# PERFORMANCE EVALUATION OF DS/CDMA COMMUNICATIONS SYSTEMS MODULATED WITH $\frac{\pi}{2}$-SHIFT BPSK OVER MULTIPATH RAYLEIGH FADING CHANNELS

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*Abstract*—**In mobile communications, power is a very important factor and nonlinear amplification of power amplifiers cannot be avoided due to their high power efficiency. This article presents the performance of $\frac{\pi}{2}$-shift BPSK modulation scheme used in DS/SS/CDMA wireless communications over multipath Rayleigh fading channel and compares the performance with the performance of conventional BPSK CDMA systems. The performance parameters: Out-of-Band power, average Bit Error Rate(BER) and Spectral Efficiency have been evaluated. In order to obtain improved performance on fading channels, a RAKE receiver has been employed. Finally it is shown that $\frac{\pi}{2} - shift\,BPSK$ outperforms conventional BPSK in the presence of nonlinear amplification.**

*Keywords*— **$\frac{\pi}{2}$-shift BPSK, DS/SS/CDMA, Soft/Hard-Limiter, Out-of-Band power, Spectral Efficiency, RAKE Receiver.**

## I. INTRODUCTION

DIRECT Sequence Spread Spectrum (DS-SS) Code Division Multiple Access(CDMA) techniques using multipath diversity over multipath fading channels have received considerable attention for mobile communications due to their obtainable satisfactory capacity. Although DS/SS/CDMA has low power spectral density, simultaneous transmissions may cause interference to the systems lying in adjacent frequency bands if the spectrum is not well trimmed out. Hence the bandwidth limitation of transmitted signal is essential.

In order to obtain high power efficiency, High Power Amplifiers(HPA) of mobile transmitters often operate near saturation regions exhibiting nonlinear distortion. The bandwidth limitation by a filter causes large envelope variations and high power efficient amplifiers generate spectral components outside the allocated bandwidth because of their nonlinearity and thus causes interference to adjacent frequency bands. To avoid the effect of nonlinear distortion, modulation methods which do not introduce large fluctuations in the envelope after filtering are highly appreciated [1] [2]. $\frac{\pi}{2}$-shift BPSK signals are considered to have smaller envelope fluctuations than the conventional BPSK, QPSK and $\frac{\pi}{4}$-shift QPSK signals because it has smaller maximum phase transition than the others. The signal constellations together with phase transitions of these modulation schemes are shown in Fig. 1.

The simplicity of the circuitry and thus less hardware requirement to realize $\frac{\pi}{2}$-shift BPSK system (compared to other QPSK schemes including offset QPSK) makes it a potential modulation method.

Differential detection has been chosen because of its robustness in the presence of multipath fading. In order to improve the BER performance on multipath fading channels by means of diversity technique, a RAKE receiver is used.

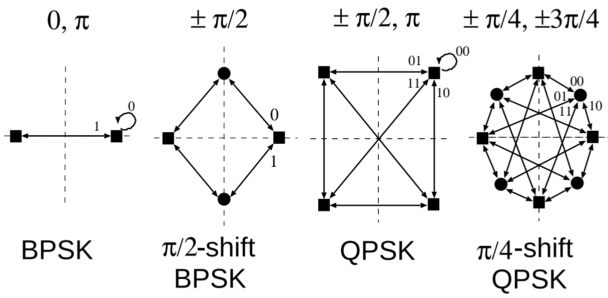


Fig. 1. Signal constellations of different modulation schemes

In this paper we evaluate the Out-of-Band power emission, BER performance and Spectral Efficiency of DS/SS/CDMA systems using $\frac{\pi}{2}$-shift BPSK modulation scheme over multipath Rayleigh fading channels and examine the effect of the soft-limiter characteristic on the performance. We also show the performance of conventional BPSK CDMA systems for comparison purpose.

## II. SYSTEM MODEL

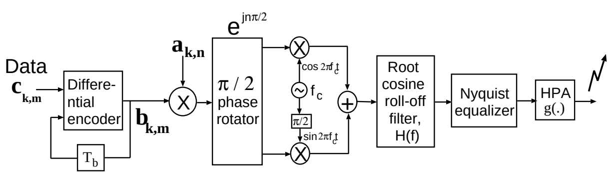


Fig. 2. Transmitter for $\frac{\pi}{2}$-shift BPSK CDMA system

### A. Transmitter Model

A.1 Generation of $\pi/2$-Shift BPSK Signal

Figure 2 illustrates the block diagram of the $\frac{\pi}{2}$-shift BPSK transmitter of the k-th user. Due to differential detection at the receiver end the incoming information $c_{k,m}$ is first differentially encoded at the transmitter to generate $b_{k,m}$ $(+1\,or\,-1)$ so that

$$b_{k,m} = c_{k,m} \oplus b_{k,m-1} \qquad (1)$$

The symbol stream $b_{k,m}$ is spread by the corresponding spreading sequence $a_{k,n}$ $(+1\,or\,-1)$. The subscript $k$ stands for the $k$-th user where $k$ = 1, 2, ...., K and the subscripts $m$ and $n$ denote the $m$-th symbol after differential encoding and the $n$-th chip of the spreading sequence, respectively. The spreading sequences used are Gold sequences of length N which has the value equal to the ratio of symbol duration $T_s$ and chip duration $T_c$.

The spread signal is then phase shifted by a $\frac{\pi}{2}$ radian phase rotator operating at the clock rate $1/T_c$. The equivalent baseband $\frac{\pi}{2}$-shift BPSK modulated signal can be expressed as

$$u_k(t) = b_{k,m}(t)a_{k,n}(t)\, e^{(j\theta_n)} \cdot p(t-(mN+n)T_C); \quad (mN+n)T_c \le t < (mN+n+1)T_C \tag{2}$$

where $\theta_n = (mN+n)\pi/2 + \phi$ with $\phi$, the initial phase and

$$p(t) = \begin{cases} 1; & |t| \le T_C/2 \\ 0; & otherwise \end{cases} \tag{3}$$

A.2 Bandwidth Limitation

Because of the rectangular shape of the chips the bandwidth of the transmitted signal becomes wide. In order to suppress the out of band emission and thus utilize valuable bandwidth efficiently, we employ a root cosine roll-off filter followed by a Nyquist equalizer. The equivalent low pass frequency responses of the root cosine shaping filter, $H(\omega)$ which is evenly divided between the transmitter and receiver and that of the Nyquist equalizer, $E(\omega)$ are expressed as [3]:

$$H(\omega) = \begin{cases} 1 & (|\omega| \le \frac{\pi}{T_c}(1-\beta)) \\ \frac{1}{2}[1-\sin(\frac{T_c}{2\beta}(\omega - \frac{\pi}{T_c}))] & (\frac{\pi}{T_c}(1-\beta) \le \omega \le \frac{\pi}{T_c}(1+\beta)) \\ \frac{1}{2}[1+\sin(\frac{T_c}{2\beta}(\omega + \frac{\pi}{T_c}))] & (-\frac{\pi}{T_c}(1+\beta) \le \omega \le -\frac{\pi}{T_c}(1-\beta)) \\ 0 & (|\omega| \ge \frac{\pi}{T_c}(1+\beta)) \end{cases} \tag{4}$$

$$E(\omega) = \begin{cases} [Sa(\frac{\omega T_c}{2})]^{-1} & (|\omega| \le \frac{\pi}{T_c}(1+\beta)) \\ 0 & \text{(otherwise)} \end{cases} \tag{5}$$

where $Sa(x) = sin(x)/x$.

A.3 Nonlinear Amplification

After band limitation the modulated signal is amplified by a High Power Amplifier(HPA). Since the cosine roll-off filter is equally distributed between the transmitter and receiver as $root$ cosine roll-off filter on each side, for a train of pulses inter symbol interference cannot be suppressed to 0 at the out put of the transmitter filter. In an ideal case when the input to the root cosine roll-off filter is a single unit pulse then manipulation regarding the scaling factor of the filter is carried out so that the peak of the output amplitude (is also the input to the HPA) has unit value.

The CDMA signals are both amplitude distorted (AM-AM) and phase distorted (PM-PM) by the nonlinearity of the HPA. However, it is known that the AM-PM distortion of the widely used transistor amplifiers is considerably small such that its effect can be ignored. For the most part, the regrowth of sidebands occurs due to AM-AM distortion of the HPAs.

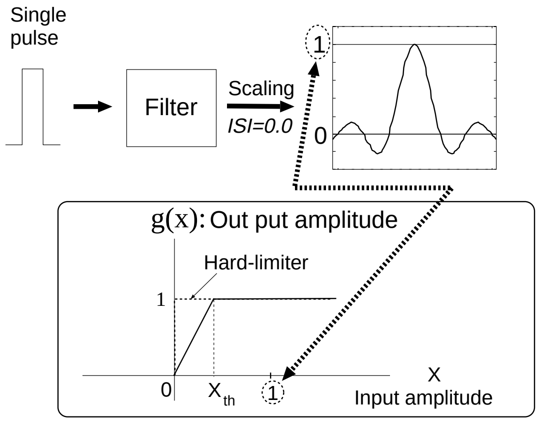


Fig. 3. Soft-limiter characteristic

Hence, the HPA is usually represented by a limiter which operates with a threshold value, $x_{th}$ (positive value) with respect to the amplitude of the input signal. Therefore by changing the value of $x_{th}$ we adjust the range of linear operation of the HPA and thus change the shape of the signal. Hence we are calling the HPA as $soft-limiter$ and when $x_{th}$ is set to zero it works as a hard-limiter. The input-output amplitude relation of the soft-limiter which is shown in Fig.(3) can be defined as

$$g(x) = \begin{cases} \frac{x}{x_{th}} & (0 \le x < x_{th}) \\ 1 & (|x| \ge x_{th}) \end{cases} \tag{6}$$

The transmitted signal from the $k$-th user can be expressed as

$$s_k(t) = Re[U_k(t)\ \exp(j2\pi f_c t)] \tag{7}$$

where $U_k(t)$ denotes the baseband signal after filtering and limiting and $f_c$ is the carrier frequency.

### B. Multipath Rayleigh Fading Channel

The received signal of the $k$-th user is affected by multipath fading and expressed as

$$r_k(t) = \sum_{l=0}^{L-1} Re[\{\gamma_l A_{k,l} \exp(j\theta_{k,l}) s_k(t-t_{k,l})\} \\ .\ \exp\{j2\pi f_c(t-t_{k,l}) + j\Theta_k\}] \tag{8}$$

where $L$ is the total number of multipath arrivals and amplitude component $A_{k,l}$ is Rayleigh distributed random variable with $E\{A_{k,l}^2\} = 1$ [4]. $\gamma_l^2$ represents the relative power of the $l$-th path with respect to 0-th path. Here it is assumed that $\gamma_0^2 = 1$. $t_{k,l}$ is the propagation delay and phase shift $\theta_{k,l}$ is uniformly distributed over $[0, 2\pi)$ and $\Theta_k$ is the carrier phase of the $k$-th user's signal. The 0-$th$ path is the path through which the signal first arrives at the receiver.

It is assumed that signal arrival through each path is so separated that correlation between paths is negligible.

In this case the signal arrivals can be approximated by Poisson distribution [5]. So the probability of $l$ arrivals during the period of $t$ is expressed as

$$f(l,t) = \exp(-\frac{\Lambda}{D_s}t)\frac{(\frac{\Lambda}{D_s}t)^l}{l!} \tag{9}$$

where $D_s$ is the maximum delay time with respect to the 0-th path and $\Lambda$ is the average number of multipath signal arrivals during $D_s$. Using the exponential distribution relation between the propagation delay and received power through paths [6], the average signal power of each path can be expressed as

$$E\{\gamma_l^2\} = \int_0^{D_s} \exp(-\frac{\alpha}{D_s}t)\exp(-\frac{\Lambda}{D_s}t)\frac{(\frac{\Lambda}{D_s}t)^{l-1}}{(l-1)!}dt \tag{10}$$

Here $\alpha$ denotes decay constant.

Considering all $K$ users are able to access asynchronously, the received signal is expressed as

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t) \tag{11}$$

where AWGN component $n(t)$ has one sided spectral density of $N_0$.

### C. Receiver model

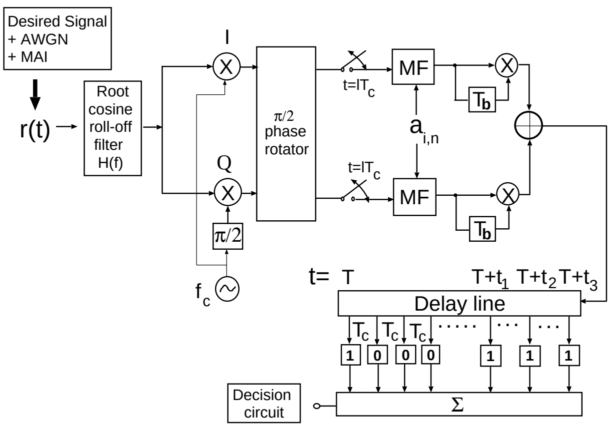


Fig. 4. RAKE receiver for $\frac{\pi}{2}$-shift BPSK CDMA system

Figure 4 shows the block diagram of the RAKE receiver for differential detection of $\frac{\pi}{2}$-shift BPSK modulated signals. The received signal which is corrupted by multipath components, AWGN component and interference from other (K-1) users is first filtered by the receiver filter with the same frequency response at the transmitter end and then multiplied by the carrier. The phase rotator at the receiver introduces a phase shift of $\pi/2$ radians to the converted baseband signal at a clock rate of $1/T_c$ which is similar to that of the transmitter but reverse in direction and this results into the regeneration of conventional BPSK signal. The outputs after despreading and summation over one symbol duration $T_s$ by matched filters (MF) are differentially demodulated and set on the tapped delay line. Finally the outputs are fed to the decision circuit to retrieve the information. When RAKE receiver is not considered the differential demodulator outputs are directly fed to the decision circuit.

## III. Derivation of Bit-Error-Rate (BER)

### A. Without RAKE Receiver

Consider the $i$-th user's receiver shown in Fig. 4. The despread outputs for $m$-th bit at the two branches are $Z_{i,m}^c$ and $Z_{i,m}^s$. Here $c$ and $s$ denote the $cos(.)$ and $sin(.)$ components, respectively. Focusing only on the upper branch and dropping $i$ from the notations, $Z_{i,m}^c$ can be expressed as

$$Z_m^c = Z_{m,D}^c + Z_{m,N}^c + Z_{m,I}^c + Z_{m,F}^c \tag{12}$$

where $Z_{m,D}^c, Z_{m,N}^c, Z_{m,I}^c$ and $Z_{m,F}^c$ denote desired component, AWGN component, interference component from other users with respect to the 0-th path and multipath interference component from both desired and undesired users, respectively.

Here we shall evaluate the performance under the following assumptions:

- Power control is carried out so that all accessed signals through the 0-th paths may arrive at the receiver at equal mean power.
- Timing between the received signal and local PN sequence is acquired perfectly.
- Time delay $t_{k,0}$ and carrier phase $\Theta_k$ of the $k$-th user signal are independent random variables uniformly distributed over $[0, NT_c)$ and $[0, 2\pi)$, respectively.
- Signal from each transmitter arrives at the receiver end with the same frequency and there is no frequency offset between the received signals and the reference carrier.

For evaluating the performance Gaussian approximation is valid in multipath fading environment [4]. Since $E\{A_{k,l}^2\} = 1$, the variance of $Z_{m,I}^c$ can be expressed as

$$var(Z_{m,I}^c) = \sum_{k=1,k\neq i}^{K} var(Z_{m,k}^c) \tag{13}$$

where $Z_{m,k}^c$ is the interference component of the $k$-th user without the effect of fading. Again the variance of $Z_{m,F}^c$ can be expressed as

$$var(Z_{m,F}^c) = \sum_{k=1}^{K}\sum_{l=1}^{L-1} E\{\gamma_l^2\} var(Z_{m,k}^c) \tag{14}$$

The band limited and despread noise component is a random variable with zero mean and its variance is given by var$(Z_{m,N}^c) = N_0/4NT_c$, where $N_0$ is one sided power spectral density of thermal noise.

Assuming $Z^c_{m,D} = Z^c_{m-1,D}$ and $Z^s_{m,D} = Z^s_{m-1,D}$ the $SNR$ after despreading is obtained as

$$SNR = \frac{A^2_{i,0}({Z^c_{m,D}}^2 + {Z^s_{m,D}}^2)}{var(Z_I) + var(Z_F) + var(Z_N)} \qquad (15)$$

In the above equation $var(Z_I) = var(Z^c_{m,I} + Z^c_{m-1,I} + Z^s_{m,I} + Z^s_{m-1,I})$ and $var(Z_F) = var(Z^c_{m,F} + Z^c_{m-1,F} + Z^s_{m,F} + Z^s_{m-1,F})$. Therefore, the bit-error-rate (BER) is obtained as

$$BER = \int_0^\infty \frac{1}{2} \exp\{-SNR\} f_{A_{i,0}}(a) da \qquad (16)$$

where $f_{A_{i,0}}(a)$ is the Rayleigh probability density function which satisfies $E\{A^2_{i,0}\} = 1$ and expressed as

$$f_{A_{i,0}}(a) = 2a \exp\{-a^2\} \qquad (17)$$

### B. With RAKE Receiver

After the despread signals are differentially demodulated, the obtained outputs are arrayed onto the tapped delay line in which intervals between two taps are set to $T_c$ or its multiple. The differentially demodulated outputs are weighed and added up at the associated taps. When differential detection is applied the tap weights can be set to either of 0 or 1 making the estimation of tap weights unnecessary [7]. In Fig. 4 we show the case where the signals through the 0-th path and three other paths have arrived at $t = T, T + t_1, T + t_2, T + t_3$, respectively. These four signals will contribute to the RAKE receiver, therefore other tap weights are set to 0.

In order to utilize the path diversity of the signals through these paths efficiently we must know the delay time for each path which can be estimated by experiments. In this paper we assume that the delay time of each path is perfectly known. From eq.(15) $SNR$ of the signal through the $r$-th path is expressed as

$$SNR_r = \frac{\gamma_r^2 A^2_{i,r}({Z^c_{m,D}}^2 + {Z^s_{m,D}}^2)}{var(Z_I) + var(Z_F) + var(Z_N)} \qquad (18)$$

If $R$ received signals are combined the $BER$ with RAKE receiver is given by the following equations [7]

$$BER = \frac{1}{2^{2R-1}} \sum_{m=0}^{R-1} m! v_m \sum_{k=0}^{R-1} \frac{\pi_k}{\overline{SNR_k}} \left( \frac{\overline{SNR_k}}{1 + \overline{SNR_k}} \right)^{m+1} \qquad (19)$$

$$\pi_k = \prod_{i=0 (i \neq k)}^{R-1} \frac{\overline{SNR_k}}{\overline{SNR_k} - \overline{SNR_i}} \qquad (20)$$

$$v_k = \frac{1}{k!} \sum_{n=0}^{R-1-k} \binom{2R-1}{n} \qquad (21)$$

## IV. Performance Comparison and Discussion

### A. Out-of-band Power

Figure 5 shows the out-of-band power normalized by total power for different threshold values of the soft-limiter. It is apparent from the result that $\frac{\pi}{2}$-shift BPSK has smaller out-of-band power than conventional BPSK and thus it is less likely to cause interference to the systems lying in adjacent frequency bands. This is because the maximum phase transition in $\frac{\pi}{2}$-shift BPSK is either of $\pm\frac{\pi}{2}$ radians where it is $\pi$ radians in conventional BPSK. As a result the former has smaller envelope fluctuations than the latter after filtering. It is worth to mention that when $x_{th} = 1.4$ the envelope of the $\frac{\pi}{2}$-shift BPSK signal falls within linear operation range whereas

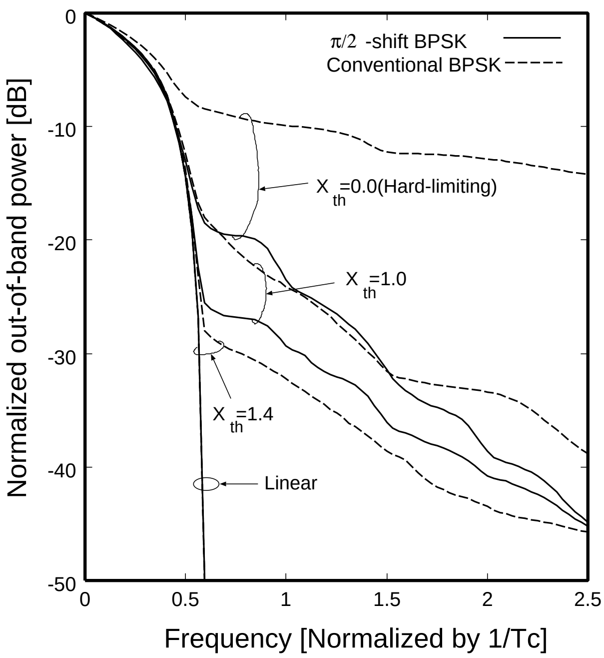


Fig. 5. Out-of-band power characteristics of $\frac{\pi}{2}$-shift BPSK and BPSK (Roll-off factor, $\beta = 0.2$)

the envelope of the conventional BPSK signal still receives certain amount of limitation by the soft-limiter. In case of linear amplification both the modulation schemes show same out-of-band power characteristic.

### B. Bit-Error-Rate Performance

Obtained numerical results of BER performance of $\frac{\pi}{2}$-shift BPSK and conventional BPSK are shown in Fig.6 and 7 respectively, for K=11 users with roll-off factor, $\beta = 0.2$. Gold sequences of length $N$=511 are used as spreading sequences. The number of fingers of the RAKE receiver considered in multipath fading case are R=1(without RAKE),2,4 and 6. About multipath fading channel the average number of signal arrivals, $\Lambda$ and attenuation factor of multipath components, $\alpha$ are set to 10 and 2, respectively.

amplification of conventional BPSK will need (25.5-20.5)=5 [dB] of extra $\frac{E_b}{N_o}$ than the linear case which is only 0.6 [dB] in the case of $\frac{\pi}{2}$-shift BPSK.

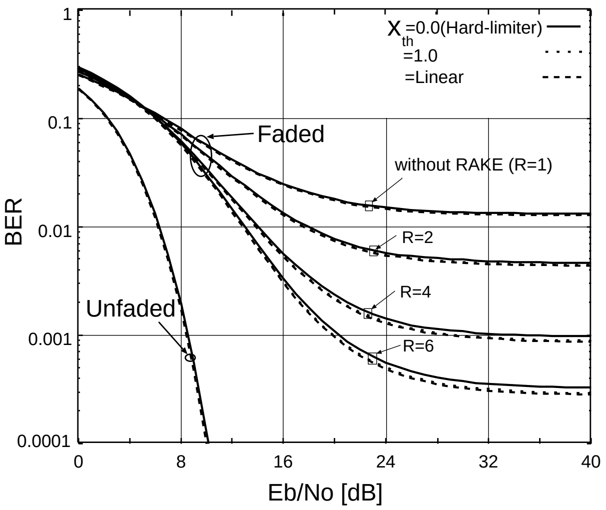


Fig. 6. BER performance of $\frac{\pi}{2}$-shift BPSK($K = 11$, $N = 511$, $\beta = 0.2$, $\Lambda = 10$, $\alpha = 2$)

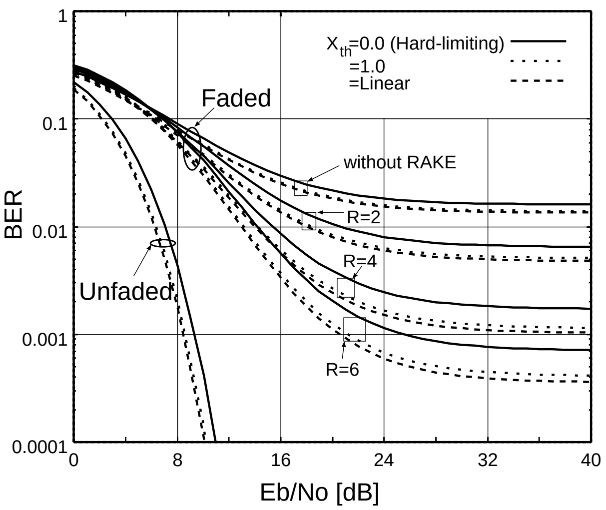


Fig. 7. BER performance of conventional BPSK($K = 11$, $N = 511$, $\beta = 0.2$, $\Lambda = 10$, $\alpha = 2$)

The BER performance degrades drastically in multipath fading environment because multiple-access interference(MAI) in multipath fading environment is more severe than that in unfaded case. Here multipath interference component $var(Z_F)$ is proportional to the number of users. Although the use of RAKE receiver improves the performance, the effect of the increase of RAKE fingers on the BER performance gradually diminishes because of the fact that the multipath signals arriving and combined later contribute with poor $SNR$ for the signal power attenuates exponentially with delay time.

For linear amplification both the modulation methods perform identically. The degradation due to non-linear amplification is noticeable in conventional BPSK (Fig.7). For a required BER of $10^{-3}$ (when R=6) non-linear

### C. Spectral Efficiency

Spectral efficiency is an important parameter for evaluating a system. It is defined as [8]

$$\eta = \frac{K_{max}.R_b}{W} = \frac{K_{max}.R_c}{N.W} \tag{22}$$

where $K_{max}$ is the maximum number of accommodable users within the bandwidth assigned for the system, $T_b$, $T_C$ and $W$ denote bit rate, chip rate and assigned bandwidth, respectively.

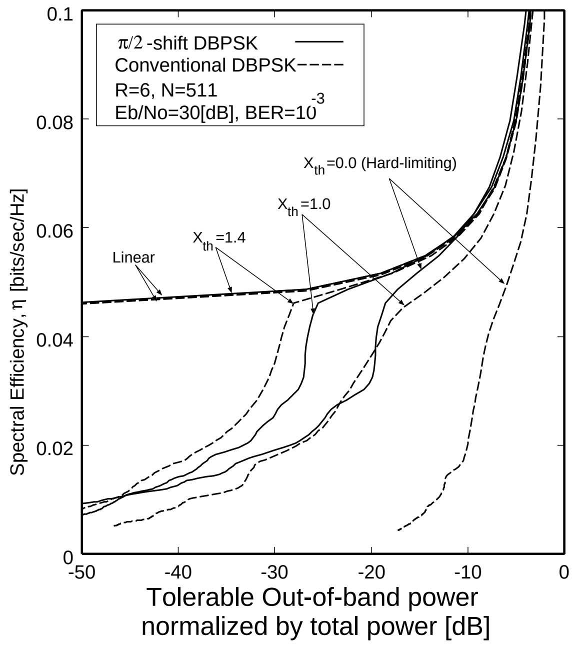


Fig. 8. Comparison of spectral efficiency between $\frac{\pi}{2}$-shift BPSK and conventional BPSK

In order to calculate the spectral efficiency, we need to know the out-of-band power as a function of bandwidth (Fig.5). For known $N, W$ and tolerable out-of-band power, maximum chip rate is determined. The maximum number of users is determined from the given $\frac{E_b}{N_o}$ and required BER. Figure 8 shows the obtained spectral efficiency for $\frac{\pi}{2}$-shift BPSK and conventional BPSK. It is apparent from the figure that in the presence of non-linearity of the HPA $\frac{\pi}{2}$-shift BPSK outperforms conventional BPSK.

## V. Conclusion

In this paper we have evaluated the out-of-band power, BER and spectral efficiency of $\pi/2$-shift BPSK CDMA under Rayleigh multipath fading environment. It is evident from the obtained results that $\pi/2$-shift BPSK modulation scheme is more robust to the nonlinearity of the HPA than conventional BPSK due to its smaller maximum phase transition. About BER performance, the RAKE receiver improves BER performance remarkably but further measures like use of longer PN sequences with smaller cross correlation, forward error correction(FEC) codes etc. should be taken into account to suppress MAI to obtain better BER performance and accommodate larger number of users.

## REFERENCES


[1] M. Sawada et al., "Effect of Nonlinear Amplifier of Transmitters in the CDMA System using Offset-QPSK", *IEICE Trans. Commun., Vol.E76-B, No.7*, Jul. 1993.

[2] Y. Akaiwa, Y. Nagata, " Highly Efficient Digital Mobile Communications with a Linear Modulation Method", *IEEE JSAC in Commun., VOL.SAC-5, NO.5*, Jun. 1987.

[3] T. Shibata et al., "Performance of Asynchronous Band-limited DS/SSMA Systems", *IEICE Trans. Commun., Vol.E76-B, No.8*, Aug. 1993.

[4] E. A. Geraniotis and M. B. Pursley, "Performance of Non-coherent Direct Sequence Spread Spectrum Communications over Specular Multipath Fading Channels", *IEEE Trans. Commun., Vol.Com-34, No.3, pp.219-226*, Mar. 1986.

[5] G. L. Turin, "Introduction to Spread-Spectrum Antimultipath Techniques and Their Application to Urban Digital Radio Communication", *Proc. IEEE , Vol.68, No.3, pp.328-353*, Mar. 1980.

[6] N. Kong, T. Eng and L. B. Milstein, "A Selection Combining Scheme for RAKE Receivers", *1995 Fourth IEEE ICUPCR,Tokyo,Japan, pp. 426-430*, Nov. 1995.

[7] J.S.Proakis, "Digital Communications", *McGraw-Hill, New York*, 1995.

[8] A.J. Viterbi,"When not to spread spectrum- A sequel", *IEEE Commun. Mag., vol.23, pp.12-17*, April 1985.