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The Optimum Received Signal-Power Distribution for CDMA Packet Communication Systems Employing Successive Interference Cancellation

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Abstract— **In CDMA packet communication systems employing Successive Interference Cancellation(SIC) scheme, the suppression of MAI depends on the received signal-power distribution, in addition to the cross-correlation characteristic of spreading sequences and the number of transmitted packets. Some works have been done about discovering the received signal-power distribution which makes MAI be suppressed effectively. This distribution minimizes the averaged Bit Error Rate(BER). However, in packet communication systems, it is dominant to the performance if packets can be transmitted successfully or not. Even if the averaged BER is high, many packets can be correctly transmitted[7]. It implies that the optimum distribution on averaged BER does not give the best performance of packet transmission. In this paper, the received signal-power distribution which gives the best performance of packet transmission is derived. Such a distribution will make clear the limit of performance improvement using SIC.**

I. INTRODUCTION

A CDMA packet communication system is often considered due to the possibility of random access and simultaneous transmission of packets with simple access procedure [1], [2]. The performances of such a system are mainly limited by Multiple Access Interference(MAI) [3], [4], which is the important issue to enhance the performances. Some papers study the application of Multiuser Detection schemes [5], [6], [7]. Successive Interference Cancellation(SIC) scheme is a form of Multiuser Detection with low complexity, simple structure and good suppression of MAI [7], [8], [9], [10].

In SIC schemes, the suppression of MAI depends on the received signal-power distribution, in addition to the cross-correlation characteristic of spreading sequences and the number of transmitted packets [7], [8]. In the case that the received signal-power is not identical, the received signals with strong signal-power can be demodulated correctly even in the presence of MAI. Since most of MAI comes from such strong signals, removing the influence of the MAI makes the signals with small signal-power be correctly demodulated. Thus, non-identical received signal-power is preferable in SIC schemes [7], [8]. This motivates researchers to discover the optimum received signal-power distribution which achieves the correct demodulation/cancellation [9], [10]. Such a distribution has been optimized about the averaged Bit Error Rate(BER).

However, in packet communication systems, packets carry information. Hence, the performance should be evaluated in terms of the packet, not of the averaged BER. Even if the averaged BER is high, many packets can be correctly transmitted [7]. It implies that the optimum distribution on averaged BER could not give the maximum performance of the packet transmission.

This paper discusses the optimum received signal-power distribution which maximizes the performance of the packet transmission in the CDMA packet communication system with SIC. Throughput is used as the performance of the packet transmission. In the discussion, we shall demonstrate that the optimum received signal-power distribution on the averaged BER cannot make the throughput be maximized. The optimum received signal-power distribution on throughput might be obtained by the maximization of the number of the simultaneously transmitted packets whose BER is enough low to success the transmission. In addition, it clearfies the utmost throughput which is achievable due to the application of SIC schemes.

This paper is organized as follows. In Section II, we describe the system model. In Section III, we sketch out the optimum received signal-power distribution on BER. The optimum received signal-power distribution on throughput is derived in Section IV. We evaluate the throughput in Section V. Finally, some conclusions are presented in Section VI.

II. SYSTEM MODEL

We treat the reverse link of DS-CDMA packet communication system as shown in Fig.1. The system consists of an infinite number of Mobile Stations(MS) and a Base Station(BS). When the request to transmit the packet occurs in a MS, the MS immediately transmits it to the BS. The number of requests (the number of the simultaneously transmitted packets) K follows a Poisson distribution with a birth rate λ, and hence, this system could be viewed as CDMA ALOHA systems [1], [2]. The probability $P_G(K)$ that K packets are simultaneously

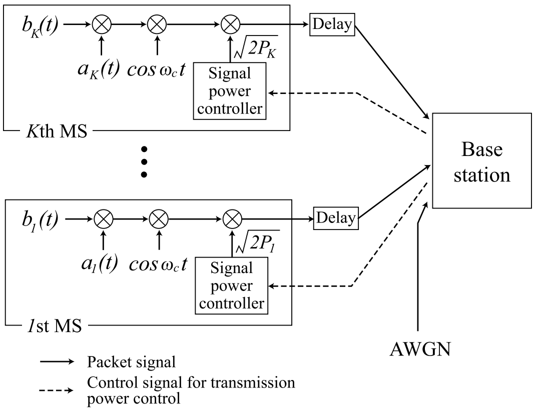

Fig. 1. Transmitter and channel model.

transmitted at the offered load G is

$$P_G(K) = \frac{e^{-\lambda T_p} \cdot (\lambda T_p)^K}{K!} = \frac{e^{-G} \cdot G^K}{K!} \tag{1}$$

where T_b is a bit duration, L is the packet length and $T_P = LT_b$ is one packet duration, and offered load $G = \lambda T_p$ is the average number of the requests during one packet duration T_P. At the BS, the received packet signals are demodulated, and then MAI is suppressed. This suppression is based on SIC scheme. Note that the BS ideally controls the signal-power of transmitted packets so that MAI could be suppressed effectively.

The MS which transmits his packet, at first, packetizes data sequence to a fixed-length packet of L bits. The packetized data is modulated by Binary Phase Shift Keying (BPSK) scheme. Then, the packet signal is spread by the uniquely assigned spreading signal. The random signature sequence of N chips is used as the spreading sequence where N is the spreading factor. Finally, the MS multiplies the packet signal by the carrier signal, and then transmits the signal to the BS in one hop. In the channel, Additional White Gaussian Noise(AWGN) is added to the packet signal. The BS receives the following signal

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k} a_k(t-\tau_k) b_k(t-\tau_k) \cos(\omega_c t + \phi_k) + n(t) \tag{2}$$

where P_k is the received signal-power, $a_k(t)$ is the spreading signal, $b_k(t)$ is the transmitted data signal, ω_c is the carrier frequency, τ_k is the transmission delay, ϕ_k is the carrier phase of the kth MS and $n(t)$ is the AWGN with double-sided power spectrum density of $N_0/2$. The carrier phase ϕ_k and the transmission delay τ_k are taken to be independent and uniformly distributed over $[0, 2\pi)$ and $[0, T_b)$, respectively.

Fig.2 shows the model of the BS [7], [8]. The BS has a Low Pass Filter(LPF) and K demodulator/cancellar blocks. In the demodulator/cancellar block assigned to each packet, the demodulation and the suppression of the MAI for the packet signal is performed. The demodulation and the cancellation for all the received packet signal is performed in order of the received signal-power in the demodulator/cancellar blocks.

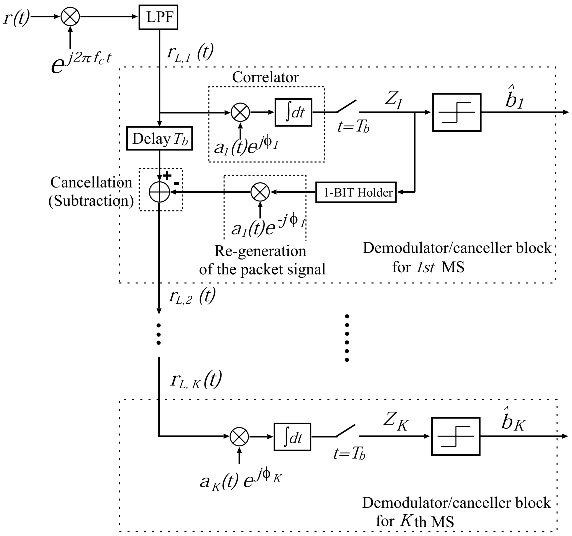

Fig. 2. Receiver.

Let us explain the demodulation and the cancellation of MAI in the demodulator/cancellar blocks. The input baseband signal $r_{L,k}(t)$ is correlated with the spreading signal in order to compute the correlation Z_k [7], [8]. Next, the data $\hat{b}_k$ is demodulated by hard-decision on the correlation Z_k. The correlation Z_k is also re-spread to generate the replica of the received packet signal. This replica is subtracted from the input signal $r_{L,k}(t)$, and then the MAI from the packet signal is reduced. We note that the transmission delay τ_k and the carrier phase ϕ_k are assumed to be known at the BS.

III. OPTIMUM RECEIVED SIGNAL-POWER DISTRIBUTION ON BER

In this section, we sketch out the optimization of received signal-power distribution in terms of averaged BER. This distribution can be derived by assuming the BERs to be identical for all packets. That is, we should consider that signal and interference to noise ratio(SINR) at the correlator output are identical for all the packets, which can be written as [9], [10]

$$\gamma_K = \frac{P_1}{\eta_1} = \frac{P_2}{\eta_2} = \cdots = \frac{P_K}{\eta_K} \tag{3}$$

where η_k is the variance of the noise and interference component of the packet signal from the kth MS.

Defining $E_b^k = P_k T_b$, Eq. (3) is expressed as

$$\gamma_K = \frac{E_b^1}{N_0} \cdot \frac{N_0}{T_b \eta_1} = \cdots = \frac{E_b^K}{N_0} \cdot \frac{N_0}{T_b \eta_K}. \tag{4}$$

From [7], [8], the variance η_k may be given by

$$\eta_{k+1} = \frac{2}{3N} \sum_{i=k+2}^{K} P_i + \frac{N_0}{T_b} + \frac{1}{3N} \sum_{s=1}^{k} \eta_s. \tag{5}$$

This variance can be transformed into the following equation [10] :

$$\frac{N_0}{T_b \eta_{k+1}} = \left\{ \frac{2}{3N} \left(Y_k - \frac{E_b^{k+1}}{N_0} \right) + 1 \right\}^{-1} \tag{6}$$

where

$$Y_k = \begin{cases} X_k + \frac{1}{3N} \sum_{l=1}^{k-1} Y_l & (k \geq 1) \\ \varepsilon_N & (k = 0) \end{cases} \tag{7}$$

$$X_k = \begin{cases} (1 + \frac{1}{3N})(\varepsilon_N - \frac{1}{N_0} \sum_{i=1}^{k} E_b^i) + \frac{k}{2} & (k \geq 1) \\ 0 & (k = 0) \end{cases} \tag{8}$$

and the term $\varepsilon_N = \sum_{k=1}^{K} \frac{E_b^k}{N_0}$ denotes the total signal to noise ratio of the received packet signal in a bit. Substituting Eqs.(6) into (4), we obtain

$$\frac{E_b^{k+1}}{N_0 \left\{ \frac{2}{3N} \left(Y_k - \frac{E_b^{k+1}}{N_0} \right) + 1 \right\}} = \frac{E_b^k}{N_0 \left\{ \frac{2}{3N} \left(Y_{k-1} - \frac{E_b^k}{N_0} \right) + 1 \right\}}. \tag{9}$$

Solving Eq. (9) for P_{k+1} yields the optimum received signal-power P_{k+1},

$$P_{k+1} = \frac{\frac{2}{3N} Y_k + 1}{\frac{2}{3N} Y_{k-1} + 1} P_k = \left(\frac{\frac{2}{3N} Y_k + 1}{\frac{2}{3N} \varepsilon_N + 1} \right)^k P_1. \tag{10}$$

Note that the value P_k can be computed by using the algorithm[10] with Eq. (10).

IV. Optimum Received signal-Power Distribution on Throughput

In this section, we optimize the received signal-power distribution for CDMA packet communication systems employing SIC. This optimization is performed by maximizing the throughput which is defined as the averaged number of successful packets during one packet duration.

The throughput could be maximized by assuming the SINR to be identical for all packets, which is the same assumption in Section III. If the SINR are not identical for all the packets, there are some packets whose SINR are lower than other packets. Such a packet degrades the throughput. However, even if the SINR are identical for all the packets, the throughput could not be maximized since many transmitted packets would degrade the BER, and then throughput would be also degraded. It implies that the optimization requires both the limitation of the number of the transmitted packets and the identical SINR.

To restrict the number, we can apply the access control scheme of Channel Load Sensing Protocol(CLSP) [12]. In CLSP, a control station (base station) senses the channel load, which is the number of ongoing transmissions. When the request to transmit the packet occurs, the transmission are allowed if the channel load is less than a certain threshold κ. In the case that the channel load is larger than the threshold, the transmission are rejected until the ongoing transmissions fall below the threshold. Note that our problem is now how many packets could be transmitted, that is, to decide the threshold κ. The SINR for κ transmitted packets

$$\hat{\gamma}_\kappa = \frac{\hat{P}_1}{\eta_1} = \frac{\hat{P}_2}{\eta_2} = \cdots = \frac{\hat{P}_\kappa}{\eta_\kappa} \tag{11}$$

shows that the optimum signal-power $\hat{P}_k$ are assigned to κ packets out of K packets, and then the κ packets are actually transmitted. Therefore, we should find the threshold κ which maximizes the throughput for the packets with the optimum signal-power.

If up to κ packets can be transmitted, the throughput is expressed as

$$S_\kappa(G) = \sum_{n=1}^{\kappa} n \left\{ 1 - Q\left(\sqrt{\hat{\gamma}_n}\right) \right\}^L P_G(n, \kappa) \tag{12}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{u^2}{2}) du \tag{13}$$

and

$$P_G(n, \kappa) = \frac{G^n / n!}{\sum_{j=0}^{\kappa} G^j / j!}. \tag{14}$$

is the probability that n packets are actually transmitted when up to κ packets are allowed to be transmitted at the offered load G [12], [13]. The maximum throughput S_{max} can be obtained by the maximization of eq. (12), which is expressed as

$$S_{max} = \max_{\kappa, G} \left[\sum_{n=1}^{\kappa} n \left\{ 1 - Q\left(\sqrt{\hat{\gamma}_n}\right) \right\}^L P_G(n, \kappa) \right]. \tag{15}$$

The optimum received signal-power distribution on throughput can be derived by solving the Eq. (11) with κ satisfying Eq.(15).

V. Performance Evaluation

We evaluate the throughput performance of the optimum received signal-power distribution. When the received signal-power follows the optimum distribution on BER, the throughput $S_B(G)$ can be expressed as [1], [2]

$$S_B(G) = \sum_{n=1}^{\infty} n \left\{ 1 - Q\left(\sqrt{\gamma_n}\right) \right\}^L P_G(n). \tag{16}$$

When the received signal-power follows the optimum distribution on throughput, the throughput is written as the Eq. (12) with κ satisfying Eq. (15) .

Fig.3 illustrates the throughput curves for L=500[bits], N=31, ε_N=30[dB]. From Eq. (15), κ=20 gives the throughput when the received signal-power follows the optimum distribution on throughput. Fig.3 also shows the two curves. One is the throughput in the case of identical received signal-power, and the other is the throughput in the case of Rayleigh-distributed received signal-power (that is, we consider Rayleigh fading environment). Note that ε_N is the same for all the cases.

In Fig.3, it can be seen that the optimum received signal-power distribution on BER could not give the best performance

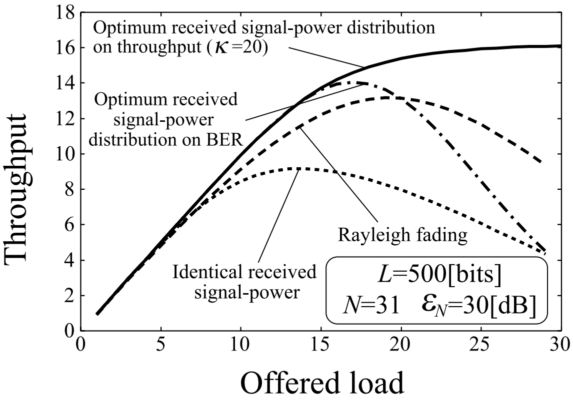

Fig. 3. Throughput.

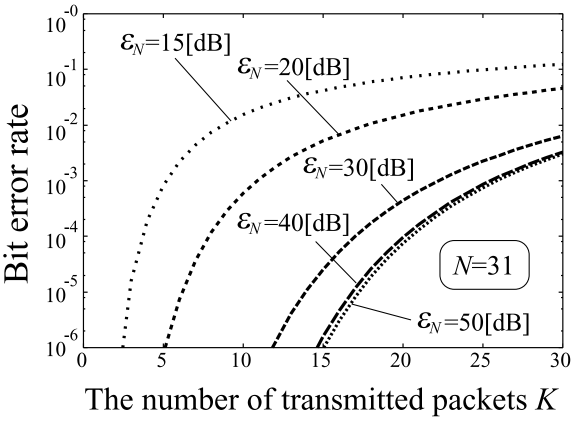

Fig. 5. Bit error rate for the packets whose received signal-power follows the optimum distribution on BER.

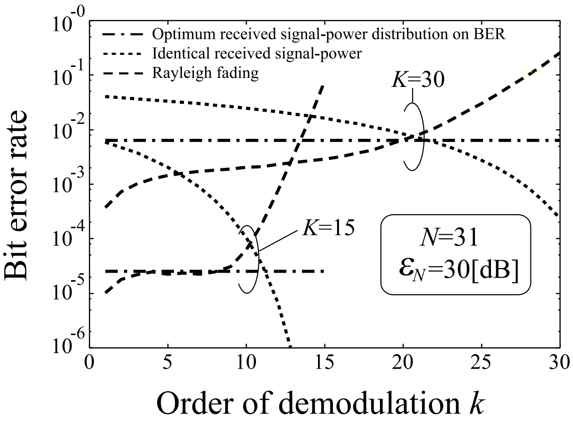

Fig. 4. Bit error rate on each packet.

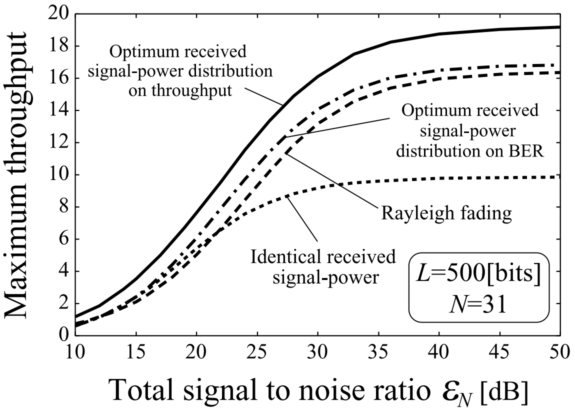

Fig. 6. Maximum throughput.

on throughput. In the small offered load, the throughput of the optimum received signal-power on BER equals the offered load. This implies that all the packets are successfully transmitted, that is, the best throughput is realized. However, in the large offered load, the throughput is not the best one since it is worse than that of Rayleigh fading. We can also find out that the throughput of Rayleigh fading is better than that of the identical received signal-power [7].

BER performance on each packet are compared in Fig.4 so as to explain the throughput curves of Fig.3. In the case of Rayleigh fading, the number of the packets whose BER is low is larger than that in the identical case. Hence, more packets can be successfully transmitted than that with the identical received signal-power. When 15 packets are transmitted, the transmission of some packets with the identical and Rayleigh-distributed received signal-power should be failed because of the poor BER. All the packets whose received signal-power follows the optimum distribution on BER have good BER so that they could be successfully transmitted. However, as more packets are transmitted, the BER for the packets with the optimum signal-power is gradually degraded as shown in Fig.5. The packets are failed to be transmitted, that is, the throughput is degraded. As shown in Fig.4, in the case of identical and Rayleigh-distributed signal-power, even if the number of transmitted packets increases such as K=30, the BER of some packets are still low. Hence, their throughput could be kept good. That is why the optimum distribution on BER does not give the best throughput.

Let us discuss the throughput when the received signal-power follows the optimum distribution on throughput. Fig.3 shows that the throughput is better than any other throughput. This is because the maximum throughput is the best as shown in Fig.6. The maximum throughput can be obtained when many packets would be successfully transmitted. However, the BER of many transmitted packets would be degraded, and then the packets are failed to be transmitted. Using CLSP keeps the identical BER for a lot of transmitted packets good enough for the packets to be successfully transmitted. For example, in Fig.3, up to 20 packets can be transmitted. From Fig.5, the identical BER of the transmitted packets are kept below about 4.4×10^{-4} (straight and dashed line). This BER is equivalent to the packet success probability of about 0.80, that is, many packets are successfully transmitted. We note that the maximum throughput for other ε_N may be achieved by restricting the number of transmitted packets so that the identical BER could be below about 4.4×10^{-4}.

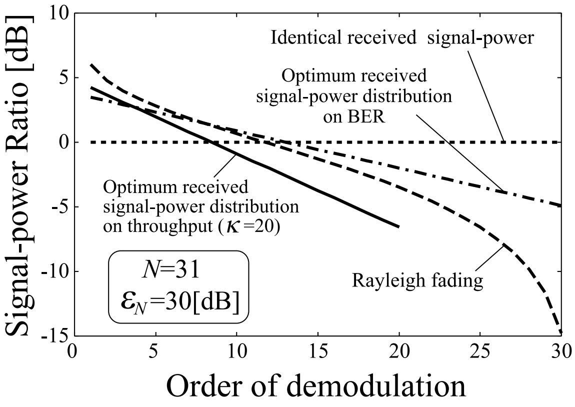

Fig. 7. Received signal-power distribution.

Fig.7 shows the optimum received signal-power distribution on throughput, in addition to the optimum one on BER, the identical one, and the one which follows the Rayleigh distribution. 30 packets are requested to be transmitted in this figure, and the signal-power is plotted for the mean value in each distribution. In the case of the optimum distribution on throughput, up to κ=20 packets are allowed to be transmitted. The curves for the optimum distribution on both throughput and BER become a straight line. However, the slope for the optimum distribution on throughput is different from the one on BER. Thus, the curve for the optimum distribution on throughput becomes straight line and might have a suitable inclination.

VI. CONCLUSION

This paper develops the received signal-power distribution which gives the maximum throughput on CDMA packet communication systems employing SIC. It is demonstrated that the maximum throughput cannot be achievable by the optimum received signal-power distribution on BER. Many transmitted packets cause the degradation of the BER, which follows that the throughput cannot be maximized. Then, we discover the optimum received signal-power distribution on throughput can be obtained by the limitation of the number of the transmitted packets and the identical SINR.

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