

OFDM Systems with Multiple Trellis Coded Modulation

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Abstract—

In an attempt to improve the performance under frequency selective fading environment, we propose in this paper an Orthogonal-Frequency-Division-Multiple-Access (OFDM) system in which MTCM (Multiple Trellis Coded Modulation) is applied to a set of multiple subcarriers. The code is designed to enlarge the summation of the Euclidean distance and the product distance. We also propose a scheme for assigning the symbols to subcarriers adaptively based on the channel state information (CSI) fed back from the reception receive end, and evaluate the performance applying these two codes and the proposed scheme under a multipath Rayleigh fading environment.

I. INTRODUCTION

Recently, Orthogonal-Frequency-Division-Multiplex (OFDM) systems have grown into becoming such important alternatives for wireless communications. Many advantages, such as high efficiency of band-width and robustness against multipath fading, have been reported. Also in OFDM systems, the inter-symbol interference (ISI), due to multipath, can be reduced by the insertion of a guard interval before each transmitted block. However, its performance is limited due to the badly attenuated subcarriers caused by frequency selective fading. Therefore, it is important to develop schemes to cope with this deterioration. A frequency interleaving technique incorporated with a forward error correcting code (FEC) may be an effective way to reduce this degradation. In this paper, we investigate the performance of OFDM systems employing multiple trellis coded modulation (MTCM) under a frequency selective fading environment.

Trellis Coded Modulation (TCM) was proposed by Ungerboeck [1], and investigated by Wei, Calderbank, Sloane and Benedetto [2],[3]. Ungerboeck extended the TCM to multidimensional codes. The procedure of set partitioning is based on the concept prescribing the Euclidean distance and maximizing the minimum free Euclidean distance between code words. Enlargement of the Euclidean distance between code words is most effective in the presence of only additive white Gaussian noise (AWGN). After this work, Divsalar and Simon proposed a new set partitioning method [7]. The set partitioning of MTCM is performed in multidimensional signals. It enables to assign all pairs of MPSK symbols to different positions in the parallel branch.

In [5]–[7], the MTCM is performed to successive symbols in time domain. It is known that the set partitioning method which enlarges a product of the Euclidean distance is robust against rapidly varying fading. In the MTCM coded OFDM system, the coding is performed to successive symbols in frequency domain and the set

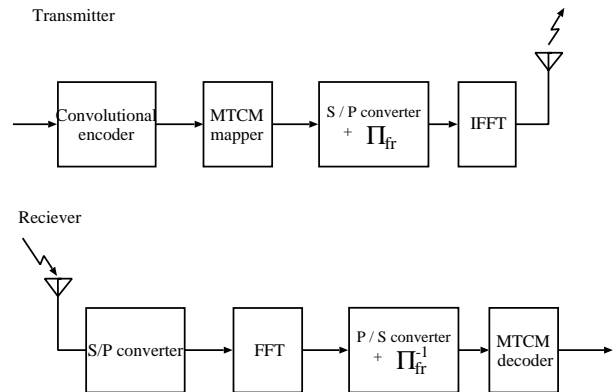


Fig. 1. MTCM coded OFDM system.

partitioning is applied to the pair of subcarriers. If we use the MTCM with the set partitioning method proposed by Divsalar, we have improvement of performance by interleaving adaptively in accordance with a channel state information (CSI).

The paper is organized as follows. After describing the MTCM coded OFDM system in the following section, the proposed adaptive interleaving schemes is developed in Section III. In Section IV, the effect of frequency selective fading is analyzed. The performance of the proposed the system is discussed in Section V through numerical examples. And finally, our conclusions are drawn in Section VI.

II. SYSTEM DESCRIPTION

The MTCM coded OFDM system is modeled as shown in Fig. 1. At the transmitter, the input data is coded by an MTCM encoder, convolutionally with a code rate R . The coded-bits are mapped into multiple complex-valued signal points, which we refer here as pair symbols. The output symbols of the MTCM encoder are fed to a serial-to-parallel converter (S/P), which converts them into K low-speed symbols: $(x_0, x_1, x_2, \dots, x_{K-1})$, each of duration T_s . Then, a frequency interleaver (Π_{fr}) assigns the symbols to each of the subcarriers, a fact that reduces the influence of error bursts. The interleaved symbols, referred to as $(X_0, X_1, X_2, \dots, X_{K-1})$, are modulated and summed up by an inverse fast Fourier transform (IFFT) [8]. The modulated and summed-up signal in the equivalent low-pass expression is written as

$$s(nT) = \frac{1}{\sqrt{N}} \sum_{k=0}^{K-1} X_k \exp\left(\frac{2j\pi kn}{N}\right), \quad (1)$$

where T denotes the sampling period and N denotes the number of samples. The signals are cyclically extended by a guard interval G , followed by a parallel-to-serial conversion (P/S), before transmission in order to prevent ISI caused by multipath spread T_m . The condition $G > T_m$ should be fulfilled.

The signal affected by frequency selective fading is received, and sampled with the same sampling period T as in the transmitter. This sampled signal, which we refer to as $r(nT)$, is then fed into an S/P converter and the guard interval is removed from it. Then it is fed into a fast Fourier transform (FFT) which divides and demodulates it. The demodulated complex-valued signal for the k -th subcarrier is:

$$Y_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r(nT) \exp\left(\frac{-2j\pi kn}{N}\right), \quad (2)$$

are fed into the frequency de-interleaver (Π_{fr}^{-1}), and then K parallel complex symbols ($y_0, y_1, y_2, \dots, y_{K-1}$) are obtained. These symbols are combined to a high-speed symbol sequence with a P/S converter and fed into an MTCM decoder. The MTCM decoder computes the Euclidean distance between the demodulated received signals and the prepared symbols at the reception end. Then, it decides the symbols which show the minimum Euclidean distance based on soft decision Viterbi algorithm, which results in providing a binary received data sequence.

III. PROPOSED ADAPTIVE INTERLEAVING

OFDM systems are robust against multipath fading, but some amount of degradation may still remain because of clustering of low-level subcarriers. In the conventional OFDM systems, symbols are assigned to subcarriers with block interleaver. It is known that TCM, including MTCM, combined with interleaving gives better performance in fading environment. Assuming that the transmitter knows perfectly CSI based on the information fed back from the reception end, in the frequency interleaver, the process of assigning symbols to subcarriers based on CSI is more effective than assigning them randomly. To reduce the effect of fluctuation among subcarriers, caused by frequency selective fading, and to avoid clustering of low-level subcarriers at the reception end, each data symbol, consisting of several bits, is assigned a pair of a high-level subcarrier and a low-level subcarrier for MTCM. For set partitioning, we adopt the methods proposed by Divsalar and Simon [7].

The MTCM proposed by Divsalar and Simon arranges all the symbol pairs into different positions. This enables to average the power of multiple symbols, by assigning each symbol pair to a low level and a high level subcarrier based on CSI.

In [7], multiple symbols are spanned over the time domain, in our scheme, we consider to span them over frequency domain. MTCM signals consist of γ symbols, which are called multiplicity coefficients. They are assigned K separate subcarriers and are transmitted simultaneously in one symbol interval. Consider the use of two PSK symbols of multiplicity $\gamma = 2$ and $K = 8$

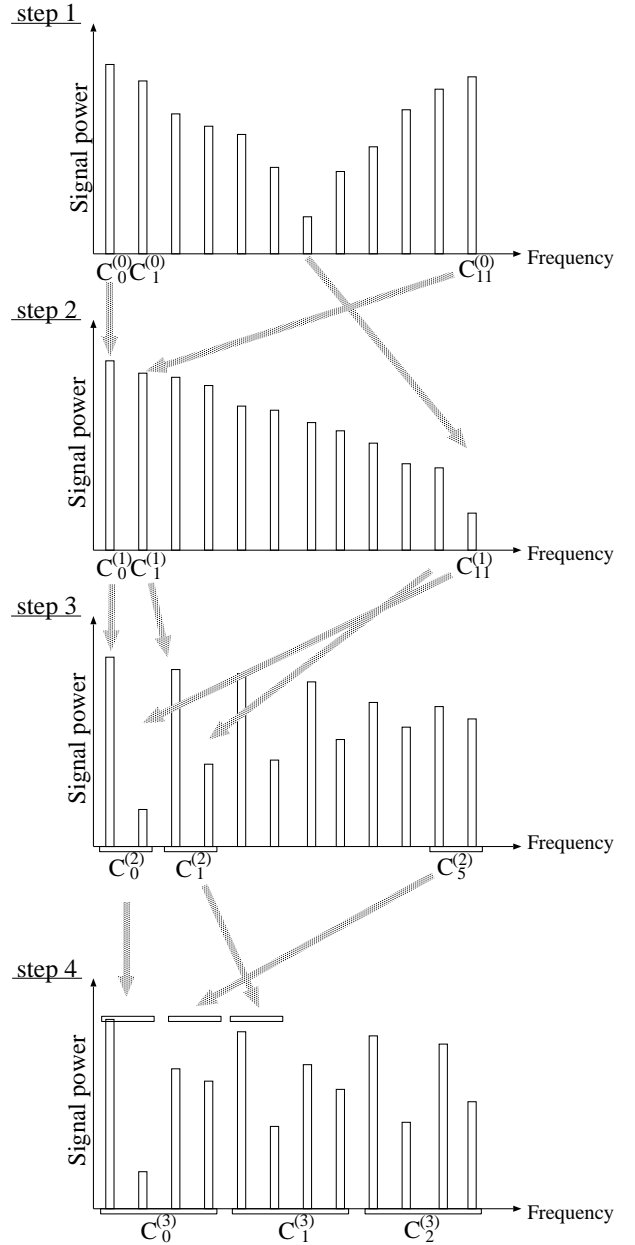


Fig. 2. The rule of adaptive interleaving in the case of 4-Dimensional signals, the number of subcarriers $K=8$.

subcarriers as a reference system. The rule of constructing the adaptive interleaver is described as follows, and its steps are illustrated in Fig. 2 (case of 4-D signals, $K = 8$).

- (1) The received powers of each subcarrier $c^{(0)} = (c_0^{(0)}, c_1^{(0)}, \dots, c_{K-1}^{(0)})$ is computed. ($k = 0, 1, \dots, K-1$)
- (2) The values of the received powers $c_k^{(0)} \forall k$ are sorted from high-level to low-level, and are labeled as $c_k^{(1)} \forall k$ such that $c_k^{(1)} > c_{k+1}^{(1)} \forall k$. ($k = 0, 1, \dots, K-1$)
- (3) The pair $c_k^{(2)} = (c_k^{(1)}, c_{K-k}^{(1)})$ is defined. ($k = 0, 1, \dots, K/2-1$)
- (4) The pair $c_k^{(3)} = (c_k^{(2)}, c_{K/2-k}^{(2)})$ is defined. ($k = 0, 1, \dots, K/4-1$)

The adaptive interleaving can be given by the inverse

process of adaptive de-interleaving. Therefore, assigning multiple symbols to low-level and high-level subcarriers at the transmitter is achieved by the inverse process of the above procedure described in Fig. 2.

IV. PERFORMANCE ANALYSIS

The mobile radio channel can be represented as a linear channel characterized by a time-varying complex baseband impulse response,

$$h(t; \tau) = \sum_{m=0}^{M-1} h_m(t) \delta(\tau - \tau_m), \quad (3)$$

where the value τ_m is referring to the time of arrival at the receiver of the m -th path of M components, and

$$h_m(t) = \rho_m e^{j\theta_m}, \quad (4)$$

represents the time-variant multipath signal component, with ρ_m denoting the attenuation factor, and θ_m the carrier phase of path m . This can be characterized as an independent zero-mean complex Gaussian process with variance:

$$E[|h_m(t)|^2] = \alpha_m^2, \quad (5)$$

where $E[\cdot]$ denotes the ensemble average. We assume that $h(t; \tau)$ is wide-sense stationary for uncorrelated scattering [10]. Its auto-correlation function is given as:

$$E[h^*(t; \tau) h(t + \Delta t; \hat{\tau})] = R_h(\Delta t; \tau) \delta(\tau - \hat{\tau}). \quad (6)$$

Considering $\Delta t = 0$ for the slowly fading channel, the auto-correlation function $R_h(0; \tau) = R_h(\tau)$ is simply the average power output of the channel as a function of delay time. And we have the root mean square (rms) delay spread τ_{rms} , defined as the square root of the second central moment [11],

$$\tau_{rms} = \sqrt{\frac{\sum_{m=0}^{M-1} (\tau_m - D)^2 \alpha_m^2}{\sum_{m=0}^{M-1} \alpha_m^2}}, \quad (7)$$

where D denotes the average delay, i.e., the centroid of arrival time delay of each path. It is expressed as

$$D = \frac{\int \tau R_h(\tau) d\tau}{\int R_h(\tau) d\tau}. \quad (8)$$

The observed OFDM signal at the receiver can be written as:

$$r(t) = \sum_{m=0}^{M-1} h_m(t) s(t - \tau_m) + n(t), \quad (9)$$

where $s(t)$ is the transmitted signal, and $n(t)$ is the AWGN with a double-sided power spectral density of $N_0/2$. Performing the Fourier transform to equation (9)

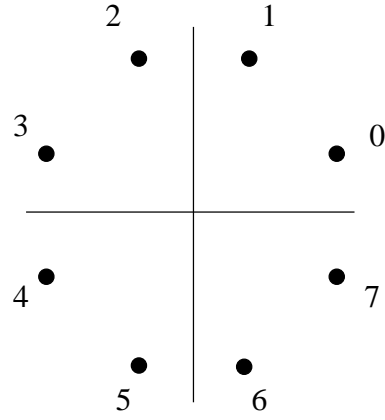


Fig. 3. Signal point constellation.

and squaring it, the received power of each subcarrier is written as

$$|R(k\Delta f)|^2 = \sum_{l=0}^{M-1} \sum_{m=0}^{M-1} \exp\{2j\pi k \Delta f \tau_{l,m} + j\theta_{l,m}\} + n_k, \quad (10)$$

where n_k denotes the component due to AWGN on the k -th subcarrier, $\Delta f = 1/T_s$, $\tau_{l,m} = \tau_l - \tau_m$ and $\theta_{l,m} = \theta_l - \theta_m$.

The MTCM decoder computes the squared Euclidean distance between the demodulated received symbol \mathbf{y} and prepared code words,

$$d^2(\mathbf{y}, \hat{\mathbf{x}}) = \sum_i |y_i - c_i \hat{x}_i|^2, \quad (11)$$

and performs the maximum likelihood sequence detection (MLSD) based on the Viterbi algorithm. c_i denotes the amplitude of the i th symbol after de-interleaving.

V. NUMERICAL EXAMPLES

In this section, the average bit error rate (BER) performance of MTCM coded OFDM systems, and that of our proposed scheme, are presented under the following conditions:

- Bit rate : 20 Mbps.
- The number of subcarriers : $N=48$.
- Symbol duration : $T_s=4.0 \mu\text{sec}$.
- Guard interval : $G = 800 \text{ nsec}$.
- Constraint length : $\nu=3$.

Let us consider the two set partitioning methods enlarging the summation and the product of Euclidean distances proposed in [4] and [7], respectively. we use these set partitioning schemes to be performed in two 8PSK symbols. To study the effects of frequency selective fading, we use the equal amplitude slowly varying 2-path profile channel model (i.e. $M = 2$) with synchronously received signals. The amplitude ρ_m of each path is subject to the uncorrelated Rayleigh fading with mean value $\alpha_m^2 = 1/2$. However the received power of each subcarrier is not independent of the adjacent subcarriers. We assume that the relative arrival time delay between the first and the second path: $\tau_0 - \tau_1$, is a random variable that is uniformly distributed in $[0, G]$.

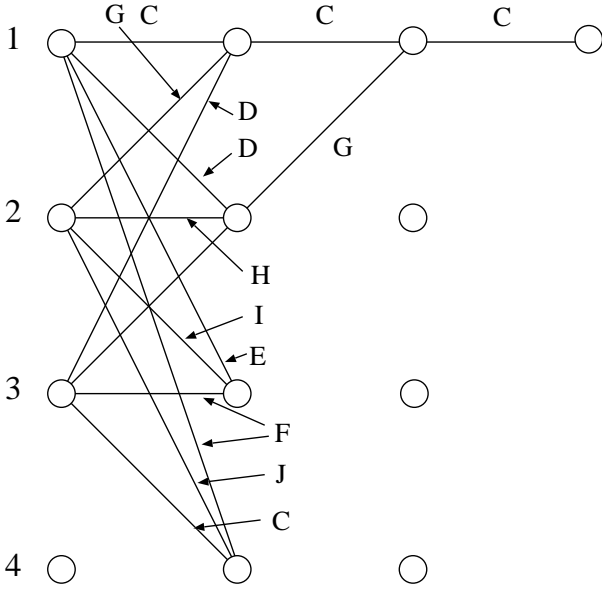


Fig. 4. Trellis diagram (constraint length $\nu = 3$).

Using the set partitioning method which enlarges the summation of Euclidean distance, the symbols assigned to each branch are expressed as:

$$\begin{aligned}
 C &= \begin{bmatrix} 00 \\ 04 \\ 44 \\ 40 \end{bmatrix} & D &= \begin{bmatrix} 22 \\ 26 \\ 66 \\ 62 \end{bmatrix} & E &= \begin{bmatrix} 02 \\ 06 \\ 42 \\ 46 \end{bmatrix} & F &= \begin{bmatrix} 20 \\ 24 \\ 64 \\ 60 \end{bmatrix} \\
 G &= \begin{bmatrix} 13 \\ 17 \\ 57 \\ 53 \end{bmatrix} & H &= \begin{bmatrix} 31 \\ 35 \\ 75 \\ 71 \end{bmatrix} & I &= \begin{bmatrix} 11 \\ 15 \\ 55 \\ 51 \end{bmatrix} & J &= \begin{bmatrix} 33 \\ 37 \\ 77 \\ 73 \end{bmatrix} . \quad (12)
 \end{aligned}$$

The numbers in the above sets are denoting the 8PSK symbol number shown in Fig. 3. Each set consists of 4 multiple symbols, so in the case of 4 state MTCM, each branch has 4 parallel paths. Using the set partitioning method which enlarges the product of the Euclidean distance, the symbols assigned to each path are expressed as:

$$\begin{aligned}
 C &= \begin{bmatrix} 00 \\ 22 \\ 44 \\ 66 \end{bmatrix} & D &= \begin{bmatrix} 15 \\ 37 \\ 51 \\ 73 \end{bmatrix} & E &= \begin{bmatrix} 04 \\ 26 \\ 40 \\ 62 \end{bmatrix} & F &= \begin{bmatrix} 11 \\ 33 \\ 55 \\ 77 \end{bmatrix} \\
 G &= \begin{bmatrix} 02 \\ 24 \\ 46 \\ 60 \end{bmatrix} & H &= \begin{bmatrix} 17 \\ 31 \\ 53 \\ 75 \end{bmatrix} & I &= \begin{bmatrix} 06 \\ 20 \\ 42 \\ 64 \end{bmatrix} & J &= \begin{bmatrix} 13 \\ 35 \\ 57 \\ 71 \end{bmatrix} . \quad (13)
 \end{aligned}$$

Figure 4 shows a 4 state trellis diagram using these set partitioning methods. The character of each path is showing a set in the equations of (12) and (13). In the simulation, it is assumed that the length of the path memory is 120 multiple symbols which is similar to the packet length, that the attenuation factor ρ_m retains the same value throughout one packet, and that the receiver has a perfect channel estimation. The interleaving is performed in one OFDM symbol.

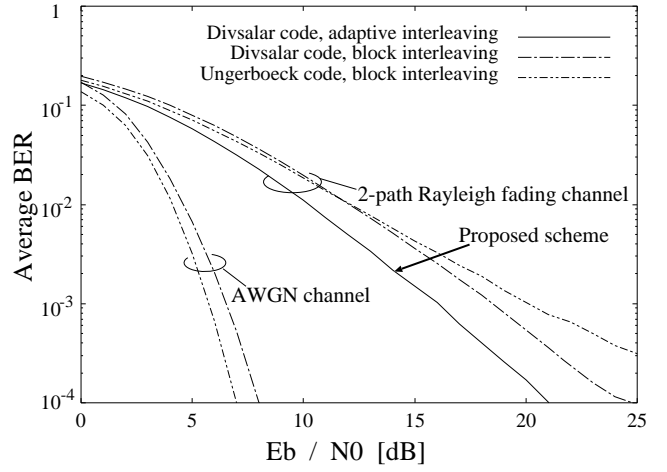


Fig. 5. The performance of MTCM coded OFDM system under AWGN and 2-path Rayleigh fading environment using block interleaving and adaptive interleaving, 48 subcarriers, constraint length $\nu=3$, multiplicity $\gamma = 2$.

Figure 5 is showing the average BER performance of the MTCM coded OFDM systems applying these set partitioning methods with block interleaving. It shows also the BER performance of the proposed adaptive interleaving scheme under AWGN and a 2-path Rayleigh fading environment given by Monte Carlo simulations. When comparing the results under AWGN channel, it can be seen that the performance applying the code which enlarges the summation of the Euclidean distance (Ungerboeck code) is better than that applying the code enlarging the product of the Euclidean distance (Divsalar code). In this case, the proposed adaptive interleaving scheme and conventional block interleaving scheme using the Divsalar code have the same performance, because it is useless to perform interleaving under such a flat channel. When comparing the results under the considered 2-path Rayleigh fading environment, we can confirm that the performance of applying the Divsalar code is better than Ungerboeck code for SNR values larger than 13 dB. As for our proposed scheme, from the figure above we can see that it gives the best performance under this 2-path Rayleigh fading environment. Note that the performance is mainly degraded due to flat fading because of the small number of paths accounted.

As a second numerical example, in order to evaluate the performance of the proposed scheme in a practical situation, we apply the typical delay profile of indoor office areas recommended by the European Telecommunications Standards Institute (ETSI). The considered delay profile is shown in Fig. 6, plotting the power mean values of each of the taps that are subject to the uncorrelated Rayleigh fading. The Average BER performance of the proposed adaptive interleaver under this environment is shown in Fig. 7. For the purpose of comparison, we introduce in the figure the performance when applying the Ungerboeck codes, and that when applying Divsalar codes. Comparing the performance of the proposed scheme using the adaptive interleaving of Section III, with that using the conventional block interleaving, one can see that for a BER=10⁻⁴ the

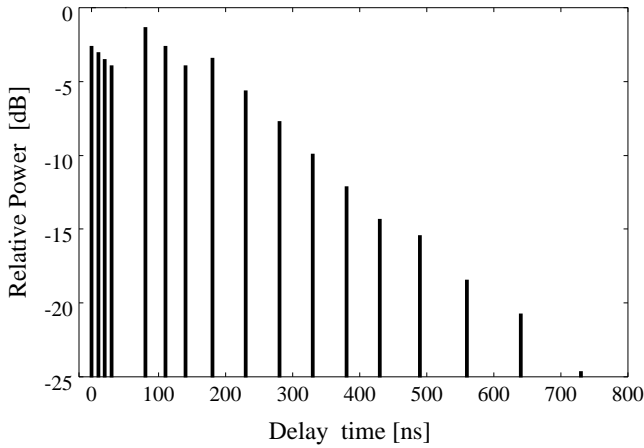


Fig. 6. The the propagation delay model of indoor office areas recommended by ETSI with delay spread $\tau_{rms}=150$ nsec.

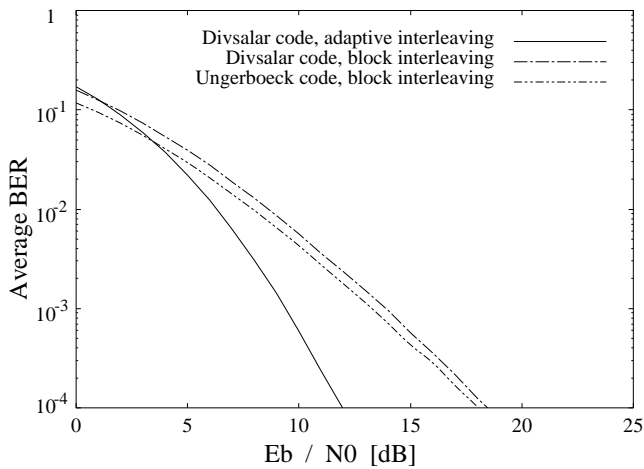


Fig. 7. The performance of MTCM coded OFDM system under the office and indoor environments with delay spread $\tau_{rms}=150$ nsec recommended by ETSI, using block interleaving and adaptive interleaving, 48 subcarriers, constraint length $\nu=3$, multiplicity $\gamma = 2$.

proposed scheme requires an SNR less than that for the conventional schemes by about 5 dB.

From these results, we can conclude that, under frequency selective environments, the proposed adaptive interleaving scheme is much more efficient than the conventional block interleaving. This is because the proposed interleaving scheme assigns symbols adaptively to subcarriers, a fact that enables equalizing the received power of pair symbols, in consequence to which the Euclidean distance is enlarged.

VI. CONCLUSION

In this paper, we have examined the performance of the MTCM coded OFDM system. As set partitioning methods, we considered the two basic schemes based on the concept of enlarging the summation and the product of the Euclidean distances. In addition to applying MTCM expanded over frequency domain, we have proposed the adaptive interleaving scheme with channel state information fed back from the reception end.

Using Monte Carlo simulations (of 1 million samples), we determined the approximated BER performances of our proposed scheme using adaptive interleaving and the schemes using the considered two set partitioning methods mentioned above. We note here that the simulation is performed for a one OFDM symbol. Comparing the above BER performances, we found that our proposed scheme using adaptive interleaving with set partitioning method enlarging the product of the Euclidean distances is more efficient than the conventional schemes using block interleaving with both the two considered set partitioning methods. We can conclude from these results that the proposed adaptive interleaving scheme is an effective way to cope with the frequency selective fading.

VII. ACKNOWLEDGMENT

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