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Performance Analysis of MC-CDMA System with and without Guard Interval in Two-Path Channel

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Abstract—**In this paper, we analyze the performance of the multi-carrier CDMA (MC-CDMA) system with and without guard interval (GI) in two-path channel. As results, we show that the BER performance of two cases are almost the same. This implies the removal of GI for the MC-CDMA system.**

I. INTRODUCTION

By the success of the indoor transmission of 100 Mbit/s based on multi-carrier code division multiple access (MC-CDMA), MC-CDMA draws much attention as a promising candidate for 4G access scheme [1]. MC-CDMA is the combination of orthogonal frequency division multiplexing (OFDM) and CDMA. It is considered for use in the downlink transmission as it holds features of OFDM, fast data transmission, and CDMA, multiple access capability.

Uplink transmission of MC-CDMA is also possible as MC-CDMA and direct-sequence CDMA (DS-CDMA) are dual system when occupied bandwidth and spreading factors are the same [2]. However, as users transmit asynchronously in the uplink, there are some interference such as inter-symbol interference (ISI) and multiple access interference (MAI). To overcome the effect of ISI, MC-CDMA systems usually adapt the guard interval (GI). Since CDMA has a potentiality of ISI and MAI resistance, the removal of GI may be possible for MC-CDMA systems.

In this paper, we derive the signal-to-noise-plus-interference ratio (SNIR) of MC-CDMA systems with and without GI. We evaluate the BER performance and consider the effect of GI in the uplink transmission. As results, we show that the case without GI shows almost the same BER performance as compared to the case with GI, while the delay time of the delayed path is less than one bit duration. This implies the removal of GI for the MC-CDMA system.

This paper is organized as follows: In Section II, the MC-CDMA system without GI is described. In Section III, we analyze its performance, and derive its SNIR. In Section IV, we consider the MC-CDMA system with GI and derive its SNIR. In Section V, numerical results are presented. Finally, in Section VI, a summary, some conclusions, and a discussion are reported.

II. MC-CDMA SYSTEM MODEL WITHOUT GI

A. Transmitter Model

Figure 1 shows the transmitter of the MC-CDMA system for user k with BPSK format. Note that we treat the baseband model throughout this paper. In the transmitter, the nth input information bit $d_k[n]$ of user k is copied and fed to N branches (corresponding to the number of subcarriers). Each branch is multiplied by one chip of the spreading code $c_k[l]$ $(l = 0, 1, \cdots, N-1)$ of user k with length N and the chip duration T_b which is the same as the bit duration, where l denotes the lth branch. Then it is modulated to a subcarrier separated from its neighboring subcarriers by $1/T_b$, and summed up. The transmitted signal of user k is written as

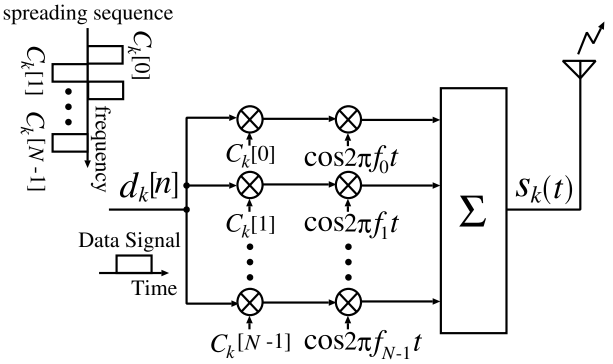

Fig. 1. Transmitter model of the MC-CDMA system for user k.

$$s_k(t) = \sum_{n=-\infty}^{+\infty} \sum_{l=0}^{N-1} d_k[n]c_k[l] \cos\left(2\pi \frac{l}{T_b} t\right) P(t - nT_b) \quad (1)$$

where $P(\cdot)$ is defined to be an unit amplitude pulse in the interval of 0 to T_b.

B. Channel Model

In a channel, we assume that K users transmit their own signals asynchronously, and there is a transmission delay $\tau_{k,\gamma} (0 \leq \tau_{k,\gamma} < T_b)$, where γ denotes the path number. The impulse response is written as

$$h_k(t) = \sum_{\gamma=1}^{\Gamma} \alpha_{k,\gamma} \delta(t - \tau_{k,\gamma}) \quad (2)$$

where Γ is the total path number, and $\alpha_{k,\gamma}$ is the γth path gain for user k which is a complex Gaussian random process with zero mean and variance σ_γ^2. It is mutually independent for different k and γ, and independent, identically distributed for different k. In this paper, we consider the uniform multipath power profile [3], that is,

$$\sigma_\gamma^2 = \frac{1}{\Gamma}. \quad (3)$$

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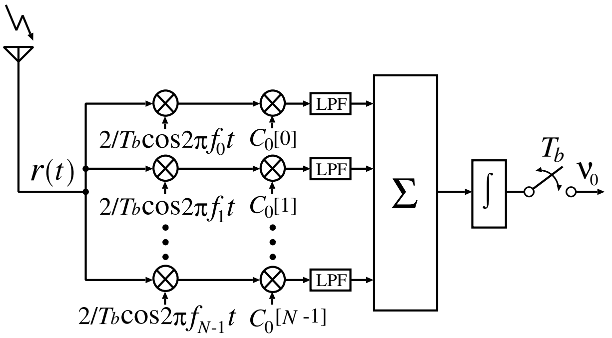

Fig. 2. Receiver model of the MC-CDMA system for user $k = 0$.

C. Receiver Model

Figure 2 shows the receiver of the MC-CDMA system for user $k = 0$. The received signal is written as

$$r(t) = \sum_{k=0}^{K-1} h_k(t) * s_k(t) + n(t) \tag{4}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=0}^{K-1} \sum_{l=0}^{N-1} \sum_{\gamma=1}^{\Gamma} \alpha_{k,\gamma} d_k[n] c_k[l] \cos\{2\pi f_l (t - \tau_{k,\gamma})\} \cdot P(t - \tau_{k,\gamma} - nT_b) + n(t). \tag{5}$$

In (4), $*$ is a convolution operation and $n(t)$ is the additive white Gaussian noise with zero mean and two-sided power spectral density of $N_0/2$.

In the receiver, the received signal is copied and fed to N branches. Each branch is demodulated and multiplied by one chip of the spreading code. Then the branches are summed up and integrated. Finally, the original data may be recovered.

III. PERFORMANCE ANALYSIS OF MC-CDMA WITHOUT GI

We consider the case that the data of user $k = 0$ is demodulated. We assume the equal gain combining (EGC), and $\alpha_{k,\gamma}$ is constant during one bit duration. If the time synchronization for user $k = 0$ is perfect ($\tau_{0,1} = 0, \tau_{0,\gamma\neq 1} \neq 0$), then the decision variable ν_0 for $n = 0$th bit is written as

$$\nu_0 = \sum_{m=0}^{N-1} \int_0^{T_b} \frac{2c_0[m]}{T_b} \cos\left(2\pi \frac{m}{T_b} t\right) r(t) dt$$

$$= S + \sum_{\gamma=2}^{\Gamma} D_{S\gamma} + \sum_{\gamma=1}^{\Gamma} I_\gamma + \eta \tag{6}$$

where S is a desired signal component from the desired user ($k = 0$) of the direct path ($\gamma = 1$), and $D_{S\gamma}$ is a signal component from the desired user of the other paths. The third term I_γ is an interference component from the other users ($k \neq 0$) of the γth path. The last term η is a noise component. Next, we show each term in detail.

A. Signal Component from the Desired User of Direct Path, S, and Noise Component, η

The first term S is written as

$$S = N\alpha_{0,1} d_0[0]. \tag{7}$$

The last term η is a zero-mean Gaussian random variable. The variance σ^2 of η is written as [2]

$$\sigma^2 = \frac{NN_0}{T_b}. \tag{8}$$

B. Interference Component from the Other Users, I_γ

The third term I_γ can be composed of two terms: $I_\gamma^{l=m}$, the same subcarrier ($l = m$) interference, and $I_\gamma^{l\neq m}$, the other subcarrier ($l \neq m$) interference. They are expressed as [2], [3]

$$I_\gamma^{l=m} = \sum_{k=1}^{K-1} \sum_{l=0}^{N-1} \frac{2\alpha_{k.\gamma}}{T_b} c_k[l] c_0[l] \left[d_k[-1] L^{l=m} + d_k[0] R^{l=m}\right] \tag{9}$$

$$I_\gamma^{l\neq m} = \sum_{k=1}^{K-1} \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \frac{2\alpha_{k,\gamma}}{T_b} \cdot c_k[l] c_0[m] \left[d_k[-1] L^{l\neq m} + d_k[0] R^{l\neq m}\right] \tag{10}$$

where

$$L^{l=m} = \int_0^{\tau_{k,\gamma}} \cos\{2\pi f_l (t - \tau_{k,\gamma})\} \cos(2\pi f_l t) dt = \frac{1}{2}\tau_{k,\gamma} \cos(2\pi f_l \tau_{k,\gamma}) \tag{11}$$

$$R^{l=m} = \int_{\tau_{k,\gamma}}^{T_b} \cos\{2\pi f_l (t - \tau_{k,\gamma})\} \cos(2\pi f_l t) dt = \frac{1}{2}(T_b - \tau_{k,\gamma}) \cos(2\pi f_l \tau_{k,\gamma}) \tag{12}$$

$$L^{l\neq m} = \int_0^{\tau_{k,\gamma}} \cos\{2\pi f_l (t - \tau_{k,\gamma})\} \cos(2\pi f_m t) dt = \frac{\sin(2\pi f_l \tau_{k,\gamma}) - \sin(2\pi f_m \tau_{k,\gamma})}{4\pi(f_l - f_m)} \tag{13}$$

$$R^{l\neq m} = \int_{\tau_{k,\gamma}}^{T_b} \cos\{2\pi f_l (t - \tau_{k,\gamma})\} \cos(2\pi f_m t) dt = \frac{\sin(2\pi f_m \tau_{k,\gamma}) - \sin(2\pi f_l \tau_{k,\gamma})}{4\pi(f_l - f_m)}. \tag{14}$$

We treat the information bit $d_k[n]$ and the delay time $\tau_{k,\gamma}$ of each user as mutually independent random variables. Then $I_\gamma^{l=m}$ and $I_\gamma^{l\neq m}$ can be considered as Gaussian random variables. Their means are zero, and variances are

$$\mathrm{Var}[I_\gamma^{l=m}] = \frac{4\sigma_\gamma^2}{T_b^2} \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{K-1} \sum_{l=0}^{N-1} \left\{(L^{l=m})^2 + (R^{l=m})^2\right\} d\tau_{k,\gamma} \tag{15}$$

$$\mathrm{Var}[I_\gamma^{l\neq m}] = \frac{4\sigma_\gamma^2}{T_b^2} \frac{1}{T_b} \int_0^{T_b} \sum_{k=1}^{K-1} \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \cdot \left\{(L^{l\neq m})^2 + (R^{l\neq m})^2\right\} d\tau_{k,\gamma}. \tag{16}$$

After some manipulations we get [2]

$$\mathrm{Var}[I_\gamma^{l=m}] = \frac{N(K-1)\sigma_\gamma^2}{3} \tag{17}$$

$$\mathrm{Var}[I_\gamma^{l\neq m}] = \frac{(K-1)\sigma_\gamma^2}{2\pi^2} \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \frac{1}{(l-m)^2}. \tag{18}$$

C. Signal Component from the Desired User of Delayed Path, $D_{S\gamma}$

The second term $D_{S\gamma}$ can also be composed of two terms: $D_{S\gamma}^{l=m}$, the same subcarrier ($l = m$) component, and $D_{S\gamma}^{l\neq m}$, the other subcarrier ($l \neq m$) component. They are expressed as

$$\begin{aligned} D_{S\gamma}^{l=m} &= \sum_{l=0}^{N-1} \frac{2\alpha_{0,\gamma}}{T_b} c_0[l] c_0[l] \Big[d_0[-1] L^{l=m}|_{k=0} \\ &\quad + d_0[0] R^{l=m}|_{k=0} \Big] \end{aligned} \tag{19}$$

$$\begin{aligned} D_{S\gamma}^{l\neq m} &= \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \frac{2\alpha_{0,\gamma}}{T_b} c_0[l] c_0[m] \Big[d_0[-1] L^{l\neq m}|_{k=0} \\ &\quad + d_0[0] R^{l\neq m}|_{k=0} \Big] \end{aligned} \tag{20}$$

where $L^{l=m}|_{k=0}$, $R^{l=m}|_{k=0}$, $L^{l\neq m}|_{k=0}$ and $R^{l=m}|_{k=0}$ are the same as (11) ~ (14) with $k = 0$, respectively.

In (19), the first term in the right-hand side is the ISI for the present data bit, $d_0[0]$, of the desired user, and the second term is the desired signal component. Since the information bit is random, $D_{S\gamma}^{l=m}$ can be considered as a Gaussian random variable that the mean and the variance are written as

$$\mathrm{E}\left[D_{S\gamma}^{l=m}\right] = \sum_{l=0}^{N-1} \frac{2\alpha_{0,\gamma}}{T_b} d_0[0] R^{l=m}|_{k=0} \tag{21}$$

$$\mathrm{Var}\left[D_{S\gamma}^{l=m}\right] = \left[\sum_{l=0}^{N-1} \frac{2\alpha_{0,\gamma}}{T_b} L^{l=m}|_{k=0} \right]^2 . \tag{22}$$

The other subcarrier component, $D_{S\gamma}^{l\neq m}$, is the interference component. Since the information bit $d_0[n]$ and the spreading code of the desired user are mutually independent random variables, each of the right-hand side of (20) can be considered as a Gaussian random variable. The mean is zero and the variance is written as

$$\begin{aligned} \mathrm{Var}[D_{S\gamma}^{l\neq m}] = \frac{4\alpha_{0,\gamma}^2}{T_b^2} \sum_{l=0, l\neq m}^{N-1} \sum_{m=0}^{N-1} \Big[\left(L^{l\neq m}|_{k=0} \right)^2 \\ + \left(R^{l\neq m}|_{k=0} \right)^2 \Big] . \end{aligned} \tag{23}$$

Let us define the interference component, Z, as

$$\begin{aligned} Z = \sum_{\gamma=1}^{\Gamma} \left\{ \mathrm{Var}[I_\gamma^{l=m}] + \mathrm{Var}[I_\gamma^{l\neq m}] \right\} \\ + \sum_{\gamma=2}^{\Gamma} \left\{ \mathrm{Var}[D_{S\gamma}^{l=m}] + \mathrm{Var}[D_{S\gamma}^{l\neq m}] \right\} . \end{aligned} \tag{24}$$

Then, the SNIR is

$$\mathrm{SNIR} = \frac{\left(S + \sum_{\gamma=2}^{\Gamma} \mathrm{E}\left[D_{S\gamma}^{l=m}\right] \right)^2}{Z + \sigma^2} . \tag{25}$$

The BER is obtained as

$$\mathrm{BER} = \frac{1}{2} \mathrm{erfc}(\sqrt{\mathrm{SNIR}/2}) . \tag{26}$$

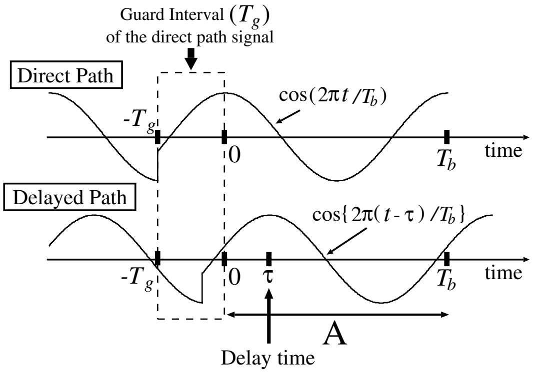

Fig. 3. Effect of ISI and InSI in the case with GI in two-path channel.

IV. PERFORMANCE ANALYSIS OF MC-CDMA WITH GI

In this section, we consider the MC-CDMA system with GI. As shown in Fig.3, in the case that the length of GI, T_g, is longer than the delay time τ of the delayed path, the effect of ISI for the desired user can be reduced by removing GI in the receiver side. But, the effect of inner-symbol interference (InSI) cannot be removed. When A part in Fig.3 is received, then the integral and dump filter may produce

$$\begin{aligned} \int_0^{T_b} \left[\left\{ \cos\left(2\pi \frac{t}{T_b}\right) + \cos\left(2\pi \frac{t-\tau}{T_b}\right) \right\} \cos\left(2\pi \frac{t}{T_b}\right) \right] dt \\ = \frac{T_b}{2} + \frac{T_b}{2} \cos\left(2\pi \frac{\tau}{T_b}\right) \end{aligned} \tag{27}$$

From the the second term in the right-hand side of (27) we can explain the InSI. Hence, (19) and (20) can be rewritten as

$$\tilde{D}_{S\gamma}^{l=m} = \alpha_{0,\gamma} d_0[0] \sum_{l=0}^{N-1} \cos(2\pi f_l \tau_{0,\gamma}) \tag{28}$$

$$\tilde{D}_{S\gamma}^{l\neq m} = 0, \tag{29}$$

the effect of InSI remains. As in (29), the other subcarrier interference of the delayed path for the desired user are removed by the orthogonality condition. Since the effect of interference from the other users cannot be removed, (9) and (10) remain the same. Thus, we can write the interference component $\tilde{Z}$ and the SNIR in this case as

$$\tilde{Z} = \sum_{\gamma=1}^{\Gamma} \left\{ \mathrm{Var}[I_\gamma^{l=m}] + \mathrm{Var}[I_\gamma^{l\neq m}] \right\} \tag{30}$$

$$\mathrm{SNIR_{GI}} = \frac{\left(S + \sum_{\gamma=2}^{\Gamma} \tilde{D}_{S\gamma}^{l=m} \right)^2}{\tilde{Z} + \sigma^2} . \tag{31}$$

V. NUMERICAL RESULTS

In this section, we demonstrate the effectiveness of above analysis results by computer simulations, which use system models of Fig.1 and Fig.2 and the channel model of (2). We evaluate the BER performance when the delay time changes.

First, we evaluate the BER performance of the MC-CDMA system without GI, where the next conditions are used.

- The number of users : $K = 1$.
- The number of subcarriers : $N = 64$.

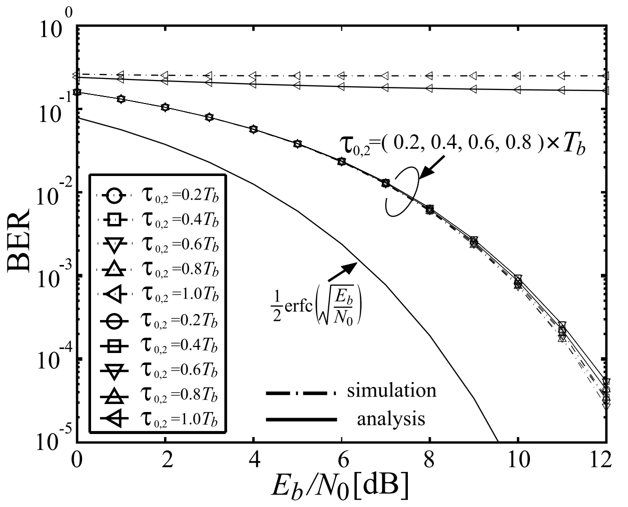

Fig. 4. Performance comparison of the MC-CDMA system without GI of simulation results (dot-dashed line) and analysis results (solid line).

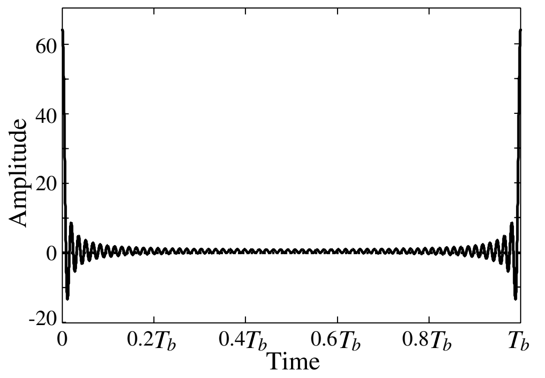

Fig. 5. MC-CDMA signal for N=64.

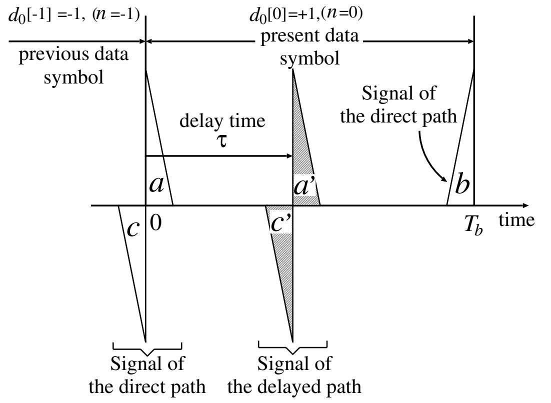

Fig. 6. Received signal of the MC-CDMA system without GI in two-path channel, where $d_0[0] = +1$ and $d_0[-1] = -1$.

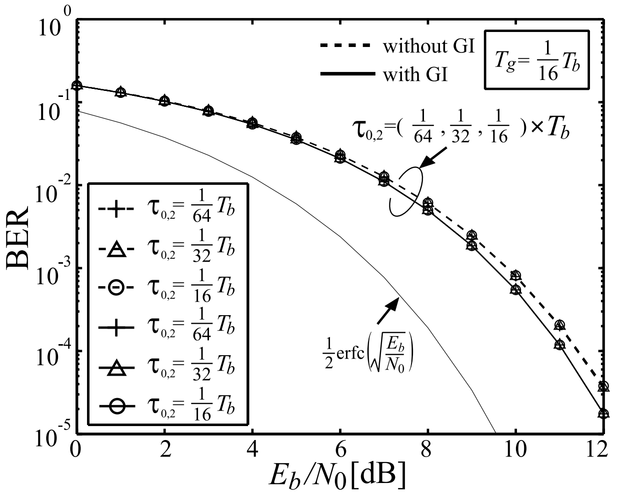

Fig. 7. Performance comparison of the MC-CDMA system with (solid line) and without (dot-dashed line) GI of analysis results.

- The number of multipaths : $\Gamma = 2$.
- The path gain : $\alpha_{0,1} = \alpha_{0,2} = 1$.

Figure 4 depicts the BER performance comparison of simulation results and analysis results for $\tau_{0,2} = (0.2, 0.4, \cdots, 1.0) \times T_b$. Analysis results agree with simulation results. It can be seen that the BER performance in the case without GI dose not change while the delay time is less than one bit duration. We can explain this reason using Fig.5 and Fig.6 as follows: The signal of MC-CDMA is composed of the superpositioning of N cosine waves which frequencies are orthogonal to each other as shown in Fig.5. There are peaks at time $t = 0$ and $t = T_b$ and the rest is much smaller amplitude, approximately zero. Figure 6 shows the received signal of the MC-CDMA system without GI in two-path channel, where the present data is $d_0[0] = +1$ and the previous data is $d_0[-1] = -1$. Here, a and b are peaks of signal corresponding to $d_0[0]$, c is a tail part of signal corresponding to $d_0[-1]$, and a' and c' are peaks of delayed path. From this figure, it is predictable that, although the delay time changes, the BER performance is almost invariant.

Next we evaluate the BER performance in the case with GI. From Fig.6, when the GI is removed, c' can be cut off, but a' remains. That is, the effect of ISI is removed but the effect of InSI still exists. In Fig.7, the BER performance in the case with and without GI of analysis results is compared for $\tau_{0,2} = (1/64, 1/32, 1/16) \times T_b$. The delay time is shorter than the length of GI, $T_g = T_b/16$. It can be seen that, although using GI, the BER performance is not satisfactory. The reason is that since MC-CDMA is robust to ISI, removing the effect of ISI dose not bring the good performance. However, when E_b/N_0 is big ($>$ 10 dB), the performance is a slightly better but the difference is almost negligible. Figure 8 represents the BER performance comparison in the case with GI of simulation results and analysis results. The delay time is the same as Fig.7. From these two results, it can be concluded that, while the delay time is less than one bit duration, the GI is not necessary for the MC-CDMA system.

In Fig.9 we show the BER performance comparison of the MC-CDMA system with and without GI for different user number (K) of analysis results. The delay time is the same as Fig.7. As shown in Fig.9, when the number of users increases, the GI dose not improve the system performance. Thus, we can say that, since the GI is not necessary in the uplink, the performance improvement techniques alternative to using GI should be considered.

Let us consider the case that the number of multipaths increases. Figure 10 shows the BER performance of the MC-CDMA system with and without GI in the case of 2 and 3 multipaths. We set $\sigma_\gamma^2 = 1/3$ and $\alpha_{0,\gamma} = \sqrt{2/3}$ and $\tau_{0,2} = T_b/64, \tau_{0,3} = T_b/32$, when the number of multipaths is three. The length of GI is $T_g = T_b/16$. It can be seen that the BER performance for 3 path channel with GI of $K = 1$ dose not change comparing with it for 2 path channel with GI of $K = 1$. When

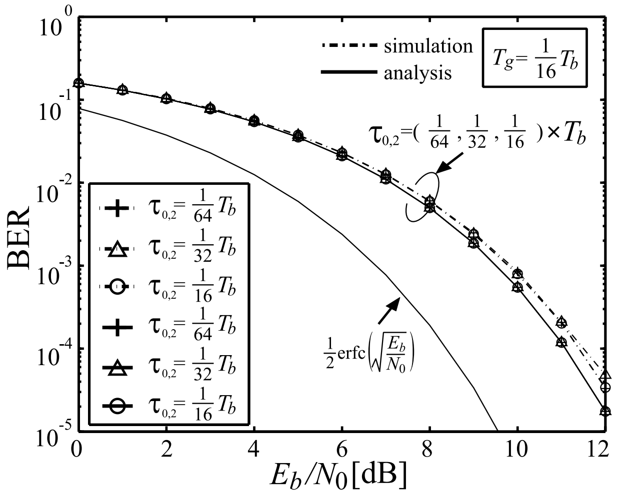

Fig. 8. Performance comparison of the MC-CDMA system with GI of simulation results (dot-dashed line) and analysis results (solid line).

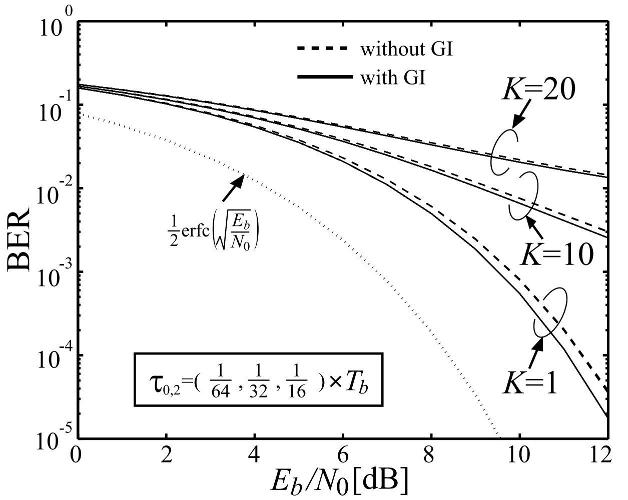

Fig. 9. Performance comparison of the MC-CDMA system with (solid line) and without (dot-dashed line) GI for different user number of analysis results.

the number of multipaths increases for $K = 10$ and $K = 20$, the BER performance without GI degrades and GI dose not improve.

From Fig.5, since the peak only appears at the beginning and the end of a symbol, we may write the signal component of the delayed path from the desired user in the case with GI, (28), as

$$\tilde{D}_S^{l=m} \simeq \begin{cases} \alpha_{0,2}d_0[0]N, & (\tau_{0,2}=0 \text{ or } \tau_{0,2}=T_b) \\ 0, & \text{otherwise.} \end{cases} \qquad (32)$$

If the number of multipaths increases, terms corresponding to the delayed paths for the desired user need to be added in (6). However those terms only take values at $\tau_{0,2} = 0$ and $\tau_{0,2} = T_b$, and zero in between. Thus, in the case $K = 1$ we can paraphrase that there are no additional terms due to the increase of number of multipaths. Therefore, it can be said that the BER performance is almost invariant although the number of multipaths increases for $K = 1$. As results, we can say that MC-CDMA is robust to the increase of multipath for $K = 1$. In the cases $K = 10$ and $K = 20$ without GI, the increase of multipaths results in producing excessive interference components from the other users, so the performance degrades more. Since the GI of the desired user cannot remove the effect of interference from the other users, the performances for 3 path channel with GI for $K = 10$ and $K = 20$ is not improved.

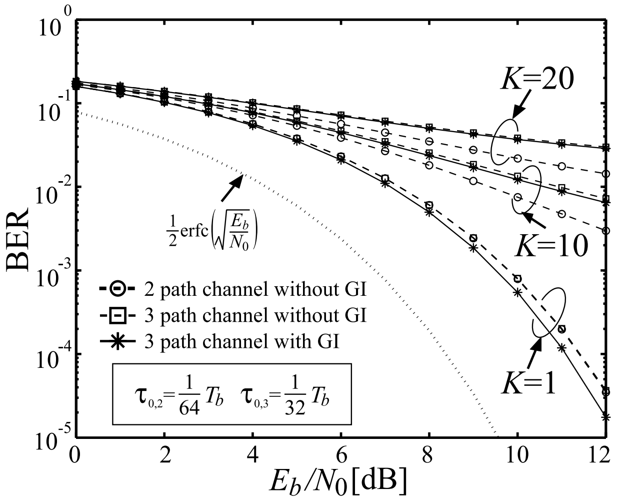

Fig. 10. Performance comparison of the MC-CDMA system with (solid line) and without (dashed line) GI for multipath numbers of analysis results.

VI. CONCLUSION

In this paper, we have analyzed the performance of the MC-CDMA system with and without GI. As results, we showed that the BER performance of two cases are almost the same.

In the uplink, when the number of users and multipaths increases, the BER performance degrades and GI dose not improve the performance. The performance improvement techniques alternative to using GI should be considered.

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