An OFDM System with Reduced Non-linear Effect

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Abstract—

In this paper, we discuss on the realization of reduced peak power transmission for the OFDM systems. Since the signals have large amplitude fluctuations in conventional OFDM systems, signals amplified by a non-linear amplifier are greatly distorted, resulting in severe performance degradation. In order to avoid this large amplitude fluctuation, we propose a scheme for reducing the non-linear distortion by using the set of the signal point series which show low Peak-to-Mean-Envelope-Power-Ratio (PMEPR) value. In this system, one symbol is transmitted with multicarriers and the received signal is detected with maximum likelihood sequence (MLS) detection.

I. Introduction

Recently, orthogonal frequency division multiplex (OFDM) technique has grown into becoming such important alternatives for wireless communications. Many advantages, such as high efficiency of band-width and robustness against multipath fading, have been reported. Also in OFDM systems, the inter-symbol interference (ISI), due to multipath, can be reduced by the insertion of a guard interval before each transmitted block. Besides these advantages, some drawbacks become apparent. One of major problems for OFDM systems is that OFDM signals which multiplexed orthogonal signals have large amplitude fluctuations, and exhibit very large Peak-to-Mean-Envelop-Power-Ratio (PMEPR) [1]. When all the sinusoidal signals of K sub-carriers are added constructively, the peak power is as large as K times the mean envelope power. This large amplitude fluctuation requires a linear high power amplifier (HPA) to avoid the irreducible bit error due to non-linear distortion.

In the mobile communication where the power consumption is strictly limited, it is difficult to use a linear HPA. Considering the use of OFDM system in the mobile communication, therefore, we have to reduce the PMEPR of OFDM signals.

Taking into account the fact that the PMEPR of OFDM signals depends on the interrelation among signal points of each sub-carriers [1] [2], we propose a novel scheme for reducing the PMEPR of orthogonal OFDM signals. In this scheme, we use a set of series that have a minimal value of PMEPR. In other words, symbol is composed of multicarriers in sequence and is received with maximum likelihood sequence detection.

The paper is organized as follows: In the following Section, we explain the PMEPR briefly. The proposed OFDM system is described in Section III. The performance of the proposed system is analized in Section IV. In Section V, the non-linear amplifier model is presented. And the performance of proposed OFDM system is discussed in Section VI through numerical examples. And finally our conclusions are drawn in Section VII.

II. PMEPR

At the OFDM transmitters, input data sequence is mapped into a complex valued signals, and then they modulate multiple carriers through an inverse fast Fourier transform (IFFT) [3]. Each sub-carrier is separated by $1/T_s$ in the frequency band, where T_s is the symbol period. In the transmitter, a complex input data series $\mathbf{C} = (c_0, c_1, c_2, \ldots, c_{K-1})$ in the frequency domain is modulated and summed up by the IFFT, and OFDM signal as the output of the IFFT is given by

$$S(n\Delta T) = \frac{1}{\sqrt{N}} \sum_{k=0}^{K-1} c_k \exp\left(\frac{j2\pi nk}{N}\right), \tag{1}$$

where ΔT denotes the sampling period, N is the number of the samples in a symbol. By an digital-to-analog (D-A) converter the discrete OFDM signal is converted into continuous signal given by

$$S(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{K-1} c_k \exp\left(\frac{j2\pi k}{T_s} t\right). \tag{2}$$

Now we consider the scalar value PMEPR. It was reported that the PMEPR is efficient scalar for evaluating the amplitude fluctuations [1], and defined by

$$\text{PMEPR} = \max_{0 \le t < T_s} \frac{|S(t)|^2}{\frac{1}{T_s}\int_0^{T_s} |S(t)|^2 dt}, \tag{3}$$

in continuous form. For the case of discrete signals the PMEPR is represented by

$$\text{PMEPR} = \max_{0 \le n < N} \frac{|S(n\Delta T)|^2}{\frac{1}{N}\sum_{n=0}^{N-1} |S(n\Delta T)|^2}. \tag{4}$$

The PMEPR depends on the peak value of OFDM signal. And the PMEPR takes lower value if the amplitude fluctuation of the signal is small. Because the non-linear distortion by use of HPA is so severe, it is desirable to reduce the PMEPR.

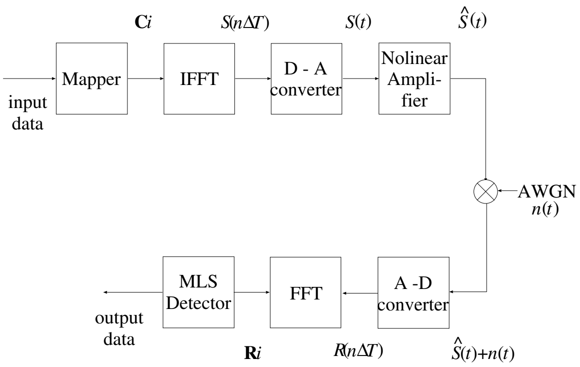

Fig. 1. System model for the proposed system.

III. Proposed System

A. The Concept of the Proposed System

In the conventional OFDM systems, one symbol is transmitted with one sub-carrier, however, in the proposed system one symbol is transmitted with multiple sub-carriers, which means that each symbol is composed of signal points on the sub-carriers. Each sub-carrier has a respective signal point selected according to the information data sequence. To avoid the performance degradation due to non-linearity, we set up the signal points which show lower PMEPR. The receiver detects the transmitted symbols based on maximum likelihood sequence (MLS) detection. According to the squared Euclidean distance, the detector selects the symbol which shows the minimum value of distance as the transmitted symbol.

The block diagram of the proposed OFDM system is illustrated in Fig. 1. At the transmitter, i-th complex signal point series including K signal points $\mathbf{C}_i = (c_{i,0}, c_{i,1}, c_{i,2}, \cdots, c_{i,K-1})$ is assigned according to input data sequence by a mapper. Each signal point series we use here have phases which are selected so as to have lower PMEPR as well as to enlarge the squared Euclidean distance with other signal point series. The output signal point series is fed into an IFFT, modulated and summed up. Then the output signal $S(n\Delta T)$ from the IFFT, we refer as OFDM signal, is converted into a continuous signal $S(t)$ through D-A converter, then it is amplified by a non-linear amplifier. The output from the non-linear amplifier is expressed as,

$$\hat{S}(t) = G(S(t)) \exp\{j(\Phi(t))\}, \tag{5}$$

where $G(\cdot)$ and $\Phi(\cdot)$ are the amplitude and phase characteristics function of the non-linear device. After the OFDM modulation, the signal is distorted with the non-linear amplification and additive white Gaussian noise (AWGN) $n(t)$ is added. The received signal is expressed as,

$$R(t) = \hat{S}(t) + n(t). \tag{6}$$

The received signal is sampled with the same sampling period ΔT as in the transmitter. And then, the sampled signals $R(n\Delta T)$ are demodulated and divided with a fast Fourier transformer (FFT),

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} R(n\Delta T) \exp(\frac{-j2\pi nk}{N}). \tag{7}$$

Then output complex signal points of the FFT are fed into a MLS detector. The MLS detector computes the squared Euclidean distances between received signal point series and locally generated signal point series, and detects the signal point set which shows the minimum distance as the transmitted one. The MLS detection scheme determines the series that minimizes the squared Euclidean distance, which is defined by

$$\mathrm{d}^2(\mathbf{C}_j, \mathbf{R}_i) = \sum_{k=0}^{K-1} |c_{j,k} - r_{i,k}|^2, \tag{8}$$

for all possible series, where $\mathbf{C}_j$ is the prepared signal point series in the reception end, and $\mathbf{R}_i$ is the output signal point series of the FFT.

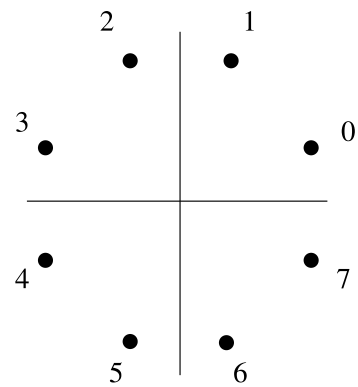

Fig. 2. The signal point constellation.

TABLE I

AN EXAMPLE OF SIGNAL POINT SERIES AND CORRESPONDING PMEPR VALUES FOR FOUR SUB-CARRIERS.

signal point series				PMEPR
0	0	0	4	1.77
0	0	4	0	1.77
0	4	0	0	1.77
0	4	4	4	1.77
2	2	2	6	1.77
2	2	6	2	1.77
2	6	2	2	1.77
2	6	6	6	1.77
4	0	0	0	1.77
4	0	4	4	1.77
4	4	0	4	1.77
4	4	4	0	1.77
6	2	2	2	1.77
6	2	6	6	1.77
6	6	2	6	1.77
6	6	6	2	1.77

B. The Relationship among the Number of Signal Point Series, PMEPR and Squared Euclidean Distance

A set of the signal point series in the case of 4 sub-carriers, each point of which corresponds to 8-PSK signal constellation shown in Fig. 2 and the PMEPR values for each series are shown in Table I. This example is obtained by computing all possible series. For a minimum PMEPR value of 1.77, we find that the number of signal point series which show a minimum PMEPR is 64 among all possible series 8^4. Among these signal point series, 16 signal point series with the minimal squared Eulidean distance of 8.0 is obtained. In this case, the selected points on each sub-carrier results in being coincident with the points in the QPSK constellation.

Actually there exists a relationship among the value of PMEPR, the squared Eulidean distance, and the number of available signal point series. We have to select an appropriate value for each parameters. The PMEPR vs. the number of available signal point series normalized by 8^4 (i.e. the number of all possible series) is shown in Fig. 3. The available signal point series are those whose PMEPR value is below a certain threshold. This threshold should be determined as a function of the permissible signal's peak power.

IV. PERFORMANCE ANALYSIS

The available signal point series is limited by acceptable PMEPR values and the minimum squared Eulidean distance.

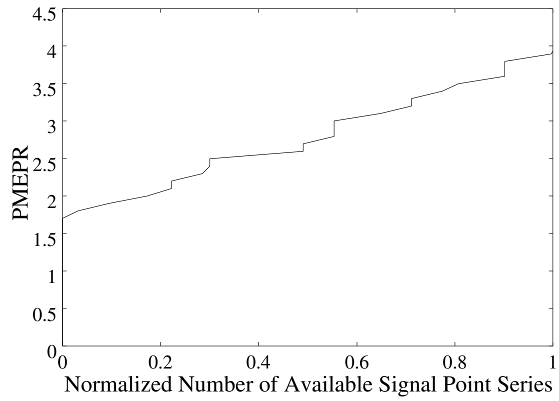

Fig. 3. The number of available signal point series normalized by the number of all possible series in the case of 4 sub-carriers.

In the proposed system, the receiver detects the transmitted symbol by comparing the squared Euclidean distance between all possible symbols and received symbol expressed as Eq. (8). Received signal point series is distorted by the non-linear amplifier and has a noise component. The detector may select an incorrect symbol when the squared Euclidean distance between the received symbol and one of the other symbols is less than transmitted symbol, and it can be expressed as,

$$\mathrm{d}^2(\mathbf{C}_j, \mathbf{R}_i) - \mathrm{d}^2(\mathbf{C}_i, \mathbf{R}_i) < 0, \quad i \neq j \tag{9}$$

where $\mathbf{C}_i$ is the correct signal point series, $\mathbf{C}_j$ is the incorrect signal point series and $\mathbf{R}_i$ is the received and demodulated signal point series. Defining $\mathbf{E}_i$ as the distorted signal point series by the non-linear amplifier at the transmitter, we can express the received signal point series in Eq. (8) as,

$$\mathbf{R}_i = \mathbf{E}_i + \mathbf{n}, \tag{10}$$

where $\mathbf{n}$ denotes noise component vector due to AWGN. Developing squared Euclidean distance in Eq. (9), we get following equation,

$$\mathrm{d}^2(\mathbf{C}_j, \mathbf{E}_i) - \mathrm{d}^2(\mathbf{C}_i, \mathbf{E}_i) + \mathbf{n}(\mathbf{C}_i - \mathbf{C}_j)^* + \mathbf{n}^*(\mathbf{C}_i - \mathbf{C}_j) < 0 \tag{11}$$

where $(\cdot)^*$ denotes the complex conjugate. From Eq. (11), the probability that one signal point series $\mathbf{C}_i$ is transmitted and MLS detector selects the incorrect series $\mathbf{C}_j$ is expressed,

$$P(\mathbf{C}_j|\mathbf{C}_i) = \mathrm{Q}\left(\sqrt{\frac{\{\mathrm{d}^2(\mathbf{C}_j, \mathbf{E}_i) - \mathrm{d}^2(\mathbf{C}_i, \mathbf{E}_i)\}^2}{4\mathrm{d}^2(\mathbf{C}_j, \mathbf{C}_i)\sigma^2}}\right), \tag{12}$$

$$= \mathrm{Q}\left(\sqrt{\frac{\{\mathrm{d}^2(\mathbf{C}_j, \mathbf{E}_i) - \mathrm{d}^2(\mathbf{C}_i, \mathbf{E}_i)\}^2}{2\mathrm{d}^2(\mathbf{C}_j, \mathbf{C}_i)} \frac{E_s}{N_0}}\right), \tag{13}$$

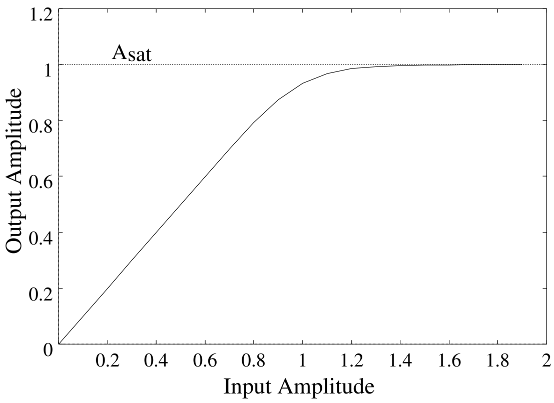

Fig. 4. The SSPA characteristics: Normalized AM/AM conversion. ($A_{sat} = 1$)

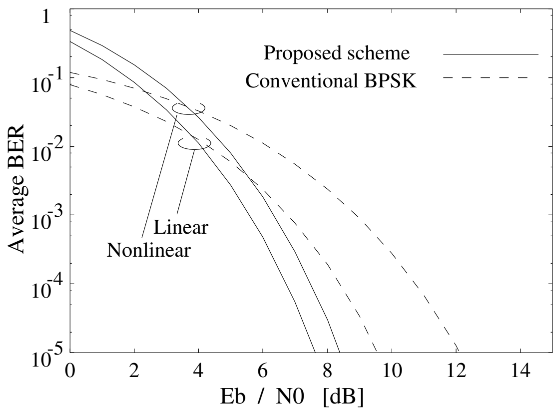

Fig. 5. Performance of proposed system for SSPA and linear HPA. (IBO=1.0dB)

where σ^2 and E_s/N_0 denote the variance of AWGN and symbol energy-to-noise spectral density radio, respectively. Assuming that all symbols are equally likely, we get union bound of symbol error probability by summing up Eq. (13) for all probable series. That is,

$$P_s < \frac{1}{N_c} \sum_{i=0}^{N_c-1} \sum_{\substack{j=0 \\ j \neq i}}^{N_c-1} P(\mathbf{C}_j|\mathbf{C}_i), \tag{14}$$

where N_c denotes the number of available signal points series. Here we define $d_h(i,j)$ as a Hamming distance between symbol $\mathbf{C}_j$ and $\mathbf{C}_i$. From Eq. (14), the bit error probability becomes:

$$P_b < \sum_{i=0}^{N_c-1} \sum_{\substack{j=0 \\ j \neq i}}^{N_c-1} \frac{d_h(i,j)}{N_c \log_2 N_c} P(\mathbf{C}_j|\mathbf{C}_i). \tag{15}$$

V. Non-linear Amplifier Models

The non-linear HPA can be modeled as a memoryless device. We define HPA input signals $g(t)$ and output signals $G(t)$. Considering Solid State Power Amplifier (SSPA), the amplitude characteristic function $G(t)$ in Eq. (5) is given as [4],

$$G(t) = \frac{g(t)}{(1 + (g(t))^{10})^{\frac{1}{10}}}, \tag{16}$$

where the input and the output amplitudes are normalized by the saturation amplitude A_{sat}. Phase can be assumed to be linear. This kind of amplifier due to no "one-to-one" AM/AM conversion, and the SSPA adds no phase distortion. The normalized characteristics of the SSPA is illustrated in Fig. 4.

The non-linear amplifier is often characterized by the input back-offs (IBO) and output back-offs (OBO) and they are defined as

$$IBO = \frac{A_{sat}^2}{E[|S(t)|^2]}, \tag{17}$$

$$OBO = \frac{A_{sat}^2}{E[|S(t)|^2]}, \tag{18}$$

where $E[\cdot]$ denotes the mean value.

VI. Numerical Results

The performance of the proposed system is presented in Fig. 5 with SSPA non-linearity. The signal point series used in the numerical example is the same as those shown in Table I, and IBO is set at 1.0 dB. For comparison purpose we consider the conventional OFDM system that each sub-carrier is modulated with BPSK, because of the equality of transmitted band width. In this case bit energy E_b is similar to symbol energy E_s.

As shown in this figure, the proposed system can achieve about 2.0dB gain at BER=10^{-5} relative to a conventional BPSK.

For SSPA, the degradation of non-linear HPA for proposed system is smaller than that of the conventional BPSK. The total degradation of the system with BPSK at BER=10^{-5} is 2.5dB. That of proposed system is 0.8dB, and it is much lower than the degradation in the system with BPSK.

VII. Conclusions

In this paper, we have proposed a scheme for improving the detect-ability with the OFDM signals and at the same time, for reducing the effect of non-linearity. A numerical result has shown that the gain of E_b/N_0 is about 3.7 dB and 1.0 dB compared with the conventional BPSK, in the case at the BER of 10^{-5} in the presence of non-linearity, while it is about 2.0 dB in the linear case.

This is due to the lower PMEPR of proposed system and having large squared Euclidean distances more than those of conventional systems, and we have shown that the proposed scheme is very effective against non-linear distortion.

For the future study, we consider increasing in the number of sub-carriers and seek for the signal point series which show lower PMEPR and large squared Euclidean distance.

Acknowledgment

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