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A New Viterbi Algorithm with Adaptive Path Reduction Method

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SUMMARY A new Viterbi algorithm with adaptive path reduction method is presented. The proposed system consists of the pre-decoder and reduced path Viterbi decoder. The pre-decoder separates the mixed channel noise from the received sequence [6]-[9]. The number of errors in the pre-decoded error sequence is counted and the path reduction is implemented by the number of errors in pre-decoded error sequence. The path reduction is implemented as a function of channel condition because the errors in the pre-decoded error sequence can be considered as the channel error sequence. Due to the reduction of the path, the number of ACS (add compare select) operations can be reduced, which occupies the dominant part in Viterbi decoding. The ACS reduction ratio for the proposed system achieves up to 30% for the case of (2, 1, 2) Ungerboeck code without degradation of the error performance.

key words: *Viterbi algorithm, reduced computation method, adaptive decoding system*

1. Introduction

Trellis structure pervades digital communication signalling. Two of the most common sources of this structure are the application of trellis codes for forward error correction [1], [2] and trellis coded modulations [3], which impress a trellis structure on the transmitted signal. A pragmatic approach to trellis coded modulation was proposed by Viterbi et al.[4]. The principle advantage of this approach is that a single rate 1/2 encoder-decoder, with only moderate modifications to handle parallel branch decoding, can provide this performance for a wide variety of trellis codes.

The Viterbi algorithm [1], [2] is well-known as the optimal (sequence) estimator for trellis-structured signals. Its computational complexity, however, is proportional to the width of the trellis, which is in turn exponential in the memory length of the coder, modulator, or channel response. Viterbi decoding can therefore become impractical with more powerful codes and modulations. Because of this computational problem with Viterbi algorithm, alternative reduced-computation algorithms (that are necessarily sub-optimal) have been proposed in the literature [5]-[12], including the parallel hardware implementation of Viterbi decoder [10], [11]. Reduced-path Viterbi decoding (RPVD) [5] was proposed by Yashima et al. using asymmetrical characteristics of the optical channel. Kubota et al. have introduced the SST (Scarce State Transition) type Viterbi algorithm which simplifies the Viterbi algorithm by introduction of a pre-decoder [6], [7]. The pre-decoder separates the mixed channel noise from the received sequence. The Viterbi decoder is only used to decode the pre-decoded error sequence which is generated by the pre-decoder. One of the unique character of the decoding system with the pre-decoder is that the pre-decoded error sequence can be considered as channel error sequence therefore, the distribution of the pre-decoded error sequence is non-uniform where it is uniform for conventional Viterbi algorithm. Focusing on this character, SST-type Viterbi algorithm solves the power conservation problem which is the main problem of realization of CMOS LSIC [6], [7]. GVA (Generalized Viterbi Algorithm) has been proposed to achieve high speed decoding by eliminating some of decoding states [8]. Probability Selecting State Viterbi decoder reduces the decoding states of which the probability is nearly zero [9].

In this paper, we propose a new Viterbi decoding algorithm with adaptive path reduction method. The path reduction is implemented by counting the errors in the pre-decoded error sequence which is generated by the pre-decoder. The path reduction is based on a fact that the correct path exists with the number of errors which are nearby the number of errors in the pre-decoded error sequence. Therefore, the system searches the correct path from the sets of paths in which the number of errors is nearby the number of errors in the pre-decoded error sequence. Moreover, the number of (searching) paths increase as the number of error increases and the path with few errors is a mostly correct path. Therefore, the number of ACS (add compare select) operations can be reduced, which occupies the dominant part in Viterbi decoding. As the computation effort of the proposed system changes by the number of the channel errors, the system can be considered as an adaptive system. In order to clarify the benefit of the proposed system, rate 1/2 (2, 1, 2) Ungerboeck code is considered as an example. The path reduction ratio and simulated error performance are presented and compared with the conventional

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Viterbi decoder and the PSS-type Viterbi decoder under same encoder condition.

2. System Model

Figure 1 shows the block diagram of the proposed system. Figure 2 shows the rate 1/2 convolutional encoder and the general structure of the pre-decoder. The received signal is firstly pass through the pre-decoder. The pre-decoder consists of two circuits, an

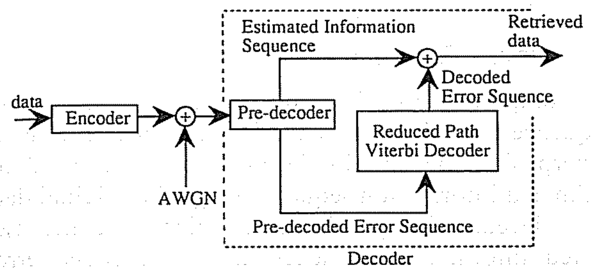


Fig. 1 Block diagram of the proposed system.

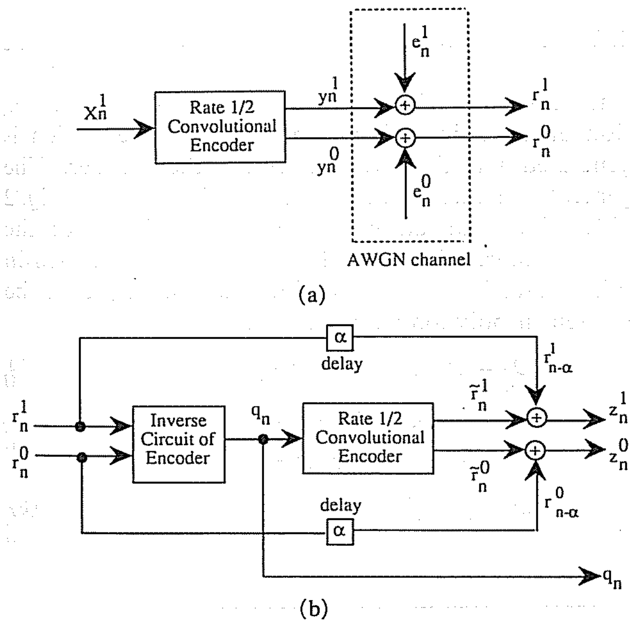


Fig. 2 (a) Rate 1/2 convolutional encoder.
(b) General structure of pre-decoder (inverse circuit of encoder and rate 1/2 convolutional encoder).

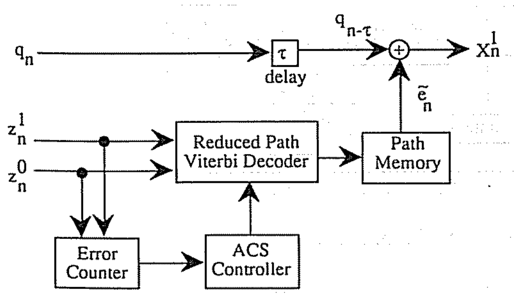


Fig. 3 Reduced path Viterbi decoder.

inverse circuit of the encoder and the encoder. The received sequences are first generated into an estimated information sequence by the inverse circuit of the encoder. No error correcting is applied at the pre-decoder. Then the estimated information sequence is put through same encoder to generate pre-decoded error sequence by taking exclusive-or operation with the received sequences and the encoder outputs. The reduced path Viterbi decoder decodes this pre-decoded error sequence only. Figure 3 shows the reduced path Viterbi decoder. The path reduction is based on a fact of the non-uniform property of the pre-decoded error sequence, and it is implemented by counting the number of errors in pre-decoded error sequence. Finally, the original data is retrieved by superposing the decoded error sequence on estimated information sequence. The unique character of the proposed decoding system is determined by the pre-decoder, which removes the mixed noise sequence from the received sequence. The implementation of the pre-decoder depends on realization of generating estimated information sequence. QLI (quick look in) code is very suitable for this type of decoding system that estimated information sequence can easily be made by superposing of received sequences. SST-type Viterbi decoder and GVA permit only the use of QLI code. However, QLI code is inferior to the optimum code in distance property. PSS-type Viterbi decoder permits the use of the non QLI code of (147, 135) which is almost as good as optimum code, however, the implementation of the inverse circuit for any rate 1/2 convolutional code has not been shown. Thus, it is very important to define the condition of the inverse circuit of the code.

In this section, we first show the implementation of the pre-decoder by taking an example of rate 1/2 (2, 1, 2) Ungerboeck code. The condition to implement the inverse circuit for the rate 1/2 convolutional code is also discussed.

2.1 Pre-Decoder Implementation

An (2, 1, m) convolutional encoder with 1 input, 2 output and m shift registers is considered. Figure 2(a) shows the rate 1/2 convolutional encoder. The information sequence $\boldsymbol{X}^1=(x_0^1, x_1^1, x_2^1, \cdots)$ enters the encoder one bit at a time. The two encoder output sequences $\boldsymbol{Y}^0=(y_0^0, y_1^0, y_2^0, \cdots)$ and $\boldsymbol{Y}^1=(y_0^1, y_1^1, y_2^1, \cdots)$ are put through the noise corrupted channel. Let $\boldsymbol{R}^0=(r_0^0, r_1^0, r_2^0, \cdots)$ and $\boldsymbol{R}^1=(r_0^1, r_1^1, r_2^1, \cdots)$ are put through the noise corrupted channel, the noise sequences are described as $\boldsymbol{E}^0=(e_0^0, e_1^0, e_2^0, \cdots)$ and $\boldsymbol{E}^1=(e_0^1, e_1^1, e_2^1, \cdots)$. The block diagram of the proposed system for the case of Ungerboeck code of (2, 1, 2) is shown in Fig. 4. For the case of (2, 1, 2) Ungerboeck code, generator polynomials are $g^0(D)=D$ and $g^1(D)=1+D^2$, where D is the delay operand and the power of D denoting the number of time units a bit is delayed. The encoding equations are therefore, written as follows;

$$Y^0 = DX^1$$
$$Y^1 = (1+D^2)X^1 \quad (1)$$

Let us express the encoded sequences at time n which is written as follows;

$$y_n^0 = x_{n-1}^1$$
$$y_n^1 = x_n^1 + x_{n-2}^1 \quad (2)$$

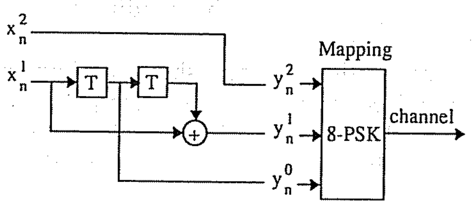
The encoder of (2, 1, 2) Ungerboeck code is shown in Fig. 4(a). The white Gaussian noise is added at the channel and the received sequences are given as follows;

$$r_n^0 = y_n^0 + e_n^0 = x_{n-1}^1 + e_n^0$$
$$r_n^1 = y_n^1 + e_n^1 = x_n^1 + x_{n-2}^1 + e_n^1 \quad (3)$$

Let $\boldsymbol{Q} = (q_0, q_1, q_2, \cdots)$ be the estimated information sequence. The estimated information sequence is generated by the inverse circuit of the encoder. $\boldsymbol{R}^0$ is delayed for a unit time and superposing on $\boldsymbol{R}^1$. The estimated information sequence is calculated as

$$q_n = r_{n-1}^0 + r_n^1$$
$$= x_{n-2}^1 + e_{n-1}^0 + x_n^1 + x_{n-2}^1 + e_n^1$$
$$= x_n^1 + \tilde{e}_n \quad (4)$$

where $\tilde{e}_n = e_{n-1}^0 + e_n^1$. Let us define the estimated error sequence as $\tilde{\boldsymbol{E}} = (\tilde{e}_0, \tilde{e}_1, \tilde{e}_2, \cdots)$. The estimated information sequence is then, put thought the encoder to



(a) Ungerboeck convolutional encoder (m=2) for 8-PSK

generate the pre-decoded error sequences. Let $\tilde{R}^0 = (\tilde{r}_0^0, \tilde{r}_1^0, \tilde{r}_2^0, \cdots)$ and $\tilde{R}^1 = (\tilde{r}_0^1, \tilde{r}_1^1, \tilde{r}_2^1, \cdots)$ be the re-encoded received sequences, which are written as follows;

$$\tilde{r}_n^0 = x_{n-1}^0 + \tilde{e}_{n-1}$$
$$\tilde{r}_n^1 = x_n^1 + \tilde{e}_n + x_{n-2}^1 + \tilde{e}_{n-2} \quad (5)$$

The pre-decoded error sequence is derived by modulo 2 addition of re-encoded received sequences and received sequences, which are written as follows;

$$z_n^0 = r_n^0 + \tilde{r}_n^0 = \tilde{e}_{n-1} + e_n^0$$
$$z_n^1 = r_n^1 + \tilde{r}_n^1 = \tilde{e}_n + \tilde{e}_{n-2} + e_n^1 \quad (6)$$

As comparing Eqs. (2) and (6), the estimated error sequence is equivalent to the input sequence, thus by superposing the Viterbi decoded sequence of $\tilde{\boldsymbol{E}}$ on estimated information sequence of $\boldsymbol{Q}$, the original data is retrieved. The pre-decoder, which generates estimated information sequence and pre-decoded error sequence, is shown in Fig. 4(b) together with reduced path Viterbi decoder.

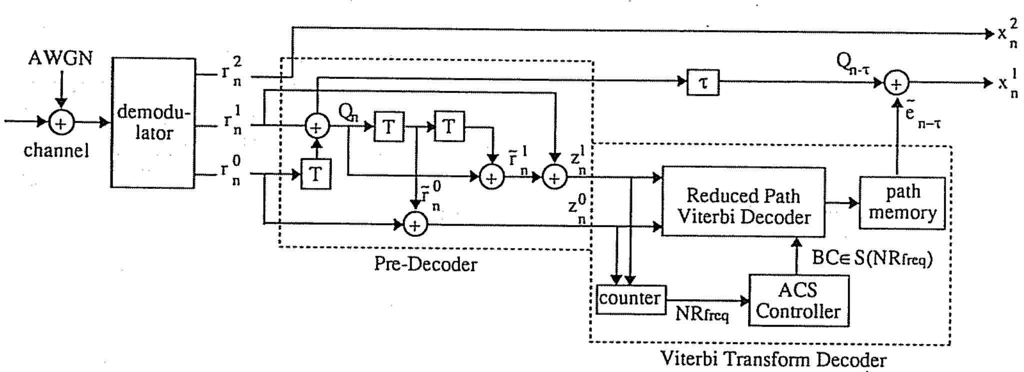
2.2 Inverse Circuit of Rate 1/2 Convolutional Code

The pre-decoder implementation depends on realization of the estimated information sequence, which is generated with the inverse circuit of the encoder. The general structure of the pre-decoder is shown in Fig. 2 (b). The realization of the inverse circuit of the encoder of rate 1/2 convolutional code is discussed in this subsection. Let the information sequence be written in polynomial representation as

$$X^1(D) = x_0^1 + x_1^1 D + x_2^1 D^2 + \cdots \quad (7)$$

and the output sequence can be represented as

$$Y^0(D) = y_0^0 + y_1^0 D + y_2^0 D^2 + \cdots$$
$$Y^1(D) = y_0^1 + y_1^1 D + y_2^1 D^2 + \cdots. \quad (8)$$



(b) Pre-decoder and reduced path Viterbi decoder

Fig. 4 The block diagram of the proposed system for the case of (2, 1, 2) Ungerboeck code.

The rate 1/2 (2, 1, m) convolutional code can be represented by generator polynomials of

$$g^0(D)=g_0^0+g_1^0D+g_2^0D^2+\cdots+g_m^0D^m$$
$$g^1(D)=g_0^1+g_1^1D+g_2^1D^2+\cdots+g_m^1D^m \quad (9)$$

with the m-time unit memory. The encoding equation can be written as follows,

$$Y^0(D)=X^0(D)*g^0(D)$$
$$Y^1(D)=X^1(D)*g^1(D) \quad (10)$$

where * denotes the discrete convolution and all operations are modulo-2. The inverse circuit must satisfy following condition,

$$D^{\alpha}=[f^0(D), f^1(D)]\begin{bmatrix} g^0(D) \\ g^1(D) \end{bmatrix} \quad (11)$$

where

$$f^0(D)=f_0^0+f_1^0D+f_2^0D^2+\cdots+f_m^0D^m$$
$$f^1(D)=f_0^1+f_1^1D+f_2^1D^2+\cdots+f_m^1D^m \quad (12)$$

are the inverse polynomials, and α is a delay coefficient of re-generated information signal. Inverse polynomials can also be found when the generator polynomials satisfy following condition,

$$GCD[g^0(D), g^1(D)]=D^{\alpha} \quad (13)$$

where GCD denotes the greatest common divisor. If the inverse polynomials are found for a certain code, that code can be applied at the system with the pre-decoder. The QLI code can be defined by

$$g^0(D)+g^1(D)=D^{\alpha} \quad (14)$$

therefore, the inverse polynomials are always found as $f^0(D)=1$ and $f^1(D)=1$. This is the case that no inverse circuit is needed. SST-type Viterbi algorithm and GVA only permit the usage of QLI code for their systems, because of the easy implementation of the inverse circuit.

For example, for the case of QLI code [6] of $g^0(D)=1+D+D^2$ and $g^1(D)=1+D^2$, the code satisfies Eqs. (11) and Eq. (13) as

$$g^0(D)+g^1(D)=D^1, \ GCD[g^0(D), \ g^1(D)]=1 \quad (15)$$

therefore, inverse polynomials are $f^0(D)=1$ and $f^1(D)=1$. This satisfies Eq. (11) as

$$D^1=[1, 1]\begin{bmatrix} 1+D+D^2 \\ 1+D^2 \end{bmatrix}. \quad (16)$$

For the case of Ungerboeck code [3] of $g^0(D)=D$ and $g^1(D)=1+D^2$, inverse polynomials are $f^0(D)=D$ and $f^1(D)=1$, and satisfy Eq. (11) shown as follows,

$$D^0=[D, 1]\begin{bmatrix} D \\ 1+D^2 \end{bmatrix} \quad (17)$$

therefore, estimated information sequence can be generated without any delay ($\alpha=0$). For the case of the convolutional code of constraint length, $K=7$, the generator polynomials are $g^0(D)=1+D^2+D^3+D^4+D^5$ and $g^1(D)=1+D+D^2+D^3+D^6$, which is used for pragmatic code [4]. The inverse polynomials are $f^0(D)=1+D$ and $f^1(D)=1$, and satisfy Eq. (11) shown as follows,

$$D^3=[1+D, 1]\begin{bmatrix} 1+D^2+D^3+D^4+D^5 \\ 1+D+D^2+D^3+D^6 \end{bmatrix} \quad (18)$$

therefore, estimated information sequence can be generated with 3 delay ($\alpha=3$).

3. Reduced Path Viterbi Decoder

In Viterbi decoding, the path metrics of survival paths are calculated and new survival paths are selected by add-compare-select (ACS) operations on the merged states. The ACS operations occupy dominant part of decoding, therefore, reduction of the number of ACS operations is useful to realize fast decoding. Reduced-path Viterbi decoding (RPVD) using asymmetrical characteristics of the optical channel [4] was first proposed by Yashima et al, and we extend their work in Gaussian channel by the path reduction method discussed previously. In this section, reduced path Viterbi decoding algorithm is presented. We first define the optimum decision criterion for our proposed system. Error performance is discussed by taking an example of (2, 1, 2) Ungerboeck code for $\mu=5$. We compare the system with the conventional Viterbi decoder and PSS-type Viterbi decoder. Although PSS-type eliminates the states of which the occurrence probabilities are nearly zero, the proposed system may have advantage due to the adaptive path reduction method which makes full use of maximum likelihood property.

3.1 Distribution of Pre-Decoded Error Sequence and Adaptive Path Reduction Method

The path reduction is implemented in the proposed system. The number of errors in pre-decoded error sequence is counted by the error counter, where the pre-decoded error sequence is considered as μ-tuple sequence, defined as $\tilde{\boldsymbol{Z}}_{\mu}=(\tilde{z}_0, \tilde{z}_1, \tilde{z}_2, \cdots \tilde{z}_{\mu-1})$. $\tilde{z}_n$ represents the pre-decoder output symbol at time n, which is given as $\tilde{z}_n=(z_n^0, z_n^1)$. z_n^0 and z_n^1 are the binary representation of pre-decoded error sequence given by Eq. (6), as shown in Fig. 2(b). As there are 4 ($=2^2$) symbols in (2, 1, m) convolutional code, one of them (usually "0" symbol) is considered as "correct" pre-decoded symbol and the other is considered as "error" symbol. The number of errors in the pre-decoded error sequence is given as $N(\tilde{Z}_\mu)$. Let us consider the set of the pre-decoded error sequences, $\tilde{Z}_\mu$, in which all members have same number of errors. The set is described as Ω_μ^φ, where φ is the number of errors, $N(\tilde{Z}_\mu)=\varphi$. For example, the set of the pre-decoded error sequence with no error is the case with $\tilde{z}_0=\tilde{z}_1=\tilde{z}_2\cdots\tilde{z}_{\mu-1}=0$, which can be written as $\Omega_\mu^0=\{(0,0,0,\cdots)\}$. The number of members in Ω_μ^0 is given as $N(\Omega_\mu^0)$ and for this case, $N(\Omega_\mu^0)=1$, $\varphi=0$. For the case of the pre-decoded error sequence of $\tilde{Z}_5=(0,0,1,0,0)$, where $\mu=5$ and $N(\tilde{Z}_5)=1$, the sequence belongs to the set, Ω_5^1, where $N(\Omega_5^1)=5\times 3$ as there are 3 error symbols. In general, $N(\Omega_\mu^\varphi)$ can be written as follows;

$$N(\Omega_\mu^\varphi)=3^\varphi\times\binom{\mu}{\varphi} \tag{19}$$

where $\binom{x}{y}$ describes the combination of x taken over y. Since the pre-decoded error sequence can be defined as channel noise sequence, the occurrence probability of Ω_μ^φ is written as follows;

$$P(\Omega_\mu^\varphi)=\binom{\mu}{\varphi}P_{ec}^\varphi(1-P_{ec})^{\mu-\varphi} \tag{20}$$

where P_{ec} denotes the probability of error of the channel and $P(\Omega_\mu^\varphi)$ denotes the occurrence probability of Ω_μ^φ. The distribution of pre-decoded error sequence is shown in Fig. 5, as a function of number of the errors in the pre-decoded error sequences.

The path reduction is implemented by counting the number of errors, $N(\tilde{Z}_\mu)$, in pre-decoded error sequence. The proposed system searches the correct sequence from the sets in which the correct sequence belongs. This is based on a fact that the correct error sequence belongs to the set of which the number of errors is nearby the number of errors in pre-decoded error sequence. $N(\tilde{Z}_\mu)$ increases when $\tilde{z}_n$ does not equal to "0." This happens neither $(z_n^0, z_n^1)=(0,0)$ nor $(1,1)$. Since the other than "0" symbol represents "error" symbol, we must consider the case of $(z_n^0, z_n^1)=(1,1)$, that $N(\tilde{Z}_\mu)$ does not increase for this case. Therefore, $N(\tilde{Z}_\mu)$ must be considered as that it varies for a certain value. This value is called decoding margin which is represented as β. The correct sequence belongs to the one of the sets of $\{\Omega_\mu^{\varphi-\beta}, \Omega_\mu^{\varphi-\beta+1},\cdots,\Omega_\mu^\varphi,\cdots,\Omega_\mu^{\varphi+\beta-1},\Omega_\mu^{\varphi+\beta}\}$, where $\varphi=N(\tilde{Z}_\mu)$. Let Λ_μ^φ be the set which can be written as $\Lambda_\mu^\varphi=\Omega_\mu^0\cup\Omega_\mu^1\cup\Omega_\mu^2\cup\cdots\cup\Omega_\mu^\varphi$, where $\cup$ describes union of sets. For those sets, each set, Λ_μ^φ, satisfies as follows;

$$\Lambda_\mu^0\subseteqq\Lambda_\mu^1\subseteqq\Lambda_\mu^2\subseteqq\cdots\subseteqq\Lambda_\mu^{\mu-1}\subseteqq\Lambda_\mu^\mu \tag{21}$$
$$N(\Lambda_\mu^0)\leqq N(\Lambda_\mu^1)\leqq N(\Lambda_\mu^2)\leqq\cdots\leqq N(\Lambda_\mu^{\mu-1})\leqq N(\Lambda_\mu^\mu)$$

where the number of member in set, Λ_μ^φ, is defined as $N(\Lambda_\mu^\varphi)$. The proposed system searches the correct path from the member of $\Lambda_\mu^{\varphi+\beta}$, where $\varphi=N(\tilde{Z}_\mu)$. As μ can be considered as the minimum length of the code, μ must be set longer than the constraint length of the code. The path reduction is implemented by counting the errors in the estimated error sequence for the proposed system, and while counting the errors, the Viterbi decoding is not implemented. Therefore, μ should not be set very long. For the case of Ungerboeck code, we set $\mu=5$. β can be at most μ, this is the case that no path reduction is implemented. The highest distribution is given by $\Lambda_\mu^0(=\Omega_\mu^0)$. β should be set to which the distribution of Λ_μ^β occupies almost 100%. Table 1 shows the distribution of (2, 1, 2) Ungerboeck code, for the case of $\varphi=5$. As Λ_5^2 occupies 99.9% at $P_{ec}=10^{-2}$, we set $\beta=2$.

The computation effort of the system can be reduced due to path reduction. The path reduction ratio, η, is defined as

$$\eta=\frac{P(\Lambda_\mu^\beta)N(\Lambda_\mu^\beta)+\sum_{i=\beta+1}^{\mu}P(\Omega_\mu^i)N(\Lambda_\mu^i)}{N(\Lambda_\mu^\mu)} \tag{22}$$

where $P(\Lambda_\mu^\varphi)$ and $N(\Lambda_\mu^\varphi)$ are the occurrence probability of Λ_μ^φ and the number of member in Λ_μ^φ. $N(\Lambda_\mu^\mu)$ equals to the number of all paths, as it is the union set. $P(\Lambda_\mu^\varphi)$ is given as follows;

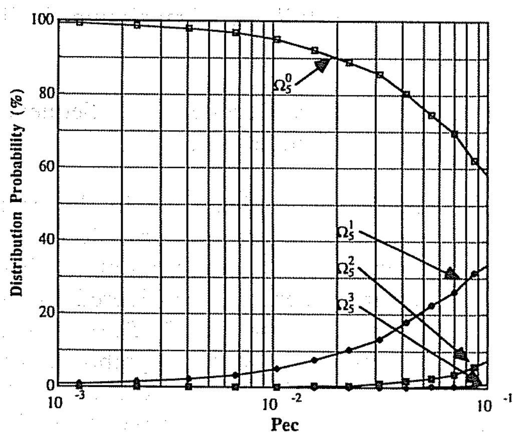


Fig. 5 The distribution of pre-decoded error sequence shown as a function of number of the errors in the pre-decoded error sequences.

Table 1 The distribution of Ω_μ^φ and for the case of (2, 1, 2) Ungerboeck code, $m=5$.

	$\varphi=0$	$\varphi=1$	$\varphi=2$	$\varphi=3$	$\varphi=4$	$\varphi=5$
Distribution of Ω^φ_μ at $Pec=10^{-1}$	59.010%	32.852%	7.2748%	0.8158%	0.0456%	0.0009%
Distribution of Λ^φ_μ at $Pec=10^{-1}$	59.010%	91.862%	99.136%	99.952%	99.998%	100.00%
Distribution of Ω^φ_μ at $Pec=10^{-2}$	95.069%	4.8318%	0.00974%	0.0102%	1e-6%	1e-7%
Distribution of Λ^φ_μ at $Pec=10^{-2}$	95.069%	99.900%	99.909%	100.00%	100.00%	100.00%

$$P(\Lambda_\mu^\varphi) = \sum_{i=0}^{\varphi} P(\Omega_\mu^\varphi) \quad (23)$$

The one of the unique character of our proposed system is that the path reduction is implemented by counting the number of errors in pre-decoded error sequence, which can be considered as a function of channel error. Therefore, the proposed system adaptively changes its ability of decoding by the channel condition.

3. 2 The Optimum Decision Criterion

The Viterbi algorithm selects one survivor path at each state which has best maximum likelihood (ML) metric. However, ML criterion is optimum only when the distribution of decoding sequence is uniform [9]. In the implementation of the decoder, the decoded sequence $\boldsymbol{X}_i$ should satisfy

$$\Pr(\boldsymbol{X}_i|\boldsymbol{R}) = \mathrm{Max}\{\Pr(\boldsymbol{X}_j|\boldsymbol{R})\} \quad (24)$$

where $\boldsymbol{R}$ is the received sequence and $\Pr(X|Y)$ is conditional probability, and called maximum a posteriori (MAP) criterion [1]. In accordance with signal detection theory, this is the optimum detection for a random digital signal. By taking natural logarithm and applying Bayes formula, Eq. (24) can be written as follows;

$$\mathrm{Log}\Pr(\boldsymbol{R}|\boldsymbol{X}_i) = \mathrm{Max}\{\mathrm{Log}\Pr(\boldsymbol{R}|\boldsymbol{X}_j) + \mathrm{Log}\, C_r\} \quad (25)$$

where C_r is called decision threshold which equals to

$$C_r = \frac{\Pr(\boldsymbol{X}_j)}{\Pr(\boldsymbol{X}_i)}. \quad (26)$$

When the distribution of $\boldsymbol{X}$ is uniform, $C_r = 1$ (Log $C_r = 0$) and MAP decision turns into ML decision. The proposed system searches the path ($\boldsymbol{X}_i$ in Eqs. (25) and (26)) from the set of $\Lambda_\mu^{\varphi+\beta}$. As $\boldsymbol{X}_i$ is the esimated error sequence, $P(\boldsymbol{X}_i) = P(\Omega_\mu^i)$ where $\boldsymbol{X}_i \in \Omega_\mu^i$. The decision threshold of the proposed system is given as follows;

$$C_r = \frac{P(\Omega_\mu^j)}{P(\Omega_\mu^i)} \quad X_i \in \Omega_\mu^i,\ X_j \in \Omega_\mu^j. \quad (27)$$

When $i < j$, the occurrence probabilities of Ω_μ^i and Ω_μ^j can be calculated by Eq. (20) and they satisfy as $P(\Omega_\mu^i) > P(\Omega_\mu^j)$. In another word, as the proposed system searches the path from the set which has high probability, for most of time, the decision threshold of the system always stays bigger than the other set. Thus, near optimum decoding can be applied with the proposed system even some of paths are eliminated. Moreover, the adaptability of path reduction method of the proposed system enables more use of maximum likelihood property than PSS-type Viterbi decoder, in which maximum likelihood property is fixed.

3. 3 Integrated Trellis Structure

The Viterbi algorithm estimates the decoding path which has best maximum likelihood (ML) metric. This estimation is taken as the state transition of path in trellis diagram [1]. Trellis diagram limits the transitions only to those are the possible encoding paths. Since the proposed system selects the path from the set $\Lambda_\mu^{\varphi+\beta}$, we consider the integrated trellis diagram. The state transition in trellis diagram is integrated for μ-unit time. In integrated trellis diagram, the number of path is $N(\Lambda_\mu^\mu) = 2^\mu \times 2^m$, as 2 paths are diverging from each state (total number of states is 2^m) for μ-step in trellis diagram. Note that, this is exact number of all possible decoding paths. The number of possible error sequence, which is derived from the Eq. (19), is the case of all possible combinations of error sequence. Although the number of paths in integrated trellis is much less than that derived from Eq. (19), path reduction method discussed previously is still applicable with a great efficiency. Figure 6 shows the integrated trellis diagram for the case of (2, 1, 2) Ungerboeck code. Here, trellis is integrated for 5 steps ($\mu = 5$) and total number of paths is $N(\Lambda_5^5) = 2^5 \times 2^2 = 128$. The eliminated branch (depicted in dashed line) is also shown in the figure as a function of φ. ACS operation is only implemented at the state where more than one path merges. Therefore, the number of ACS operation can be reduced for the proposed system. The conventional trellis structure is same as the one with $\varphi > 4$.

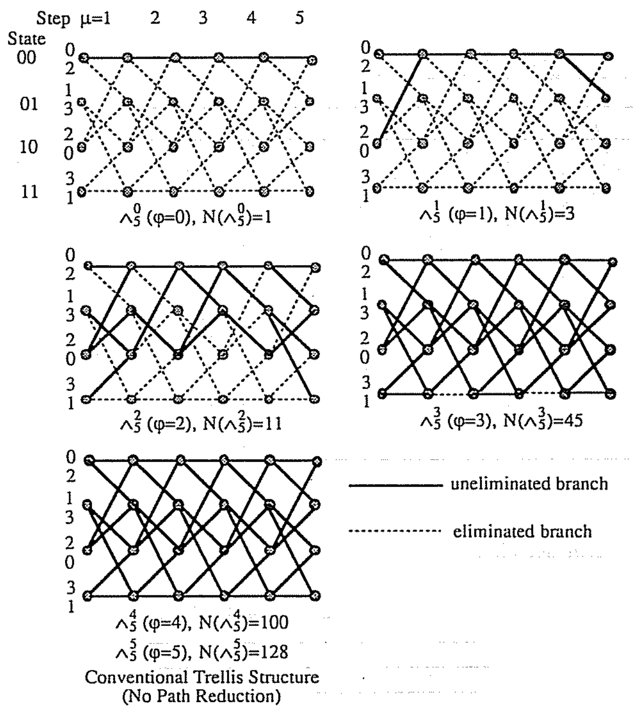


Fig. 6 Integrated trellis diagram for the case of (2, 1, 2) Ungerboeck code, $\varphi = 5$.

The number of path in Ω_μ^φ is summarized in Table 2, together with number of path in Λ_μ^φ. The representation of paths for each set of Ω_μ^φ is also listed in Table 2. For the case of (2, 1, 2) Ungerboeck code with $\beta=2$, the path reduction ratio, η, can be up to 11.636% for the proposed system, where $N(\Lambda_5^2)=11$ for the proposed system and $N(\Lambda_5^2)=128$ for the conventional Viterbi decoder. The members in Λ_5^2 appears at most, for this case. The proposed system eliminates the transitions of path which do not belong to the set selected by the ACS Controller, as shown in Fig. 4. The ACS reduction ratio can be derived with a little modification of Eq. (22) which is written as follow;

$$\eta_{ACS}=\frac{P(\Lambda_\mu^\beta)\,\chi_\mu^\beta+\sum_{i=\beta+1}^{\mu}P(\Omega_\mu^i)\,\chi_\mu^i}{\chi_\mu^\mu} \qquad (28)$$

where χ_μ^φ is the number of ACS unit in Λ_μ^φ integrated trellis and $\chi_\mu^\mu(=\mu\times 2^m=5\times 2^2=20)$ is the case of the conventional Viterbi decoder with no path reduction. The number of ACS and ACS reduction ratio at high SNR are in Table 3. For the case of (2, 1, 2) Ungerboeck code, the ACS reduction ratio is up to 30% for the proposed system, thus a fast decoding can be realized.

3.4 Performance Comparison

Figure 7 shows the ACS reduction ratio for the case of $\varphi=5$ and $\beta=2$, along with P_{ec} (uncoded 8-PSK).

Table 2 The number of path in Λ_μ^φ and Ω_5^φ, and the member of Ω_5^φ.

	φ=0	φ=1	φ=2	φ=3		φ=4			φ=5	
$N(\Lambda^\varphi_\mu)$	1	3	11	45		100			128	
$N(\Omega^\varphi_5)$	1	2	8	34		55			28	
The member of Ω^φ_5	00000	0002	00021	00212	00233	02122	03113	33033	21221	31111
		2000	00023	00231	02120	02313	32021	31320	21223	22122
			00210	02101	02103	21202	32210	31303	23322	22332
			12000	02330	21200	21033	30332	20212	23132	22313
			20002	21010	12002	23320	30311	20231	23113	22311
			01200	10120	10101	23303	13012	22101	23111	32212
			01010	10103	10330	12021	13031	22330	12212	32233
			32000	33200	33010	12210	11301	01223	12233	32231
				20021	20023	10332	11130	03132	12231	13221
				20210	01202	10311	02332	03111	33221	13223
				01012	01033	33012	02311	32023	33223	11322
				01031	03320	33031	21012	30122	31322	11132
				03301	03303	31301	21031	30313	31132	11113
				03130	32002	31130	23301	13202	31113	11111
				30120	30101	20233	23130	13033		
				30103	30330	22120	12023	11320		
				13200	13010	22103	10122	11303		
						01221	10313			
						03322	33202			

Table 3 χ_5^φ and η_{ACS} at high SNR for the case of reduced path Viterbi decoder. $\varphi=5$.

φ	χ^φ_5	η_{ACS} (%)
0	0	0
1	1	5
2	6	30
3	18	90
4	20	100
5	20	100

Here, the channel error probability is given as the error rate of 8-PSK. The probability of error of 8-PSK is given as

$$Pe_{8\text{-PSK}}=\mathrm{erfc}(\sin\pi/8\,\sqrt{SNR}) \qquad (29)$$

where erfc is the complementary error function.
For performance comparison, the conventional Viterbi decoder and PSS-type Viterbi decoder are considered. As the occurrence probability of state "11" is nearly 0, state "11" is eliminated for PSS-type Viterbi decoder. ACS reduction ratio is 75% for PSS-type Viterbi decoder and 30% for the proposed system. Figure 8 shows the simulated results for the case of the reduced path Viterbi decoder, the conventional Viterbi decoder and PSS-type Viterbi decoder. The error performance is almost as same as that of PSS-type Viterbi decoder and it slightly degrate from the conventional Viterbi decoder. However, the proposed system is superior to PSS-type Viterbi decoder on ACS reduction ratio and the adaptability of path reduction method enables more use of maximum likelihood property than PSS-type Viterbi decoder.

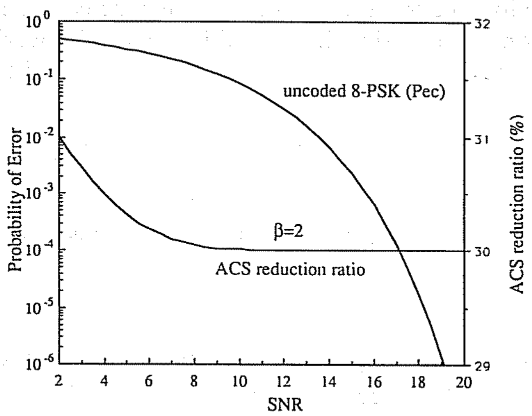


Fig. 7 ACS reduction ratio for the case of $\mu=5$ and $\beta=2$, along with P_{ec} (uncoded 8-PSK).

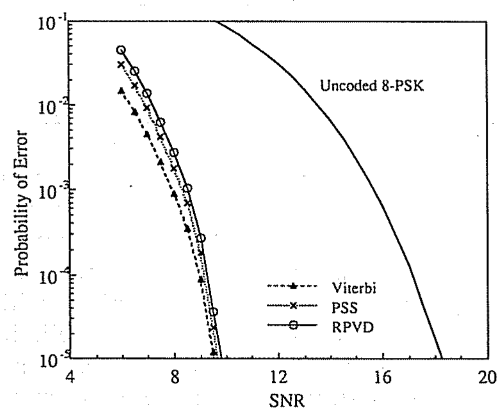


Fig. 8 Error performance of reduced path Viterbi docoder, PSS-type Viterbi decoder and conventional Viterbi decoder along wihth P_{ec} (uncoded 8-PSK).

4. Conclusions

A new Viterbi algorithm with adaptive path reduction method has been proposed. The error performance and the ACS reduction ratio are compared with the PSS-type Viterbi decoder for the case of (2, 1, 2) Ungerboeck code. The proposed system adaptively reduces paths by counting the errors in pre-decoded sequence. ACS reduction ratio can be up to 30% for the case of the proposed system, which is 75% for PSS-type Viterbi decoder. As pre-decoded sequence can be considered as the channel noise, the proposed system changes its decoding ability as a function of channel condition which enables more use of maximum likelihood property than PSS-type Viterbi decoder. The error performance of the proposed system is almost same as PSS-type Viterbi decoder which slightly degrates from the conventional Viterbi decoder. Since the error performance and ACS reduction ratio much depend on the system parameters of μ and β, these relations must be investigated.

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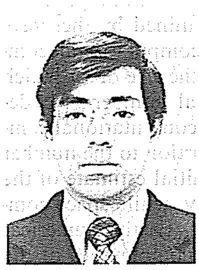

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