CDMA Unslotted ALOHA Systems with Finite Buffers

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Abstract—**CDMA unslotted ALOHA system with finite size of queueing buffers is discussed. We introduce an analytical model in which the system is divided into two Markov chains; one is in user part, and the other is in channel part. We analyze the system performance using this analytical model, and evaluate the effect of buffer capacity in terms of the throughput, the rejection probability and the average delay.**

I. Introduction

CDMA ALOHA systems have drawn much attention for satellite and mobile communications because of the features such as random access capability and potentiality of high throughput performance. Moreover, CDMA unslotted ALOHA (CDMA U-ALOHA) systems have the advantage of no need to synchronize the packet transmissions so that they initiate at the beginning of a slot. Many works have been made so far aiming at improving the system performance [1]–[3].

If each user station is equipped with a certain size of queueing buffers, it brings about not only the reduction of the rejection of the packet transmission but also the possibility of the autonomous control of the packet transmission. Therefore, we can expect an improvement in the grade of service in terms of higher throughput and lower rejection probability for the packet transmission.

In this paper, we consider the CDMA U-ALOHA system equipped with a certain size of queueing buffers so as to improve the grade of service.

Incidentally, various approaches and approximations have been studied on conventional (unspread) slotted ALOHA systems [4]–[7]. On the CDMA slotted ALOHA (CDMA S-ALOHA) system, we have studied the queueing analysis [8]. In all of previous discussions, however, both CDMA and conventional unslotted ALOHA systems have never been studied in consideration of the effect of packet queueing buffers.

We analyze the performance of the CDMA U-ALOHA system with finite size of queueing buffers by using the Markov chain analysis. The packets arriving at the user station with empty buffer are transmitted immediately to reduce the delay time, while other arriving packets and unsuccessful packets queue at each user's buffer and are transmitted from the top of the queue with a transmission rate p. In order to analyze the system performance, we introduce the analytical model in which the system is divided into two Markov chains. We evaluate the system performance in terms of the throughput, average delay and rejection probability, and clarify the effect of buffer capacity.

II. System Model

Figure 1 shows the system model of the CDMA U-ALOHA with finite size of buffers. The system consists of a single hub station and symmetric K user stations, each

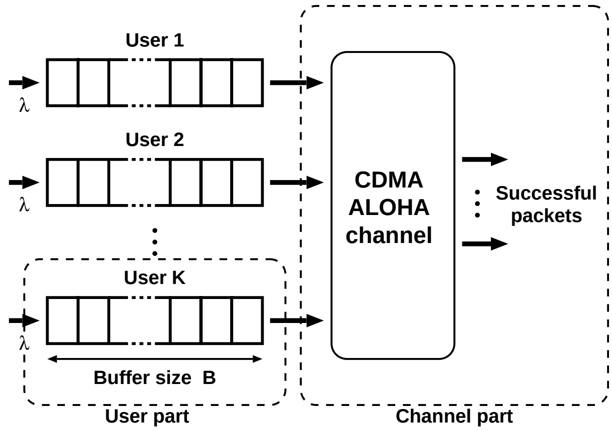

Fig. 1. System model of CDMA ALOHA system with finite size of buffers.

with a finite buffer capacity of B packets. Every user station transmits a packet to the hub station by one hop, and we consider only the packet access on the up-link. Each packet has a fixed length of a packet time duration $T_p = L/R$, where R is a data rate, L [bit] is a bit length of the packet.

The packet flow at each user station is shown in Fig.2. Every user station generates a packet following the Poisson process with a birth rate λ. We define a busy station as a station with a nonempty buffer, and an idle station as a station with an empty buffer. If a packet arrives at a user station with a full buffer, this packet is rejected. Otherwise the packet arriving at a busy station is stored at his buffer, while the packet arriving at an idle station is transmitted immediately to the channel to reduce the time delay and stored at his buffer as well. After successful transmission, the packet is removed from the buffer. Packets stored in a user station are served on a first-in-first-out (FIFO) discipline. Busy user stations attempt to transmit the packet asynchronously at the top of queue with a packet transmission rate p (transmitting interval is exponentially distributed with average $1/p$). The packet transmission rate is identical among the user stations. When the packet is transmitted to the channel and fails to be received correctly by the hub station, unsuccessful packet is retransmitted with a rate p.

Every packet is spectrum-spread with a uniquely assigned random signature sequence. We assume that all packets are received with the equal power and all data bit errors are caused by the effect of multiple access interference (MAI) and additive white Gaussian noise (AWGN). The bit error rate $P_b(k)$ is expressed as [9],

$$P_b(k) = \frac{2}{3}\,Q\left[\left(\frac{k}{3N} + \frac{N_0}{2E_b}\right)^{-0.5}\right]$$

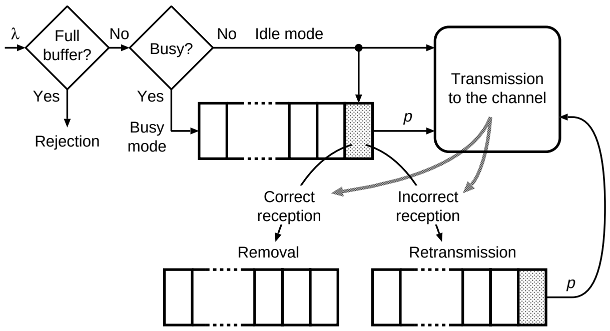

Fig. 2. Schematic of packet flow at each user station.

$$+\frac{1}{6}\,Q\left[\left(\frac{k\cdot N/3+\sqrt{3}\sigma}{N^2}+\frac{N_0}{2E_b}\right)^{-0.5}\right]$$
$$+\frac{1}{6}\,Q\left[\left(\frac{k\cdot N/3-\sqrt{3}\sigma}{N^2}+\frac{N_0}{2E_b}\right)^{-0.5}\right], \quad (1)$$

with

$$\sigma^2 = k\left\{N^2\frac{23}{360}+N\Big(\frac{1}{20}+\frac{k-1}{36}\Big)-\frac{1}{20}-\frac{k-1}{36}\right\} \quad (2)$$

where N is the number of chip per bit, k is the number of interfering packets, E_b is the bit energy of the signal, $N_0/2$ is two-sided spectral density of AWGN, and

$$Q[x] = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-u^2/2)\,du. \quad (3)$$

III. Queueing Analysis

A. Two Markov Chains

The packet success probability depends on the birth rate for actually transmitted packets. The birth rate is determined by the ratio of the number of the idle user stations to the number of all users. In other words, it is determined by the steady state probability of having no packets at each user station. The behavior of the user station depends on the elapsed time from transmitting a packet for the first time until receiving the packet correctly at the hub station. This elapsed time is determined by the packet success probability and the packet transmission rate p.

Let P_j be the steady state probability of having j packets at a certain user station. We assume that the system condition changes slowly enough to regard the steady state probability P_j and the packet success probability Q_S as constant. This assumption allows us to construct the analytical model in which the system is divided into two parts; one is in the user part, and the other is in the channel part, as shown in Fig.1.

In the user part, we can consider only a certain user's behavior because of symmetry of the system. In the user station, packets are generated according to the Poisson process, the number of outgoing channel is 1, and the number of stored packets which include a packet on service is B. Thus, we model the queueing behavior of the user station as a M/G/1/B queue taking account of the influence from other stations through the service time distribution. The service time is defined as the time duration from the instant that a packet arrives at the top of the queue until the successful transmission of its packet.

In the channel part, birth interval is exponentially distributed with average $1/\lambda$ at the idle user station or with average $1/p$ at the busy user station, service time (= packet length) is fixed, and the number of simultaneously transmitted packets is not limited. Thus, we can regard the number of simultaneously transmitted packets as an $\mathrm{M}_1+\mathrm{M}_2$/D/$\infty$//$K$ queue.

B. User Part

At first, we calculate the probability density function (pdf) of service time for packets. The user station which is initially in an idle mode transmits the new packet as soon as it arrives, while the user station which is initially in a busy mode transmits the stored packet with a rate p. Thus, the pdf of service time for the idle user station is different from that of the busy user station.

For the case of the user station which is initially in the idle mode, newly arriving packet is transmitted immediately. If the packet is transmitted successfully at first, the elapsed time is T_p and its probability becomes Q_S. Otherwise user station retransmits the packet until the packet transmission succeeds. After being transmitted a total of $m+1$ times, the elapsed time is $(m+1)T_p$ plus sum of m exponentially distributed transmitting intervals, and its probability is $(1-Q_S)^m Q_S$. The distribution of the sum of k exponentially distributions is k-Erlangian distribution. Therefore, the pdf of the service time for this case is

$$d_I(t) = Q_S\cdot\delta(t-T_p) + \sum_{m=1}^{\infty}(1-Q_S)^m Q_S\cdot E_m(t-(m+1)T_p;p/m) \quad (4)$$

where $\delta(t)$ is a delta function, $E_k(t;\nu)$ is the pdf of k-Erlangian distribution with an average $1/\nu$, defined as,

$$E_k(t;\nu) \equiv \begin{cases}\dfrac{(k\nu t)^{k-1}}{(k-1)!}k\nu e^{-k\nu t} & (t\geq 0)\\ 0 & (t<0)\end{cases}. \quad (5)$$

By averaging $d_I(t)$, the average service time $1/\mu_I$ is derived as

$$1/\mu_I = T_p + \frac{T_p+1/p}{Q_S}(1-Q_S). \quad (6)$$

For the case of the user station which is initially in the busy mode, the packet is transmitted with a rate p. Each packet transmission occurs after exponentially distributed transmitting interval. After being transmitted a total of m times, the elapsed time is mT_p plus sum of m exponentially distributed transmitting intervals for m transmissions, and this probability is $(1-Q_S)^{m-1}Q_S$. Thus, the pdf of the service time for this case is

$$d_B(t) = \sum_{m=1}^{\infty}(1-Q_S)^{m-1}Q_S\cdot E_m(t-mT_p;p/m), \quad (7)$$

and the average service time $1/\mu_B$ is derived as

$$1/\mu_B = \frac{T_p+1/p}{Q_S}. \quad (8)$$

To calculate the steady state probability of having j packets at the user station, we refer to the analytical method of a steady state probability for an M/G/1/B queue [11]. Let $D_I(s)$ and $D_B(s)$ be the Laplace-Stieljes Transform (LST) of the $d_I(t)$ and $d_B(t)$, respectively. These are obtained as,

$$D_I(s) = \sum_{m=0}^{\infty}(1-Q_S)^m Q_S \cdot e^{-(m+1)T_p s}\left(\frac{p}{s+p}\right)^m \quad (9)$$

$$D_B(s) = \sum_{m=1}^{\infty}(1-Q_S)^{m-1} Q_S \cdot e^{-mT_p s}\left(\frac{p}{s+p}\right)^m \quad (10)$$

Let p_{Ij} be the probability that j packets are arriving at the user station which is initially in an idle mode during the service time. This probability p_{Ij} is obtained as [11],

$$p_{Ij} = \frac{1}{j!}\lim_{z\to 0}\frac{\partial^j}{\partial z^j}D_I([1-z]\lambda). \quad (11)$$

Similarly, the probability that j packets are arriving at the user station which is initially in the busy mode during the service time is

$$p_{Bj} = \frac{1}{j!}\lim_{z\to 0}\frac{\partial^j}{\partial z^j}D_B([1-z]\lambda). \quad (12)$$

From (11) and (12), the probability of having j packets at a certain user station immediately after completion of a packet transmission is derived as the asymptotical equation:

$$\Pi_{j+1} = (\Pi_j - p_{Ij}\Pi_0 - \sum_{m=1}^{j} p_{Bj-m+1}\Pi_m)p_{B0}^{-1} \quad (j=0,1,\cdots,B-2) \quad (13)$$

and the normalized condition:

$$\sum_{j=0}^{B-1}\Pi_j = 1. \quad (14)$$

In (13), we set $C_j = \Pi_j/\Pi_0$, and obtain the following asymptotical equations:

$$C_{j+1} = (C_j - p_{Ij} - \sum_{m=1}^{j} p_{Bj-m+1}C_m)p_{B0}^{-1} \quad (j=0,1,\cdots,B-2) \quad \text{(15-a)}$$

$$C_0 = 1 \quad \text{(15-b)}$$

We also set

$$C = 1 + \sum_{j=1}^{B-1} C_j. \quad (16)$$

The steady state probability P_j is expressed in terms of Π_j [11] as the following equations:

$$P_j = \begin{cases} \dfrac{\Pi_j}{\Pi_0 + a} & (j=0,1,\cdots,B-1) \\ 1 - \dfrac{1}{\Pi_0 + a} & (j=B) \end{cases} \quad (17)$$

where a is the traffic intensity at the user station, expressed as

$$a = \lambda \cdot \{\Pi_0/\mu_I + (1-\Pi_0)/\mu_B\}. \quad (18)$$

From (15)–(18), we obtain the following equations:

$$P_j = \begin{cases} \dfrac{C_j}{1+aC} & (j=0,1,\cdots,B-1) \\ 1 - \dfrac{C}{1+aC} & (j=B) \end{cases} \quad (19)$$

where aC is expressed as

$$aC = \frac{\lambda}{\mu_I} + \frac{\lambda}{\mu_B}\cdot(C-1). \quad (20)$$

By setting $j = 0$ in (19), the ratio of the number of the idle user stations to the number of all users is

$$P_0 = \frac{C_0}{1+aC} = \frac{1}{1+aC}. \quad (21)$$

C. Channel Part

In the channel part, we can regard the number of simultaneously transmitted packets as an $\mathrm{M}_1 + \mathrm{M}_2$/D/$\infty$//$K$ queue. The steady state probability of an $\mathrm{M}_1 + \mathrm{M}_2$/D/$\infty$//$K$ queue is equal to that of an M/M/∞//K queue [11]. We analyze the packet success probability by using the analytical method of CDMA U-ALOHA system with fixed packet length and finite population assumptions [3].

Let P_0^* be the ratio of the number of the users which transmit the packet with rate λ to the number of all users. We define the initially on-transmitting user as the user which is on transmitting the packet for the first time when it was initially in an idle mode. The user station which transmits the packet with rate λ is both idle user station and initially on-transmitting user station. The ratio of the number of the idle user stations is P_0. We assume that the probability of having j packets at the user station is independent of time. Under this assumption, the probability of having more than one packet at user station when it was initially in an idle mode is $(1-P_0)P_0$. The ratio of transmitting time for the first attempt to the time spent in transmitting the packet successfully is

$$b = \sum_{m=0}^{\infty}\frac{T_p}{T_p + m(Tp + 1/p)}(1-Q_S)^m Q_S. \quad (22)$$

The ratio of the number of the initially on-transmitting user stations is $(1-P_0)P_0 \cdot b$. Thus, P_0^* is derived as

$$P_0^* = P_0 + (1-P_0)P_0 \cdot b. \quad (23)$$

The birth rate for actually transmitted packets λ_c is expressed as

$$\lambda_c = \lambda P_0^* + p(1-P_0^*). \quad (24)$$

Let G_{sys} be the average number of packets transmitted to the channel. From the steady state probability of an M/M/∞//K queue [10], we obtain G_{sys} as

$$G_{sys} = \sum_{m=0}^{K} m \cdot \frac{(\lambda_c/\mu_c)^m \binom{K}{m}}{(1+\lambda_c/\mu_c)^K} = \frac{G_c}{1+G_c/K} \quad (25)$$

where $\mu_c = 1/T_p$ and $G_c = K\cdot\lambda_c\cdot T_p$.

We assume that the channel load is constant over a tiny duration Δt, where we set Δt a bit interval. Moreover, because of Poisson packet arrival, two or more packets hardly arrive simultaneously, we assume that the channel load changes by ± 1 at most between adjacent Δt's.

Let $P_S(k,i)$ be the probability that the packet is transmitted successfully from the first bit to the $(i-1)$th bit, and the number of interfering packets on the ith bit is k.

Case $i = 1$; By using the steady state probability of an M/M/∞//K queue, we obtain $P_S(k,i)$ as,

$$P_S(k,i) = \begin{cases} \dfrac{(\lambda_c/\mu_c)^k \begin{pmatrix} K-1 \\ k \end{pmatrix}}{(1+\lambda_c/\mu_c)^{K-1}} & ;\text{if } k < K \\ 0 & ;\text{otherwise} \end{cases} \tag{26}$$

Case $i > 1$; We obtain $P_S(k,i)$ as the following.

(a) $k < K-1$

$$\begin{aligned} P_S(k,i) &= P_S(k,i-1) \\ &\cdot\{1 - k\mu_c\Delta t - (K-1-k)\lambda_c\Delta t\} \cdot \{1 - P_b(k)\} \\ &+ P_S(k+1,i-1) \cdot (k+1)\mu_c\Delta t \cdot \{1 - P_b(k+1)\} \\ &+ P_S(k-1,i-1) \cdot (K-k)\lambda_c\Delta t \cdot \{1 - P_b(k-1)\} \end{aligned} \tag{27-a}$$

(b) $k = K-1$

$$\begin{aligned} P_S(k,i) &= P_S(k,i-1) \cdot \{1 - k\mu_c\Delta t\} \cdot \{1 - P_b(k)\} \\ &+ P_S(k-1,i-1) \cdot (K-k)\lambda_c\Delta t \cdot \{1 - P_b(k-1)\} \end{aligned} \tag{27-b}$$

(c) $k > K-1$

$$P_S(k,i) = 0 \tag{27-c}$$

The throughput, which is defined as the mean number of successful packets in a packet time duration, is

$$S = G_{sys} \sum_{k=0}^{\infty} P_S(k,L) \cdot (1 - P_b(k)). \tag{28}$$

Accordingly, the packet success probability Q_S is

$$Q_S = S/G_c. \tag{29}$$

D. Combination of the User Part and the Channel Part

We solve the simultaneous equations (21) and (29) derived in the user part and channel part, respectively. If the packet success probability Q_S is 0, the steady state probability of having no packets at a user station P_0 will become 1 because all packets are not transmitted successfully and not removed from each user's buffer. If Q_S is 1, P_0 will come close to 0 because all packets are transmitted successfully and not stored at each user's buffer. Accordingly, the equations (21) and (29) must have some solutions. If the equations have more than one solution, we use the solution with the smallest value of the throughput by the analytical tool called Equilibrium Point Analysis (EPA) [12]. In such case, the system exhibits the bistable behavior and the performance curves change discontinuously.

We substitute (21) into (29), and solve the simultaneous equations by the appropriate algorithm, such as the bisection.

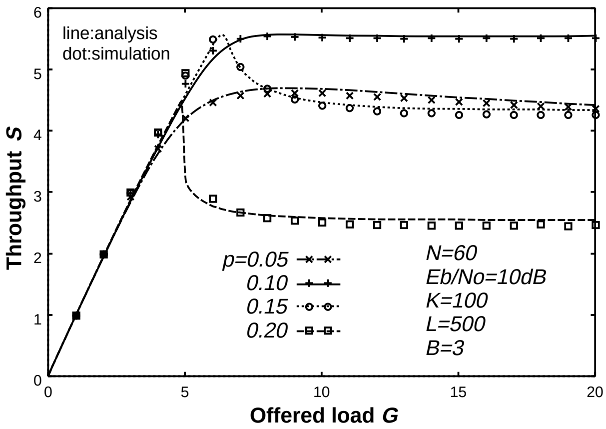

Fig. 3. Throughput versus offered load curves for $B = 3$.

By using this solution, we can obtain the system performance. The throughput is derived as (28). Rejection probability Q_R, which is the probability that a new packet arriving at a user station is rejected because its buffer is full, is

$$Q_R = P_B = 1 - \frac{C}{1+aC}. \tag{30}$$

By Little's formula, the average delay is derived as

$$D = \frac{\overline{Q}}{S/K} \tag{31}$$

where $\overline{Q}$ is the average queue length, derived as

$$\overline{Q} = \sum_{j=1}^{B} j \cdot P_j. \tag{32}$$

IV. Numerical Examples

Figure 3 shows the throughput versus offered load with the parameters of $N = 60$, $E_b/N_0 = 10$ [dB], $K = 100$, $L = 500$ [bits], and $B = 3$. Offered load G is defined as the average number of packets arriving in the system during one packet time duration, expressed as $G = K \cdot \lambda \cdot T_p$. The packet time duration T_p is normalized to 1. Simulated results are also plotted in these figures. In the simulations, based on the system model described in Sect.II, K user stations generate the packets. Those are stored at their own buffers and transmitted to the channel. We can find that analytical results almost agree with simulated results.

For $p \leq 0.1$, the throughput gradually and monotonously increases. The larger the transmission rate p becomes, the higher the throughput is. But, for $p > 0.1$, the throughput curves are in the shape of a convex cap. This tends to be more remarkable for larger p. These reasons are as follows. The throughput depends on the birth rate for actually transmitted packets λ_c in the channel part. The throughput curve as a function of λ_c is in the shape of a convex cap [3], i.e. there is the maximum value of the throughput S_{max}. In this condition, $S_{max} = 5.6$ when $\lambda_c = 0.096$. From (24), λ_c varies from λ to p with P_0 decreasing. We also expect that P_0 will decrease from 1 to 0 with G increasing because large offered load brings about the degrade of the packet success probability. Thus, λ_c comes close to p with the offered

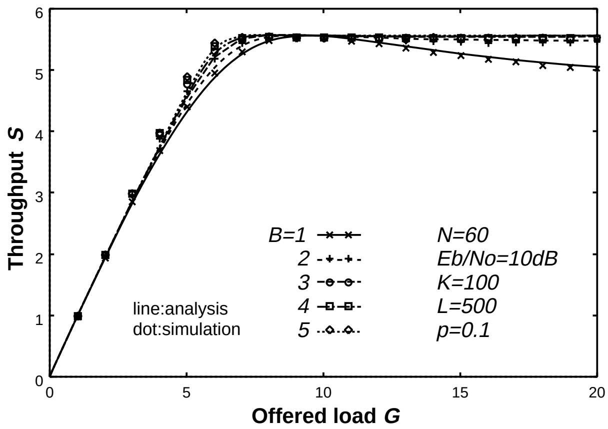

Fig. 4. Throughput versus offered load curves for $p = 0.1$.

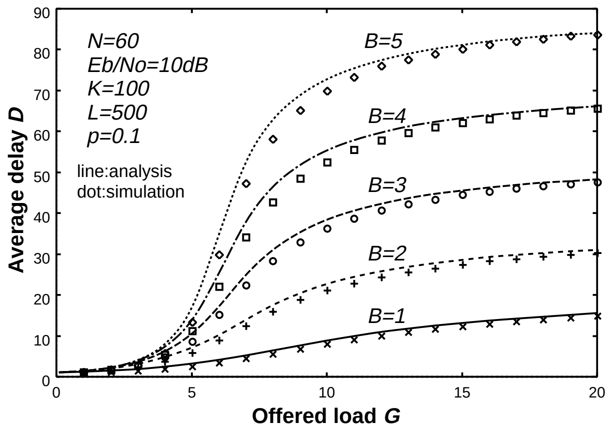

Fig. 6. Average delay versus offered load curves for $p = 0.1$.

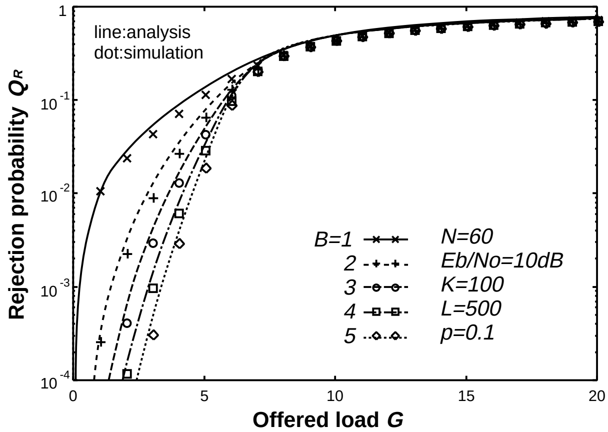

Fig. 5. Rejection probability versus offered load curves for $p = 0.1$.

load increasing. Within the region $p \leq 0.096$, packets are more rapidly processed for larger p, because λ_c will be less than 0.096 and the throughput will not be more than S_{max}. Within the region $p > 0.096$, however, because user stations attempt to transmit packets over the channel capacity, the number of retransmitted packets is increasing and system performance is rapidly degraded.

Figures 4-6 show the throughput, the rejection probability and the average delay versus offered load with $B =$ 1-5 and $p = 0.1$. The larger buffer size a user station has, the more rapidly the throughput is increasing for the region of small offered load. For the case $p = 0.1$, if many user stations become in the busy mode, λ_c will come close to 0.1 and the throughput will become near to the maximum value. The number of busy user stations increases by increasing the buffer size, because each user station can keep more packets. It can be seen from Fig.5 that the rejection probability is improved by increasing the buffer size. From Fig.6, large buffer size causes an increase of the average delay. It is, however, not serious because the average delay increases in compensation for reduction of the rejection of the packet transmission.

V. Conclusions

CDMA U-ALOHA system with finite size of queueing buffers has been discussed. We introduced the analytical model in which the system was divided into two Markov chains and analyzed the system performance. Based on these results, we clarified the effect of buffer capacity in terms of the throughput, the average delay and the rejection probability.

When we set the transmission rate so that the throughput takes the maximum value at large offered load, increase in the buffer size brings about an improvement of the throughput and lower rejection probability. Large buffer size causes an increase of the average delay. It is, however, not serious because the average delay increases in compensation for reduction of the rejection probability.

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