congestion window and the exponential increase of the timeout back-off. Finally, we focus on the bulk throughput performance analytically, and compare it of different versions of the TCP in the presence of random bit errors on a wireless link.

1 Introduction

With the advance of wireless communication technologies, the need to access to the Internet via wireless networks is expected to increase more and more in the future. In the Internet, the Transmission Control Protocol (TCP) is widely used to support applications such as telnet, ftp and http.

The TCP has been designed for wired networks, in which main segment losses are caused by congestion, and link error rate is very low. On the other hand, main segment losses are caused by bit errors in wireless links because wireless links have much more bit errors than wired ones. Therefore, segment losses in wireless networks occur more frequently and randomly than those in wired ones. In the case of many segment losses, performance of the TCP is very poor. To study the performance of the TCP in wireless networks, several simulations and experiments have been performed, and several modifications have been proposed to improve it in wireless environments [1].

Recently, to study the basic behavior of the TCP, some analytical models have been proposed in wired networks [3] or wireless networks [2, 4]. In analytical studies, researchers have modeled the congestion window which controls the transmission rate of the TCP as a Markov chain. To analyze the TCP's behavior, some researchers assume that the congestion window control is simplified to the linear increase from exponential increase, and use an average number of transmission segments and an average value of the transmission period [3]. While other researchers consider the exponential increase of the congestion window, but sim-plify timeout back-off mechanisms, and use a steady state distribution of the congestion window during the transmission period between occurrences of the segment loss [2]. However, the size of a congestion window which controls the number of transmitted segments changes dynamically at each Round Trip Time (RTT), and the transmission period between segment losses is various lengths because of the timeout or the fast retransmit. In particular, the change of the congestion window size might occur frequently due to bit errors in a wireless link. The use of the average value is not enough to express these dynamic changes exactly. So, we should consider the change of the congestion window size every RTT, and model the exponential increase of the congestion window, the fast retransmit, the fast recovery and the timeout back-off.

In this paper, we propose a new analytical model which includes basic TCP mechanisms, and consider the different versions of the TCP. In our analysis, the exponential increase of the congestion window, fast retransmit mechanisms, fast recovery mechanisms and the exponential increase of the timeout back-off are considered by representing a state transition diagram of the TCP every RTT as a Markov chain. And we obtain the throughput of a bulk transfer TCP flow (i.e., a large amount of data to be sent, such as FTP).

2 Analytical Approach

In this section, we propose an analytical model of each TCP versions. We assume that a segment is lost with probability p, and losses are independent like as [2] because we consider only the effect of errors in wireless links. This assumption allows us to model mechanisms of the TCP as a Markov chain. Furthermore, we assume that Acknowledgment (ACK) segments never lost, because they are relatively smaller in size than data segments (40 bytes versus $500 - 1500$ bytes).

It is important to note that these simplifications do not spoil characteristics of TCP mechanisms.

2.1 State Transition Diagrams

In this section, we express the fluctuation of TCP mechanisms in the state transition diagram in order to model the TCP behavior as a Markov chain.

In order to make descriptions simply, we use the simple model of the TCP where the congestion window control is simplified to the linear increase and the multiple increase of the timeout duration is limited to 4 times.

In this section, we describe the state transition diagram of the TCP Reno. But it is easy for the TCP Tahoe and the TCP New Reno to describe them same as the TCP Reno. Figure 1 shows the state transition diagram of the TCP Reno with the maximum advertised window size equals to W segments. This diagram is divided into 3 parts according to its status such as a transmission, a fast retransmit, and a timeout.

First, states R_n $(1 \leq n \leq W)$ are transmission status. In these states, the congestion window size equals n, and the TCP transmitter transmits n segments. The initial state of the state transition diagram is the state R_1. If the transmission of the segment succeeds, the congestion window size will increase by transiting from the state R_1 to the state R_2, from R_2 to R_3, $\cdots$, and it transits to R_W finally.

Second, states R_n $(W + 1 \leq n \leq W + F)$ are fast retransmit fast recovery status with one segment loss, where F is the number of fast retransmit with one segment loss status and may be expressed as

$$F = \lceil (W - 3)/2 \rceil. \tag{1}$$

At the state R_W, if one segment is lost and the TCP transmitter receives more than 3 duplicate ACKs, a fast retransmit with single segment loss will occur by transiting from the state R_W to the state R_{W+F}, and he retransmits the lost segment. If the retransmitted segment is transmitted successfully, he will start transmitting segments with the congestion window size equals to $\lfloor W/2 \rfloor$ by transiting from the state R_{W+F} to the state $R_{\lfloor W/2 \rfloor}$. Otherwise, a timeout will occur by transiting from the state R_{W+F} to the state R_{W+F+2}.

Third, states between R_n $(W + F + D(1) - 1 \leq W + F + D(3))$ are timeout status and the state R_{W+F+1} is fast retransmit status with multiple segment losses which leads the timeout procedure, where $D(i)$ is the number of $i - th$ timeout status and may be expressed as

$$D(i) = \sum_{j=1}^{i} (2^{j-1} + 1) + 1. \tag{2}$$

If some segments are lost and the TCP transmitter receives less than 3 duplicate ACKs, a timeout will occur by transiting to the state $R_{W+F+D(1)-1}$. At the state $R_{W+F+D(1)}$, he retransmits the lost segment. If the retransmitted segment is transmitted successfully, he start transmitting segments with the minimum congestion window size by transiting to the state R_2. Otherwise, he waits for the twice period by transiting from the state $R_{W+F+D(1)}$ to the state $R_{W+F+D(2)-2}$, and retransmits the lost segment again at the state $R_{W+F+D(2)}$. If some segments are lost and the TCP transmitter receives more than 3 duplicate ACKs, a fast retransmit with multiple segment losses will occur by transiting to the state R_{W+F+1}, and retransmits the first lost segment. But he may not receive 3 duplicate ACKs for the second lost segment, a timeout will occur by transiting from the state R_{W+F+1} to the state $R_{W+F+D(1)-1}$.
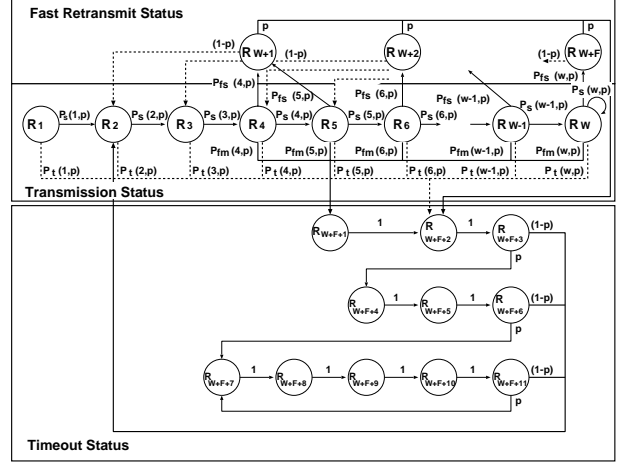

Figure 1: State transition diagram for the TCP Reno.

2.2 Transition Probability Matrix

Transition probabilities are derived from the probability that transmission of a segment has succeeded $P_s(n, p)$, the probability that a fast retransmit with a single loss occurs $P_{fs}(n, p)$(for the TCP Tahoe and the TCP Reno), the probability that a fast retransmit with multiple losses occurs $P_{fm}(n, p)$(for the TCP Tahoe and the TCP Reno), the probability that a fast retransmit occurs $P_f(n, p)$(for the TCP New Reno), and the probability that a timeout occurs $P_t(n, p)$, where n is the congestion window size.

Let $\mathbf{M}$ be the transition probability matrix of state transition diagrams, and the element $m_{i,j}$ be the transition probability from the state i to the state j. Elements $m_{i,j}$ may be expressed as the following;

(1) Transmission status $(1 \leq i \leq W)$

For $1 \leq i \leq 3$, only a successful transmission or a timeout occurs, so the transition probability for the successful transmission is,

$m_{i,i+1} = P_s(i, p)$

and the transition probability for the timeout is,

$m_{i,W+F+1} = P_t(i, p)$.

For $4 \leq i \leq W - 1$, a successful transmission, a timeout or a fast retransmit occurs, so the transition probability for the successful transmission is,

$m_{i,i+1} = P_s(i, p)$,

the transition probability for the timeout is,

$m_{i,W+F+1} = P_t(i, p)$

and the transition probability for the fast retransmit with a single loss or multiple losses are,

$m_{i,W+\lfloor (i-2)/2 \rfloor} = P_{fs}(i, p)$

$m_{i,W+F+1} = P_{fm}(i, p)$.

For $i = W$, a successful transmission, a timeout or a fast retransmit occurs, however, the transition probability for the successful transmission is different from that for $4 \leq i \leq W - 1$. so the transition probability for the successful transmission is,

$$m_{i,i} = P_s(i,p),$$
the transition probability for the timeout is,
$$m_{i,W+F+1} = P_t(i,p)$$
and the transition probability for the fast retransmit with a single loss or multiple losses are,
$$m_{i,W+\lfloor(i-2)/2\rfloor} = P_{fs}(i,p)$$
$$m_{i,W+F+1} = P_{fm}(i,p).$$

(2) Fast retransmit status ($W + 1 \leq i \leq W + F + 1$)

For $W + 1 \leq i \leq W + F$, the fast retransmit with a single segment loss occurs. If the retransmitted segment is transmitted successfully, the TCP transmitter starts transmitting segments, so the transition probability for the successful retransmission is,
$$m_{i,i-W+1} = 1 - p$$
and if the retransmitted segment is lost, the timeout occurs. So the transition probability for the failure retransmission is,

$$m_{i,W+F+2} = p.$$

For $i = W + F + 1$, fast retransmit with multiple segment losses occurs. The TCP Tahoe and the Reno does not have a solution for multiple segment losses, the timeout occurs after the fast retransmit with multiple segment losses. So the transition probability for the fast retransmit with multiple segment losses is,
$$m_{W+F+1,W+F+2} = 1.$$

(3) Timeout status ($W+F+D(1)-1 \leq i \leq W+F+D(3)$)

For $i = W + F + D(1) - 1$, the waiting for the first timeout occurs, so the transition probability of the waiting for the first timeout is,
$$m_{i,i+1} = 1$$
and for $i = W + F + D(1)$, the first timeout occurs and the TCP transmitter retransmits the lost segment, so the transition probability of the first timeout is,
$$m_{i,2} = 1 - p$$
$$m_{i,i+1} = p.$$

For $i = W + F + D(2) - 2, W + F + D(2) - 1$, the waiting for the second timeout occurs, so the transition probability of the waiting for the second timeout is,
$$m_{i,i+1} = 1$$
and for $i = W + F + D(2)$, the second timeout occurs and the TCP transmitter retransmits the lost segment, so the transition probability of the second timeout is,
$$m_{i,2} = 1 - p$$
$$m_{i,i+1} = p.$$

For $W + F + D(3) - 4 \leq i \leq W + F + D(3) - 1$, the waiting for the third timeout occurs, so the transition probability of the waiting for the third timeout is,
$$m_{i,i+1} = 1$$
and for $i = W + F + D(3)$, the third timeout occurs and the TCP transmitter retransmits the lost segment, so the transition probability of the third timeout is,
$$m_{i,2} = 1 - p$$
$$m_{i,i-4} = p.$$

We show the state transition matrix of the TCP Reno. While, the TCP Tahoe does not support the fast recovery mechanism, so elements $m_{i,j}$ ($W + 1 \leq i \leq W + F, 1 \leq j \leq W + F + D(3)$) aggregate to the element $m_{i,j}$ ($i = W+1, 1 \leq j \leq W + F + D(3)$). In the TCP New Reno, the element $m_{i,j}$ ($i = W + F + 1, 1 \leq j \leq W + F + D(3)$) for the fast retransmit with multiple segment losses is deleted, because the TCP New Reno has a solution for this situation.

2.3 Transition Probability

Probability of the Successful Transmission

When the transmitter receives n acknowledgments (ACKs) for n transmitted segments, he can increase the congestion window size. Under the assumption that an ACK is never lost, the transition probability $P_s(n,p)$ obtains from the probability that transmission of all segments succeeds. Hence $P_s(n,p)$ is derived as
$$P_s(n,p) = (1 - p)^n. \tag{3}$$

Probability of the Fast Retransmit with a single loss

When one segment is lost, the ACK may be duplicate. The condition of the fast retransmit occurrence is that the TCP transmitter receives more than three duplicate ACKs. Since the maximum number of duplicate ACKs is two in the case of the advertised window size $n \leq 3$, the fast retransmit never occur in the case of $n \leq 3$. In the case of $3 < n$, when the TCP transmitter receives more than three duplicate ACKs, a fast retransmit occurs. Then $P_{fs}(n,p)$ is derived as

$$P_{fs}(n,p) = \sum_{i=1}^{n}(1-p)^{i-1}p(1-p)^{n-i}$$
$$\times \sum_{\substack{j=0;(n-i)\leq 3 \\ j=(3-w+i);3<(n-i)}}^{i-1} \binom{i-1}{j} p^{(i-1-j)}(1-p)^j \quad (3 < n). \tag{4}$$

Probability of the Fast Retransmit with multiple losses

When some segments are lost, the ACK may be duplicate same as a single segment loss. However the second lost segment is not retransmitted by a fast retransmission, but is retransmitted by a timeout. It is because the TCP transmitter may not receive three duplicate ACKs after the first lost segment.

In the case of $3 < n$, when the transmitter receives more than three duplicate ACKs, a fast retransmit procedure occurs. Then $P_{fm}(n,p)$ is derived as

$$P_{fm}(n,p) = \sum_{i=1}^{n}(1-p)^{i-1}p(1-p)^{n-i}$$
$$\times \sum_{j=1}^{n-i} \binom{n-i}{j} (1-p)^{(n-i-j)}p^j$$
$$\times \sum_{\substack{k=0;(n-i-j)\leq 3 \\ k=3-(n-i-j));3<(n-i-j)}}^{i-1} \binom{i-1}{k} p^{(i-1-k)}(1-p)^k (3 < n). \tag{5}$$

Probability of the Fast Retransmit for TCP New Reno

In the TCP New Reno, when the TCP transmitter receives partial ACKs, he retransmits the lost segment without waiting for the timer timeout. Hence, when the transmitter receives three duplicate ACKs, the fast retransmit procedure will occur. Then $P_f(n,p)$ is derived as

$$P_f(n,p) = \sum_{i=1}^{n}(1-p)^{i-1}p$$
$$\times \sum_{j=3}^{n-1} \binom{n-1}{j} p^{n-1-j}(1-p)^j \quad (3 < n). \tag{6}$$

Probability of the Timeout

When some segments are lost, the ACK may be duplicate. When the transmitter receives more than three duplicate ACKs, the fast retransmit procedure occurs. So the condition of the timeout occurrence is that the TCP transmitter receives less than 2 duplicate ACKs. To obtain the condition that less than 2 duplicate ACKs occur, we consider two cases;

- The congestion window size $n \leq 3$
  When some segments are lost in the case of $n \leq 3$, the timeout procedure must occur. Then $P_t(n,p)$ in the case of $n \leq 3$ is derived as

$$P_t(n,p) = \sum_{i=1}^{n} \binom{n}{i} p^i (1-p)^{n-i} \quad (n \leq 3).$$

(7)

- $3 <$ The congestion window size n
  The maximum number of duplicate ACKs is $n-1$ in the case of $3 < n$. The condition of the timeout occurrence is that the TCP transmitter receives less than 2 duplicate ACKs. $P_t(n,p)$ in the case of $3 < n$ is derived as

$$P_t(n,p) = \sum_{i=1}^{n} (1-p)^{i-1} p$$
$$\times \sum_{j=0}^{2} \binom{n-1}{j} p^{n-1-j} (1-p)^j \quad (3 < n).$$

(8)

2.4 A Calculation of the TCP Throughput

We calculate the steady state distribution from the transition probability matrix, and obtain the TCP throughput from this steady state distribution.

For example, we show the derivation of the TCP Reno for the simple model.

Let $\Pi = (\pi_1, \pi_2, \cdots, \pi_{W+F+D(3)})$ be the steady state distribution vector. π_n means that the distribution probability of n-th state in the steady state. Generally, we can obtain it by solving the following equations.

$$\begin{cases} \Pi = \Pi \cdot M \\ \sum_{i=1}^{W+F+D(3)} \pi_i = 1 \end{cases}$$

(9)

For example, when the multiple increase of timeout duration is limited to 4 times, the parameter of function $D()$ is set to 3.

We can obtain steady state distributions of the TCP Tahoe and the TCP New Reno same as the TCP Reno.

Let $S = (s_1, s_2, \cdots, s_{W+F+D(3)})$ represent the vector whose element is the expected number of the good segment in each state. This vector is obtained as follows,

(1) Transmission status ($1 \leq i \leq W$)

For $1 \leq i \leq W$, the expected number of successful transmitted segments is,

$$s_i = \sum_{j=1}^{i} j \binom{i}{j} (1-p)^j \times p^{(i-j)}.$$

(2) Fast retransmit status ($W + 1 \leq i \leq W + F + 1$)

For $W+1 \leq i \leq W+F+1$, the TCP transmitter retransmits the lost segment when the fast retransmit occurs. So the expected number of successful transmitted segments is,

$$s_i = 1 \times (1-p).$$

(3) Timeout ($W + F + D(1) - 1 \leq i \leq W + F + D(3)$)

For $i = W + F + D(1) - 1$, the waiting for the first timeout occurs, so the TCP transmitter does not transmit a segment. So the expected number of successful transmitted segments is,

$$s_i = 0$$

and for $i = W + F + D(1)$, the first timeout occurs and the TCP transmitter retransmits the lost segment. So the expected number of successful transmitted segments is,

$$s_i = 1 \times (1-p).$$

For $i = W+F+D(2)-2, W+F+D(2)-1$, the waiting for the second timeout occurs, so the expected number of successful transmitted segments is,

$$s_i = 0$$

and for $i = W + F + D(2)$, the second timeout occurs, so the expected number of successful transmitted segments is,

$$s_i = 1 \times (1-p).$$

For $W + F + D(3) - 4 \leq i \leq W + F + D(3) - 1$, the waiting for the third timeout occurs, so the expected number of successful transmitted segments is,

$$s_i = 0$$

and for $i = W + F + D(3)$, the third timeout occurs, so expected number of successful transmitted segments is,

$$s_i = 1 \times (1-p).$$

Let $T = (t_1, t_2, \cdots, t_{W+F+D(3)})$ represent the vector whose element is the value of the time duration period in each state. This vector is obtained as,

(1) Transmission and fast retransmit status

For $1 \leq i \leq W + F + 1$, the value of the time duration period is RTT (Round Trip Time), so it is,

$$t_i = RTT.$$

(2) Timeout status

For $i = W + F + D(1) - 1, W + F + D(1)$, the value of the time duration period is the multiple of AMT (Average Minimum Timeout), so it is,

$$t_i = 1 \times AMT.$$

For $i = W + F + D(2) - 2, W + F + D(2)$, the value of the time duration period is,

$$t_i = 2 \times AMT.$$

For $i = W + F + D(3) - 4, W + F + D(3)$, the value of the time duration period is,

$$t_i = 4 \times AMT.$$

We can obtain the TCP throughput from the following equation.

$$B = \frac{\Pi \cdot S}{\Pi \cdot T}$$

(10)

3   Numerical Result

We assume that a round trip time (RTT) is 100 [ms], and a size of TCP segment is 536 [bytes] which is the default size in the TCP implementation. In the situation of accessing to the Internet, all TCP segments are capsuled as IP datagrams, so a size of IP datagram which includes a TCP segment is 576 [bytes] . In the TCP procedure, it is set only in multiples of a timer granularity; for example, BSD-based systems have a timer granularity of 500 [ms]. In most implementations such as BSD-based systems, a minimum timeout duration is set for 2 timer ticks, implying an average minimum timeout (AMT) of 750 [ms] [2].

Figure 2 shows the throughput of the TCP Tahoe, the Reno and the New Reno. These curves are obtained from our analytical model or simulation with the advertised window size W equals to 16 [segment], and RTT equals to 100[ms]. In this result we consider the exponential increase of congestion window and the multiple increase of timeout duration is limited to 64 times. In this simulation, we calculate throughput by network simulator ns [5] with link bandwidth equals 10 [Mbps]. Figures 2 shows these results according to both analysis and simulation are quite close.

We observe that throughput of the TCP Reno and the New Reno has similar performance where the segment loss probability is about 10^{-3}. This is because the TCP Reno and the New Reno have a fast recovery mechanism, so the performance of the TCP Reno and the New Reno is better than that of the TCP Tahoe. While throughput of the TCP Reno and the Tahoe has similar performance where the segment loss probability is about 10^{-2}. This is because the TCP Reno and the Tahoe cannot solve the problem that performance is degraded when multiple segments are lost in a window. So the performance of the TCP New Reno is better than those of the Reno and the Tahoe. From these results, the TCP throughput is effected by fast retransmit mechanisms when the segment loss probability is about 10^{-3}, and it is effected by timeout mechanisms when the segment loss probability is more than 10^{-2}.

4   Conclusion

In this paper, we have proposed new analytical model which includes basic TCP mechanisms, and consider the different versions of the TCP. In our analysis, we focus on the random errors in a wireless link, the exponential increase of the congestion window, fast retransmit mechanisms, fast recovery mechanisms, and the exponential increase of timeout backoff are considered by representing a state transition diagram of the TCP every RTT (Round Trip Time) as a Markov chain.

We have clarified TCP throughput characteristics analytically. We have observed that the throughput of the TCP New Reno is the best performance, and that of the TCP Tahoe is the worst performance. But the difference of these throughput is small. And, the throughput of TCP is degraded rapidly as segment loss probability increases. These reason are the effect of the fast recovery mechanism and probability of the fast retransmit and the timeout occurrence. Finally, we compare the result from our analytical model with the result
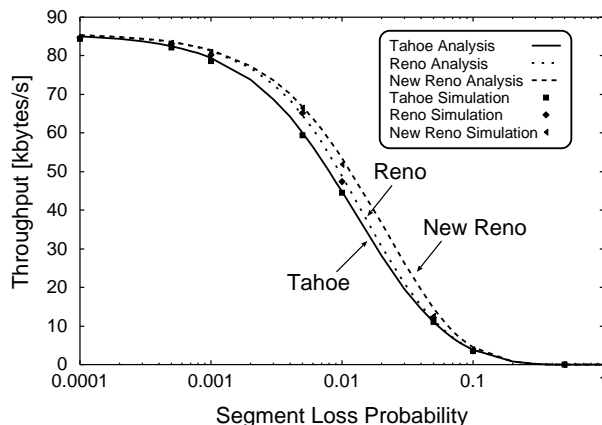

Figure 2: TCP throughput (W=16, RTT=100ms).

from the simulation and we confirmed that these results are quite close.

Acknowledgment

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