Convolutionally Coded DS/CDMA System using Multi-Antenna Transmission

K.Ban†, M.Katayama†, W.E.Stark††, T.Yamazato†, and A.Ogawa†
† Dept. of Information Electronics, Nagoya University, Japan.
†† Electrical Engineering and Computer Science Dept., University of Michigan, U.S.A.

Abstract— In this paper, we discuss the use of convolutional codes with a multi-antenna transmission scheme for DS/CDMA systems. The binary input data to a rate $1/M$ encoder produces M coded bits, which, in turn, are assigned to M different antennas and transmitted from each antenna simultaneously. An intentional delay of several chips duration is introduced at each antenna before transmission, which enables a receiver to distinguish the signals from different antennas. Because the proposed scheme utilizes spatial and time domains for coding, it can achieve not only implicit time-diversity through the use of coding with interleaving, but also space-diversity through the transmission from multiple antennas. Multi-antenna schemes with convolutional codes can perform better than conventional single antenna schemes with the same codes and transmission diversity technique with the same number of transmitting antennas, especially when a fading is relatively slow and interleaving size is limited.

I. INTRODUCTION

Recently, there has been increased interest in the use of direct-sequence spread spectrum multiple access (DS/SSMA), or code division multiple access (DS/CDMA), systems for indoor wireless local area network (LAN), since it has the potential advantage of inherent diversity against multipath fading. In indoor communications, however, the delay spread is often very small, and path diversity is not always available. In [1] a diversity technique which employs multiple transmitting antennas (Tx antennas) to combat frequency nonselective fading was considered. In this technique, the transmitter sends a signal bearing the same information from multiple antennas at the same time with intentional differential time delays, which allows a receiver to distinguish and combine the signals from different transmitting antennas. A very similar concept of the above multi-antenna scheme, distributed antenna, was formally proposed in [2]. These transmission diversity techniques are alternatives to conventional receiving antenna diversity and path diversity.

Besides the above diversity techniques, the use of forward error correcting (FEC) codes is another method to improve the performance in DS/CDMA systems. In the case of using FEC codes in fading channels, statistical behavior of each coded symbol should be independent to achieve the full potential error correction capability. Most work dealing with FEC codes in fading channels assumes sufficiently large interleaving depth to make the fading experienced by each coded symbol independent of the fading of other code symbols. In indoor channels, however, a fading process is often very slowly changing and interleaving techniques are impractical when there are stringent delay constraints. Hence, FEC codes will not be as effective for an indoor communication systems with delay constraints.

Another method to realize statistical independence in each coded symbol is a multichannel transmission technique, where each component channel is distinguished by frequency slot and/or code. In the case of multiple codes, e.g. multicode CDMA[3], an unique orthogonal code is assigned to each channel and every channel shares the same time and frequency. Hence, additive background noise in each channel can be treated as independent, but a fading process is common for every channel. In the case of multiple frequencies , e.g. multicarrier CDMA[4], it requires a separation of more than the coherent bandwidth between two carriers to realize independent fading channels. In general, the coherent bandwidth in indoor channels is on the order of 10 MHz or more[5], which makes it almost impossible to achieve independent fading under frequency limited situations.

The basis of the transmission diversity technique is that the signals sent from different antennas have different fadings. Thus another solution to obtain statistically independent fading channels is multi-antenna transmission. In this new scheme, the coded bits are sent in spatial domain rather than time, code or frequency domain, and it makes the utilization of FEC codes in indoor fading channels possible. Note that the standard transmission diversity scheme can be regarded as a system with a trivial repetition code as the FEC code.

In this paper, we consider the combination of the multi-antenna transmission scheme and rate $1/M$ convolutional codes in DS/CDMA systems. The introduction of the convolutional codes promises additional coding gains in comparison with repetition codes. In this system, the M coded bits are assigned to M different antennas and transmitted from each antenna simultaneously. We can expect some coding gain even if the interleaving is not used because the signal from the different antenna experiences an independent fading. In the analysis, we shall consider the very slow fading channel, meaning the assumption that the channel state is stationary during the decoding span. For reference, we shall consider a memoryless fading channel, which is equivalent to the assumption of infinite (ideal) interleaving situations.

II. SYSTEM MODEL

We consider an asynchronous DS/CDMA system consisting of K users transmitting asynchronously over a frequency nonselective Rayleigh fading channel. The transmitter of the kth user in the proposed scheme is illustrated in Fig. 1. Each transmitter employs M Tx antennas. In this figure, $a^k(t)$ represents a spreading signal for the kth user. The binary data b_i^k of duration T enters a rate $1/M$ convolutional encoder at the ith data bit interval, and produces M coded bits $\{x_{i,1}^k, ..., x_{i,M}^k\}$ in the transmitter of the kth user. Note that a coded bit duration and a data bit duration is the same, which differs from conventional convolutionally coded schemes which employs a single Tx antenna. Each of the M coded bits is assigned to different antenna respectively, and then transmitted from mth antenna of kth user's transmitter with an intentional delay d_m^k to distinguish the signals form different antennas. We define, without loss of generality, $d_1^k = 0$ and $d_1^k < d_2^k < ... < d_M^k < T$ as in [1]. Figure 2 shows the structure of the receiver for the kth user. The receiver demodulates a received signal coherently and decodes using Viterbi-Algorithm with perfect channel state information (CSI).

Let us consider the situation that the receiver demodulates the signal sent from the nth antenna of the 1st user's transmitter. The correlator output corresponding to this signal at some bit interval i becomes

$$Y_{i,n} = \sqrt{E_s}\{\beta_{i,n}^1 x_{i,n}^1 + S_{i,n} + I_{i,n}\} + \eta_{i,n}, \qquad (1)$$

where E_s is the energy of a coded bit and then the energy of a data bit is given by $E_b = ME_s$. The first term in (1) is the desired signal component and the last term is the zero-mean Gaussian random variable due to additive white Gaussian noise(AWGN) with two sided power spectral density of $N_0/2$. The second term in (1) denotes the $M-1$ self-interference terms due to the signals from the other antennas of the 1st user's transmitter and given by

$$S_{i,n} = \frac{1}{T}\sum_{\substack{m=1\\ m\neq n}}^{M} \beta_{i,m}^1 \cos\phi_m^1 \cdot \left[x_{i-1,m}^1 R_{1,1}(\Delta_{m,n}^1) + x_{i,m}^1 \hat{R}_{1,1}(\Delta_{m,n}^1)\right]. \qquad (2)$$

The third term in (1) is for the multiuser interference from the $K-1$ other simultaneous users with M antennas and given by

$$I_{i,n} = \frac{1}{T}\sum_{k=2}^{K}\sum_{m=1}^{M} \beta_{i,m}^k \cos\phi_m^k \cdot \left[x_{i-1,m}^k R_{k,1}(\Delta_{m,n}^k) + x_{i,m}^k \hat{R}_{k,1}(\Delta_{m,n}^k)\right], \qquad (3)$$

where $\beta_{i,m}^k$ and ϕ_m^k is a path gain and a phase which corresponds to the signal from the mth antenna of the kth user's transmitter at ith data bit interval respectively. We assume that $\beta_{i,m}^k$ is a Rayleigh random variable normalized to satisfy $E[|\beta_{i,m}^k|^2] = 1$ and ϕ_m^k is uniform in $[0, 2\pi)$. The R_{k_1,k_2} and $\hat{R}_{k_1,k_2}$ are even and odd continuous-time partial cross(auto)-correlation function defined in [6]. The term $\Delta_{m,n}^k$ in (2)-(3) is defined as $\Delta_{m,n}^k = \tau^k + d_m^k - d_n^1$, where τ^k is the propagation delay of the kth user's signal and assumed to be uniformly distributed in $[0, T)$. Note that the term $\Delta_{m,n}^k$ for $k \neq 1$ is a random variable on the basis of the fact that τ^k for $k \neq 1$ is uniform in $[0, T)$. The $\Delta_{m,n}^1$ in the self-interference term, however, is deterministic. The good selection of the delays $\{d_1^1, ..., d_M^1\}$ has been discussed in [1].

III. PERFORMANCE ANALYSIS

A. Pairwise Error Probability

For simplicity, we shall drop off the superscript identifying each user, and we assume that the interference including the self-interference and the multiuser interference is Gaussian random variable with variance σ_Z^2.

Under this assumption and the conditioning on the CSI, the pairwise error probability $P(\mathbf{x} \to \tilde{\mathbf{x}})$ that the Viterbi-decoder decides in favor of an erroneous sequence $\tilde{\mathbf{x}}$ instead of the transmitted coded bit sequence $\mathbf{x}$ is given by

$$P(\mathbf{x} \to \tilde{\mathbf{x}} | \{\beta_{i,m}\}) = Q\left(\sqrt{2\gamma_s \sum_{(i,m)\in\nu} (\beta_{i,m})^2}\right), \qquad (4)$$

where $\gamma_s = \left(\frac{N_0}{E_s} + 2\sigma_Z^2\right)^{-1}$ and ν is the set of (i, m) such that $x_{i,m} \neq \tilde{x}_{i,m}$.

A.1 Very Slow Fading Channel

The fading process in an indoor environment is often very slow so that the coherent time may last for a long bit interval. In practical systems, interleaving size is limited, and thus the long duration of very low signal-to-noise ratio caused by deep fading can't be scattered in time. In the multi-antenna schemes, however, a signal transmitted from each antenna experiences different fading. For simplicity, we assume that the path gain is constant during the decoding span of a sequence sequence $\mathbf{x}$, and we shall drop off the subscript i in the path gain $\beta_{i,m}$.

Define the number of different coded bits which are assigned to mth antenna between $\mathbf{x}$ and $\tilde{\mathbf{x}}$ as $H_m = \sum_i |x_{i,m} - \tilde{x}_{i,m}|/2$. This H_m represents the Hamming Distance in the mth antenna. Consequently, the sum of H_m becomes the Hamming distance H between $\mathbf{x}$ and $\tilde{\mathbf{x}}$. Substituting H_m into (4) yields

$$P(\mathbf{x} \to \tilde{\mathbf{x}} | \{\beta_m\}) = Q\left(\sqrt{2\gamma_s \sum_{m=1}^{M} (\beta_m)^2 H_m}\right). \qquad (5)$$

Averaging (5) over $\{\beta_m\}$ gives [7]

$$P(\mathbf{x} \to \tilde{\mathbf{x}}) = \frac{1}{2}\sum_{m=1}^{M} \pi_m \left[1 - \sqrt{\frac{H_m\gamma_s}{1 + H_m\gamma_s}}\right], \qquad (6)$$

where

$$\pi_m = \prod_{\substack{i=1\\ i\neq m}}^{M} \frac{H_m}{H_m - H_i}. \qquad (7)$$

When $\gamma_s >> 1$, (6) can be approximated as follows

$$P(\mathbf{x} \to \tilde{\mathbf{x}}) \simeq \binom{2M^{'}-1}{M^{'}} \left(\frac{1}{4\gamma_s}\right)^{M^{'}} \prod_{m=1}^{M^{'}} \frac{1}{H_m}, \qquad (8)$$

where $M^{'}$ is the number of non-zero H_m and $M^{'}$ means the diversity order. To achieve the Mth order diversity by M-antenna schemes, every H_m must not be zero in each error event.

In the case of a single antenna scheme, (6) can be replaced by

$$P(\mathbf{x} \to \tilde{\mathbf{x}}) = \frac{1}{2}\left[1 - \sqrt{\frac{H\gamma_s}{1+H\gamma_s}}\right]. \qquad (9)$$

A.2 Memoryless Fading Channel

The assumption of memoryless fading channel is tantamount to assuming that the interleaving size is infinite. Under this assumption, the $\beta_{i,m}$'s are independent random variables, and hence, taking the expectation of (4) with respect to $\{\beta_{i.m}\}$ gives [7]

$$P(\mathbf{x} \to \tilde{\mathbf{x}}) = \left[\frac{1}{2}\left(1 - \sqrt{\frac{\gamma_s}{1+\gamma_s}}\right)\right]^H \cdot \sum_{k=0}^{H-1} \binom{H-1+k}{k} \left[\frac{1}{2}\left(1 + \sqrt{\frac{\gamma_s}{1+\gamma_s}}\right)\right]^k. \qquad (10)$$

B. Bit Error Rate Performance

An approximation of the bit error probability can be obtained by summing the pairwise error probabilities as follows

$$P_b \simeq \sum_{\tilde{\mathbf{x}} \neq \mathbf{x}} w(\mathbf{x}, \tilde{\mathbf{x}}) P(\mathbf{x} \to \tilde{\mathbf{x}}), \qquad (11)$$

where $w(\mathbf{x}, \tilde{\mathbf{x}})$ is the number of bit errors associated with each error event, and the summation is taken over the set of dominant (or most probable) error events. If there are any error events which have zero H_m, i.e. $M^{'} < M$ in (8), the error events having smallest $M^{'}$ are most dominant. Hence, the smallest value of $M^{'}$ means the achievable diversity order of the code. It should be noted that one should avoid the codes which cannot achieve the Mth order diversity by M-antenna, because repetition codes always achieve the Mth order diversity. However, it is easy to show that almost all so-called good codes satisfies the condition $M^{'} = M$ for every error event.

Under the condition that the diversity order of the code is M in M-antenna schemes, it is apparent from (8) that error events having small $\prod_{m=1}^{M} H_m$ are dominant when a fading process is very slow and no interleaving is used. The transfer function $T(D_1, ..., D_M, N)$ can provide us with Hamming Distance in each antenna H_m(and product of them $\prod_{m=1}^{M} H_m$) in each error event[4].

In the case of single-antenna schemes, memoryless fading channels or AWGN channels, it is clear from (9), (10) that the error events having small Hamming Distance are dominant.

IV. NUMERICAL RESULTS

In this section, the bit error rate(BER) performance of convolutionally coded DS/CDMA systems with multi-antenna transmission scheme are presented. The convolutional codes we have examined are maximum free distance codes, which are presented in [7]. For the purpose of a comparison, the BER performance of uncoded(or repetition coded) systems with multi-antenna transmission, which means transmission diversity technique, and conventional convolutionally coded systems with single-antenna transmission are also presented.

The analytical results of 4-state convolutional codes obtained from (11) with (8) or (9) are shown in Fig. 3. In the analysis, we assumed the self-interference to be zero because it can be negligible small by proper selection of the delays at the transmitter[1]. Figure 3 shows two extreme cases: one is the very slow fading where the randomization by the interleaver cannot be expected and the other is the memoryless channel which models the ideal infinite interleaving size. Comparing the results of these two cases, we can confirm that the performance under the very slow fading is worse than that of memoryless channel. As the number of Tx antennas increases, however, the performance improves by the space-diversity effect through the multi-antenna transmission even if the fading is very slow. When $M = 1$ or 2, the employment of convolutional codes provides us with energy loss in comparison with the uncoded cases. This is because convolutional codes require statistically independent fading for each coded bit to achieve full potential error correction capability. When $M = 4$, however, we can expect the performance improvement with the use of convolutional codes.

To examine the required number of Tx antennas for convolutionally codes to outperform repetition codes when a fading process is very slow and no interleaving is used, the simulation results for various number of Tx antennas with convolutional or repetition codes are shown in Fig.4, 5. For reference, the simulation result of rate 1/2 code with a single-antenna is also presented. The spreading sequences are Gold sequences of length 63 for multi-antenna transmission scheme throughout the simulations. In this case, the length of Gold sequences for rate 1/2 coding with single-antenna scheme must be half of that for multi-antenna scheme to maintain the same data rate and bandwidth. However, there is no Gold sequence of length $63/2 = 31.5$, and thus we use Gold sequences of length 31 instead.

In the simulations, we have assumed that a fading is relatively slow and normalized Doppler frequency $f_d T$ is 0.0005, which corresponds a movement at a velocity of 3.6km/h if the data rate is 16kbps and the carrier frequency is 2.4GHz, where f_d is Doppler frequency.

Figure 4 compares the performance of 4-state codes and repetition codes. From this figure, we can confirm that the multi-antenna scheme outperforms conventional single-antenna scheme. As in Fig. 3, the employment of convolutional codes with small number of Tx antennas degrades the performance more than repetition codes. When the number of Tx antennas M is 3 or 4, convolutional code and repetition code shows almost the same performance. When $M \geq 6$, convolutional codes outperform repetition codes in the range of the BER of interest. This fact indicates that one should not use convolutional codes with small number of Tx antennas when a fading process is very slow and little randomization by the interleaver can be expected.

Figure 5 compares the performance of 4-state and 16-state codes in the same conditions in Fig.4. From this figure, we can find that complex 16-state codes shows worse performance than simple 4-state codes when M is 1 or 2, where 4-state code was worse than repetition code. When $M \geq 3$, the 16-state codes shows much better performance than the 4-state codes. From these facts, we can conclude that the system with a convolutional code and at least 3 Tx antennas outperforms the uncoded system even if a fading process is very slow and no interleaving is used. However, the convolutional codes we have examined are not optimized for the fading channel under our consideration. It may be possible to achieve more coding gains with the same complexity than the above convolutional codes even if the number of Tx antennas is smaller than 3.

Figure 6 compares 2-antenna and single-antenna schemes with 4-state rate1/2 code when block interleaving is used and the multiuser interference exists. In this figure, the BER performance is plotted as a function of the interleaving depth. The interleaving span in each Tx antenna is fixed to 16 and 32 coded bits for 2-antenna and single-antenna schemes respectively to maintain the same interleaving size and delay. For reference, the performance of repetition codes are also presented. As the interleaving depth increases, the BER performance of convolutionally coded systems improves, while the BER performance of uncoded (repetition coded) systems is invariant regardless of the interleaving depth. Although the convolutional codes show worse performance than the repetition codes when the interleaving depth is small, the convolutional codes with a moderate size of interleaving can outperform the repetition codes. It is obvious that the BER performances of the convolutionally coded systems with single- and 2-antenna schemes become closer as the interleaving size becomes large. However, the multi-antenna schemes require less interleaving depth than the single-antenna schemes to achieve the same BER performance. Note that the BER performance of the coded systems is strongly dependent on a normalized Doppler frequency f_dT when interleaving is used. If f_dT is smaller, larger interleaving size is needed to achieve the same BER performance.

The interference in the multi-antenna schemes is different from that in the conventional single-antenna scheme. Although the self-interference exists in the multi-antenna scheme, it can be suppressed to be negligible small by proper selection of the delays at the transmitter[1]. Hence, the main factor needing consideration is the multiuser interference. One may ask which multiuser interference in the two schemes is crueler to the BER performance ? We can know the answer intuitively without complex calculation. It should be noted that the total average power of multiuser interference is constant regardless of the number of Tx antennas. The factors inducing the differences in the multiuser interference are as follows. Firstly, the period of the spreading sequences in the M-antenna schemes is longer by a factor of M, resulting in better cross-correlation properties than single antenna schemes. Secondly, the $M(K-1)$ signals compose the multiuser interference term in the M-antenna schemes, while the $K-1$ signals do in the single antenna schemes, meaning that the probability density function of the interference in the multi-antenna scheme is nearer to Gaussian from the central limit theorem. It is well known that the Gaussian approximation, which uses only average power of the multiuser interference to calculate the BER, becomes very optimistic when the number of simultaneous users is small and bit error rate is low[8]. These facts suggests that the nearer the distribution of the multiuser interference becomes Gaussian, the lower the BER becomes. Hence, we can conclude that the multi-antenna schemes are less sensitive to the multiuser interference than conventional single-antenna schemes.

V. CONCLUSION

In this paper, we have discussed the combination of the multi-antenna transmission scheme and convolutional coding for DS/CDMA systems. In this scheme, the M coded bits for a data bit interval are assigned to M different antennas, and transmitted from each antenna with an intentional delay in parallel. The results of this study have shown that the multi-antenna schemes perform better than conventional single antenna schemes with the same convolutional codes, and that the convolutional codes with at least 3 Tx antennas can outperform the transmission diversity technique with the same number of Tx antennas, even if a fading is very slow and little randomization by the interleaver is expected. We have pointed out that multi-antenna transmission has another advantage of decreasing the effective multiuser interference.

REFERENCES

[1] K.Ban, M.Katayama, T.Yamazato, and A.Ogawa, "The DS/CDMA System using Transmission Diversity for Indoor Wireless Communications," PIMRC'96 Conf.Proc., vol.3,pp.803-812,1996.

[2] A.J.Viterbi and Roberto Padovani, "Implications of Mobile Cellular CDMA," IEEE Commun. Mag., vol.30,No.12,pp.38-41,Dec.1992.

[3] F.Adachi, K.Ohno, A.Higashi, T.Dohi, and Y.Okumura, "Coherent Multicode DS-CDMA Mobile Radio Access," IEICE Trans. Commun., vol.E79-B,pp.1316-1324,Sept.1996.

[4] D.N.Rowitch and L.B.Milstein, "Convolutional Coding for Direct Sequence Multicarrier CDMA," MILCOM'95 Conf.Proc., pp.55-59,1995.

[5] A.A.M.Saleh and R.V.Valenzuela, “A Statistical Model for Indoor Multipath Propagation,” IEEE J.Selected Areas in Commun., vol.SAC-5,pp.128-137,Feb.1987.
[6] M.B.Pursley, “Performance Evaluation for Phase Coded Spread Spectrum Multiple Access Communication-Part I:System Analysis,” IEEE Trans. Commun., vol.COM-25,pp795-803,Aug.1977.
[7] J.G.Proakis, Digital Communications, New York: McGraw-Hill, 1983.
[8] R.K.Morrow.JR and J.S.Lehnert, “Bit-to-Bit Error Dependence in Slotted in DS/SSMA Packet Systems with Random Signature Sequences,” IEEE Trans. Commun., vol.COM-37,pp.1052-1061,Oct.1989.

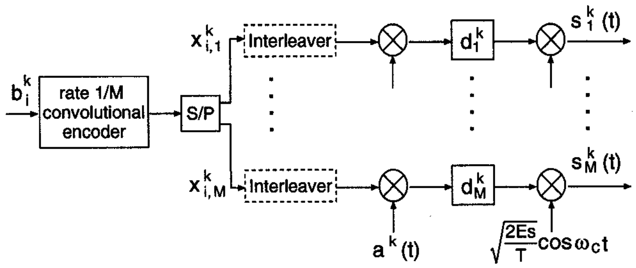

Fig. 1. The kth user's transmitter with multi-antenna

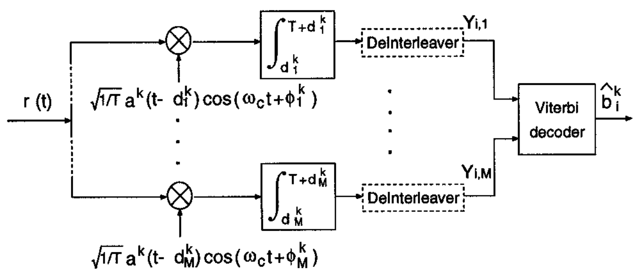

Fig. 2. The receiver for the kth user

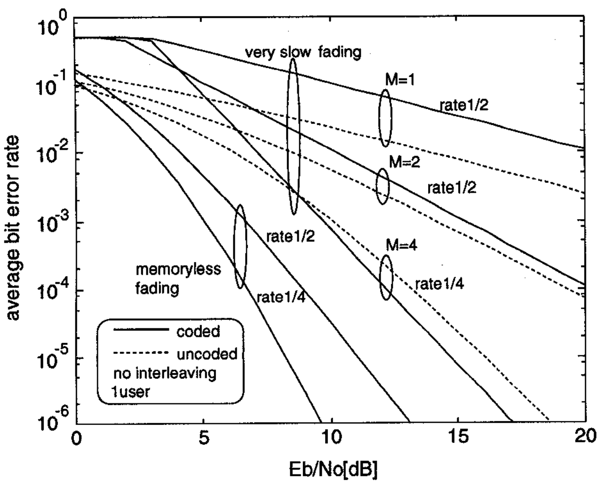

Fig. 3. analytical results for 4-state codes ($K = 1$)

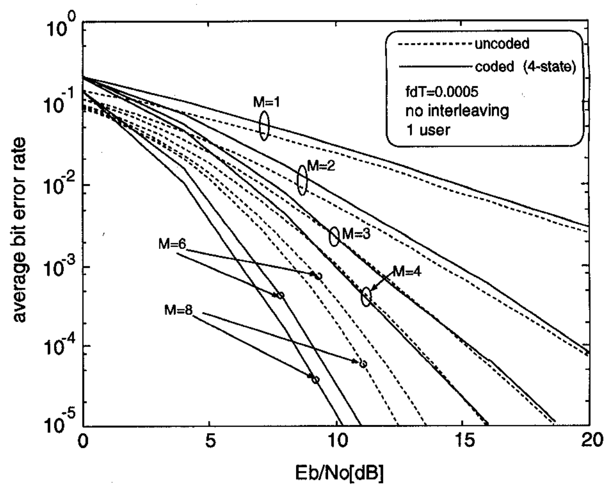

Fig. 4. performance comparison of 4-state codes and repetition codes($fdT = 0.0005$, $K = 1$)

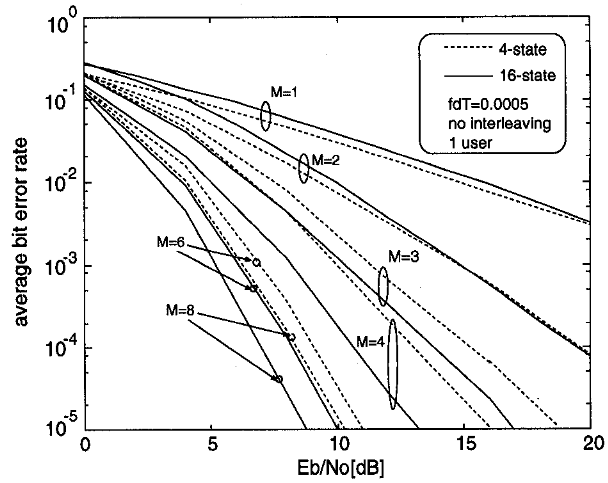

Fig. 5. performance comparison of 16-state and 4-state codes ($fdT = 0.0005$, $K = 1$)

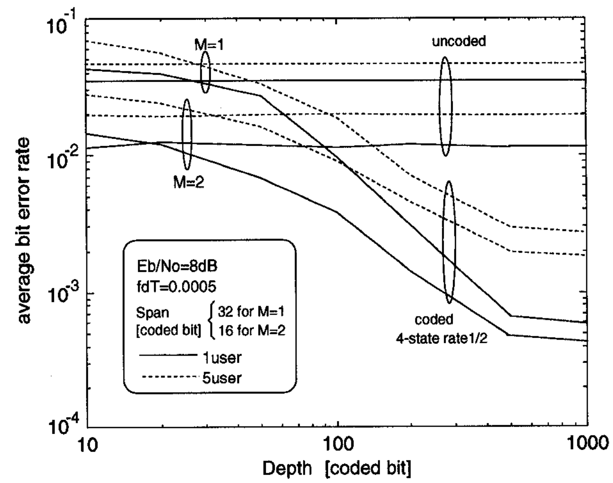

Fig. 6. the BER versus the interleaving depth for 4-state rate1/2 code ($E_b/N_0 = 8dB$, $fdT = 0.0005$)