# POSSIBILITIES OF MATHEMATICALY PREDICTING TIMBER PRODUCTION IN COMMERCIAL FORESTS

by

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経済林における木材生産の科学的予測の可能性

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# POSSIBILITIES OF MATHEMATICALLY PREDICTING TIMBER PRODUCTION IN COMMERCIAL FORESTS

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**Summary**

This contribution begins with a description of the relationship among economic processes, ecology, and forest production along with a presentation of some basic principles of forestry. The principle of sustained yield in forestry and the efforts toward attaining ecological stability are discussed. A stochastic growth model for even-aged forest stands is presented, in which a stochastic differential equation of Itô-type is used and verified. The model was verified by graphical comparisons in the marginal distribution functions and correlation functions of the process.

## 1. Introduction

The growing energy shortage and the environmental problems force all scientific disciplines – including forestry and general economics – to reconsider, redefine, or supplement their aims and fields of study on the basis of a changing situation. Even among well-known political economists, the conception of the earth as a space-ship earth and economic processes as dynamic irreversible processes is increasingly becoming accepted. The results of these processes consist not only in the material goods that have ensured the survival of mankind under different degrees of prosperity, but also in the thoughtless exploitation and resulting shortage of natural resources and non-renewable energy resources (component of negative entropy). Besides these irreparable consequences, an increasing amount of energy is being decomposed to form an accumulation of waste (component of entropy).

The belief that our ingenuity will open up ever more effective sources of energy to maintain our standard of lining cannot change the fact that our natural resources are limited. It is inevitable that pollution as an excess of non-removable entropy will keep on increasing.

Analogously to the description of ecology by Haeckel, Stugren states (1974) as the theory of the economics of nature, that economic processes are explained – according to Georgescu-Roegen (1971) – by the thermodynamics of irreversible processes. Ecosystems are open systems in which a continuous input of solar energy from outside enables processes of life, and in which a conversion of energy by means of food chains takes place between many forms of life. The entropy of these processes can, for example, be found in the biomass of ecosystems.

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The paper is dedicated to Professor Suzuki, Nagoya Univ. The research for this paper was supported by DFG.

During the long history of the earth, mineralization processes have transformed biomass into stores of "hard energy resources," such as coal, oil, etc., which seem to be the only "hard currency" today. Being the primary producers forest ecosystems, particularly the forest stands, transform solar energy into assimilates. One of these is wood which can be used both as a resource and as a source of energy. During this process, other vital products are set free, but they will not be considered further here.

### 1.1 Economic Systems and Ecosystems

The human economic systems based to a great extent on secondary non-renewable energy resources (e.g. oil, coal) impair the function and existence of ecosystems which are dependent upon life-giving solar energy, due to their increasing amount of waste. The serious consequences of basing economic growth on oil and coal are already becoming apparent. For example, it is assumed that in the FRG forests die as a result of $SO_2$ emissions [Ulrich, (1980, 1981)]. These unpleasant but inevitable results of the so-called second theorem of thermodynamics lead to a decrease in the diversity of species and to the instability of ecosystems and social systems with all the consequences liable to result in a restriction of the fundamentals of life. In order to prevent the worst or at least to postpone it, rethinking is necessary in all spheres of life and by every human being. The short-term economic and political profits resulting from the wasteful production of goods with the well-known consequences cannot be the aim. The protection of the foundations of life by guaranteeing the stability of ecosystems and social system which is secured by life-affirming primary energy, must be considered as essential additional requirements when formulating economic objectives. This is a very difficult task which will continue to contain many contradictions in itself as long as the homo-economicus-behavior in the classical sense is regarded as the basis of economic processes. The reason for this is often the ignorance of mankind, whose level of education is quite low as far as ecology is concerned. This subject should be introduced in the schools and should accompany almost all courses of study, including those of economic sciences, so that the development of concentrated thought and action in society can take the place of those emotional protests we now witness and which cannot achieve much.

### 1.2 Forests as Ecosystems and Producers of Natural Resources

The above considerations should not be misunderstood as a general request or protest. This is rather a plea in a forester's very own interest. His profession and sustained productivity are not only endangered, but in some areas are already hampered by the accumulation of entropy from economic processes. In the FRG, Prodan (1977, 1981) and Ulrich (1980, 1981) in particular have dealt with the subject from the point of view of forestry and have drawn significant conclusions. In the sector of political economy, the work of the Swiss national economist Kapp (1971) should be pointed out especially. Kapp gives a critical review of the relationship between political economy and forestry under the aspect of the environmental risks and social function of the forests.

Foresters manage and administer not only natural resources, but also economically-motivated and socially-obliged biological communities, with the aim of securing the continued benefits for mankind while watching over the ecological stability of the systems.

Prodan (1977) states: "With the development of an organized forestry in the Germanspeaking region basic principles arose which have been maintained over the course of centuries and have only been reformulated again and again. These are: the principles of sustained yield – continuity and long-range planning – large-scale management. They are the pillars of an efficient forestry. As they are very complex concepts, their meaning is also influenced by actual developments. For example, the permanent guarantee of the fulfillment of the social function of the forest is incorporated into the principle of sustained yield."

Of course, these principle can be extended to include the achievement of ecological stability. Prodan demands that these general principles be taken into consideration in general economic theory, so that comprehensive thinking will more strongly influence decisions made on a small level, beginning with the individual. The present stage of development could well be defined as an "ecologization phase" of the classical aims and subject matter of forestry and forestry sciences. The production of the raw material wood will certainly maintain its outstanding importance in the affairs of mankind, and special attention will continue to be directed on this product of energy exchange in the ecosystem.

### 1.3 The Process of Timber Production in Ecosystems

The basis of wood production are the forest trees. They are a medium for the exchange of energy between atmosphere and the earth. For this reason Stugren (1974) speaks of "soil-tree-atmosphere-continuities." Forest trees should be considered as collectives of such continuities. These collectives show a special dynamics, imposing an individual environmental situation and competitive behavior (social rank) on each member. Forest stands are special dynamical systems whose members have a social behavior which is characteristic of the stand. The functioning of these systems is no *perpetuum-mobile.* The accumulation of assimilates (growth of wood mass) takes place under steady input of solar energy and conversion (exploitation) of soil nutrients. Soil nutrients can be replenished only by circulation of material in ecosystems (this demands food chains rich in species) or by fertilization (negative entropy from the outside). It is known that natural forests have a well-balanced economy of nutrient circulation [Stugren (1974)]. Unfortunately these natural forests, while rich in species, do not perform efficiently enough in the sense of an optimal net yield for man. Additional negentropy in the form of human labor was necessary to clear natural forests and to establish and tend commercial forests (almost exclusively monocultures). The "fragile" stability and productivity has to be paid for constantly with an input of negentropy in terms of human and mechanical labor, fertilization, and chemical measures of protection. Comparing the energy balance of a classical commercial forest – production + negentropy from outside – with a natural forest (human need for special assortment will be neglected here), one would probably have to decide in favor of the natural forest. Such calculations are at the present very unreliable, but the Scandinavian colleagues are contributing very intensively towards answering such questions. The SWECON-project will certainly throw much light on the economy of pine forests. Here, in Göttingen, the current "terrestrian Ecosystems" project should contribute to the evaluation of beech forests on calcareous soils, in particular.

Foresters must surely bear the blame to some extent for the irreversible transformation of natural forests into unstable but profitable monocultures. They have tried, however, by using their knowledge of biological processes, to prevent total devastation as a result of overly-strict application of the principle of sustained yield, at least in the central European region. The adoption of the wide range normal-forest-structure [Baader (1933)] secured a balanced distribution of all age classes on forest land with respect to the species-specific rotation periods. In addition to this "ecological contribution," this type of forest management has so far secured, at least for the large areas, sustained yields and possibilities for long-term utilization planning. This standard forest management principle is a macro-concept which can be simulated quite well. The corresponding mathematical models can be explained as macro-models for predicting sustained production for one management class. The models which describe the growth of a stand as a collective can be defined as micro-models.

As the main point of my speech I will present a special stochastic micro-model or stand growth model. For the sake of thoroughness, a few introductory remarks on macro-models and inventories are given at this point.

## 2. Macro-models and Inventories

### 2.1 Inventories

For political economists, large-scale or national inventories are of great importance. The Swedish foresters are pioneers in this field. The beginnings date back to the year 1911, when in Värmland for the first time growing stocks, age classes, and species distribution were recorded for all forests of one province. Since 1923, the Swedish National Survey has become a standard term and has been based on mathematical-statistical procedures. It has been permanently established since 1953 [Hägglund and Bengtsson (1980)].

For 1983 a change in method is planned. Some of the aims are to compile information about marketing and multiple uses of production as well as to diminish costs through use of new techniques.

The Swedish inventory receives scientific assistance from the biometrist B. Matérn, who has acquired a distinguished reputation with his work in the field of "spatial variation" and sampling theory [Matérn (1961)]. In connection with the Swedish forest inventory the names of Pettersson and Näslund should also not be forgotten. In the FRG, the Bavarian large-scale inventory [Franz and Kennel (1973)] and the wood production prognosis based on it [Franz (1976)], as well as the computed inventory of forest waste potential, i.e., of the unused biomass, by Dauber (1979) and Kramer (1981), deserve special mention. The comprehensive information about the current state of national inventories in other parts of the world can be found in the IUFRO-collection National Inventories, Bukarest 1978.

### 2.2 Macro-Models and Normal Forest-Theory

An organic and elegant formal simulation of the processes in the normal forest and the accompanying theory were, to my knowledge, first developed in Japan by Suzuki (1959) and subsequently extended by him [Suzuki (1972)].

Suzuki describes the process of change in the areal distribution of age classes (age class vector) as a homogeneous Markow-chain with a transition matrix similar to the Leslie-matrix. For determining the average rotation period and the felling area after n years he uses the basics of the renewal theory (Gentan-probability). He shows that under the conditions that the chain of transitions is homogeneous and the transition matrix is homogeneous and "ergodic" (this is the case with the practical cutting policy), the process converges to a stable = normal age class distribution in one management class ($\lim_{n\to\infty} \omega \cdot P^n = \tilde{\omega}$). For the prediction of the yield and for the development of a continuous regional or national forest resources plan, he uses linear optimization in which special yield-determining functionals in the age space are maximized. This method is already used to a great extent internationally. Kouba (1973, 1974, 1978), for example, adapted this method in the CSSR for a production prognosis on the basis of a national forest inventory. In this application, certain objective difficulties arising from the assumption of homogeneity for the Markow-chain cannot be overlooked. We shall now deal with the individual stand growth prediction in greater detail.

## 3. A Stand-Oriented Model for Growth Prediction

To enable a more exact and differentiated production prognosis of a stand, i.e., a collective of soil-tree-atmosphere-continuities, growth model for the individual stand is needed. I shall initially limit considerations to the model conception of an even-aged stand of one tree species. (In the FRG about 85% of forest land is covered by such stands). For age-inhomogeneous and mixed stands, these concepts must be modified accordingly. Because the basic outlines of this method can well be applied to general questions of social system development, a systematic approach illustrated by examples has been chosen.

### 3.1 Heuristic Motivation and Objectives

A stand observed over the time interval $[t_0, T]$ can be viewed as the set of patterns of growth development of the individual trees. We can identify this set with an abstract probability space $(\Omega, \mathfrak{A}, P)$. The space $\Omega$ is allowed to be infinite. By means of a continuous measuring process over the course of time $t \in [t_0, T]$ of certain, say $d$, tree variables, we obtain an $\mathbf{R}^d$-valued (d-dimensional) stochastic process (stand growth process)

$$X_T : (\Omega, \mathfrak{A}, P) \to (R^{d \cdot [t_0, T]}, \mathfrak{B}^{d \cdot [t_0, T]}, P^{X_T})$$

We will restrict ourselves at first to the description of one variable ($d = 1$), for example tree diameter at a height of 1.3 m (DBH). (Mass production is strongly correlated with this variable). Death of a tree at time $t_1$ is modeled as a jump to $x_t = 0$. Thus the realization of a certain "tree" is a stepwise continuous function. With fixed $t$, $X_t(\cdot)$ is a real-valued random variable.

In figures 1a, 1b and 1c the growth development of tree diameters in the research areas Büren and Westerhof is shown. It should be noted that for practical forestry the performance at the end of production (crop trees), (that is, a subset of $\Omega$ with continuous paths) is of special interest.

For a quantitative dynamic description of growth, the sequence of marginal frequency distributions for $X_t$ over time can be used (Fig. 1c). The illustrations show a

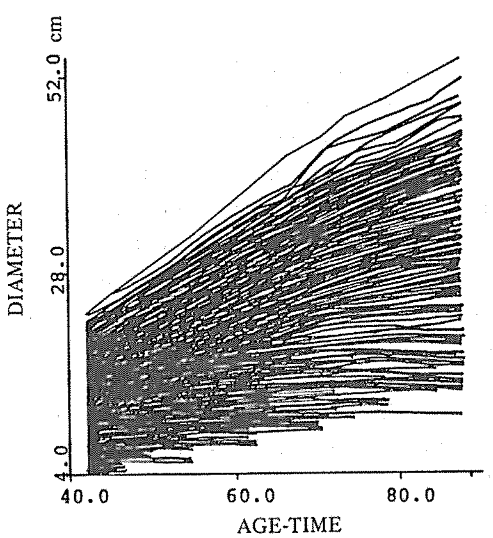



Fig. 1a.

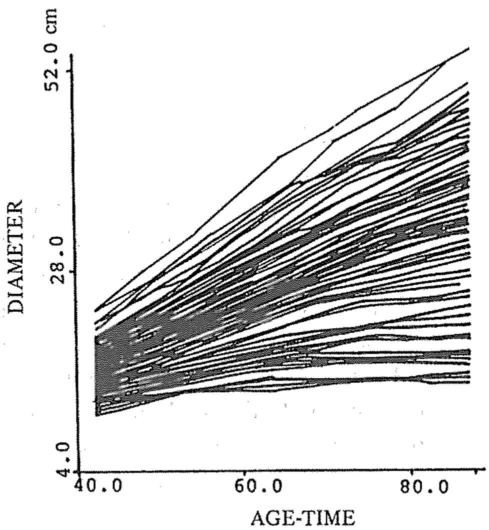



Fig. 1b.

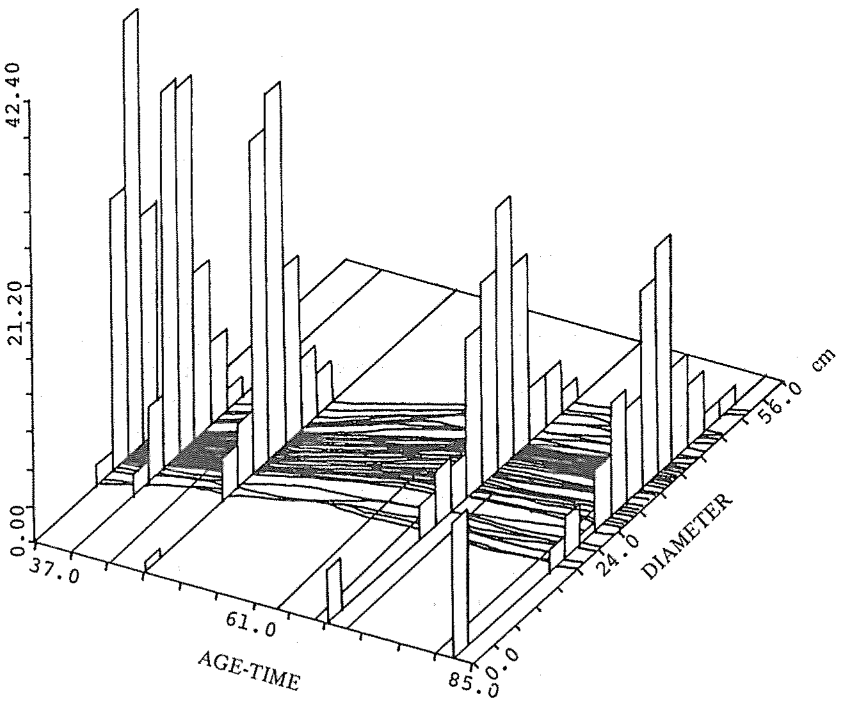



Fig. 1c.

Fig. 1. A example for the empirical basis for the formulation of stochastic growth models of forest stand (DBH)

Fig. 1a. Plot Büren All trees
Fig. 1b. Plot Büren All living trees
Fig. 1c. Dynamic marginal of change of diameter frequencies

strong ranking structure. This corresponds to the social factum "rich stays rich at least with a certain probability." In stochastic terms, this means a high intercorrelation in the process.

Our objectives lead to the following results:

a) We are looking for a dynamic model for projecting the momentaneous state of the forest stand $X_{t_0}$ or its probability distribution $P(X_{t_0})$ into the future $t > t_0$. An acceptable agreement between model and reality should be ensured.

b) The stochastic model we are looking for should contain input functions which can be fixed with the help of data from the present and a part of the past of the stand.

c) The intercorrelative behavior of the realizations (rank conservation structure) of the stand should be reflected by the model in a realistic manner.

### 3.2 Transition Probability or Control Function of the Growth Process

In order to analytically describe the movement of single trees in the state space, the transition probability or conditional probability $p(t, x, \tau, B) \fallingdotseq p(X_\tau \in B \mid X_t = x)$ as used by Suzuki (1973) or the associated transition density or control function of the growth process $p(t, x, \tau, y)$ with

(1) $$p(t, x, \tau, y)\,dy \fallingdotseq p(X_\tau \in [y, y + dy] \mid X_t = x)$$

can be employed (fig. 2a, 2b). For $\tau \downarrow t$ this function has the form of a Dirac-Finction in $x$, that is $p(\tau, x, \tau, y) = \delta(y - x)$.

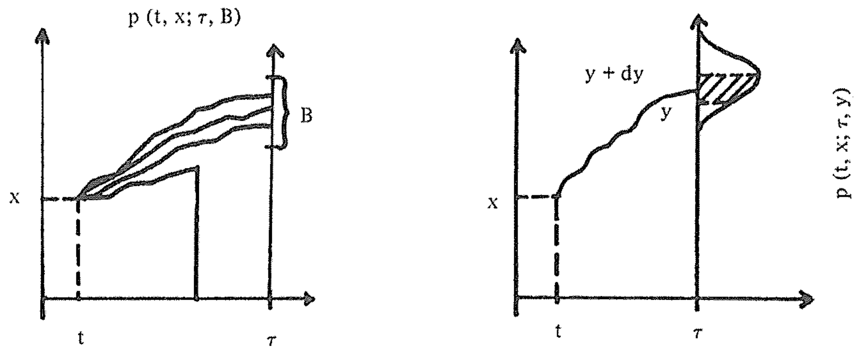



Fig. 2a. Fig. 2b.

Fig. 2. Transition Probability

**Remark:** It is obvious that the control function p can be approximated only *ex post facto,* if at all, through the relative frequencies, which requires knowledge of the whole process. This would, of course not be an advantageous concept for prediction. We will try to develop more practical "input functions" by making further restrictions on the assumptions concerning the process $X_t$.

We can consider and define the following three functions as being related to the "diffusion field of growth" (fig. 1a). We assume for the time being that these input functions are easy to derive from the small amount of empirical data available.

a) **Drift field: (local growth rate)**

(2) $$\beta(t, y) := \lim_{\tau \downarrow t} \frac{1}{\tau - t} \int_{|z-y| \leq \epsilon} (z - y)\, p(t, y, \tau, z)\, dz$$

b) **Diffusion field: (local variance rate)**

(3) $$\alpha(t, y) := \lim_{\tau \downarrow t} \frac{1}{\tau - t} \int_{|z-y| \leq \epsilon} (z-y)^2\, p(t, y, \tau, z)\, dz$$

c) **local death rate**

(4) $$\gamma(\tau, y) := \lim_{\tau \downarrow t} \frac{1}{\tau - t} \int_{|z-y| > \epsilon} p(t, y, \tau, z)\, dz \qquad \text{(Fig. 2b)}$$

The functions $\beta$ and $\alpha$ represent cut off moments and $\gamma(\tau, y)$ corresponds to the probability that a tree of diameter $y$ and age $\tau$ suddenly dies. That means singularity for $z = 0$.

(5) $$p(t, y, \tau, 0) = \gamma(t, y) \cdot (\tau - t) + o(\tau - t)$$

We assume further that the stand growth proceeds approximately according to a Markow process or is "without memory," so that we consider the Chapman-Kolmogorow-equation as being valid for $p(t, x, \tau, y)$ that is

(6) $$p(t, x, \tau, y) = \int_{(0, \infty)} p(t, x, s, z)\, p(s, z, \tau, y)\, dz$$

If we assume the existence of the necessary partial derivatives for $\beta, \alpha, \gamma$ and $p(t, x, \tau, y)$, then the following Kolmogorow-Suzuki forward equation [Suzuki (1973), Sloboda (1977)] is true for $p(t, x, \tau, y)$

(7) $$\frac{\partial p}{\partial \tau} = -\frac{\partial}{\partial y}[\beta \cdot p] + \frac{1}{2} \frac{\partial^2}{\partial y^2}[\alpha \cdot p] - \gamma \cdot p$$

Of interest is p with $\lim_{\tau \downarrow t} p(t, x, \tau, y) = \delta(y - x)$ and

$$\int_R p(t, x, \tau, y)\, dy + p(t, x, \tau, 0) = 1$$

If the state of the stand at the starting point $t_0$ is given by the probability density $\varphi(t_0, x)$, then we have for the forecast of $\varphi(\tau, y)$ for a future time $\tau$ the following

$$(8) \qquad \varphi(\tau, y) = \int_{(0,\infty)} \varphi(t_0, x) \cdot p(t_0, x, \tau, y)\, dx \qquad \text{(Fig. 1c)}$$

According to (8) $\varphi(\tau, y)$ satisfies (7) in $\varphi$ with the initial condition $\varphi(t_0, x)$ [Suzuki (1973), Sloboda (1977)].

$$(9) \qquad F(\tau, y) = \int_{-\infty}^{y} \varphi(\tau, \xi)\, d\xi + P(\tau, 0)$$

is the predicted distribution function at the future time $\tau$.

### 3.3 Growth Process of the Remaining Portion of the Stand

It is not simple to deal in practice with the process of tree dying or the removal of trees by thinning. Long-term observations are not available and growth parameters will have to be estimated in the course of experiments which have yet to be laid out. Further restrictions on the assumptions will be made in order to obtain a reasonable result.

$$(10) \qquad \gamma(t, x) = c(t) \qquad \text{(the death rate is dependent only on time)}$$

Furthermore, we choose the approach

$$(11) \qquad q(t, x, \tau, y) = e^{c(t)(\tau - t)}\, p(t, x, \tau, y)$$

Then q is the control function of the growth process with the mortaility of trees being excluded [Sloboda (1977)].

Then the growth process of the remaining part of the stand will be approximately controlled by the function $q(t, x, \tau, y)$. This is a fact of great practical value. The sub-population of crop-trees will be selected at a relatively early age $t_0 \fallingdotseq 10 \sim 15$ years and will be supported by influencing its individual environmental situation. This elite sub-population is of superior growth and is almost independent of other members of the stand. This fact justifies the simplification of the model. The term q in (11) satisfies the Kolmogorow-forward equation

$$(12) \qquad \frac{\partial q}{\partial \tau} = -\frac{\partial}{\partial y}[\beta \cdot q] + \frac{\partial^2}{\partial y^2}[\alpha \cdot q]; \quad q(\tau, x, \tau, y) = \delta(y - x)$$

As in (8) and (9) we obtain the distribution density or -function for the growth of the remaining part of the stand, which corresponds to a diffusion process.

$$(13) \qquad \varphi(\tau, y) = \int_{R} \varphi(t_0, x)\, q(t_0, x, \tau, y)\, dx \quad \text{or} \quad \int_{-\infty}^{y} \varphi(\tau, \xi)\, d\xi$$

### 3.4 Solving the Problem by a Stochastic Differential Equation

It has been shown in the theory of stochastic differential equations that if a stochastic process $X_t$ – a diffusion process $X_t$ – has the initial distribution $\varphi(t_0, x)$ and if its control function q satisfies (12) (i.e., it is a Markow process with continuous paths), then it can be solved by Itô's differential equation (14) [Arnold (1973)].

This transition enables a better understanding of the parameters of the growth process and facilitates their estimation. [Sloboda (1977)]

(14) $$dX_t = \beta(t, X_t)\, dt + \alpha(t, X_t)\, dW_t$$

with the random initial value $X_{t_0} \sim \varphi(t_0, x)$.

Here $W_t$ is the Wiener-Levy process, which is stochastically independent of $X_{t_0}$. $\alpha$ and $\beta$ are the same input functions as in (12) according to (2) and (3).

The concrete data yields the result that $\beta$ and $\alpha$ can be approximated by the following

(15) $$\beta(t, x) := A(t)\, X + a(t) := \frac{Lke^{-kt}}{1 - Le^{-kt}}\, X; \quad a(t) = 0$$

(16) $$\alpha(t, x) := B(t) := pe^{qt}$$

**Justification choosing $\alpha$ and $\beta$ according to (15) and (16)**

a) For $a(t) = 0$ the mean value equation $EX_t$ and the integrals of $\dot{x}_t = \beta(t, x)$ yield linear differential diagrams, i.e.

$$x_{t+1} = ax_t + b$$

This property can be verified empirically. [Suzuki (1973), Sloboda (1977)]

b) Little is known about the rate of variance $\alpha$, but it can be rightly assumed that it has a monotone decreasing tendency.

c) This very simple linear approach yields reasonably good agreement between the forecast and the data, as will be shown later. This is true for the distribution functions as well as for the autocorrelation function of the process $R(s, t) = E(X_s - EX_s) \cdot (X_t - EX_t)$, which approximately represents the tendency of paths to maintain their ranking. A further justification can be deduced from the properties of the solution of the stochastic differential equation.

### 3.5 Discussion of the Solution of the Linear Approach of the Growth Process

The solution of the linear stochastic differential equation

(17) $$dX_t = [A(t)\, X_t + a(t)]\, dt + B(t)\, dW_t$$

with the initial state $X_{t_0} \sim \varphi(t_0, x)$ is

$$(18)\qquad X_t = \Phi(t)\cdot X_{t_0} + \Phi(t)\int_{t_0}^{t} \Phi^{-1}(s)\,a(s)\,ds + \Phi(t)\int_{t_0}^{t}\Phi^{-1}(s)\,B(s)\,dW_s,$$

or

$$(19)\qquad X_t = \Phi(t)\,X_{t_0} + c(t) + Y_t$$

where $\Phi(t) := \exp\left\{\int_{t_0}^{t} A(s)\,ds\right\}$, $c(t)$ is a deterministic function, and $Y_t$ is a Gauß-process with $EY_{t_0}\,Y_t = 0$ and variance

$$(20)\qquad \mathrm{Var}Y_t = \Phi^2(t)\cdot\int_{t_0}^{t}\frac{B^2(s)}{\Phi^2(s)}\,ds =: \Psi^2(t),\ \text{i.e.}\qquad \mathfrak{V}(Y_t) = \mathfrak{N}(0,\Psi^2)$$

The solution of (17) according to (19) represents the sum of the linear function of the initial value $X_{t_0}$ and the stochastic Gauß-process $Y_t$, which is stochastically independent of $X_{t_0}$, as a random fluctuation. For the moments of $X_t$ from (18) or (19), it holds that

$$(21)\qquad EX_t = \Phi(t)\cdot EX_{t_0} + c(t)$$

$$(22)\qquad \mathrm{Var}X_t = \Phi^2(t)\cdot\mathrm{Var}X_{t_0} + \Psi^2(t)$$

The autocovariance function $K(t_0, t)$ or the standardized autocorrelatic function $R(t_0, t)$ of $X_t$ for $t_0 < t$ is obtained by a lengthy calculation

$$(23)\qquad K(t_0, t) = \mathrm{Var}X_{t_0}\cdot\Phi(t);\qquad R(t_0, t) = \left(1 + \frac{\Psi^2(t)}{\mathrm{Var}X_{t_0}\cdot\Phi^2(t)}\right)^{\frac{1}{2}}$$

### 3.6 Possibilities of Representing the Process through $\Phi$, c, $\Psi^2$

In order to verify the complete model, the parameters of the input functions $\alpha$ and $\beta$ will not be estimated directly, but rather through the integrals $\Phi(t_0, t)$ and $c(t_0, t)$ and the residual variance $\Psi^2(t_0, t)$ of $Y_t$. (Fig. 3)

This will be accomplished for the fixed times $t_1, t_2, t_3, \ldots, t_k$ on the basis of linear regression. If there are $n$ independent measurements of tree developments at each of the times $t_1, t_2, t_3, \ldots, t_k$, the regression estimation of $\Phi$, c, $\Psi^2$ according to the least squares method can be performed in these cases. One obtains $\hat{\Phi}(t_i)$, $\hat{c}(t_i)$, $\hat{\Psi}^2(t_i)$, $i = 1, \ldots, k$.

The trend will be extrapolated and smoothly approximated by the simple functions $\hat{\hat{\Phi}}(t)$, $\hat{\hat{c}}(t)$, $\hat{\hat{\Psi}}^2(t)$. An especially simple but in many cases sufficiently precise representation of the actual situation is obtained nicely by the approach $a(t) = 0$, i.e., in (18) and (19) $c(t) = 0$.

The prolonged smoothed functions $\hat{\hat{\Phi}}(t)$ and $\hat{\hat{\Psi}}(t)$ result in the following control function q of the approximated growth process [Arnold (1973)].

$$(24)\qquad q(t, x, \tau, y) = \frac{1}{\sqrt{2\pi\Psi^2(t,\tau)}}\exp\left\{-\frac{[y - x\cdot\hat{\hat{\Phi}}(t) - \hat{\hat{c}}(t)]^2}{2\hat{\hat{\Psi}}^2(t)}\right\}$$

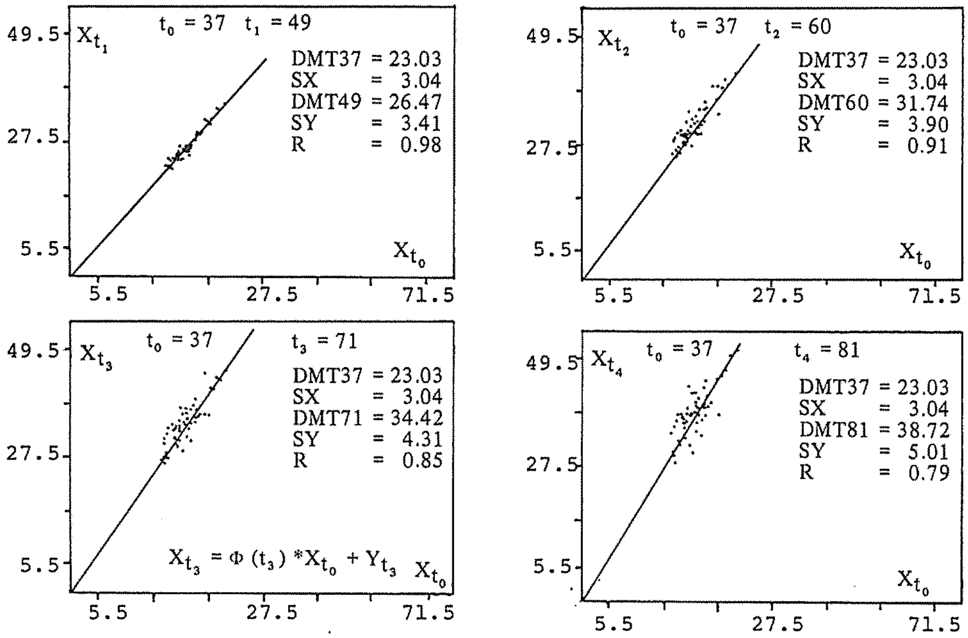



Fig. 3. Graphical verification of model $X_t = \Phi(t) X_{t_0} + c(t) + Y_t$ as solution of stoch. differential equation $dX_t = [A(t) X_t + a(t)]\, dt + B(t)\, dW_t$

The distribution function of the process at time $\tau$ with the initial density $\varphi(t_0, x)$ is obtained as an integral [Frohn (1978)]

$$(25) \qquad \int_{-\infty}^{y} \varphi(\tau, z)\, dz = \int_{R} \varphi(t_0, x) \cdot F\left(\frac{y - \hat{\hat{\Phi}}(t) \cdot x - \hat{\hat{c}}(t)}{\hat{\hat{\Psi}}^2(t)}\right) dx$$

F $(x)$ symbolizes the Gaussian distribution function in $x$.

### 3.7 Verification of the Model – an Example

A large sample of diameters $x_1, x_2, \ldots, x_n$, at the initial time $t_0$ and the input functions $\hat{\hat{\Phi}}(t)$, $\hat{\hat{\Psi}}(t)$, $c(t) = 0$ are given. We construct the "$\frac{1}{n}$ Dirac" functions in $x_1, x_2, \ldots, x_n$ and insert them in (25) for $\varphi(t_0, x)$ as an estimation of density.

$$(26) \qquad \varphi(t_0, x) := \frac{1}{n} \sum_{i=1}^{n} \delta(x_i - x) \Rightarrow \int_{-\infty}^{y} \varphi(\tau, z)\, dz = \frac{1}{n} \sum_{i=1}^{n} F\left(\frac{\hat{\hat{\Phi}}(t_i) \cdot X_i}{\hat{\hat{\Psi}}(t_i)}\right)$$

One obtains an overlaying of the Gaussian S-curves as the predicted distribution function for the future $\tau$.

Fig. 4a and Fig. 4b show the graphic comparison of the empirical and theoretical distribution function according to model (26) for the two research areas Büren and Westerhof where long term observations have been made.

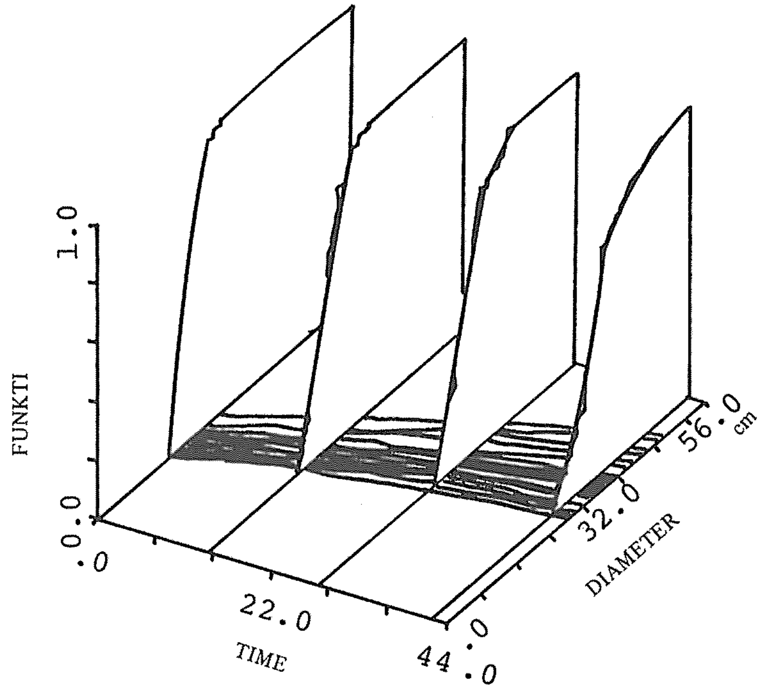



Fig. 4a.

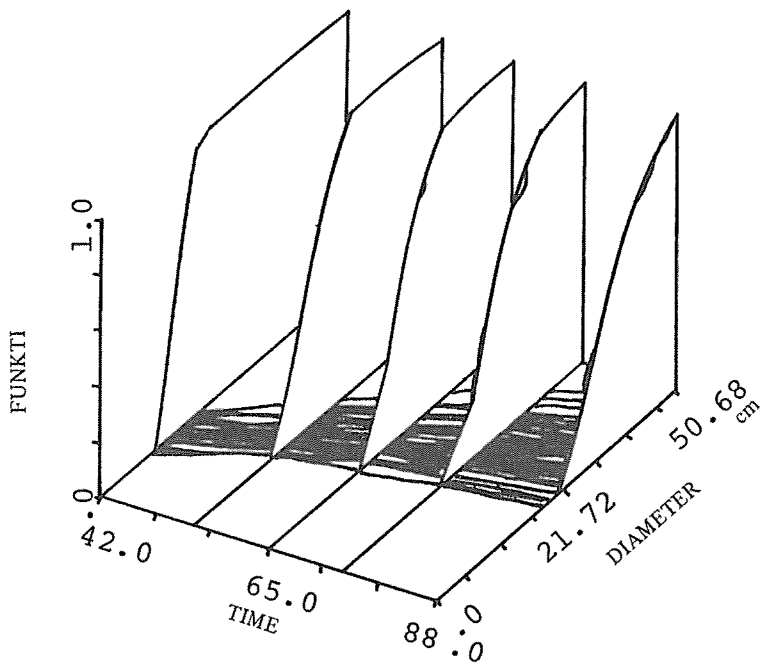
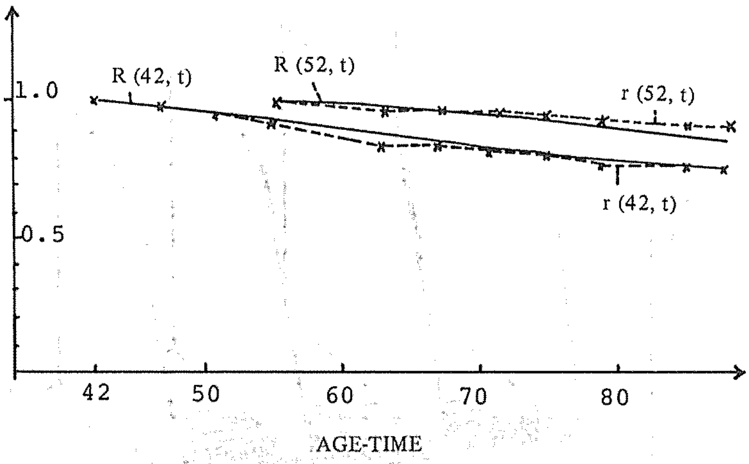



Fig. 4b.

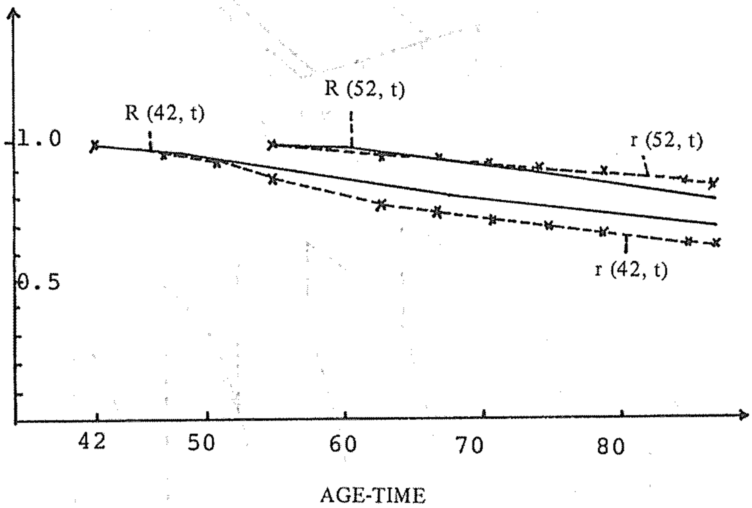
Fig. 4. Comparison between theoretical and empirical distribution function

Fig. 4a. Plot Westerhof Crop trees

Fig. 4b. Plot Büren Crop trees




Plot Büren All living trees




Plot Büren All living trees

Fig. 5. Comparison between theoretical and empirical correlation functions

Fig. 5 gives a graphical illustration of the comparison of the empirical and theoretical correlation functions for the area of Büren. The empirical and adjusted theoretical processes show a good correspondence (No statistical testing was done here). Comparisons with other existing data confirm this tendency. In the case of Westerhof, the observations were made during the age interval 37 ~ 81 years and in Büren during the interval 42 ~ 88 years.

Knowledge of the predicted distribution function enables a more differentiated prediction of the assortment yield for a stand. This is not possible with conventional yield-tables.

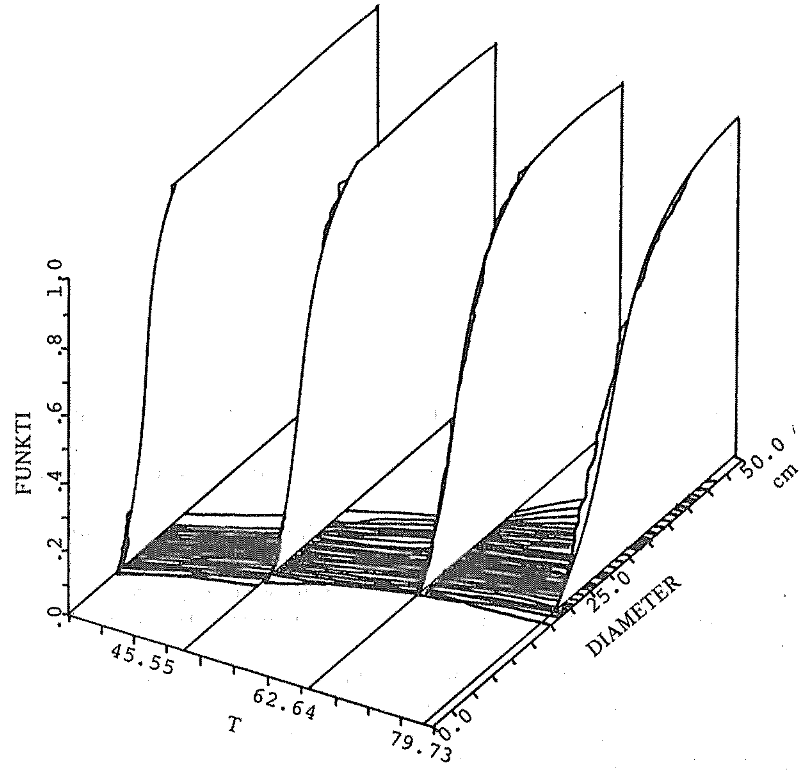



Fig. 6. Plot Hauersteig 3 all trees

## Conclusions

a) With this procedure, it becomes possible to transform the biological complex "forest stand" into a space which can be dealt with mathematically. At first, this was performed with regard to diameter growth. The stand system could thus be approximately described by a four-dimensional vector $[(k, L, p, q) \in \mathbf{R}^4]$ and by its definite initial distribution. This is one step closer to the possibility of quantitatively predicting individual stand growth. The theory of stochastic differential equations allows a better insight with regard to the choice of models and their use than does the classical analytic theory of stochastic processes.

b) The application of similar procedures is also possible for higher-dimensional growth processes. This theory can even be used in functional spaces. The dynamics of the entire morphology of the stem taper (assortment dynamics) can be described even more exactly by means of it. Here, especially important progress can be expected.

c) The improvement of models for quantitative prediction in recording the autocorrelation (shifting behavior, rank conservation structure) can be achieved with stochastic differential equations of second order. This is of special importance when the Markow-property of the growth process is less valid.

d) The application of these as well as higher-dimensional models for individual stand prognosis is becoming more and more realistic. The introduction of EDP-techniques in forest superintendents offices and in small private enterprises makes it possible to apply such procedures. Because of their stand-relatedness, these procedures come closer to reality than supra-regional yield-tables.

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