International Symposium on Information Theory and its Applications, ISITA2006
Seoul, Korea, October 29–November 1, 2006

Iterative Joint Channel-Decoding Scheme Using the Correlation of Transmitted Information Sequences in Sensor Networks

Kentaro KOBAYASHI†, Takaya YAMAZATO‡, Hiraku OKADA†† and Masaaki KATAYAMA‡

† Department of Electrical Engineering and Computer Science,
Graduate School of Engineering, Nagoya University,
Furo-cho, Chikusa-ku, Nagoya, 464-8603, JAPAN
E-mail: kobayasi@katayama.nuee.nagoya-u.ac.jp

‡ EcoTopia Science Institute, Nagoya University,
Furo-cho, Chikusa-ku, Nagoya, 464-8603, JAPAN
E-mail: {yamazato,katayama}@nuee.nagoya-u.ac.jp

†† Center for Transdisciplinary Research, Niigata University,
8050 Nino-cho, Ikarashi, Niigata, 950-2181, JAPAN
E-mail: hiraku@ie.niigata-u.ac.jp

Abstract

In this study, we consider joint channel decoding of Turbo code for multiple correlated data that are observed by sensor nodes densely deployed in a sensor field. We focus on the correlation properties of observation data and try to reduce decoding error by an iterative procedure. An approach to use practical channel codes for more than two correlated data is still not presented. A problem in the extension to cases of more than two sensor nodes is how to use the information of correlation obtained from observation data. In this study, we propose an iterative channel decoding scheme that uses them with weighting. We show that when the number of sensor nodes is increased, decoding performance improvement cannot be achieved by simple weighting, and so a more appropriate weight is needed. We find the optimum weight that minimizes the bit error rate from the analytical formula for uncoded BPSK and apply it to the case of Turbo code.

1. INTRODUCTION

In sensor networks, a large number of sensor nodes are densely deployed in a sensing field of interest. The sensor node should be small and long-lived with a wireless communication function to transmit the observation data.

Each sensor transmits observation data over wireless noisy channel to a central fusion center, where appropriate processings and analyses of collected observation data are performed and required information are extracted. Thus, detailed observation can be achieved by jointly processing the information about one sensing field from a large number of sensor nodes. Note the difficulty of observing the sensing field closely without overlapping the sensing ranges of each sensor node. Therefore, the observation data obtained by each sensor node are correlated and collected at a fusion center.

We intend to reduce channel error by using the correlation of observation data at the fusion center. Related works on channel encoding and decoding schemes using the correlation of observation data can be separated into two types: one considers the problem of compressing the redundancies of transmitted information [1, 2, 3], and the other considers the problem of channel error correction by using them [4, 5]. We focus on the latter, especially joint channel decoding that uses the correlation of transmitted information sequences. In particular, we are interested in the joint decoding scheme presented in [4, 5]. Based on an iterative procedure, this decoding scheme improves decoding performance by exchanging information of correlation between each decoder. However, it is only presented for cases of two sensor nodes.

In this study, we extend the joint decoding scheme presented in [4, 5] to cases of more than two sensor nodes. A problem in such extension is how to use the information of correlation obtained from other sequences. We propose an iterative joint decoding scheme that uses them with weighting.

The proposed decoding scheme requires weight that minimizes bit error rate. In this study, we show that when the number of sensor nodes is increased, decoding performance improvement cannot be achieved by simple weighting. For more appropriate weight, we find the optimum weight that minimizes bit error rate from an analytical formula derived for uncoded BPSK and apply it to the case of Turbo code. We show that the weight that minimizes BER for uncoded BPSK is effective in the case of Turbo code, too.

In Section 2, we describe the proposed iterative joint channel decoding system using the correlation of transmit-

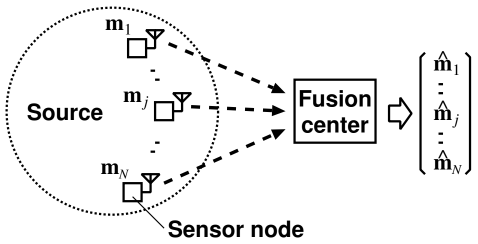

Figure 1: Sensor network.

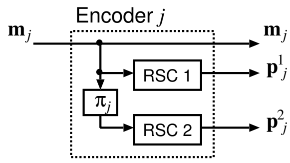

Figure 2: Turbo encoder for the j-th information bit sequence.

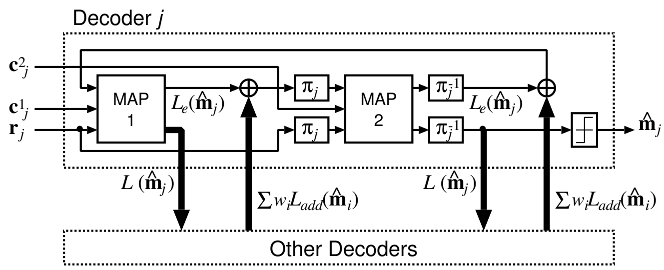

Figure 3: Iterative joint decoder of Turbo code for the j-th information bit sequence.

ted information sequences. We evaluate the performance of the proposed joint decoder and discuss the results in Section 3. Finally, conclusions are presented in Section 4.

2. SYSTEM MODEL

We consider N sensor nodes deployed in a sensing field, as depicted in Fig. 1.

Each sensor node independently observes and transmits the observed data to the fusion center. We assume that the observed data is Turbo encoded and transmitted over an independent AWGN channel to the fusion center, where the received signal sequences from the sensor nodes are demodulated and channel decoded. In a traditional communication system, received signal sequences are independently demodulated and channel decoded. In this study, we propose a channel decoding scheme that iteratively and jointly decodes received signal sequences by using the correlation of information bit sequences.

In the following, we consider an environment where sensor nodes with equal transmitting powers are densely deployed and the fusion center is placed far from them. Therefore, every E_b/N_0 of each sensor node is assumed to be equal.

2.1. Correlation of Transmitted Information Sequences

Let $\mathbf{m}_j = \{m_{j,1}, \cdots, m_{j,k}, \cdots, m_{j,K}\}$ be the transmitted information bit sequences of the j-th sensor node for $j = 1, \cdots, N$, and $m_{j,k}$ takes a value 1 or 0 with the same probability. The number of sensor nodes is N, and the length of the information bit sequences is K.

First, we define the correlation parameter by

$$\rho = \frac{w_H(\mathbf{m}_i \oplus \mathbf{m}_j)}{K} \tag{1}$$

where $W_H(\mathbf{m}_i \oplus \mathbf{m}_j)$ denotes the Hamming weight of the XOR of sequences $\mathbf{m}_i$ and $\mathbf{m}_j$ [4]. The correlation coefficient, which is the standard measure of correlation, can be defined by $1 - 2\rho$, and correlation parameters $0.5 \sim 0.0$ are equivalent to correlation coefficients $0.0 \sim 1.0$.

2.2. Turbo Encoder

Figure 2 shows a block diagram of the Turbo encoder. This is a standard Turbo encoder with coding rate $R_c = 1/3$ [6]. For each encoder, information bit sequence $\mathbf{m}_j$ is encoded using a Turbo code to produce parity bit sequences $\mathbf{p}_j^1$ and $\mathbf{p}_j^2$. Two component codes are used to encode the same input bit sequence, but interleaver π_j is placed between the component encoders. Recursive Systematic Convolutional (RSC) codes are used as component codes.

2.3. Proposed Iterative Joint Decoder

Figure 3 shows a block diagram of our proposed iterative joint decoder. For decoding, soft-in/soft-out maximum a posteriori (MAP) decoders are associated with each RSC encoder [6]. The proposed iterative joint decoder in Fig. 3 is focused on the decoding of j-th information bit sequence $\mathbf{m}_j$, and the decoding of other information bit sequences $\mathbf{m}_i$, $(i = 1, \cdots, N,\ i \neq j)$ are similarly performed in parallel. The deinterleaver, denoted as π_j^{-1}, corresponds to interleaver π_j in Fig. 2.

Given received signal sequence $\mathbf{y}_j$ (i.e., received information sequence $\mathbf{r}_j$ and received parity sequences $\mathbf{c}_j^1$ or $\mathbf{c}_j^2$), the MAP decoder outputs decoded bit $m_{j,k} = 1$ or 0 to maximize a posteriori probability $P(m_{j,k} \mid \mathbf{y}_j)$ for each information bit $m_{j,k}$. In other words, the MAP decoder calculates the log likelihood ratio (LLR) of a posteriori probability

$P(m_{j,k} \mid \mathbf{y}_j)$, i.e.,

$$\begin{aligned} &L(\hat{m}_{j,k}) \\ &= \ln \frac{P(\hat{m}_{j,k}=1)}{P(\hat{m}_{j,k}=0)} = \ln \frac{P(m_{j,k}=1 \mid \mathbf{y}_j)}{P(m_{j,k}=0 \mid \mathbf{y}_j)} \\ &= L_c \cdot r_{j,k} + L_a(m_{j,k}) + L_e(\hat{m}_{j,k}) \ , \ \left(L_c = 4R_c \frac{E_b}{N_0} \right) . \end{aligned} \quad (2)$$

Finally, the decoder make a hard decision: if $L(\hat{m}_{j,k}) \geq 0$, decoded bit $\hat{m}_{j,k} = 1$, and otherwise, decoded bit $\hat{m}_{j,k} = 0$.

The thick line in Fig. 3 shows the difference with standard Turbo decoding. For iterative decoding, the joint decoder uses extrinsic information $L_e(\hat{m}_{j,k})$ and additional information $L_{add}(\hat{m}_{i,k})$ obtained from the correlated information bit sequences as a priori information $L_a(m_{j,k})$.

In this study, as defined by (3), we extend a priori information $L_a(m_{j,k})$ of each information bit sequence to the case of N sensor nodes.

$$L_a^{(2)}(m_{j,k}) = L_e^{(1)}(\hat{m}_{j,k}) + \sum_{i=1, i \neq j}^{N} w_i \cdot L_{add}^{(1)}(\hat{m}_{i,k}) \quad (3)$$

where the superscript denotes the respective MAP decoder. For the above case, input from first MAP decoder (MAP 1) is fed to the second MAP decoder (MAP 2). A priori information passing from MAP 2 to MAP 1 takes the same form. Note that additional information $L_{add}(\hat{m}_{i,k})$ obtained from the correlated information bit sequences does not exist in standard Turbo decoding. Parameter w is a weight for additional information $L_{add}(\hat{m}_{i,k})$.

Based on the definition of correlation parameter ρ, additional information $L_{add}(\hat{m}_{i,k})$, obtained from the correlated information bit sequences, can be defined by [4]

$$L_{add}(\hat{m}_{i,k}) = \ln \frac{(1-\hat{\rho})\ P(\hat{m}_{i,k}=1) + \hat{\rho}\ P(\hat{m}_{i,k}=0)}{(1-\hat{\rho})\ P(\hat{m}_{i,k}=0) + \hat{\rho}\ P(\hat{m}_{i,k}=1)} \ . \quad (4)$$

Estimated correlation parameter $\hat{\rho}$ can be calculated by (1) using MAP decoded bit sequences $\hat{\mathbf{m}}_i$ and $\hat{\mathbf{m}}_j$. We assume that the correlation parameters of all information bit sequences are equally ρ.

2.3.1. Optimum weight

For simplicity, all correlation parameters are assumed to be equal. In this case, Eq. (3) can be treated as

$$L_a^{(2)}(m_{j,k}) = L_e^{(1)}(\hat{m}_{j,k}) + w \cdot \sum_{i=1, i \neq j}^{N} L_{add}^{(1)}(\hat{m}_{i,k}) \ . \quad (5)$$

Let us focus on Eq. (5) and find the weight that minimizes bit error rate.

We first consider the case of $w = 1$, where increasing the number of sensor nodes may give overmuch value of $L_{add}(\hat{m}_{i,k})$. Thus it may not be effective for a large number of N. Let us denote the weight of this case as w_{add}. On the other hand, if $w = \frac{1}{N-1}$, which corresponds to the average of all additional information $L_{add}(\hat{m}_{i,k})$, then the contribution of the additional information becomes too small for a large number of N. Therefore, it may not improve performance especially in low SNR. Let us denote the weight of this case as w_{avg}.

It is difficult to derive an analytical formula of the bit error rate of Turbo code, and it also greatly depends on the constraint length, the decoding algorithm, and the number of iterations, etc. For this reason, we find the optimum weight that minimizes bit error rate from the analytical formula for uncoded BPSK.

We assume bit sequence $\mathbf{m}_s$ that has correlation parameter q between each information bit sequence. It can be treated as a case where each sensor node observes source output $\mathbf{m}_s$ and bit flips occur at probability q due to observation noise. In this case, the analytical formula of bit error rate can be summarized relatively simply.

Let us start with three sensor nodes. Now assume that sensor node 1 transmits bit $m_1 = 1$. At the fusion center, based on the MAP decision given by Eq. (6), we decide that $\hat{m}_1 = 1$ if $L_1 > 0$, otherwise $\hat{m}_1 = 0$.

$$L_1 = \ln \frac{P(m_1 = 1 \mid r_1)}{P(m_1 = 0 \mid r_1)} = L_c \cdot r_1 + L_a \quad (6)$$

The additional information obtained from the correlation between m_1 and m_2, and between m_1 and m_3 can be used as a priori information L_a. From Eqs. (4) and (5), a priori information L_a is obtained by

$$\begin{aligned} L_a(r_2, r_3) &= w \cdot \sum_{i=2}^{3} \ln \frac{(1-\rho)\ P(\hat{m}_i = 1) + \rho\ P(\hat{m}_i = 0)}{(1-\rho)\ P(\hat{m}_i = 0) + \rho\ P(\hat{m}_i = 1)} \\ &= w \cdot \sum_{i=2}^{3} \ln \frac{(1-\rho) \cdot e^{L_c \cdot r_i} + \rho}{(1-\rho) + \rho \cdot e^{L_c \cdot r_i}} \ . \end{aligned} \quad (7)$$

In this case, the probability that $m_1 = 1$ is transmitted from sensor node 1 and is erroneously read (i.e., $\hat{m}_1 = 0$) is obtained by

$$\begin{aligned}
P(E) &= P(\hat{m}_1 = 0 \mid m_1 = 1) \\
&= P(m_2 = 1, m_3 = 1 | m_1 = 1) \\
&\quad \cdot P\left(r_1 + \frac{L_a(r_2, r_3)}{L_c} < 0 \,\middle|\, m_1 = 1, m_2 = 1, m_3 = 1\right) \\
&+ P(m_2 = 1, m_3 = 0 | m_1 = 1) \\
&\quad \cdot P\left(r_1 + \frac{L_a(r_2, r_3)}{L_c} < 0 \,\middle|\, m_1 = 1, m_2 = 1, m_3 = 0\right) \\
&+ P(m_2 = 0, m_3 = 1 | m_1 = 1) \\
&\quad \cdot P\left(r_1 + \frac{L_a(r_2, r_3)}{L_c} < 0 \,\middle|\, m_1 = 1, m_2 = 0, m_3 = 1\right) \\
&+ P(m_2 = 0, m_3 = 0 | m_1 = 1) \\
&\quad \cdot P\left(r_1 + \frac{L_a(r_2, r_3)}{L_c} < 0 \,\middle|\, m_1 = 1, m_2 = 0, m_3 = 0\right) .
\end{aligned} \tag{8}$$

Focusing on bit m_s, the first term of Eq. (8) can be expanded as

$$\begin{aligned}
&\text{The first term of Eq. (8)} \\
&= \big(P(m_2 = 1 \mid m_s = 1)\, P(m_3 = 1 \mid m_s = 1)\, P(m_s = 1 \mid m_1 = 1) \\
&+ P(m_2 = 1 \mid m_s = 0)\, P(m_3 = 1 \mid m_s = 0)\, P(m_s = 0 \mid m_1 = 1)\big) \\
&\cdot\ P\left(r_1 + \frac{L_a(r_2, r_3)}{L_c} < 0 \,\middle|\, m_1 = 1, m_2 = 1, m_3 = 1\right) \\
&= ((1-q)\,(1-q)\,(1-q) + q\,q\,q) \\
&\cdot \iint_{-\infty}^{\infty} Q\left(\sqrt{\frac{2E_b}{N_0}}\left(1 + \frac{L_a(r_2, r_3)}{L_c}\right)\right) \mathcal{N}(r_2 - 1)\mathcal{N}(r_3 - 1)\, dr_2 dr_3
\end{aligned} \tag{9}$$

where $\mathcal{N}(\cdot)$ is the Gaussian probability density function with variance $N_0/2$. Also, the other terms of Eq. (8) can be expanded similarly. Furthermore, the second and third terms of Eq. (8) are equal due to symmetry, and we have $\rho = 2q(1 - q)$. Therefore, bit error rate in the case of three sensor nodes can be summarized as

$$\begin{aligned}
P(E) = (1 - 1.5\rho) &\iint_{-\infty}^{\infty} f(r_2, r_3)\mathcal{N}(r_2 - 1)\mathcal{N}(r_3 - 1)\, dr_2 dr_3 \\
+ \quad \rho \quad &\iint_{-\infty}^{\infty} f(r_2, r_3)\mathcal{N}(r_2 - 1)\mathcal{N}(r_3 + 1)\, dr_2 dr_3 \\
+ \ 0.5\rho\ &\iint_{-\infty}^{\infty} f(r_2, r_3)\mathcal{N}(r_2 + 1)\mathcal{N}(r_3 + 1)\, dr_2 dr_3
\end{aligned} \tag{10}$$

$$f(r_2, r_3) = Q\left(\sqrt{\frac{2E_b}{N_0}}\left(1 + \frac{L_a(r_2, r_3)}{L_c}\right)\right) . \tag{11}$$

Similarly, bit error rate in the case of N sensor nodes can be summarized as

$$\begin{aligned}
P(E) = &\int \!\cdots\! \int_{-\infty}^{\infty} Q\left(\sqrt{\frac{2E_b}{N_0}}\left(1 + \frac{L_a(\mathbf{r})}{L_c}\right)\right) \\
&\cdot \sum_{n=1}^{N} \Bigg[{}_{N-1}C_{n-1} \left\{(1-q)^n q^{N-n} + q^n (1-q)^{N-n}\right\} \\
&\cdot \prod_{l=2}^{n} \mathcal{N}(r_l - 1) \prod_{l=n+1}^{N} \mathcal{N}(r_l + 1) \Bigg]\, d\mathbf{r}
\end{aligned} \tag{12}$$

$$L_a(\mathbf{r}) = w \cdot \sum_{i=2}^{N} \ln \frac{(1 - \rho) \cdot e^{L_c \cdot r_i} + \rho}{(1 - \rho) + \rho \cdot e^{L_c \cdot r_i}} \tag{13}$$

where $\rho = 2q(1 - q)$ and $\mathbf{r} = \{r_2, r_3, \cdots, r_N\}$. The optimum weight that minimizes the bit error rate $P(E)$ is obtained by $w_{opt} = \arg\min_w P(E)$.

3. Numerical Examples

First, we evaluate the characteristics of optimum weight w_{opt}, calculated from Eq. (12) for sensor nodes $N = 3$ and $N = 4$, as shown in Figs. 4 and 5, respectively.

The value range of w_{opt} is $\frac{1}{N-1} \leq w_{opt} \leq 1$. As shown in Figs. 4 and 5, optimum weights w_{opt} have identical characteristics, regardless of the number of sensor nodes. Notice that optimum weight w_{opt} is large in low SNR, which shows that using the correlation is effective for error correction in low SNR. Notice also that the smaller correlation parameter ρ is, the smaller optimum weight w_{opt} is. This is because the smaller correlation parameter ρ is, the bigger the absolute value of additional information $L_{add}(\cdot)$ obtained from other sequences is. However, since the diversity effect can be obtained for very large correlation, the larger weight becomes more effective (if $\rho \to 0$, $w_{opt} \to 1$ $(= w_{add})$).

Next, to evaluate the decoding performance of Turbo code using Eq. (5), we simulate the proposed scheme for different weights and consider three cases: averaging weight w_{avg} ($w = 1$), adding weight w_{add} ($w = N - 1$), and optimum weight w_{opt} obtained by (12). In all cases, the length of the transmitted information bit sequences is fixed to $K = 1000$. Each Turbo encoder includes two identical 16-state RSC encoders. We consider independent AWGN channels and BPSK modulation for the transmitter. The Max-Log-MAP algorithm is applied to decode individual Turbo codes, and the number of iterations is set to five. Correlation parameters are estimated during the decoding process, as mentioned in Section 2.3, and updated for each decoding iteration. Now, every E_b/N_0 of each sensor node is assumed to be equal, and estimations of E_b/N_0 are assumed to be perfect in the decoder.

Figures 6, 7, and 8 show, respectively, the BER performances of the proposed joint iterative decoder using averaging weight w_{avg}, adding weight w_{add}, and optimum weight

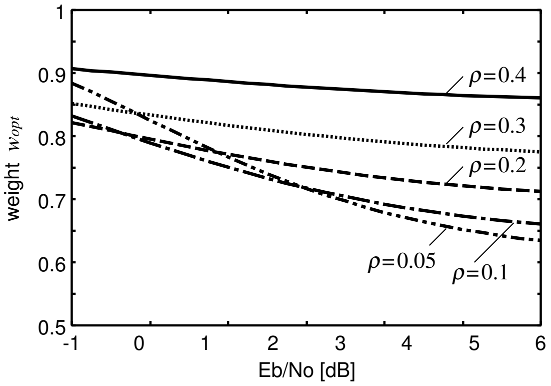

Figure 4: Weight w_{opt} that minimizes BER in the case of uncoded BPSK for sensor nodes $N = 3$.

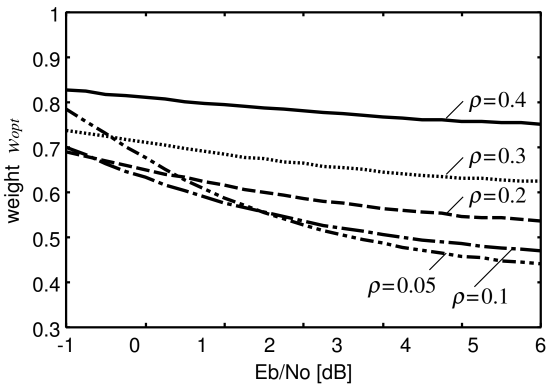

Figure 5: Weight w_{opt} that minimizes BER in the case of uncoded BPSK for sensor nodes $N = 4$.

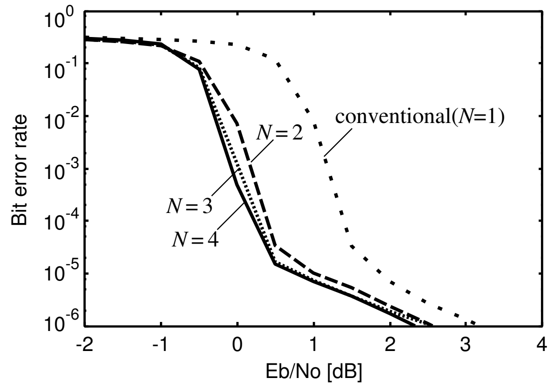

Figure 6: BER performances of proposed joint iterative decoder using averaging weight w_{avg} for correlation parameter $\rho = 0.1$.

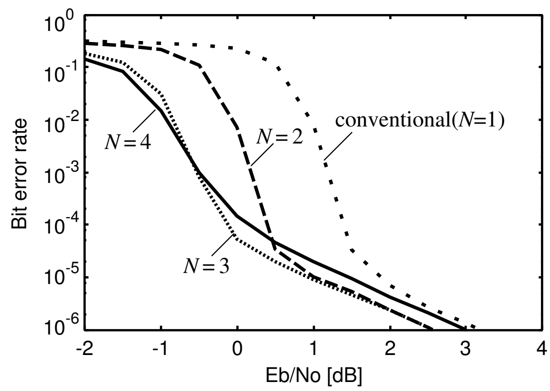

Figure 7: BER performances of proposed joint iterative decoder using adding weight w_{add} for correlation parameter $\rho = 0.1$.

w_{opt}. In these cases, correlation parameters are $\rho = 0.1$. In addition, Figs. 9 and 10, which respectively consider cases $\rho = 0.2$ and $\rho = 0.1$, compare BER performances using w_{avg}, w_{add}, and w_{opt}.

As shown in Fig. 6, when the number of sensor nodes N is increased, decoding performance improvement cannot be achieved by averaging weight w_{avg}. On the other hand, when using adding weight w_{add}, performance degradation occurs for a large number of N, as shown in Fig. 7.

Figure 8 shows that in the case of using optimum weight w_{opt}, large performance gain for a large number of N can be obtained, especially in low SNR. Note that performance degradation, such as when averaging weight w_{avg} or adding weight w_{add} is applied, is nonexistent. The best performance is obtained when weight w_{opt} is applied, as shown in both Figs. 9 and 10. Thus weight w_{opt}, which minimizes BER for uncoded BPSK, is effective in the case of Turbo code, too.

4. CONCLUSION

In this study, we extended joint channel decoding of Turbo code to cases of more than two sensor nodes. A problem in extension to cases of more than two sensor nodes is how to use the information of correlation obtained from other sequences. Specifically, it must be used with weighting to minimize bit error rate.

We presented an iterative channel decoding scheme that uses the information of correlation with weighting. The resulting performances show that when the number of sensor nodes is increased, decoding performance improvement cannot be achieved by simple weighting. For more appropriate weight, we found the optimum weight that minimizes the bit error rate from an analytical formula derived for cases of uncoded BPSK and applied it to cases of Turbo code. The resulting performances show that the weight that

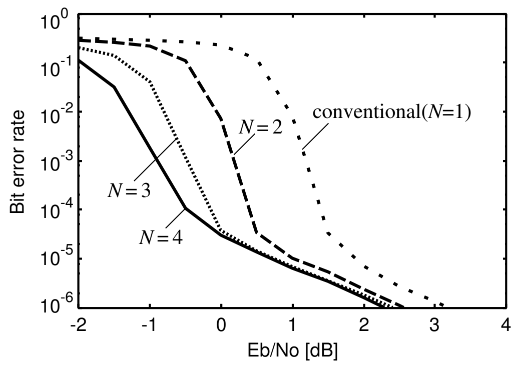

Figure 8: BER performances of proposed joint iterative decoder using optimum weight w_{opt} for correlation parameter $\rho = 0.1$.

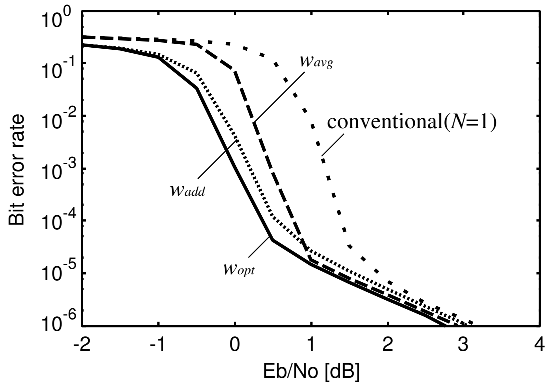

Figure 9: Comparison of BER performances of proposed joint iterative decoder using different weights: w_{avg}, w_{add}, and w_{opt} for sensor nodes $N = 4$ and correlation parameter $\rho = 0.2$.

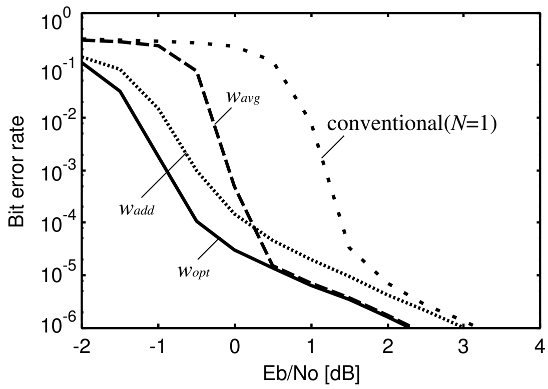

Figure 10: Comparison of BER performances of proposed joint iterative decoder using different weights: w_{avg}, w_{add}, and w_{opt} for sensor nodes $N = 4$ and correlation parameter $\rho = 0.1$.

minimizes BER for uncoded BPSK is effective in the case of Turbo code, too.

Acknowledgments

This work is supported in part by "The 21st Century COE Program by the Ministry of Education, Culture, Sports, Science and Technology in Japan" and "Japan Society for the Promotion of Science under Grant-in-Aid for Scientific Research (C)".

References

[1] S.S. Pradhan and K. Ramchandran, "Distributed source coding using syndromes (DISCUS): design and construction, " IEEE Trans. Inf. Theory, vol. 49, no. 3, pp. 626–643, Mar. 2003.

[2] A.D. Liveris, Z. Xiong and S. Cheng, "Distributed source coding for sensor networks, " IEEE Signal Process. Mag., vol. 21, no. 5, pp. 80–94, Sept. 2004.

[3] Y. Zhao and J. Garcia-Frias, "Joint estimation and compression of correlated nonbinary sources using punctured turbo codes, " IEEE Commun. Lett., vol. 53, no. 3, pp. 385–390, Mar. 2005.

[4] F. Daneshgaran, M. Laddomada and M. Mondin, "Iterative joint channel decoding of correlated sources employing serially concatenated convolutional codes, " IEEE Trans. Inf. Theory, vol. 51, no. 7, pp. 2721–2731, July 2005.

[5] J. Garcia-Frias and Y. Zhao, "Near-Shannon/Slepian-Wolf performance for unknown correlated sources over AWGN channels, " IEEE Trans. Commun., vol. 53, no. 4, pp. 555–559, Apr. 2005.

[6] L. Hanzo, T. H. Liew and B. L. Yeap, Turbo Coding, Turbo Equalisation and Space-Time Coding for Transmission over Fading Channels, New York, USA: John Wiley, IEEE Press, 2002.