A Mathematical Model of Narrowband Power-Line Noise Based on Measurements

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Abstract— **This manuscript introduces a mathematically tractable and accurate model of narrowband power-line noise based on experimental measurements. In this model, the noise is expressed as the sum of cyclostationary Gaussian processes. The non-stationary features of power-line noise are represented with a small number of parameters, and the noise waveforms generated with the model achieve good agreement with those of actually measured noise.**

Keywords: Power-Line, Narrowband System, Impulsive Noise, Non-stationary Noise, Colored Noise.

I. Introduction

Until recent active discussions on the opening of high frequency bands for wideband Power-Line Communications (PLC), the researches had been focusing on the narrow band systems, which use middle and low frequency bands, for example, below 450kHz in Japan. This paper re-focuses on the narrow-band PLC systems and discusses the modeling of the noise on power-lines.

The narrow-band PLC systems have been assumed to be used only in very low-speed and unreliable communications. However, the low speed and quality of narrowband PLC systems are not an ineluctable destiny. It is the reason of the insufficient performance of former narrowband PLC systems that they simply introduce the old-fashioned signaling schemes developed for the environment completely different from power-lines. Conversely, the employment of the sophisticated schemes may realize much higher performance than the conventional PLC systems.

The noise in power-lines is mainly caused by electric appliances connected to the lines. For design and analysis of conventional communication systems, stationary additive white Gaussian noise (AWGN) is often used as a model of noise. In PLC, however, the statistical behavior of this man-made noise is quite different from that of stationary AWGN.

Many reports claim that this non-Gaussian feature of the noise is the cause of low quality of PLC systems. On the contrary, the fact that Gaussian distribution has the largest entropy implies that communications under the non-Gaussian noise may achieve better performance than under AWGN. In fact, in [1] Miyamoto et al. show that the performance of the system under impulsive noise environment exceeds that under AWGN if proper consideration of noise statistics is made, while conventional receivers optimized for AWGN suffer large performance degradation by the impulsiveness of noise. It thus can be expected that narrow-band PLC systems still have large room for performance improvement if the behavior of power-line noise is clarified and taken into account in system design.

In 1998, the authors have proposed the model of power-line noise at the second ISPLC held in Tokyo[2], and some further considerations were presented at the fourth ISPLC in 2000[3]. This manuscript denotes the mathematical model of power-line noise based on these former studies and recent results of measurements.

II. Noise Measurement System

The system for the noise measurement is shown in Fig. 1. The system consists of a pick-up circuit followed by an A/D converter(ADC) and a computer(PC), and a signal injection circuit with a waveform generator.

The pick-up circuit is shown in Fig.2. In this figure, the mains alternating current (AC) component is attenuated by the capacitors, common-mode noise component is removed by a high frequency transformer, and balance-to-unbalance transform of the circuit is performed by a balun. The noise component is obtained at the terminal labeled "noise" in the figure. In order to eliminate aliasing effect, Low Pass Filter(0-1.9MHz) is inserted before the ADC. ADC then samples the signal with 10M[samples/s] and passes the data to the computer. As described in the following sections, rest of processing is executed with digital signal processing programs in the PC.

The pick-up circuit also outputs a down-transformed AC waveform with the peak-to-peak voltages of 2[V] at the terminals labeled "AC". This AC voltage waveform is used as a triggering signal to start a measurement. The circuit elements are shown in Table.I.

In power-line communication systems, there often occurs abrupt and large fluctuation of line impedance, which influences both noise and signal amplitudes at a receiver. In designing communication systems, noise amplitude relative to signal level is more important than the noise value itself. For this reason, in our discussion, the noise voltage divided by a reference signal level is used. The reference signal is generated by the waveform generator and injected into the power-line

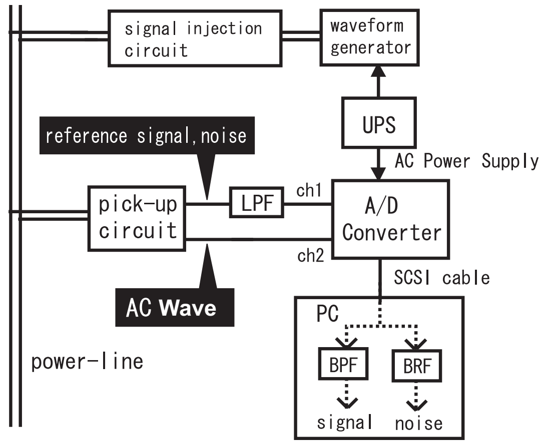

Fig. 1. Measurement System

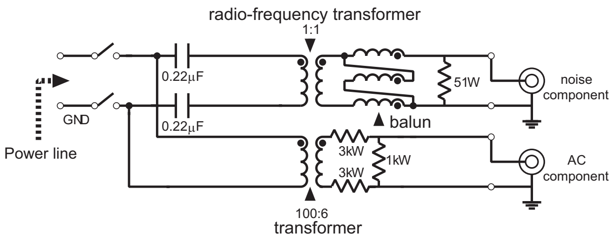

Fig. 2. Pick-up Circuit

TABLE I

CIRCUIT ELEMENTS

Component	Model	Specifications
transformer	TOYOZUMI HT-61	Ratio 100:6
RF transformer	JPC ELH-01	Turn ratio 1:1
Balun	—	5 Turns Core TDK HF70
Low Pass Filter	Min-Circuit BLP-1.9M	DC to 1.9MHz LPF
A/D Converter	Elmec EC-6800 B2CH1E01	Range ±1.28V Resolution 8bits max rate 50Msps

at the same outlet where pickup circuit is connected. The injection circuit and the pick-up circuit for "noise" have the same configuration and parameters.

III. Measured Noise

The input to the ch.1 of the ADC can be expressed as $r(t) = s(t) + n(t)$, where $s(t)$ is the reference signal component and $n(t)$ is the noise component.

Let us assume that the power of the reference signal is constant for a short duration $t_0 - \delta \le t < t_0 + \delta$. The reference signal for this time duration can be denoted as

$$s(t) = \sqrt{2P_s}\cos(2\pi f_c t + \phi), \tag{1}$$

where f_c and ϕ are the frequency and the phase of the reference noise. The power P_s is calculated by

$$P_s \simeq \frac{1}{2\delta}\int_{t_0-\delta}^{t_0+\delta} s^2(t)dt. \tag{2}$$

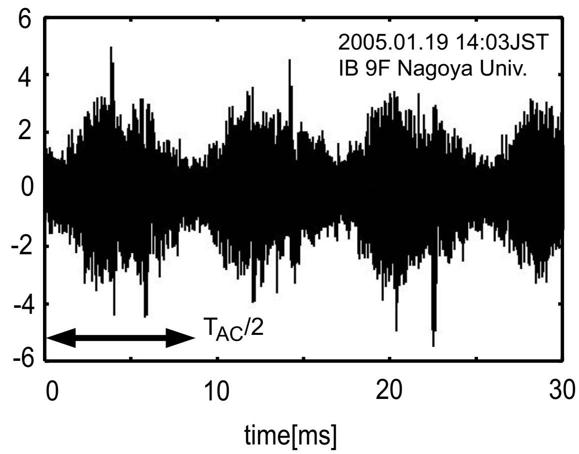

Fig. 3. a Snap-shot of a Measured Noise Waveform (Normalized)

In our measurement, $r(t)$ is sampled with the sampling rate $1/T_s = 10$[MHz], and the sample at $t = kT_s$ is $r(kT_s) = s(kT_s) + n(kT_s)$. The train of samples is stored in the memory of computer and the signal and noise components are extracted by signal processing programs, i.e.narrow-band (band-pass and band-rejection) filters of the center frequency f_c. After the extraction of $s(kT_s)$ and $n(kT_s)$ components, signal power in the vicinity of iT_s is calculated as

$$P_s(iT_s) = \frac{1}{2\Delta+1}\sum_{k=-\Delta}^{\Delta} s^2((i+k)T_s), \tag{3}$$

where $\delta = \Delta T_s$. In the numerical example shown in this manuscript, $\delta = 40 \times 10^{-6}$[s], and thus $\Delta = 800$D

In order to concentrate on the noise level relative to the signal level, each noise sample $n(iT_s)$ is divided by the $P_s(iT_s)$ and the normalized noise level, or noise to signal ratio, $\rho(iT_s)$ is obtained as follows:

$$\rho(iT_s) \quad \simeq \quad n(iT_s)\left[\frac{1}{2\Delta+1}\sum_{k=-\Delta}^{\Delta} s^2((i+k)T_s)\right]^{-1/2} \tag{4}$$

Let we have $2mK$ samples in the duration $0 \le t < mT_{AC}$, where $K = \lfloor T_c/T_s \rfloor$ and $T_c = T_{AC}/2$. Then the time-averaged variance of $\rho(iT_s)$ is calculated as

$$\gamma^{-1} \quad = \quad \frac{\sum_{j=0}^{2m-1}\sum_{i=0}^{K-1}\rho^2(iT_s + jT_c)}{2mK}. \tag{5}$$

With this value $\rho(iT_s)$ is normalized to have unity variance as

$$\eta(iT_s) = \sqrt{\gamma}\rho(iT_s). \tag{6}$$

In the followings of this manuscript, the statistical feature of this noise component $\eta(iT_s)$ is examined.

IV. Instantaneous Power and Cyclicity of Noise

Figure 3 shows an example of $\eta(iT_s)$, measured in the laboratory of the authors. From this figure, we can observe

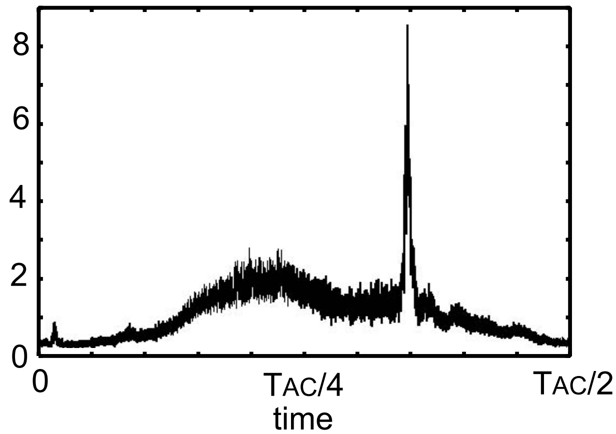

Fig. 4. an Example of Measured Noise Variance

that the noise is not stationary. Thus we have to discuss the instantaneous power (variance) of the noise,

$$\sigma^2(t) = \mathrm{E}[\eta^2(t)] = \gamma \mathrm{E}[\rho^2(t)]. \quad (7)$$

As shown in the figure, the noise in power-line has cyclic feature. Thus let us replace the ensemble average $\mathrm{E}(\cdot)$ by cyclic average with the cycle duration $T_c = T_{AC}/2$, where T_{AC} is the cycle duration of the mains' voltage (in western Japan, $T_{AC} = 1/60 = 16.66 \times 10^{-3}$). Then for $0 \le iT_s < T_c$, we have instantaneous power (variance) of $\eta(t)$ as

$$\begin{aligned} \sigma_m^2(iT_s) &= \frac{1}{2m} \sum_{j=0}^{2m-1} \eta^2(iT_s + jT_c) \\ &= \frac{\gamma}{2m} \sum_{j=0}^{2m-1} \rho^2(iT_s + jT_c), \end{aligned} \quad (8)$$

which is shown in Fig. 4. Note that $\sum_{i=0}^{K-1} \sigma_m^2(iT_s) = 1$.

V. PDF OF POWER-LINE NOISE

In order to study communications under man-made impulsive noise environment, the model of the probability distribution function (PDF) proposed by Middleton[4] is often employed. In this model, the PDF of noise is expressed as a sum of Gaussian functions with different variances. This model is good for expressing various classes of impulsive noise with a small number of parameters; however, it is difficult to express non-stationary feature of power-line noise with this model.

Thus for the power-line noise with the cyclostationary features, we have proposed a new model, first in [2]. In this model, power-line noise is assumed to be cyclostationary additive Gaussian noise whose mean is zero and the variance is synchronous to AC voltage of mains. Thus the PDF of the noise at the instance $t = iT_s$ is expressed as

$$P(\eta(iT_s)) = \frac{1}{\sqrt{2\pi\sigma^2(iT_s)}} \exp\left[-\frac{\eta^2(iT_s)}{2\sigma^2(iT_s)}\right], \quad (9)$$

TABLE II
PARAMETERS FOR $\hat{\sigma}^2(t)$ OF FIG. BUN

ℓ	A_ℓ	θ_ℓ[deg]	n_ℓ
0	0.230	-	0
1	1.38	174	1.91
2	7.17	145	1.57×10^5

where $\sigma^2(t)$ is the instantaneous variance of the noise, which is a cyclic function with the cycle duration $T_c = T_{AC}/2$. Therefore, the PDF is also a cyclic function, $P(n(iT_s)) = P(n(iT_s + jT_c))$ for any integer j.

Since the noise of this model has different variances at different phases of the AC voltage, PDF of noise samples taken without the synchronization to the AC voltage becomes the sum of Gaussian distributions with different variances. Thus the PDF of the noise in the proposed model at arbitral sampling timing agrees with the PDF model of impulsive noise proposed by Middleton.

If the noise has cyclostationary characteristics, we can expect $\lim_{m\to\infty} \sigma_m^2(iT_s) = \sigma^2(iT_s)$. The results of our experiments show that $m = 50 \sim 100$, i.e. observation of about one second, is enough for the convergence of $\sigma_m^2(iT_s)$.

Then the next problem is to approximate this time function by a simple function with small set of parameters. For this purpose, the model employs the following cyclic function to approximate $\sigma^2(iT_s)$:

$$\hat{\sigma}^2(t) = \sum_{\ell=0}^{L-1} A_\ell |\sin(\pi t/T_C + \theta_\ell)|^{n_\ell}, \quad (10)$$

where a set of $3L$ parameters A_ℓ, θ_ℓ and n_ℓ for $\ell = 0, 1, 2, ..., L-1$ denotes the characteristics of the noise. Note that the following equation should stand to keep the time-average power of $\eta(t)$ unity, i.e.,

$$\frac{1}{T_c} \int_{-T_c/2}^{T_c/2} \hat{\sigma}^2(t) dt = 1. \quad (11)$$

In order to express the detailed features of noise, large L is needed; however, often $L = 3$ is enough. In this case, the first term is used to represent the constant (or background) noise component, and the parameters θ_0 and n_0 are not needed i.e., n_1 is zero and θ_0 can be an arbitral value. The second term is used to express continuous cyclic feature of noise and the last term for cyclic impulsive noise. As the result, seven parameters are needed for (10) and additional one parameter a describes the feature of noise in frequency domain as in the following section.

In Fig. 5 an example of $\hat{\sigma}^2(t)$ is shown, which is the approximation of the measured variance in Fig. 4 with the parameters given in Table II where $L = 3$.

VI. PDS OF POWER-LINE NOISE

Following the PDF of the instantaneous value of power-line noise given in the previous section, this section discusses Power Density Spectrum (PDS) of noise. Figure 6 shows an example of PDS of the measured noise waveform, shown in

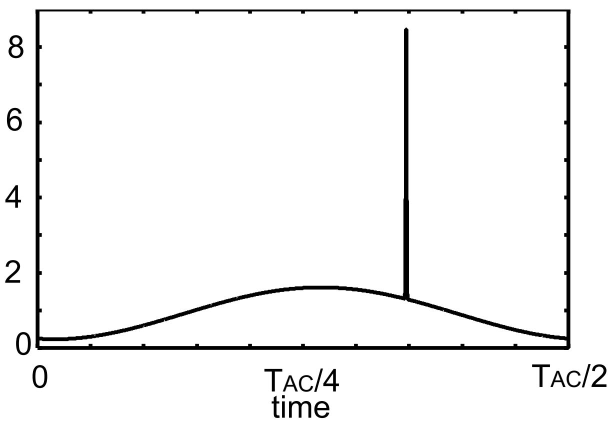

Fig. 5. Example of Measured Noise Variance

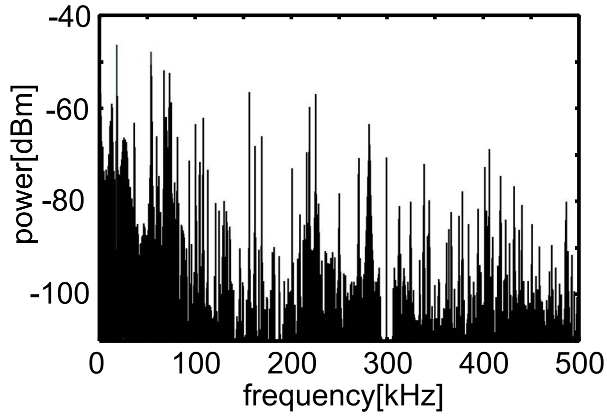

Fig. 6. Example of Power Spectrum Density of Power-line Noise

Fig. 3. From this figure, we can confirm that the noise in power-line is non-white. In addition, the noise power is time function as is discussed in the previous section. For this reason, the proposed noise model denotes the variance of power-line noise at t on the frequency f as

$$\sigma^2(t,f) = \sigma^2(t)\alpha(f), \tag{12}$$

where $\alpha(f)$ is the PDS denoted as

$$\alpha(f) = \frac{a}{2}\exp(-a|f|). \tag{13}$$

Note that the equation (13) becomes a linear function with a negative inclination in the domain of positive frequency if the power is ploted in logarithm scale as

$$\ln\alpha(f) \;=\; -af + C \quad \text{for } 0 < f, \tag{14}$$

where C is a constant. Thus the parameter a can be estimated by least-squares method. In the calculation of a, we have used averaged noise power in sub-bands of width 50kHz as shown in Fig. 7 instead of row value of noise. For an example, from this figure, a is estimated to be 1.2×10^{-5}.

Since the inverse Fourier transform of PDS is autocorrelation, the following equation denotes the correlations of noise samples taken at two different time instances.

$$\begin{aligned} \mathrm{E}\left[\eta(t)\eta(t+\tau)\right] &= \sigma^2(t)\int_{-\infty}^{\infty}\alpha(f)\exp(j2\pi f\tau)df \\ &= \frac{1}{1+(2\pi\tau/a)^2}\cdot\sigma^2(t). \end{aligned} \tag{15}$$

Note that this equation implies that the noise in different time instances has significant correlation only for small τ. For example, if $\tau > a/2 \simeq 25[\mu\text{s}]$, the correlation of two samples is less than 10%.

VII. Generation of Noise Waveform

Using the mathematical model discussed above, we can generate simulated waveforms of power-line noise as follows.

1) Determine a set of parameters for $\sigma^2(t)$.
2) Generate Gaussian noise with instantaneous variance $\sigma(t)$.

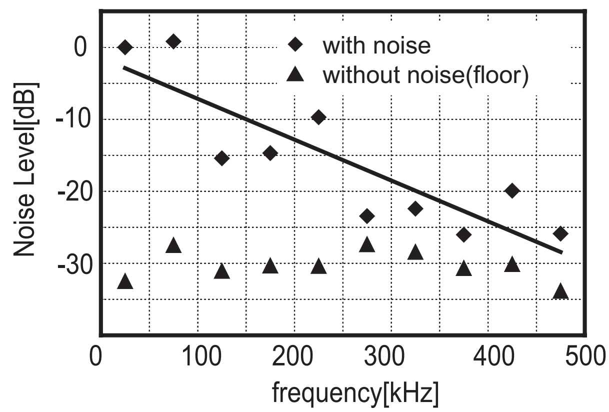

Fig. 7. Power Distribution of Power-line Noise in Subbands

3) Pass the noise to the filter with the frequency response $\sqrt{\alpha(f)}$. Parameter a is needed.

In the previous sections, we have derived parameters from the noise shown in Fig. 3 and obtained the set of parameters for $\hat{\sigma(t)}$ in Table II and the parameter in frequency domain as $a = 1.2 \times 10^{-5}$. Figure 8 shows an example of a simulated noise waveform generated by these parameters using the procedure shown above.

Comparing Fig. 8 and Fig. 3, we can confirm that the features of cyclostationary power-line noise is well represented by the computer generated noise.

VIII. Conclusions

In this manuscript, a simple mathematical representation of the noise in narrowband power-line communication systems is introduced. This model can express time variant and non-white features of the noise in power-lines with a small number of parameters. The meaning of the noise model and also the procedure to generate noise waveform from given parameters are described, and a set of parameters derived from the noise waveforms recently measured is provided.

The proposed model provides a benchmark for the design and the evaluation of communication systems under the time

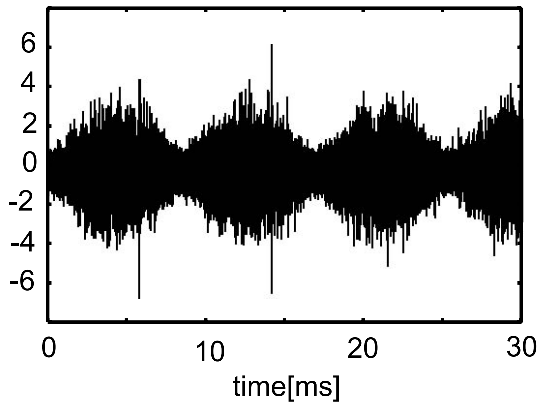

Fig. 8. Computer Simulated Power-line Noise

variant colored power-line noise environment, which cannot be represented by conventional noise models. However the importance of the proposed model lies not only in the introduction of a tractable tool for performance evaluation. Since various types of measured noise can be described with the proposed tractable equations, the model can be used as a powerful mean for the studies of man-made noise itself.

The measurement and parameter determination of noise waveforms at many locations, construction of a database of the power-line noise, and the establishment of standard sets of parameters, are important future works.

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